The Symmetries of QCD

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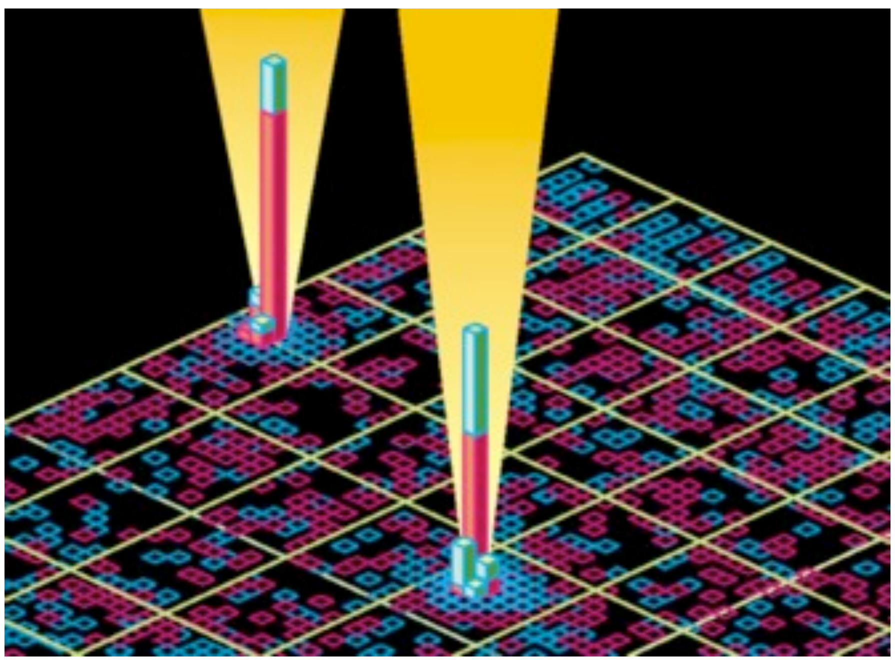
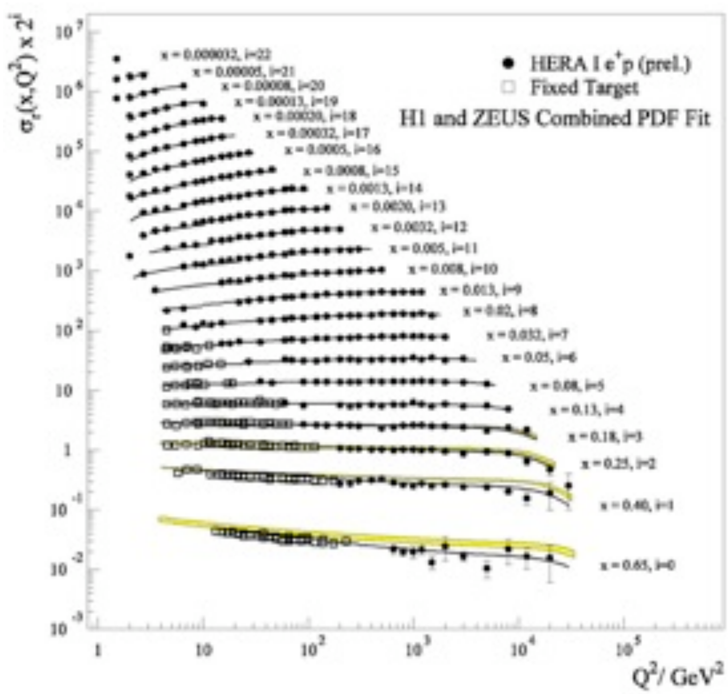
Fermilab Colloquium

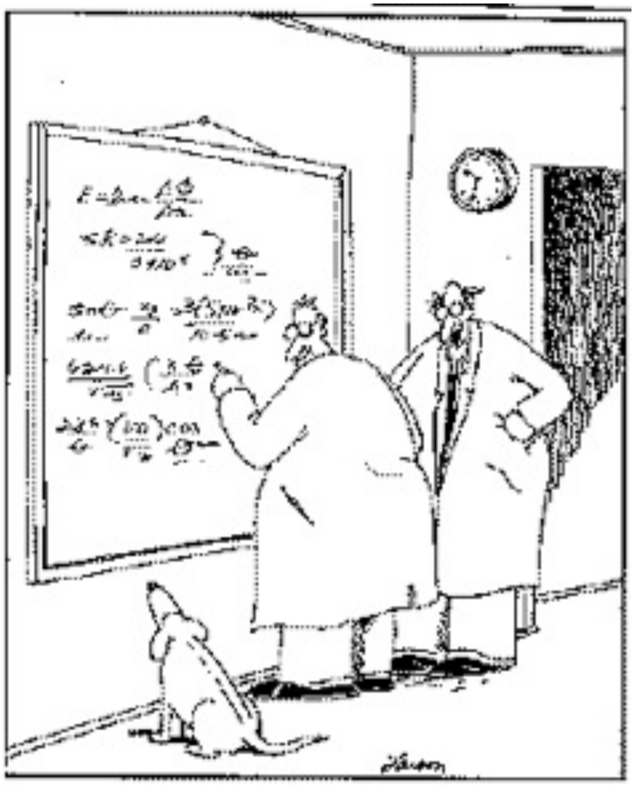




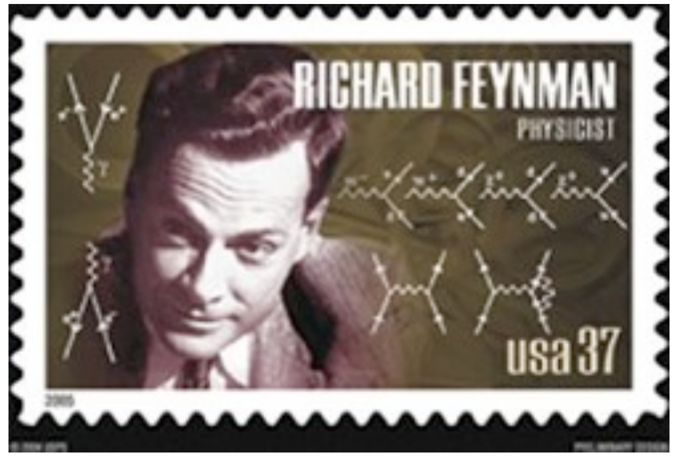
The Symmetries of QCD

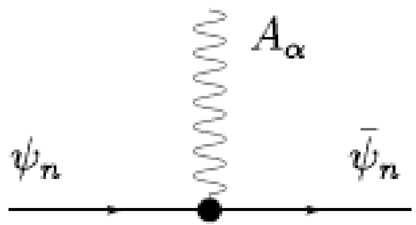
- What is QFT? Gauge Theory? QCD?
- Classical and Quantum Symmetries
- No Quantum Scale Symmetry!
- Chiral Symmetries are Broken
- Symmetry Summary
- Applications and Connections:
  - Morals for the LHC Era

Quantum Field Theory: QED

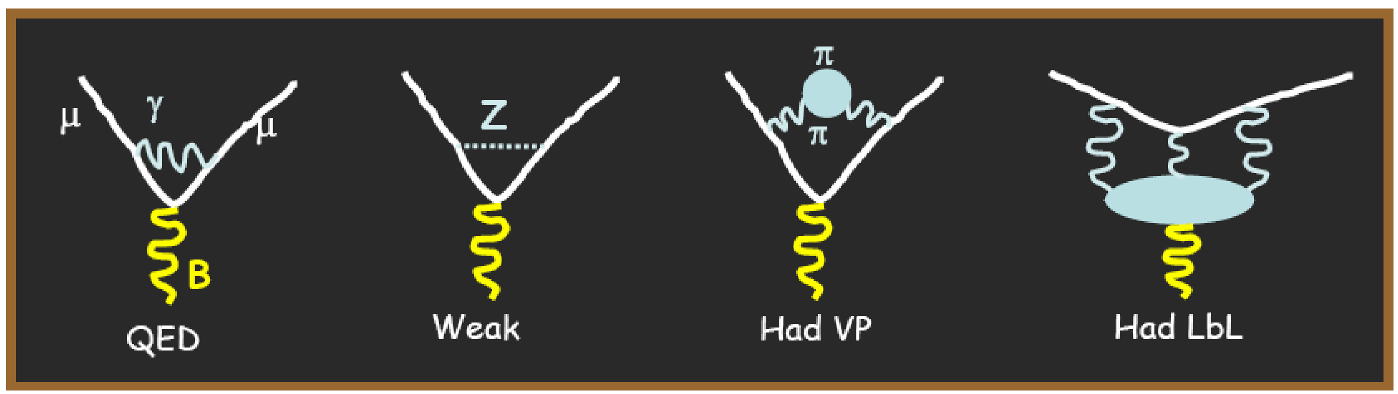


"They act so cute when they try to understand Quantum Field Theory"





$$g = 2 + \frac{\alpha}{\pi} + \ldots$$

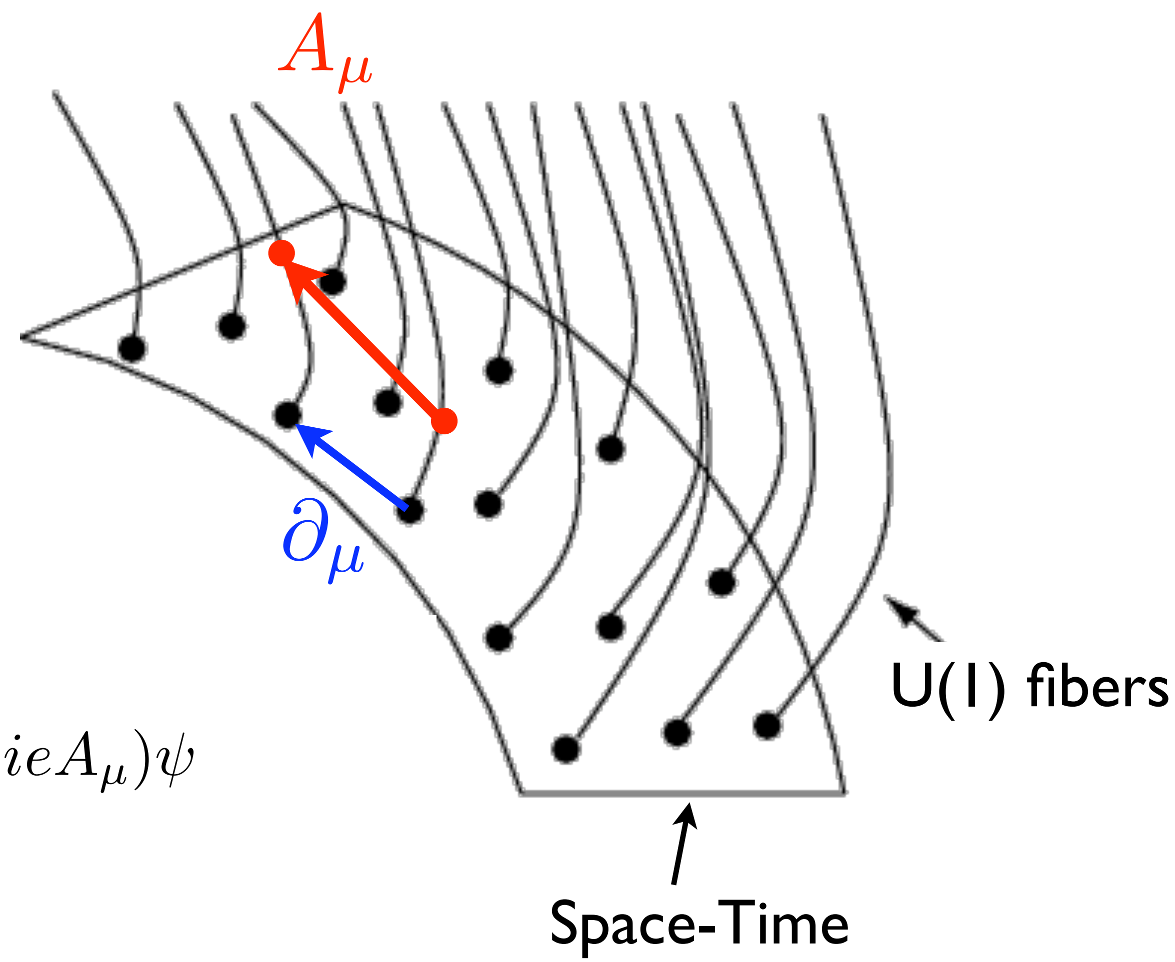


QED: Gauge Theory

Gauge "Symmetry"*

$$\psi(x) \to \exp\left(ie\theta(x)\right)\psi(x)$$

$$A_\mu(x) \to A_\mu(x) + \partial_\mu\theta(x)$$



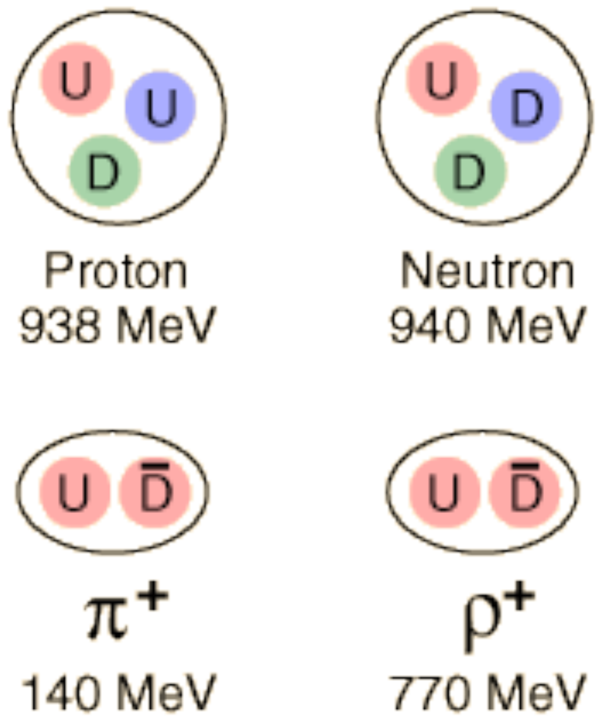
$\vec{E}^2 - \vec{B}^2$ $\vec{J}\cdot\vec{A}$

$$L_{QED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu - ieA_\mu)\psi$$

Weak Coupling

*Not a symmetry in the usual sense: rather it is a redundancy in the description of the theory!

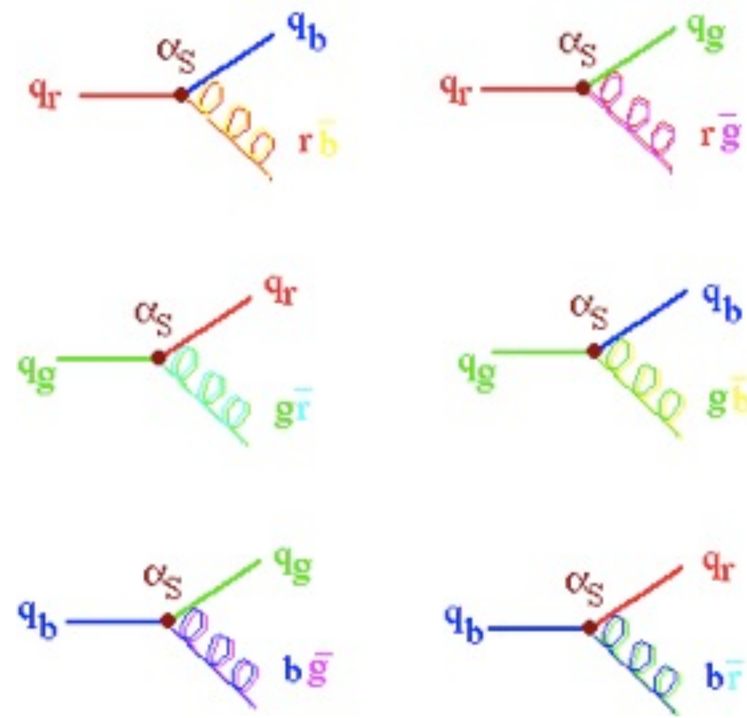
Quantum Chromodynamics: Hadrons Contain Quarks

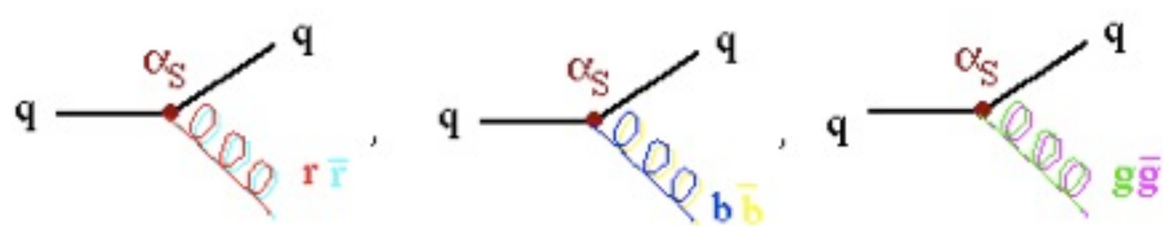


Light quarks have "masses" of 300 MeV?

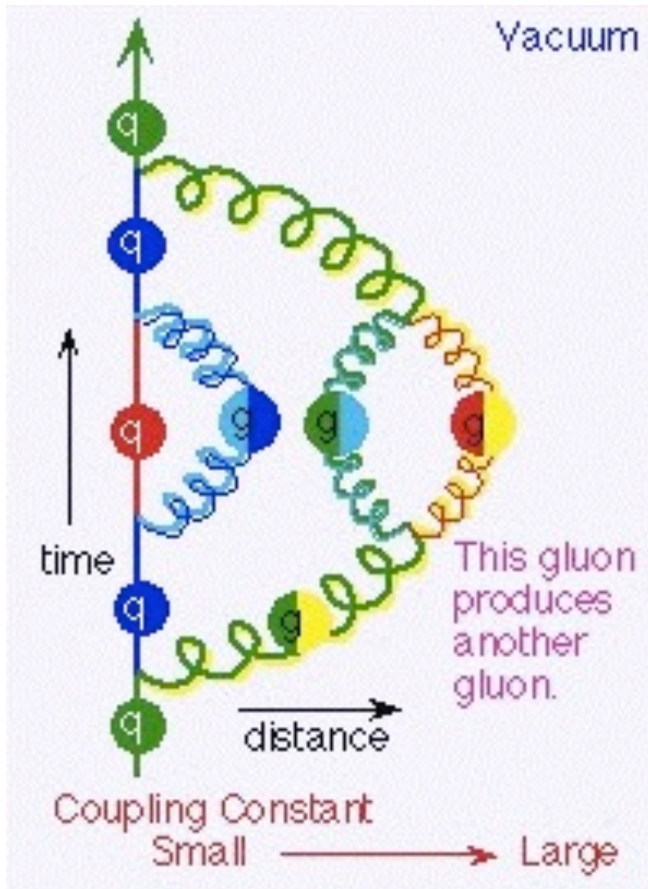
Why is the pion so light?

QCD Binds Quarks into Hadrons



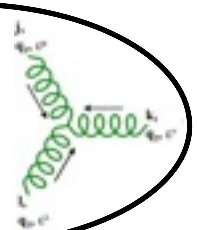


Gluons Couple to Gluons



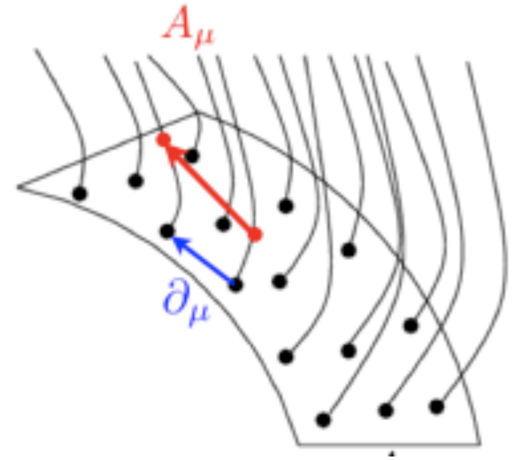
QCD Lagrangian

$\vec{E}_a^2 - \vec{B}_a^2$



$J_\mu^a \cdot A^{a\mu}$

$$L_{\rm QCD} = -\frac{1}{4} F_{\mu\nu}^{(a)} \, F^{(a)\mu\nu} + i \sum_q \overline{\psi}_q^i \, \gamma^\mu \, (D_\mu)_{ij} \, \psi_q^j$$
$$- \sum_q m_q \, \overline{\psi}_q^i \, \psi_{qi} \, ,$$
$$F_{\mu\nu}^{(a)} = \partial_\mu \, A_\nu^a - \partial_\nu \, A_\mu^a - g_s \, f_{abc} \, A_\mu^b \, A_\nu^c \, ,$$
$$(D_\mu)_{ij} = \delta_{ij} \, \partial_\mu + i g_s \sum_a \frac{\lambda_{i,j}^a}{2} A_\mu^a \, ,$$

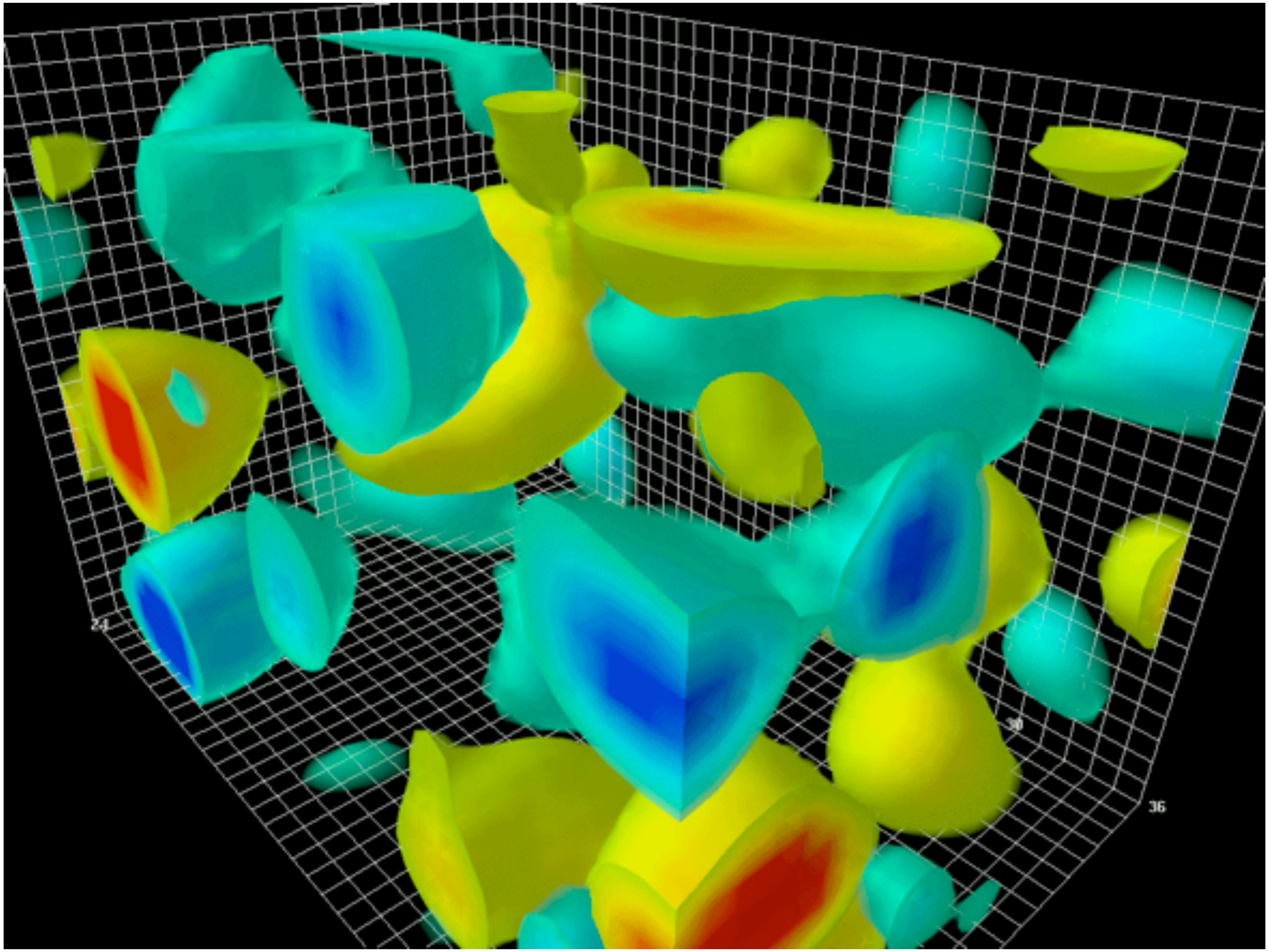


SU(3) gauge "symmetry":

$$\psi_q(x) \to U(x)\psi_q(x)$$
$$A^\mu(x) \to U(x) A^\mu(x) U^\dagger(x) - \frac{1}{g_s} U(x) \partial^\mu U^\dagger(x)$$

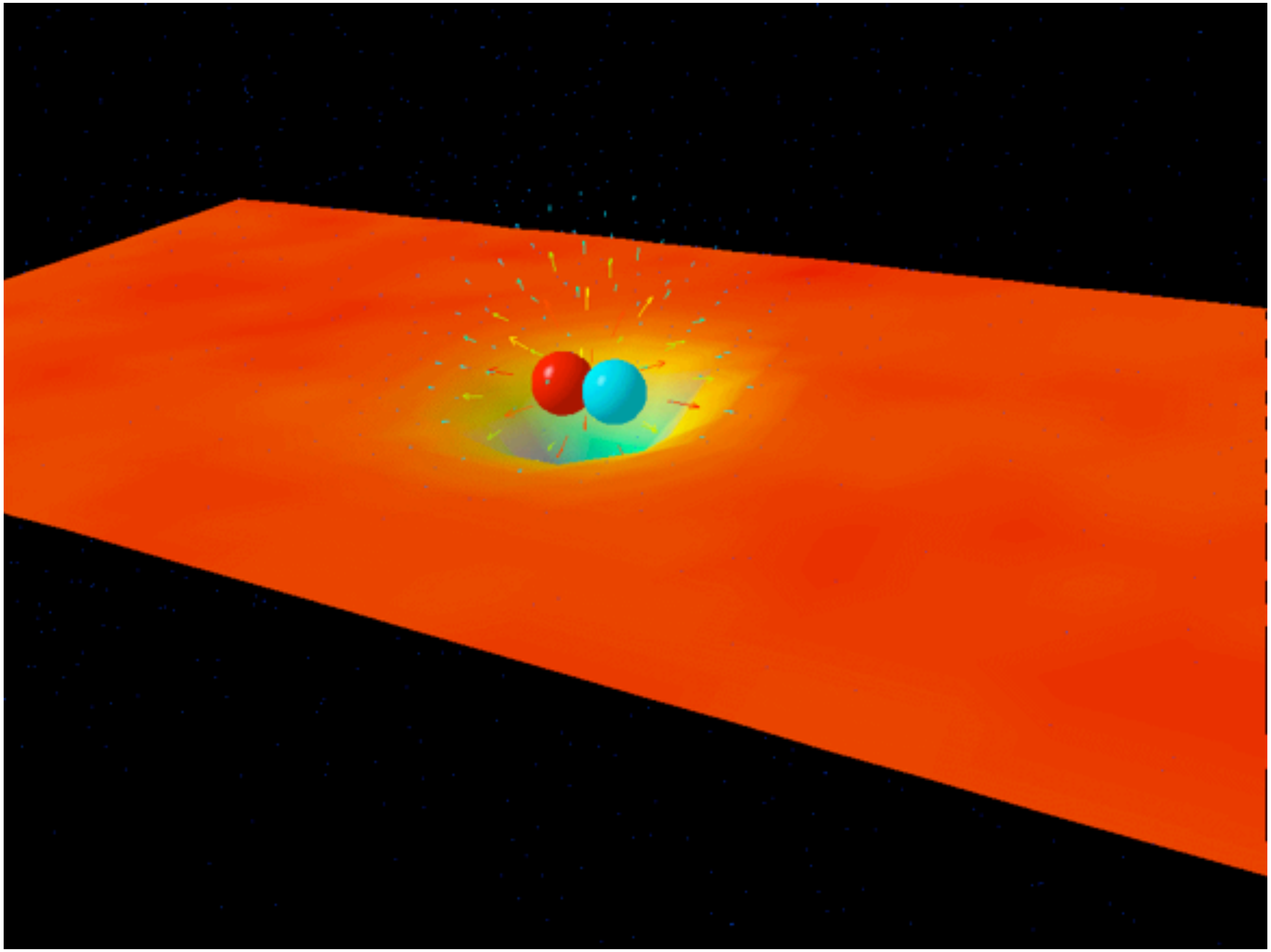
Parameters: Quark masses and QCD coupling

Lattice Simulation



Derek Leinweber: www.physics.adelaide.edu.au/theory/staff/leinweber/

Quarks are Confined!



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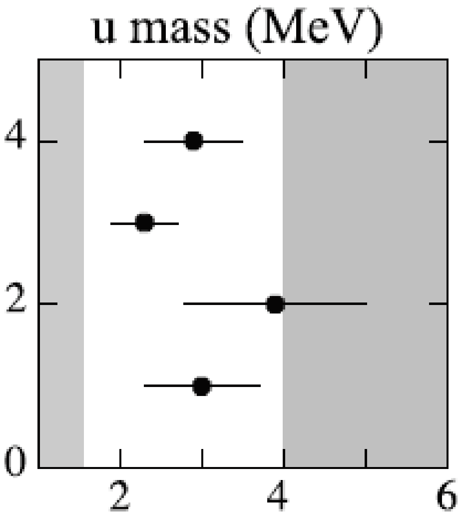
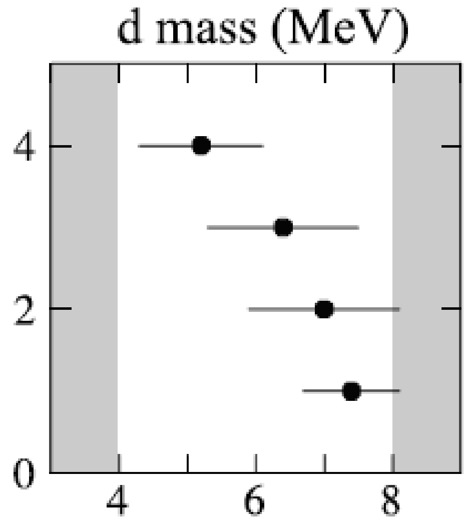
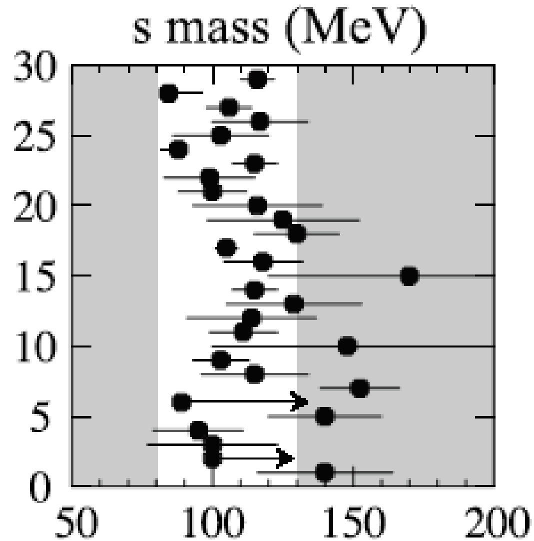
Lagrangian Quark Masses

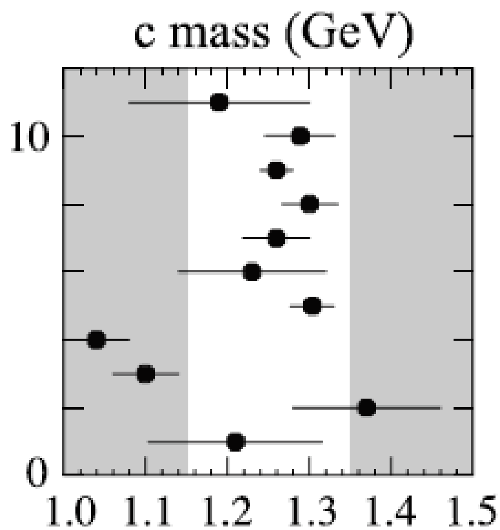


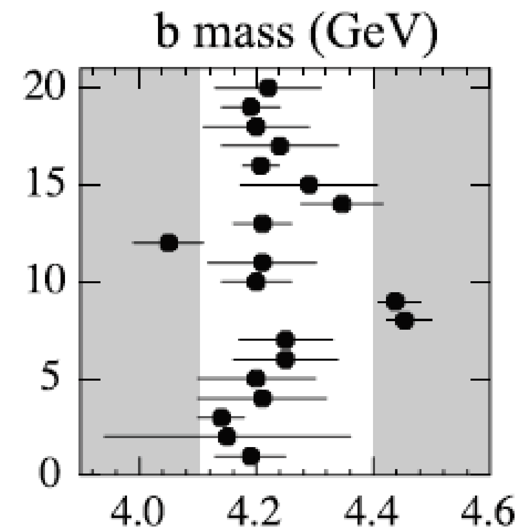




Light?

Heavy!

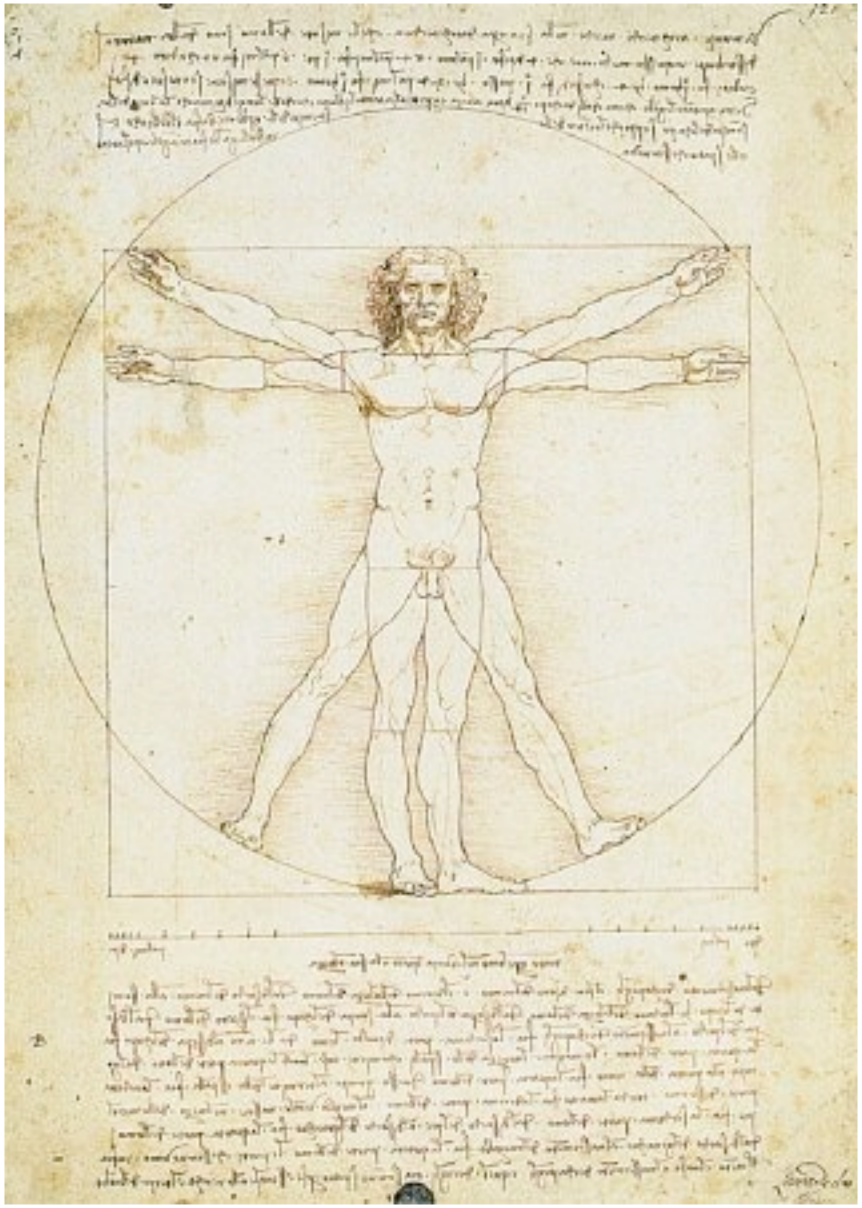




Explaining these masses is the "flavor problem"

RPP2004

Symmetries



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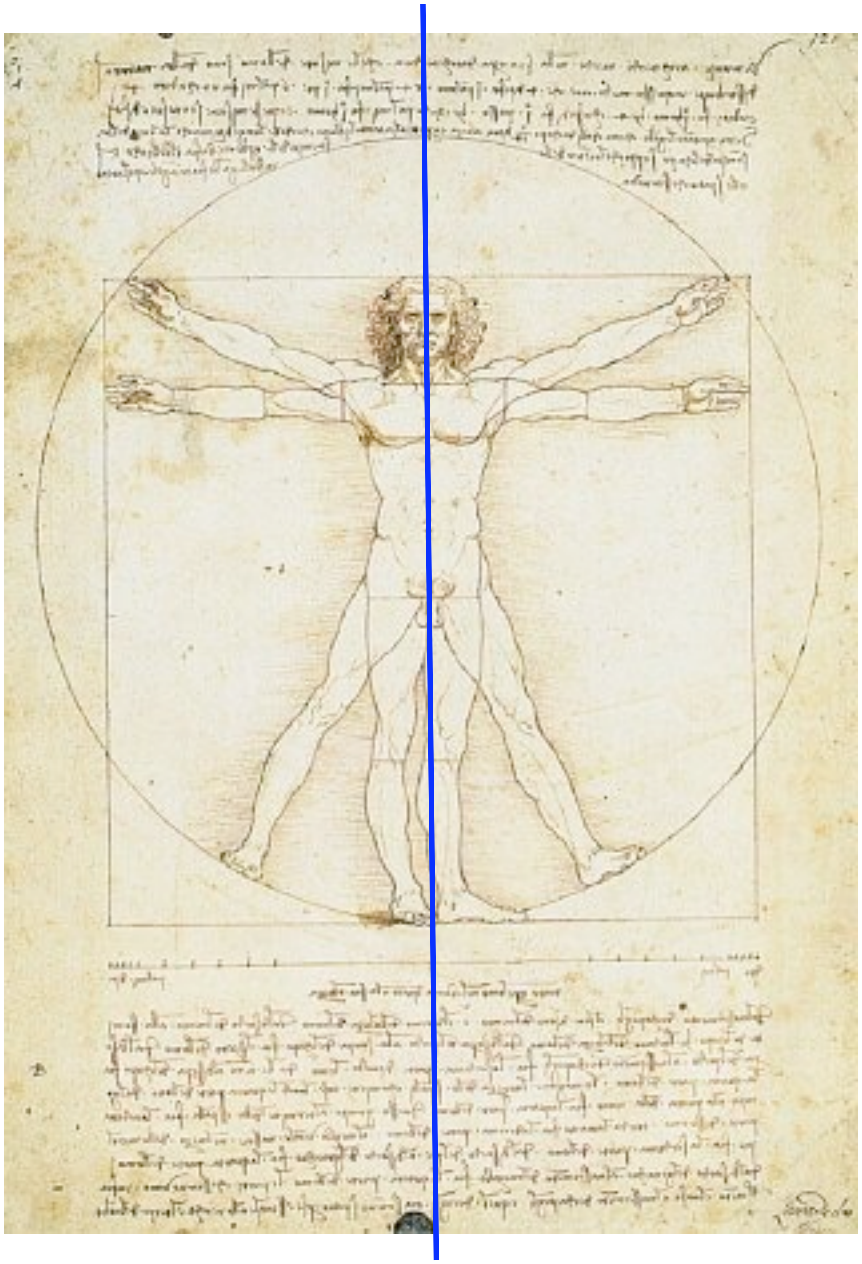
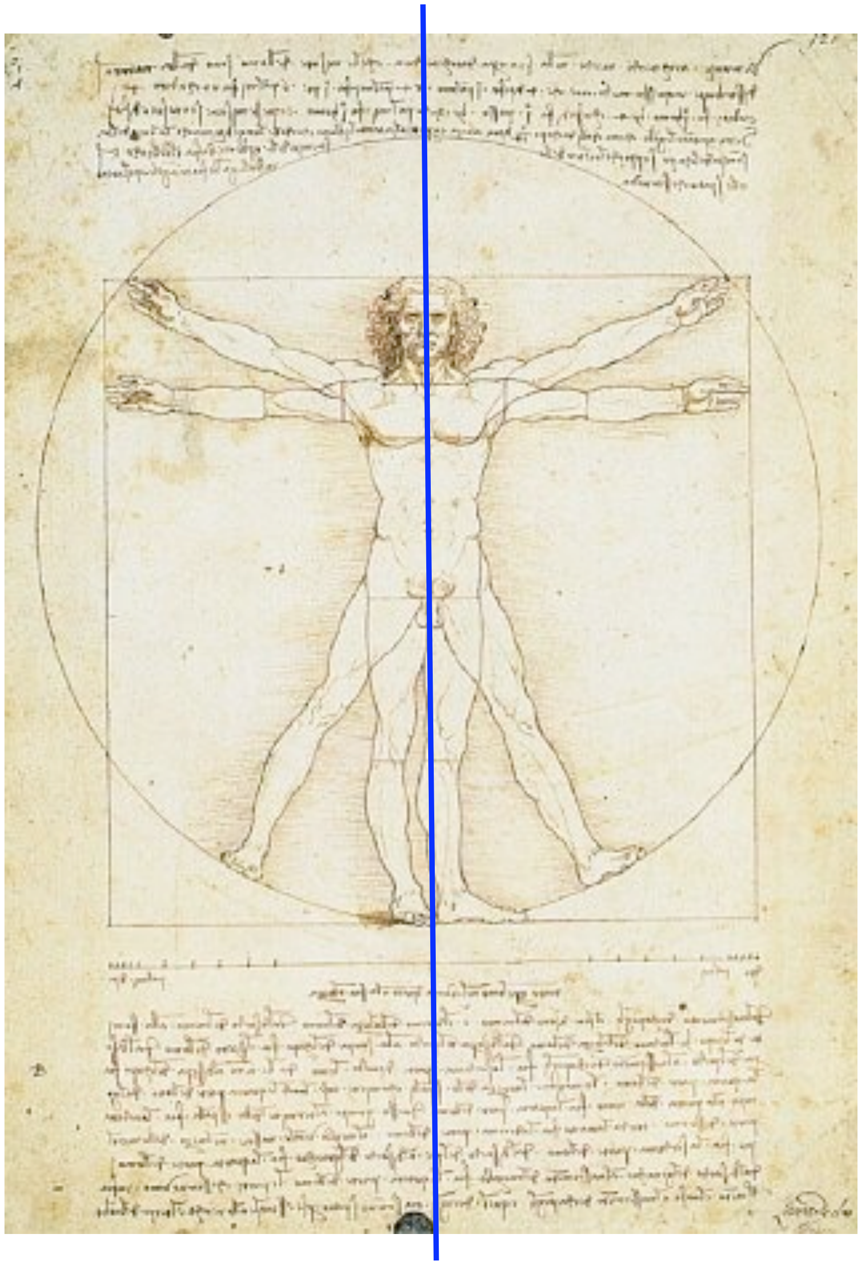
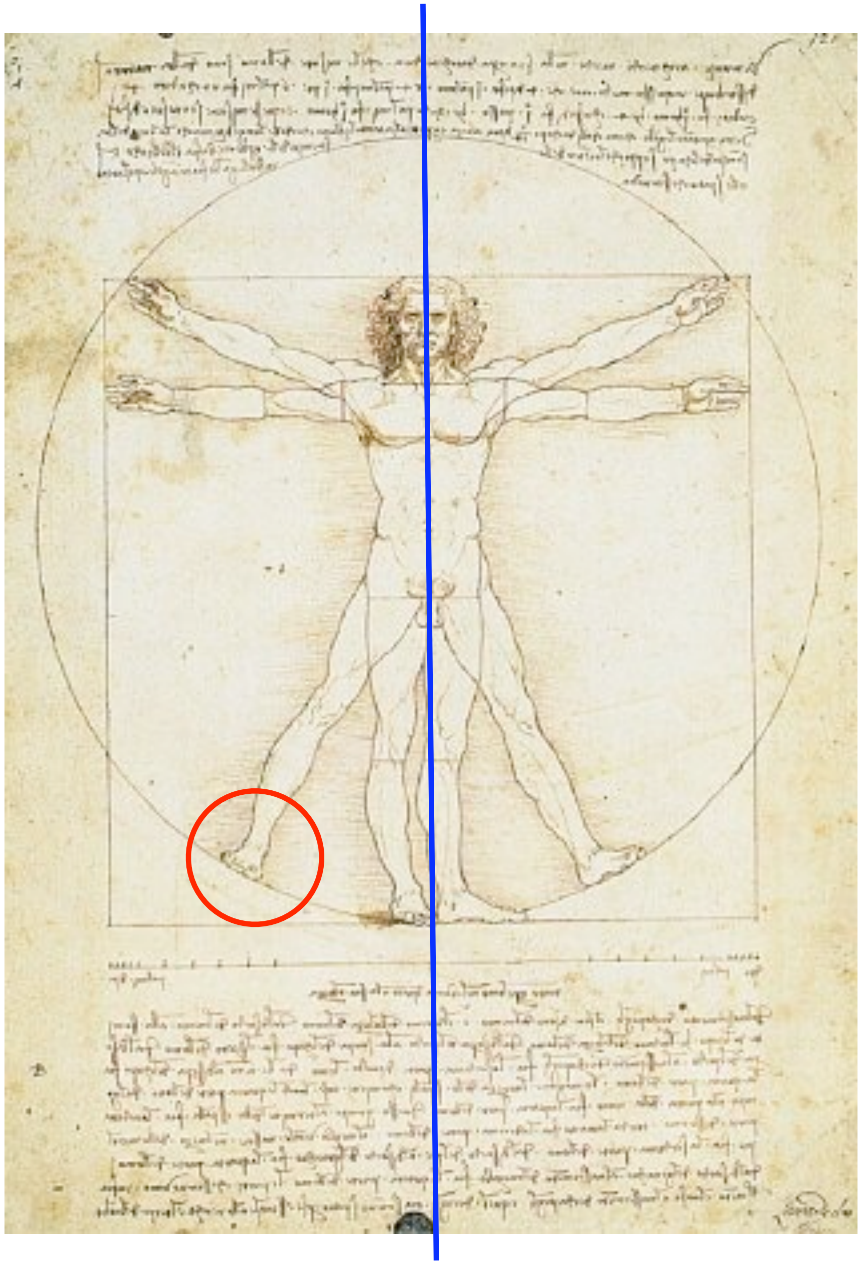


Symmetries



Bilateral Symmetry

Symmetries



Bilateral Symmetry

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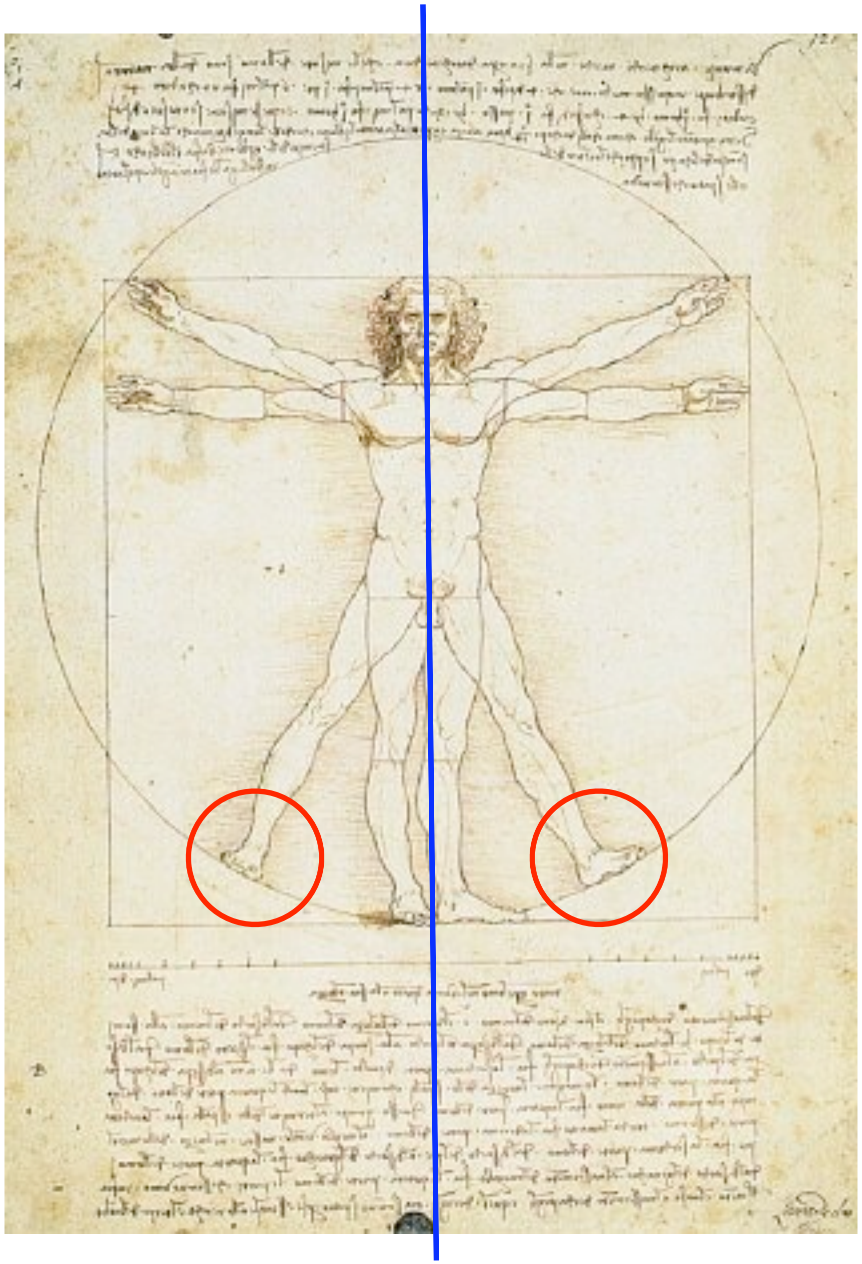
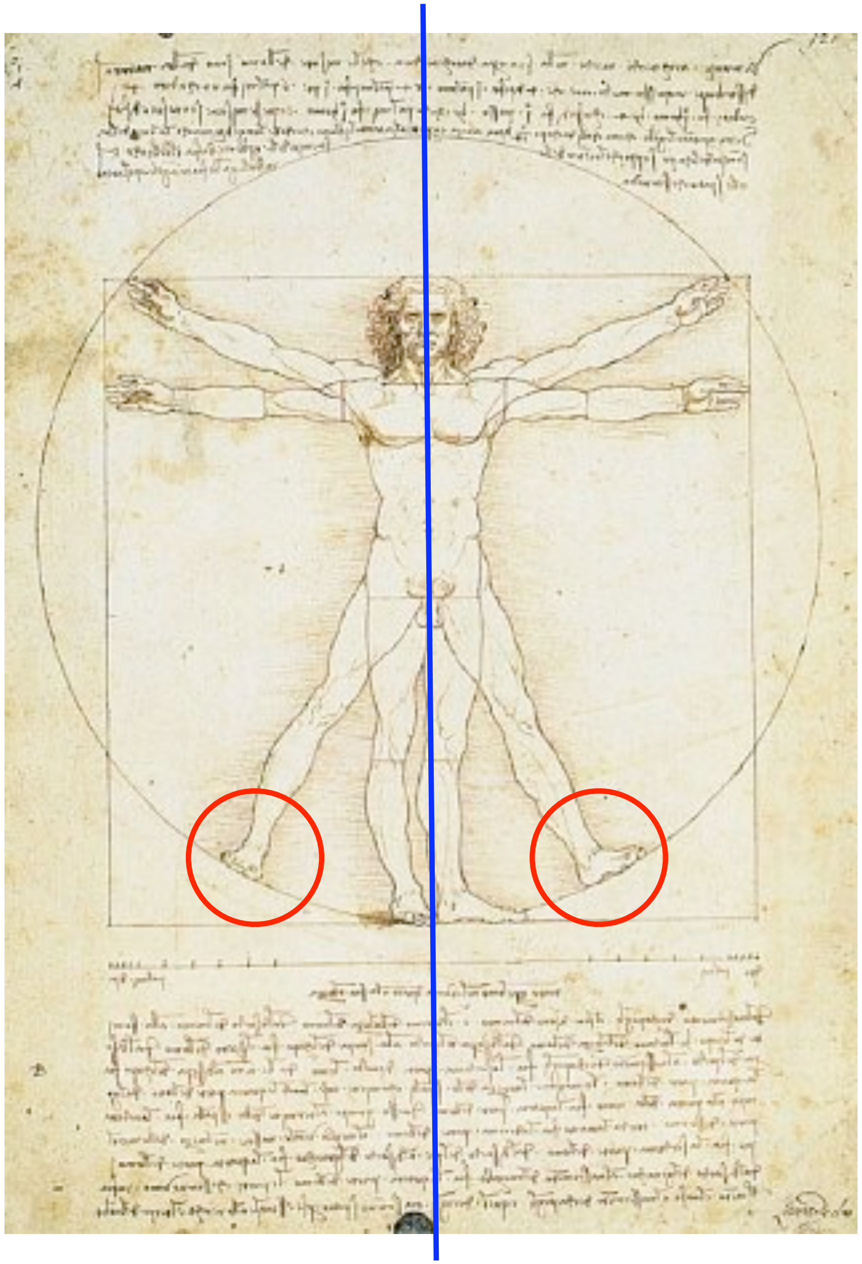


Bilateral Symmetry

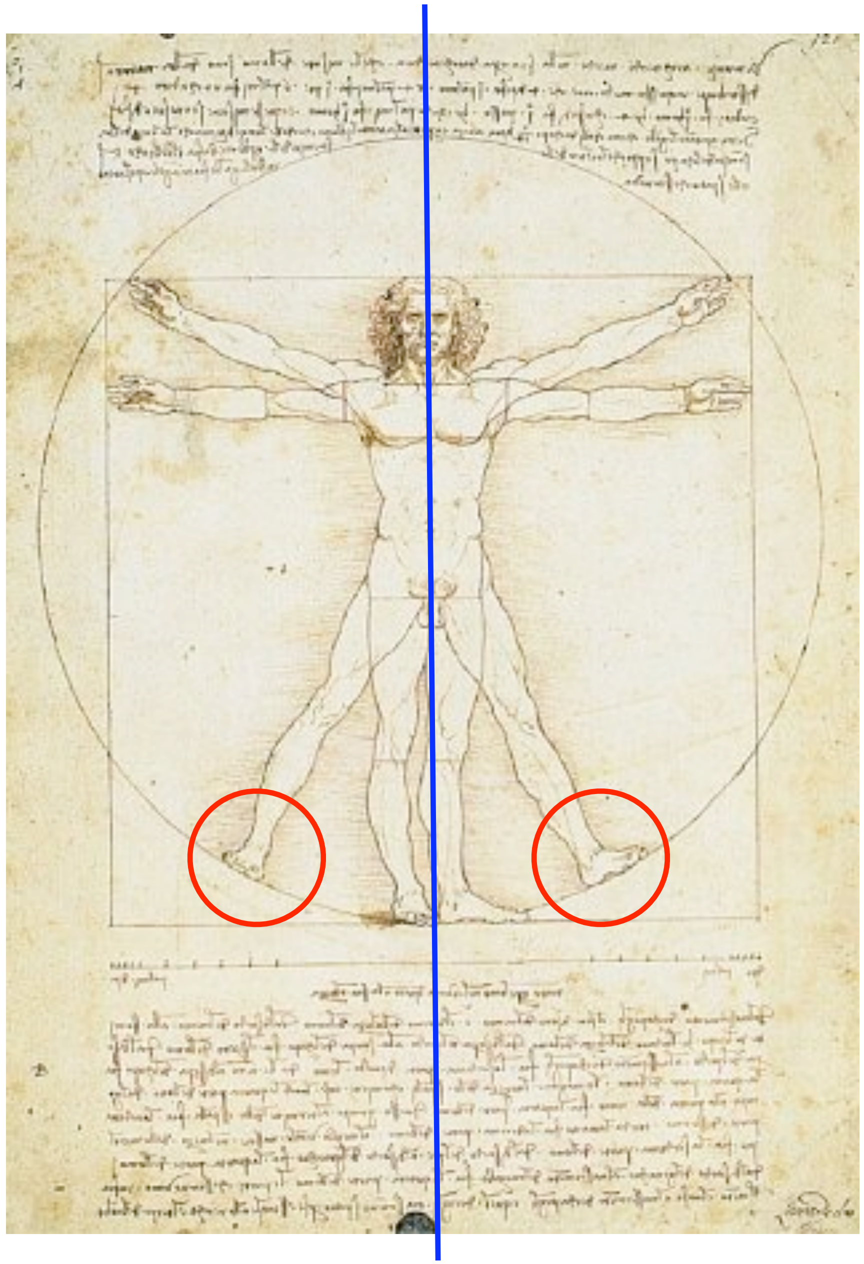
Symmetries



Bilateral Symmetry

Approximate Symmetry

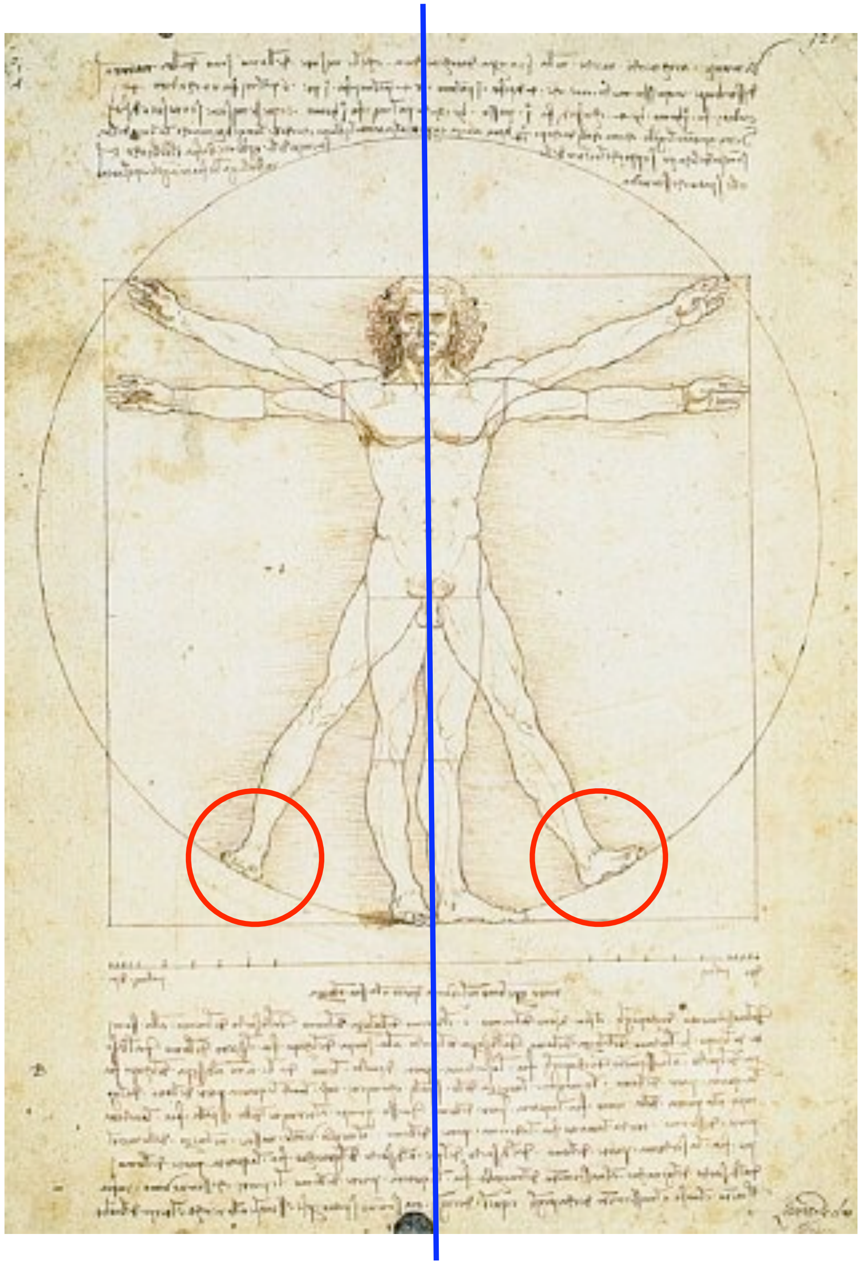
Symmetries



Bilateral Symmetry

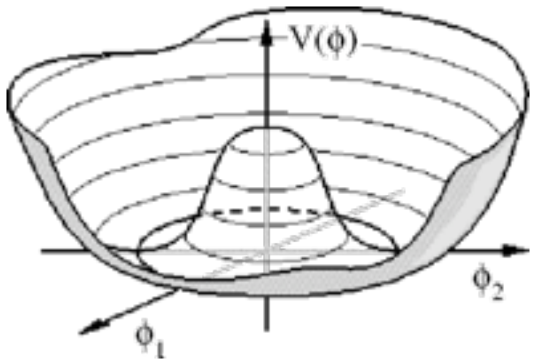
Approximate Symmetry

Symmetries

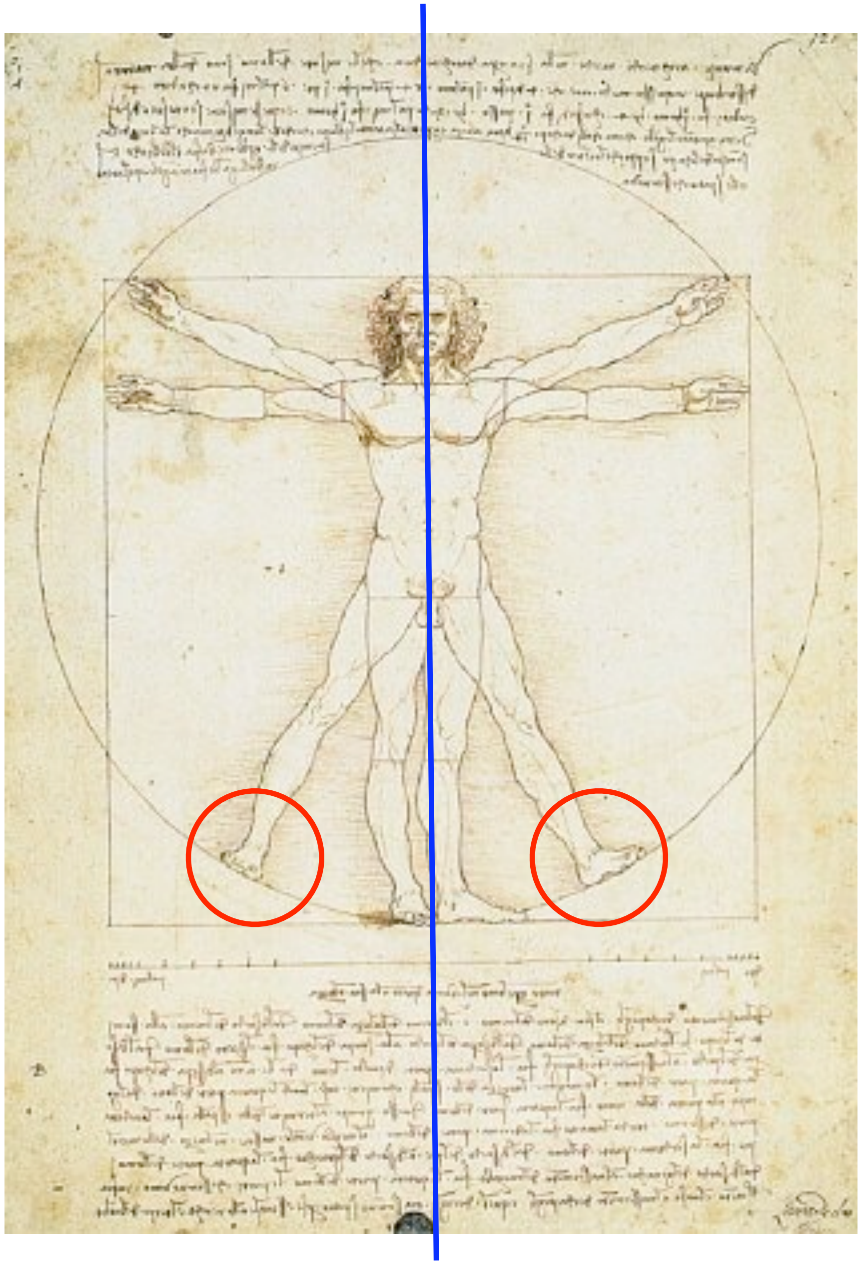


Bilateral Symmetry

Approximate Symmetry



Symmetries



Bilateral Symmetry

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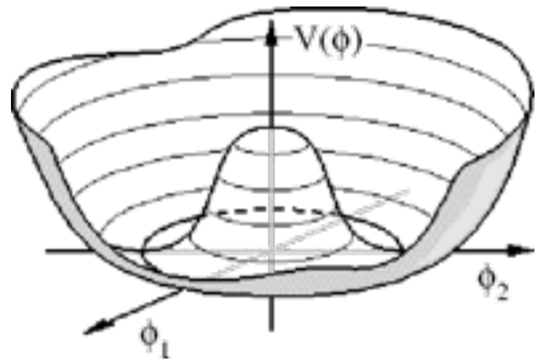
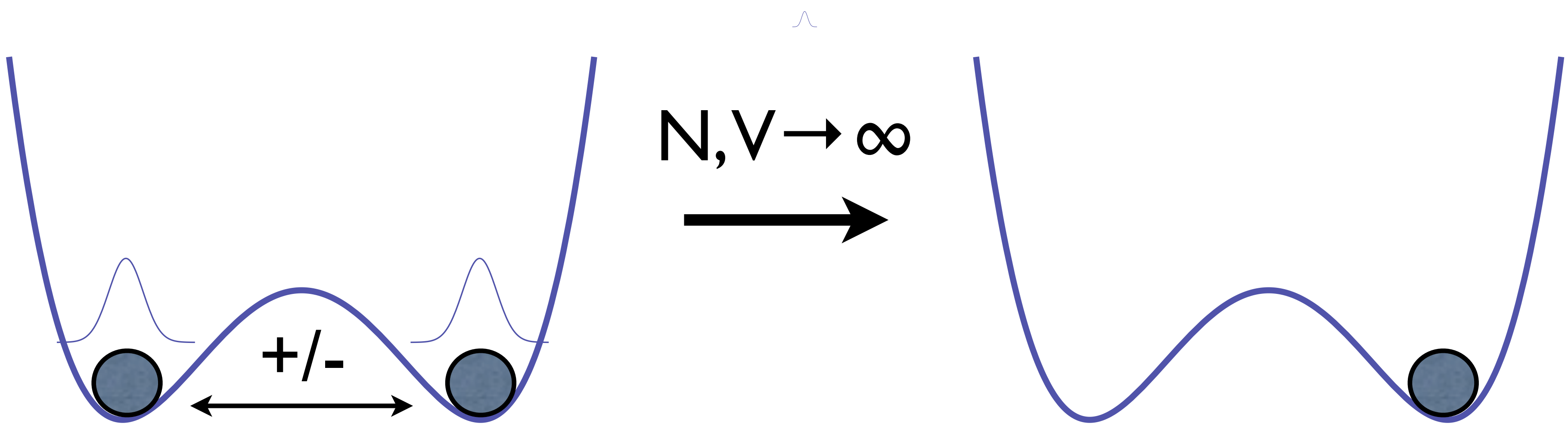


Spontaneously Broken Symmetry - By dynamics!

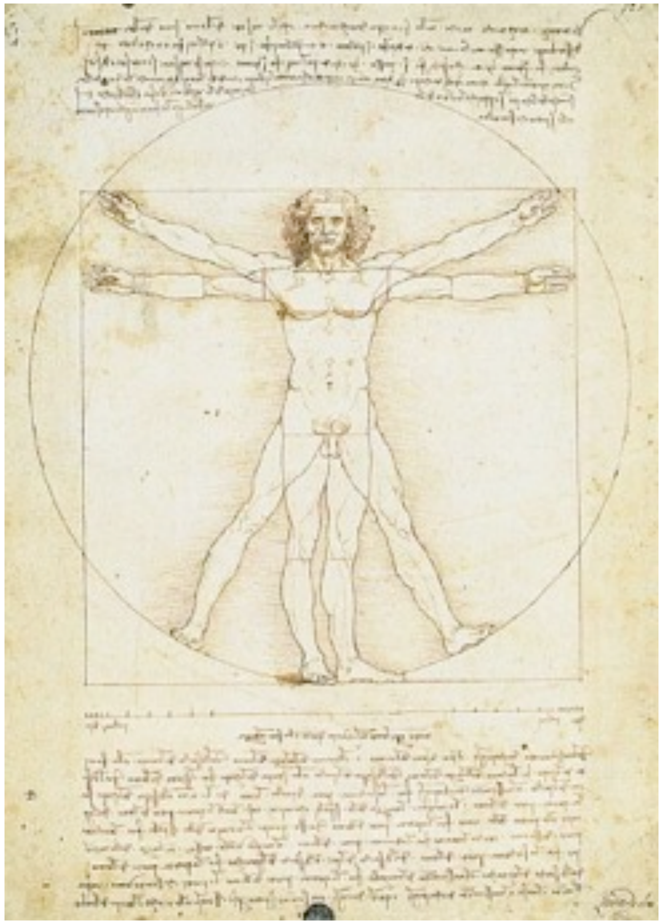
QFT differs from QM!



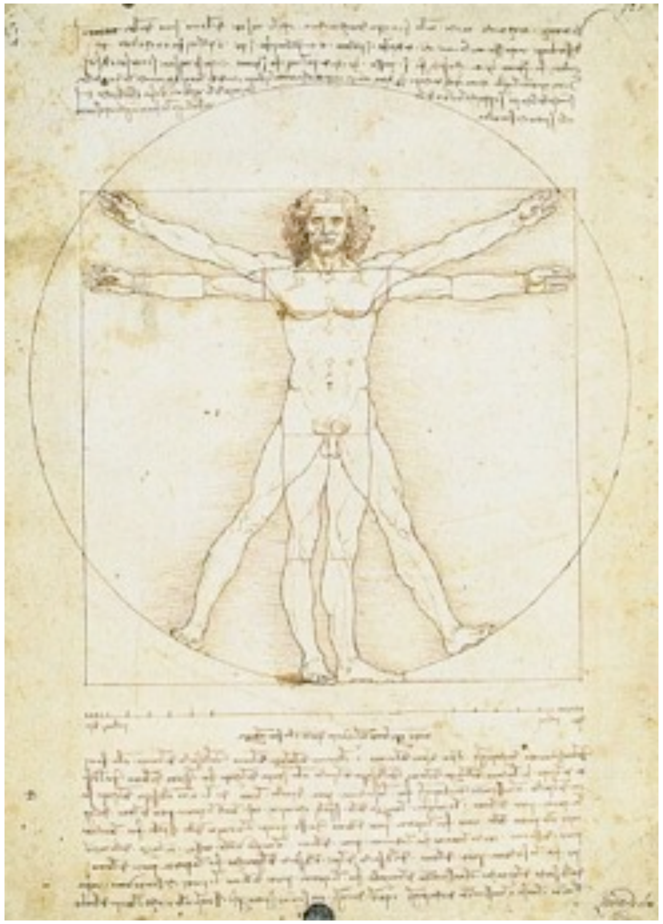
QM: Tunneling
No Symmetry Breaking

QFT: Overlap vanishes
Spontaneous Symmetry Breaking

Anomalous Symmetry

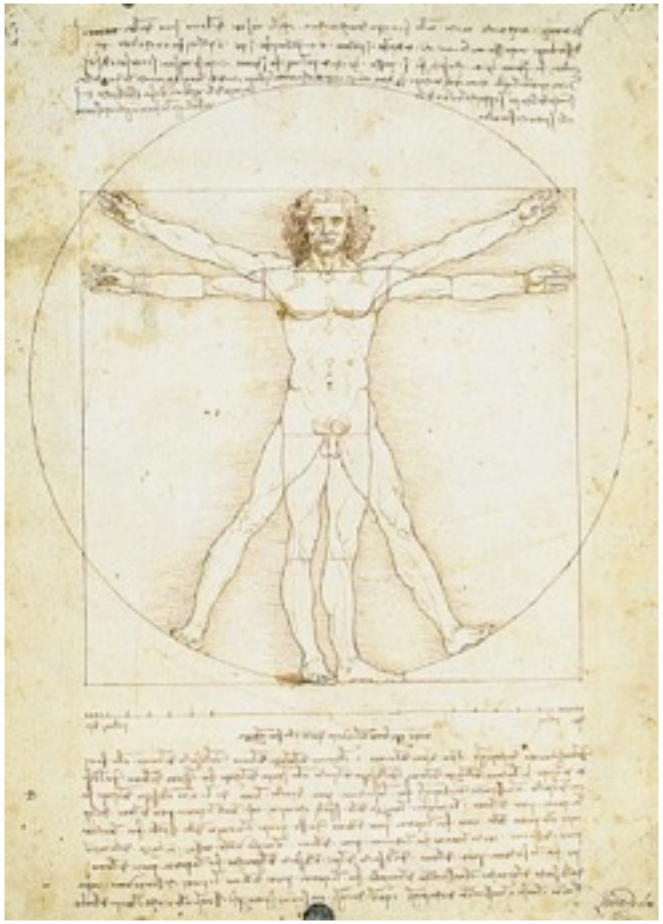


Anomalous Symmetry



Sometimes,
quantum
dynamics
eliminates
the symmetries!

Anomalous Symmetry



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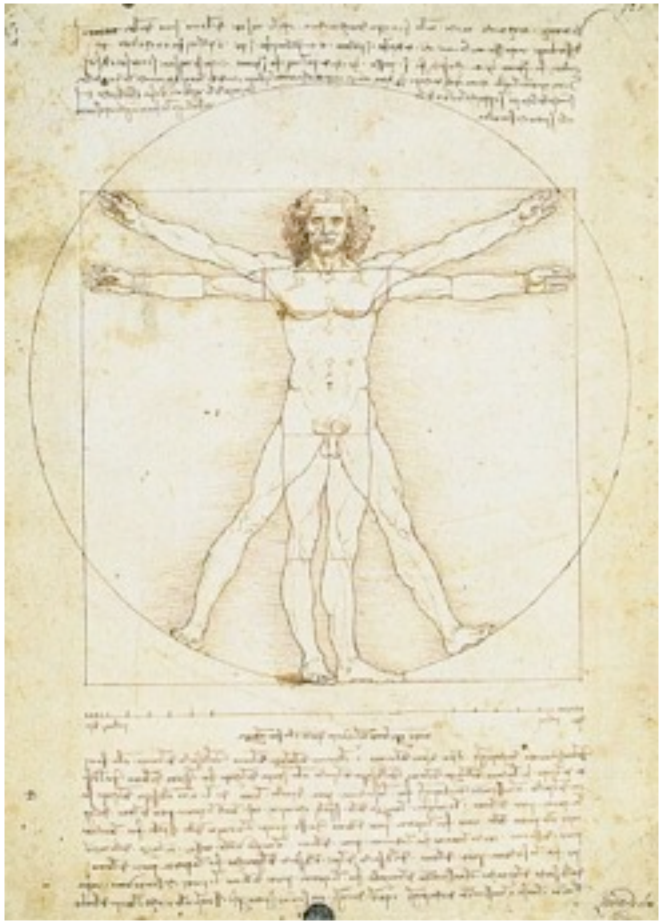

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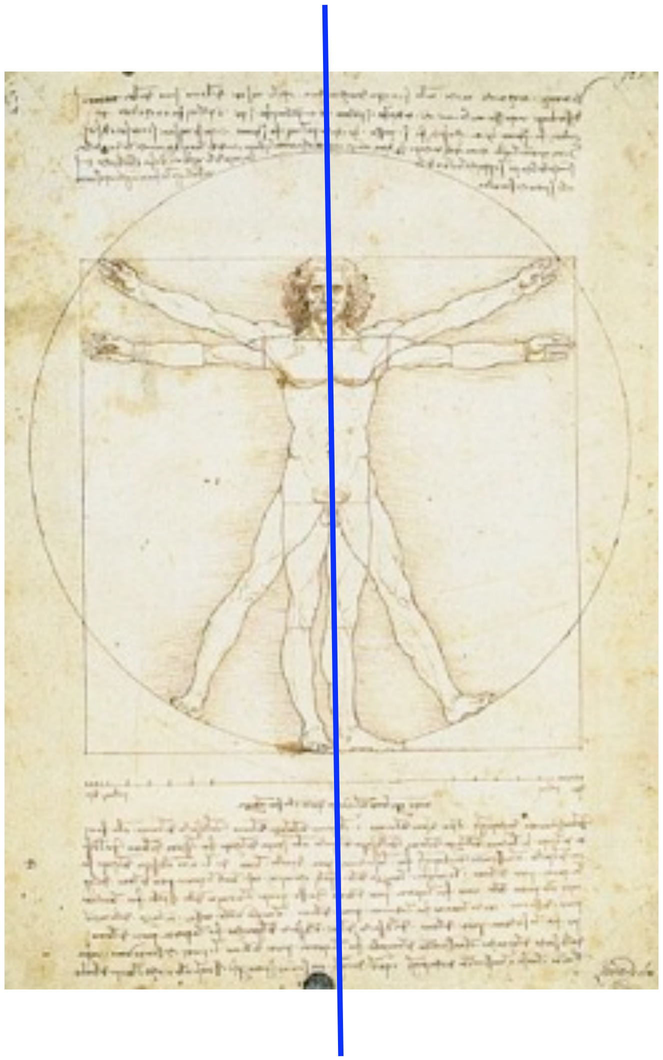
Resistance is futile!

Anomalous Symmetry



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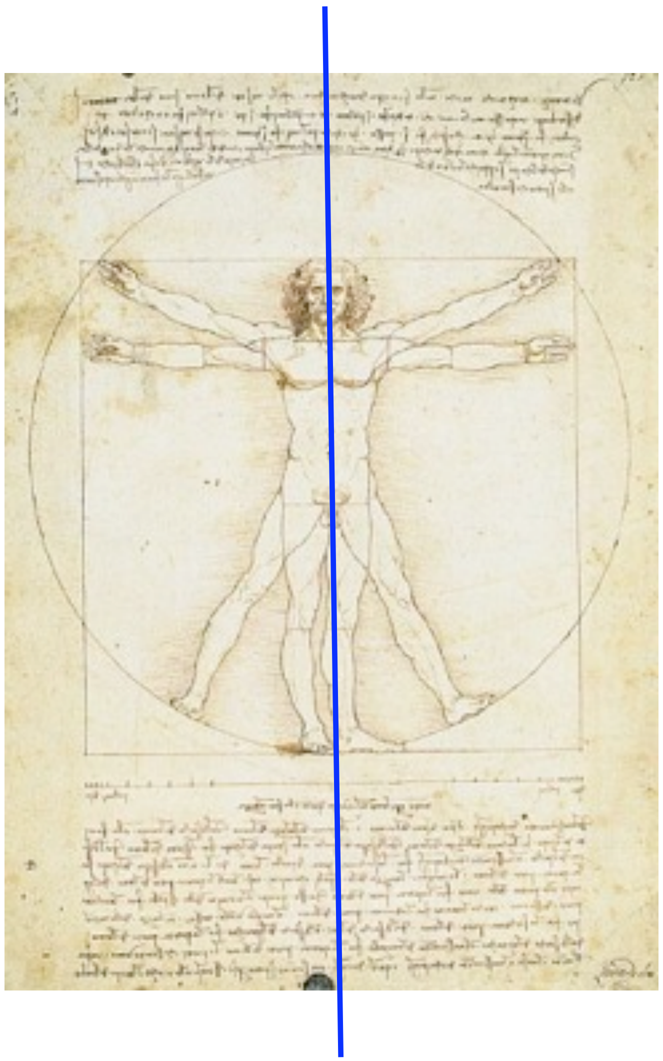
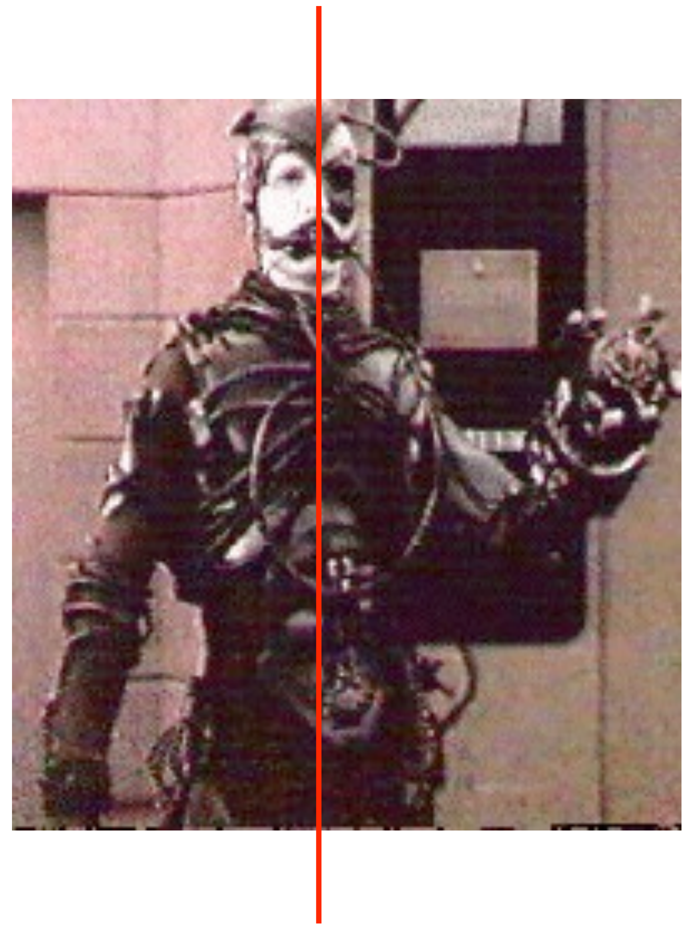


Sometimes, quantum dynamics eliminates the symmetries!



Resistance is futile!

Space-time Symmetries of Classical Chromodynamics ("~~Q~~CD")

- Poincare Invariance
- C, P, and T (as written...)
- (Approximate) Scale Invariance*

A scale transformation:

$$x^\mu \to \lambda \cdot x^\mu$$

$$\psi_q(x) \to \lambda^{3/2} \psi_q(\lambda x) \quad A_\mu^a(x) \to \lambda \, A_\mu^a(\lambda x)$$

NB: Broken by all mass terms...

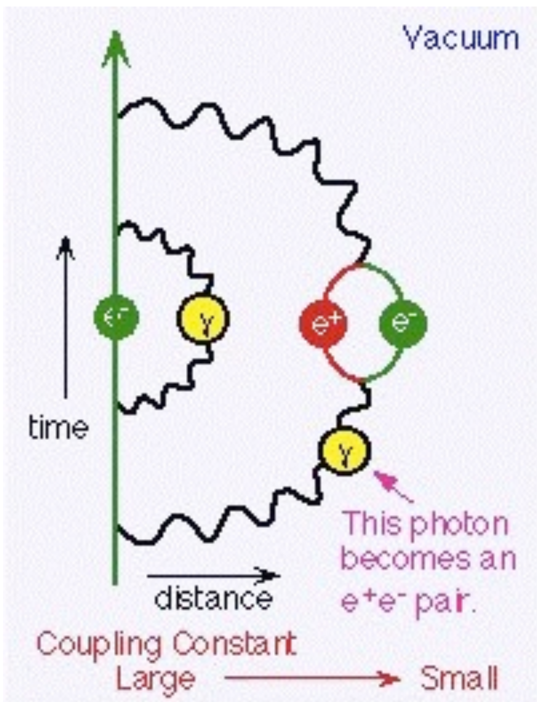
*(& Proper conformal trans.)

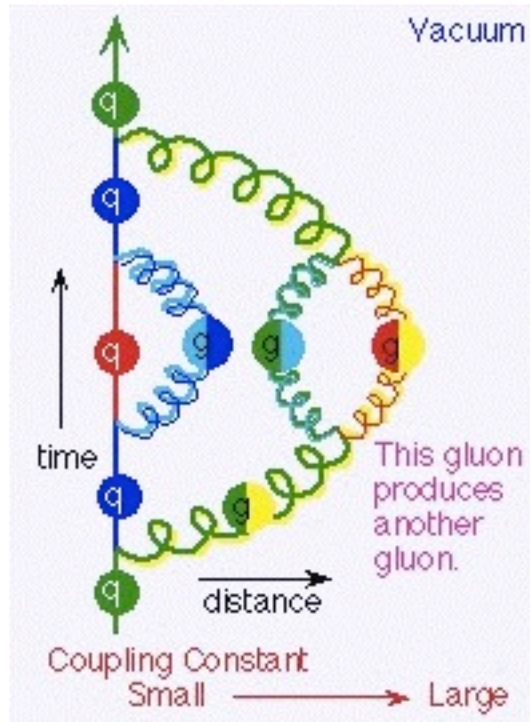
NB: Local scale invariance the reason for the name "gauge transformation" (Weyl)

No Quantum Scale Invariance!

- Quantum vacuum is a polarizable medium

QED





QCD

Fermions and Gluons make opposite contributions!

QCD Beta Function

$$\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3$$

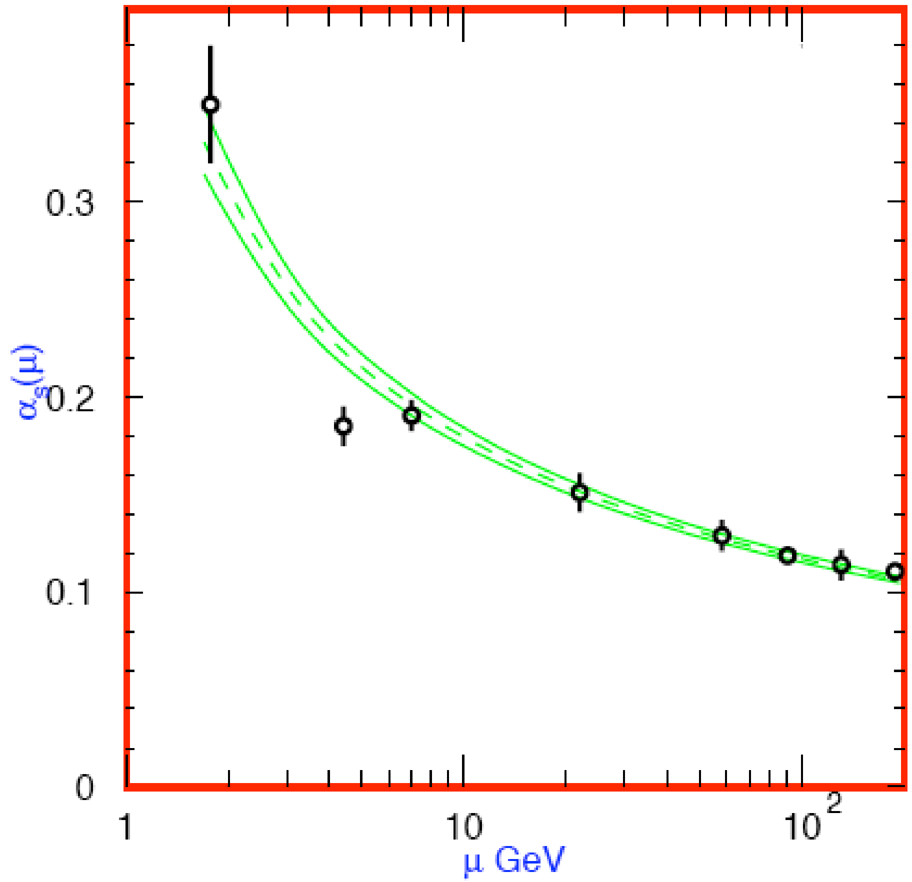
$$\beta_0 = 11 - \frac{2}{3}n_f ,$$

$$\beta_1 = 51 - \frac{19}{3}n_f ,$$

Asymptotic Freedom

vs.

Infrared Slavery



RPP2004

2004 Nobel Prize:
Gross, Politzer, Wilczek

Dimensional Transmutation

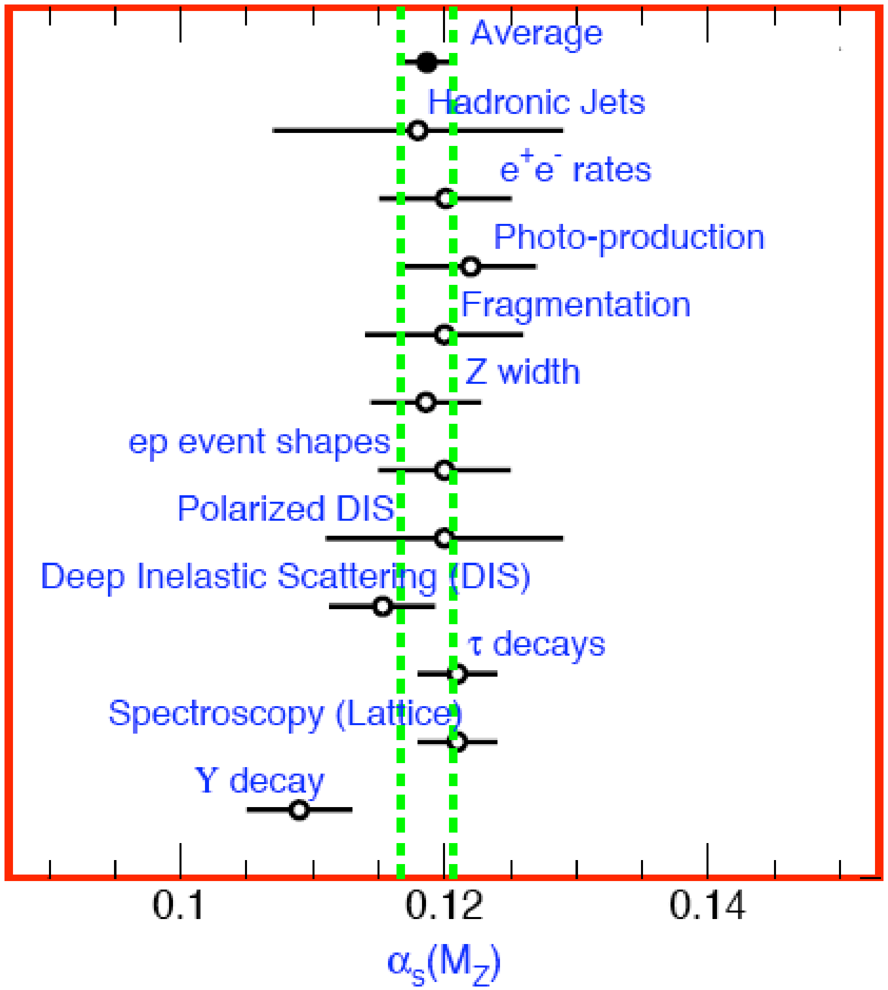
- Value of strong-coupling fixed at one scale, predicted at others.
- Alternatively, given by energy scale: Λ_{QCD}

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}\left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda^2)} \times \left(\left(\ln\left[\ln(\mu^2/\Lambda^2)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right].$$

RPP2004

Strong-interaction scale set by Λ_{QCD}

Strong Coupling



$$\Lambda^{(5)}_{\text{QCD}} = 217^{+25}_{-23}\,\text{MeV}$$

$$\Lambda^{(3)}_{\text{QCD}} \simeq 350\,\text{MeV}$$

Λ_{QCD} defines "light" and "heavy"

Why are nuclear binding energies so much smaller?

Scale Current

- Any continuous transformation defines an associated current
- A continuous symmetry has a conserved current

$$\partial_\mu s^\mu = T^\lambda_\lambda = -\frac{\beta}{2g} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + \sum_q m_q \bar{\psi}_q \psi_q + \ldots$$

Quantum Effect!

Scale Symmetry is Anomalous!

Global Quark Flavor Symmetries of "CD" (I)

- Baryon Number: $\psi_q \rightarrow e^{i\alpha}\psi_q$
- (Approximate*) Gell-Mann SU(3)v:

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Together: U(3)v

Realized "As Is" in Quantum Theory!

*Requires $m_u \approx m_d \approx m_s$

Global Quark Flavor Symmetries of "CD" (2)

- (Approximate*) Chiral SU(3)L x SU(3)R:

$$\bar{\psi}_q \, i\not{D}\psi_q \equiv \bar{\psi}_q^L \, i\not{D}\psi_q^L + \bar{\psi}_q^R \, i\not{D}\psi_q^R$$

$$\begin{pmatrix} u^{L,R} \\ d^{L,R} \\ s^{L,R} \end{pmatrix} \rightarrow U_{L,R} \begin{pmatrix} u^{L,R} \\ d^{L,R} \\ s^{L,R} \end{pmatrix}$$

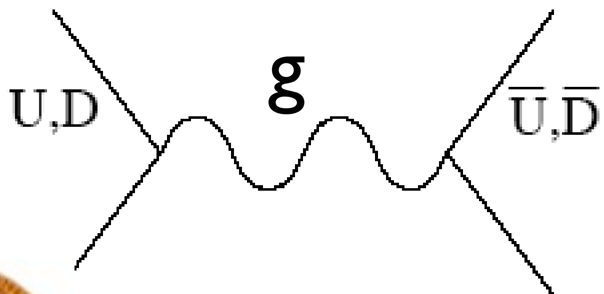
*For "light" quarks: u, d, and s(?)

$$m_q \bar{\psi}_q \psi_q \equiv m_q \bar{\psi}_q^L \psi_q^R + m_q \bar{\psi}_q^R \psi_q^L$$

Quantum Theory different...

Chiral Symmetries are Spontaneously Broken

- Chiral symmetries are good quantum symmetries, but ...
- ... strong low-energy QCD dynamics rearranges the vacuum



$$\rightarrow \langle \bar{U}_L U_R \rangle = \langle \bar{D}_L D_R \rangle \neq 0 \, .$$

2008 Nobel Prize: Nambu

Goldstone Bosons

- $SU(3)_L$ x $SU(3)_R$ breaks to Gell-Mann $SU(3)_V$
- Implies existence of eight (massless) Goldstone Bosons

$$j^i_{A\mu} = -f_\pi \partial_\mu \pi^i + \ldots$$

$$\partial^\mu j^i_{A\mu} = -f_\pi \, \partial^\mu \partial_\mu \pi^i + \ldots = 0 \Leftrightarrow m^2_\pi = 0$$

Pseudo Goldstone Bosons

- Relevant for light quarks only: $m_q \leq \Lambda_{QCD}$
- u, d, and s are not massless,
- Therefore, pions are light*:

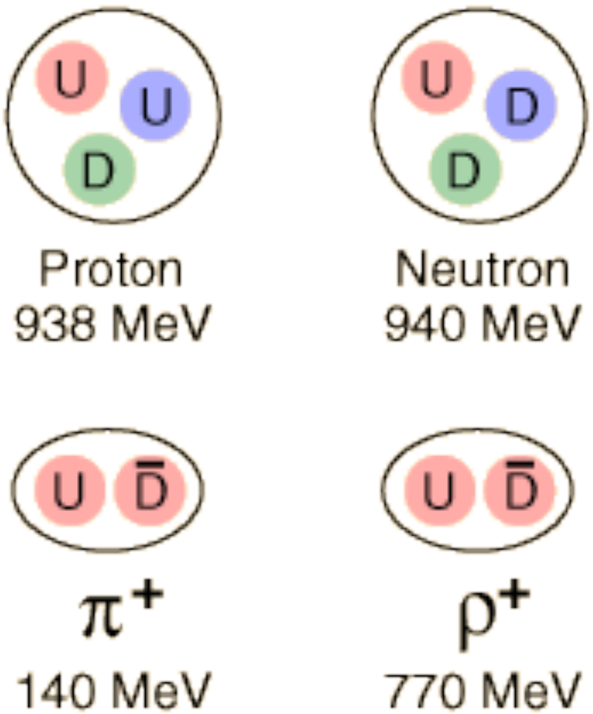
$$m_\pi^2 \propto (m_u + m_d)\Lambda_{QCD}$$

$$m_K^2 \propto (m_s + m_{u,d})\Lambda_{QCD}$$

$$m_\eta^2 \propto \frac{1}{3}(m_u + m_d + 4m_s)\Lambda_{QCD}$$

*NB: QCD Lagrangian Quark masses appear here!

Constituent Quarks



~300 MeV quark-model mass is of a "dressed" or "constituent" quark

~99% of proton and neutron "mass" due to QCD field energy!

What about U(1)$_A$?

- U(1)$_A$: $\psi_q^L \to e^{i\theta}\psi_q^L \;\; \& \;\; \psi_q^R \to e^{-i\theta}\psi_q^R$
- Broken by chiral condensate
- Where is the ninth Goldstone Boson?
- Another Quantum "Anomaly": $\vec{E}^a \cdot \vec{B}^a$

$$\partial_\mu j_A^\mu \propto \frac{g^2}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}^{(a)}F_{\alpha\beta}^{(a)}$$

Adler, Bell, Jackiw 't Hooft

Strong-CP Problem?

- But the same term could be in the QCD Lagrangian...
- ... and violates CP!
- Yields a neutron electric dipole moment
- Motivates the Axion...

't Hooft

Peccei and Quinn

QCD is the culmination almost a century of effort in QFT...

- Heisenberg, Pauli, Dirac, Jordan, Oppenheimer,...
- Schwinger, Feynman, Tomonaga, Bethe,...
- Nambu, Goldstone, ...
- Yang & Mills
- ‘t Hooft, Veltman, Lee, Ward,...
- Gross, Politzer, Wilczek

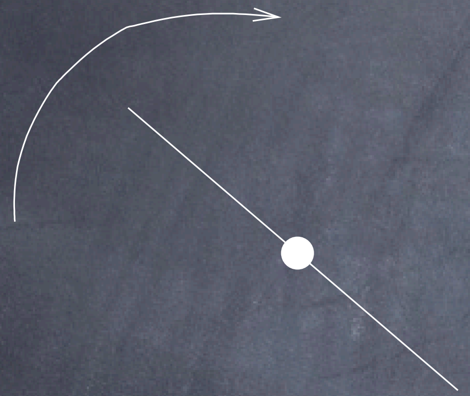
Shelter Island, 1947

Symmetries in QCD

Symmetry	Anomaly?	Fate
Color (gauged)	no	unbroken (confined)
Scale Invariance	yes	not a symmetry
$SU(3)_V \times U(1)_B$	no	unbroken
$SU(3)_A$	no	spontaneously broken
$U(1)_A$	yes	not a symmetry
CP	no?!	not a symmetry!?

New Theoretical Developments...

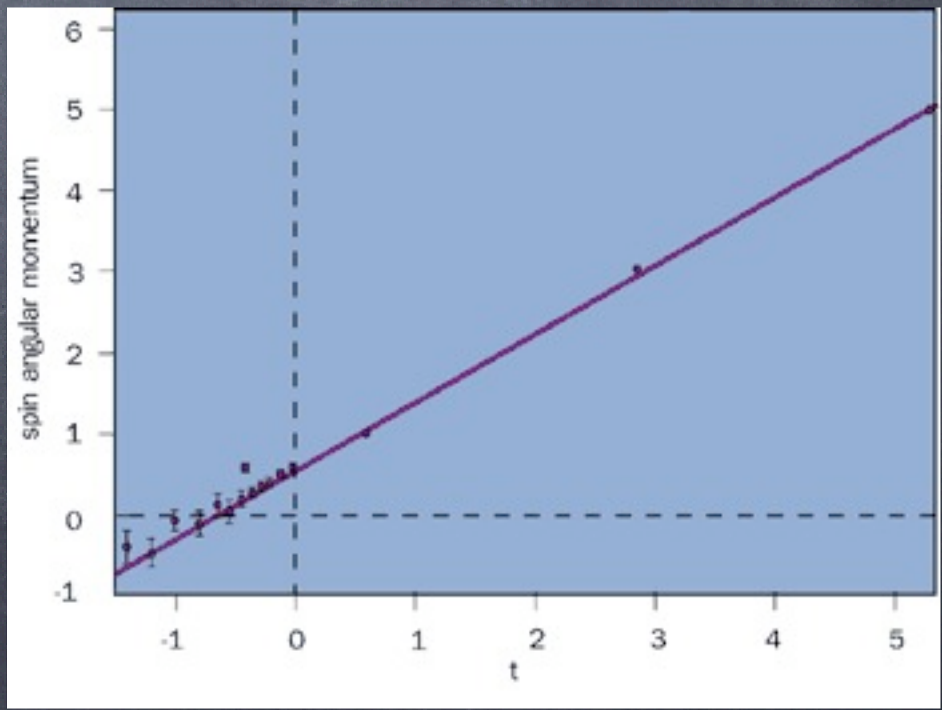
QCD and String Theory: Relativistic Classical String



$$E \propto \frac{J^2}{2I} \qquad I \propto ML^2$$

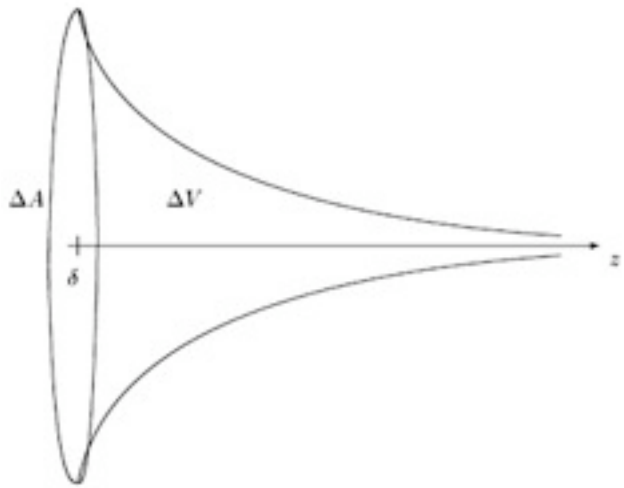
$$M, L \propto E \qquad \Rightarrow J \propto E^2$$

Regge Trajectories



AdS/CFT Duality

Conjecture: Equivalence of 5D theory in AdS and 4D CFT



$$ds^2 = \left(\frac{R}{z}\right)^2 \left[\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right]$$

$$R < z < R'$$

UV ⟶ IR

NB: Rescaling Invariance!

Strong evidence for N=4 SUSY YM string theory on AdS

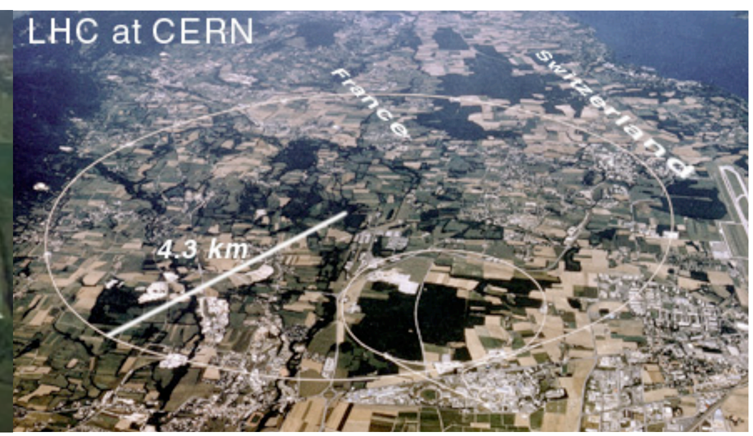
Strongly-coupled CFT ⇔ Weakly-coupled 5D Theory!

Is there an 5D/QCD duality?

Morals for the LHC Era

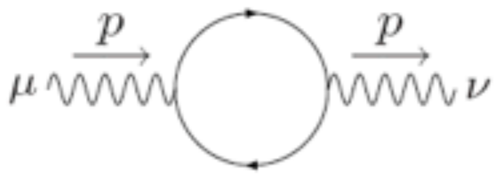
What is the origin of the Higgs Mechanism?

Why do some particles have large masses while the photon and neutrinos have puny masses?



The Higgs *Mechanism*

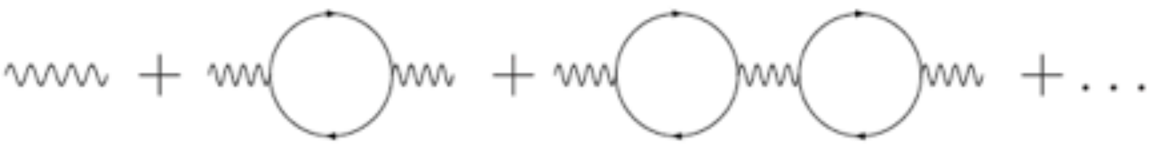
The polarization tensor $\Pi_{\mu\nu}(p)$ is defined as:



$$i\,\Pi_{\mu\nu}(p) \equiv i(p_\mu p_\nu - p^2 g_{\mu\nu})\Pi(p^2)$$

where the form of $\Pi_{\mu\nu}(p)$ is governed by gauge invariance, i.e. it satisfies $p^\mu \Pi_{\mu\nu}(p) = p^\nu \Pi_{\mu\nu}(p) = 0$.

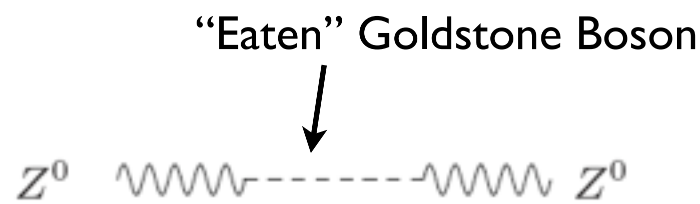
The renormalized propagator is the sum of a geometric series



$$= \frac{-i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)}{p^2[1+\Pi(p^2)]}$$

The pole at $p^2 = 0$ is shifted to a non-zero value if:

$$\Pi(p^2) \underset{p^2 \to 0}{\simeq} \frac{-g^2 v^2}{p^2}\,.$$

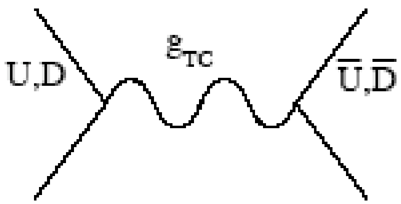


Then $p^2[1 + \Pi(p^2)] = p^2 - g^2 v^2$, yielding a gauge boson mass of gv.

Technicolor

- Use scaled-up QCD to break electroweak symmetry

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$



$$\rightarrow \langle \bar{U}_L U_R \rangle = \langle \bar{D}_L D_R \rangle \neq 0$$

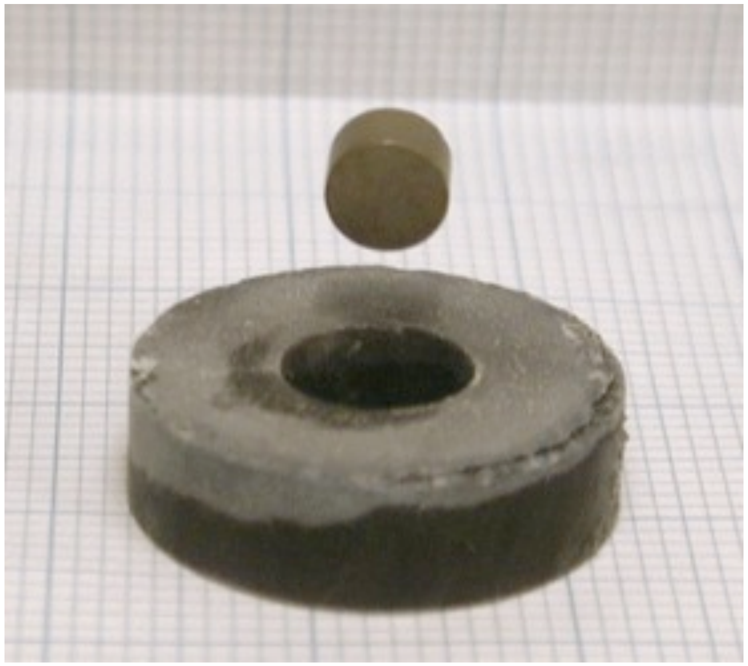
No hierarchy problem!

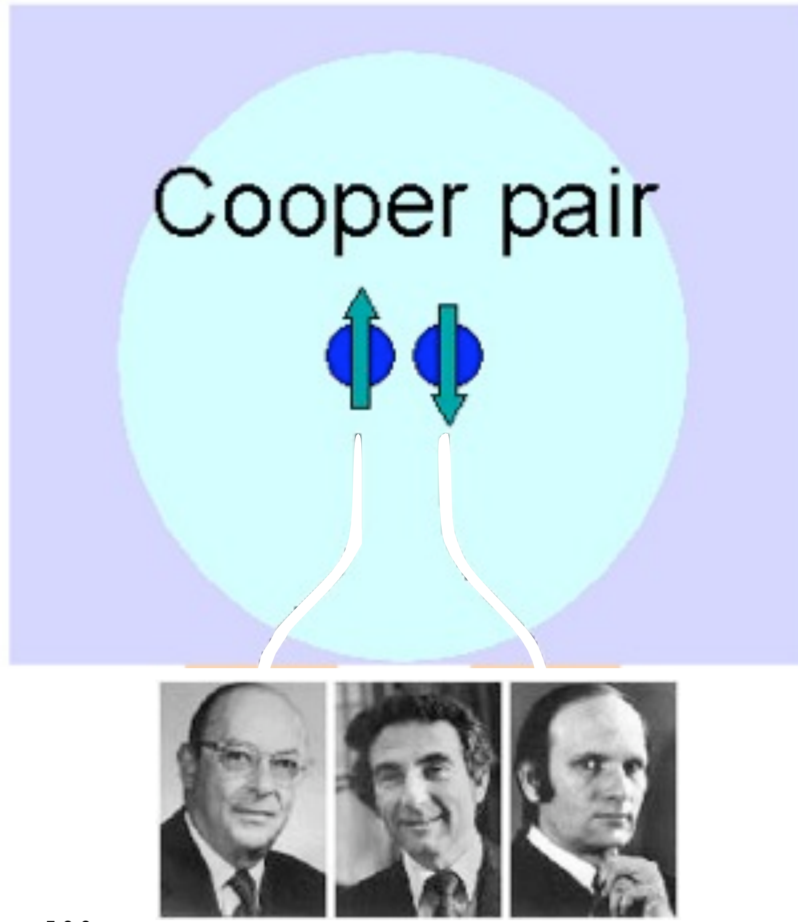
$$\pi^{\pm},\, \pi^0 \rightarrow W_L^{\pm},\, Z_L$$

$$M_W = \frac{gF_{TC}}{2} \rightarrow F_{TC} \approx 250\text{GeV}$$

S?

(“Low-Energy” Analog)





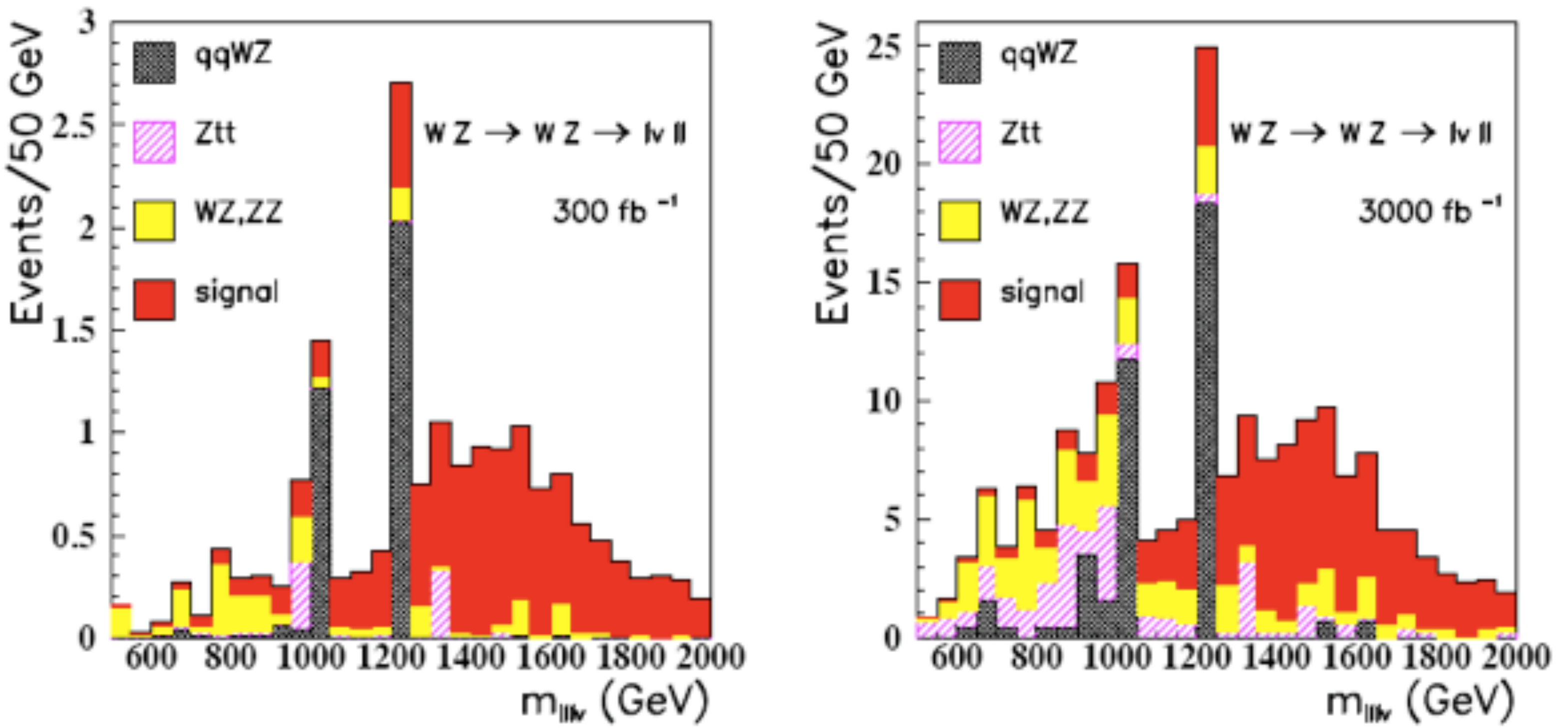
$$\langle \phi^{--} \rangle \neq 0$$

“Abelian Higgs Model”

B C S

Weinberg: “Superconductivity for Particular Theorists”

WZ Scattering at SLHC



$$p_T(\ell_1) > 150 \text{ GeV}, \quad p_T(\ell_2) > 100 \text{ GeV}, \quad p_T(\ell_3) > 50 \text{ GeV}$$

$$|m(\ell_1\ell_2) - m_Z| < 10 \text{ GeV}$$

$$E_T^{miss} > 75 \text{ GeV}$$

+ forward jets

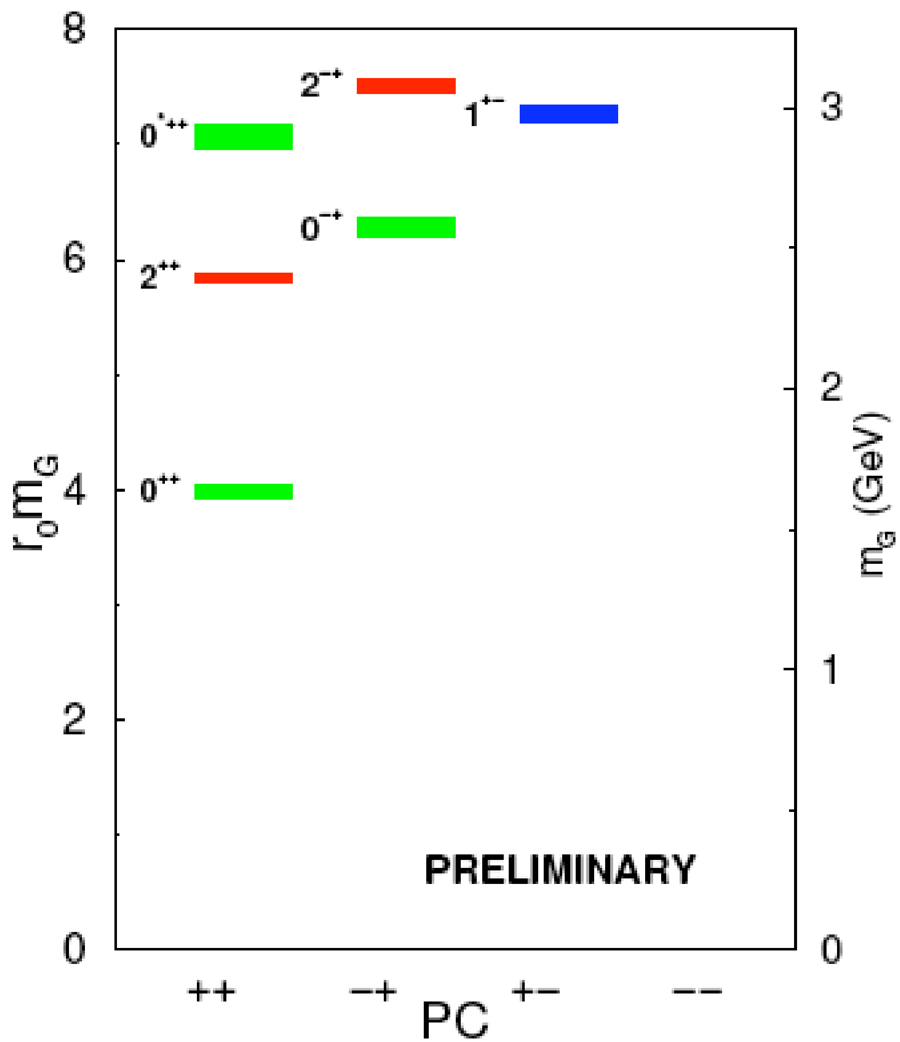
F. Gianotti, et. al., hep-ph/0204087

QCD

- Strong QCD dynamics breaks quark chiral symmetries ...
- ... yielding ~99% of the mass of ordinary matter ...
- and provides an example for models of dynamical electroweak symmetry breaking.
- QCD is the very model of a modern quantum field theory.

Extra Slides

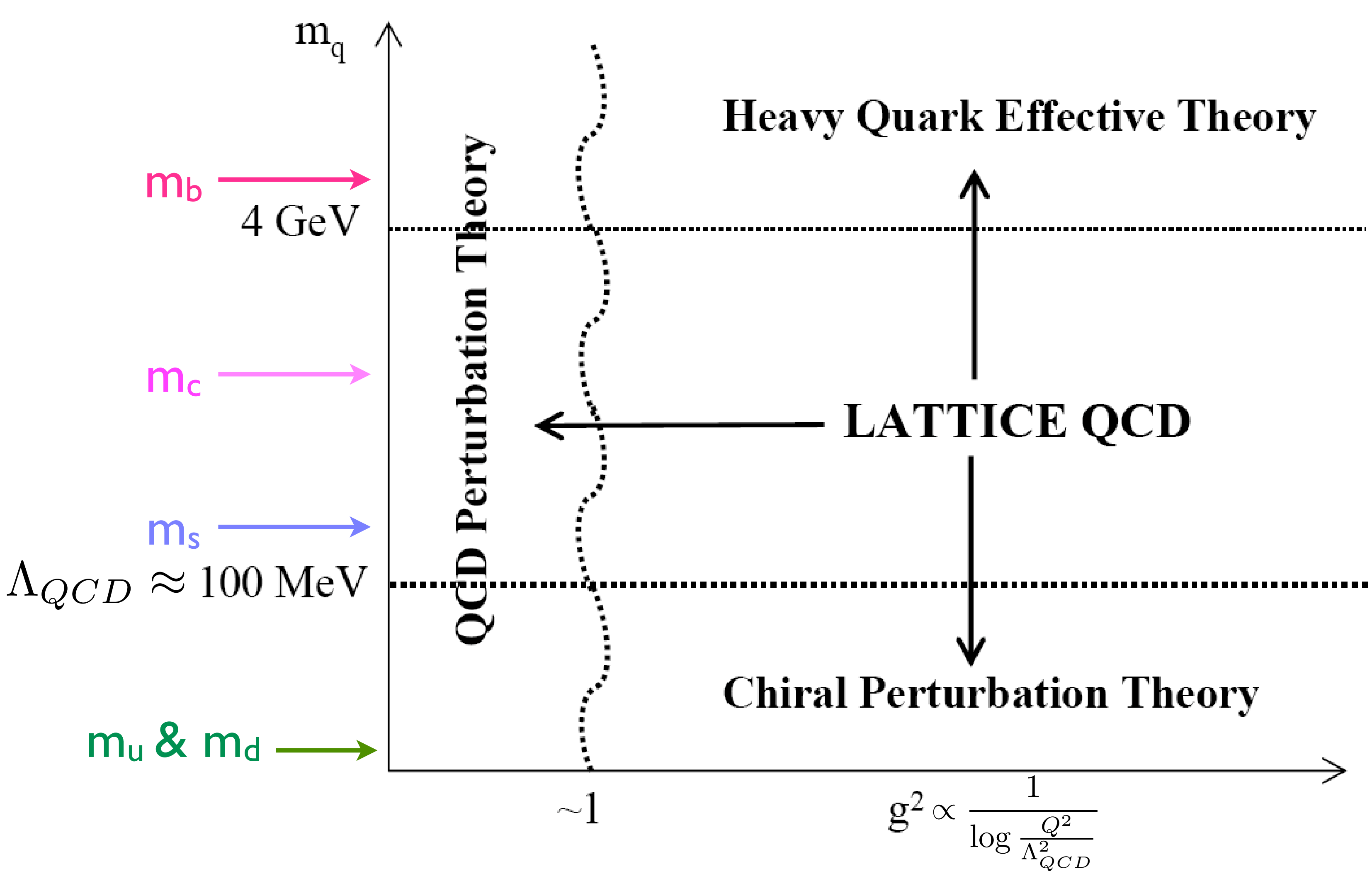
Mass-Gap in Pure Yang-Mills Theory



Glueball Spectrum

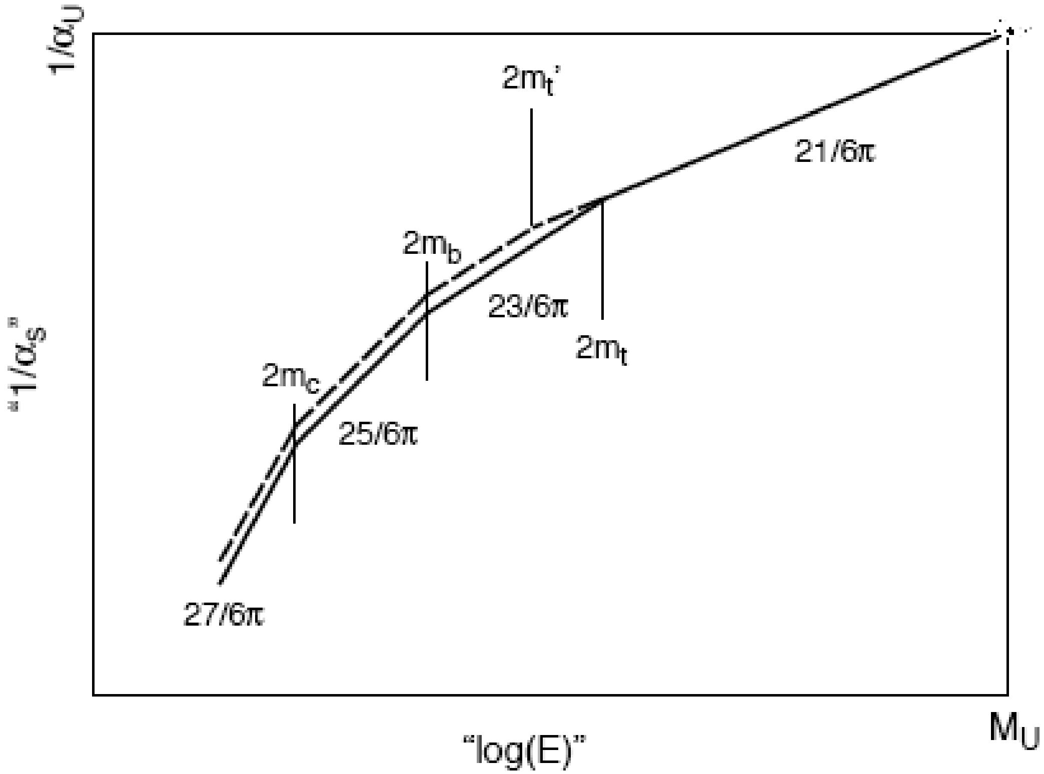
Morningstar and Peardon, nucl-th/0309068

QCD: Modern Perspective



Rajan Gupta, LANL

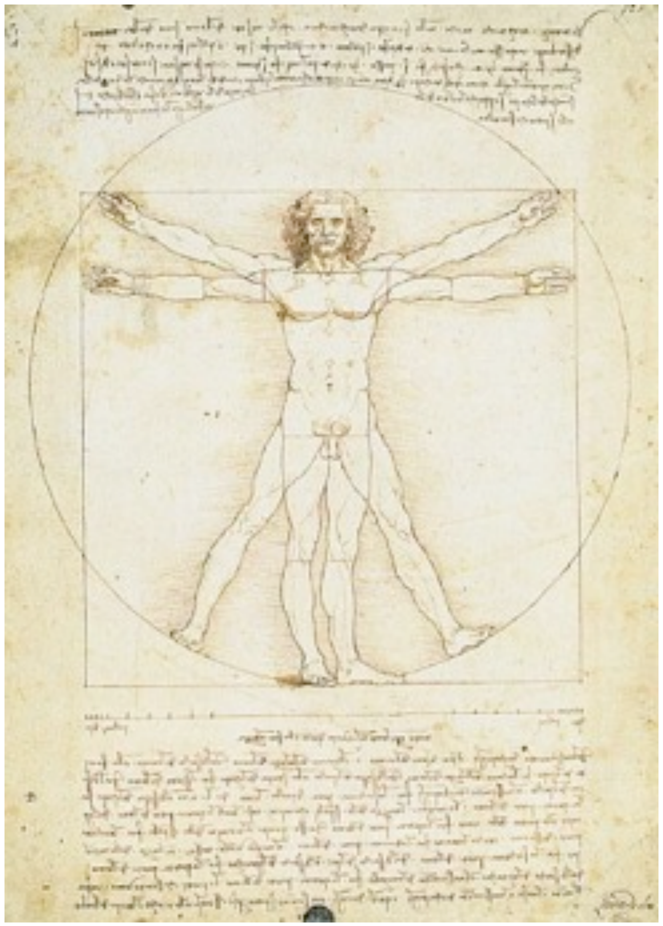
Top-Quark Matters



$$\frac{M_{\mathrm{proton}}}{1\ \mathrm{GeV}} \propto \left(\frac{m_t}{1\ \mathrm{GeV}}\right)^{2/27}$$

Quigg, hep-ph/9707508

Anomalous Symmetry



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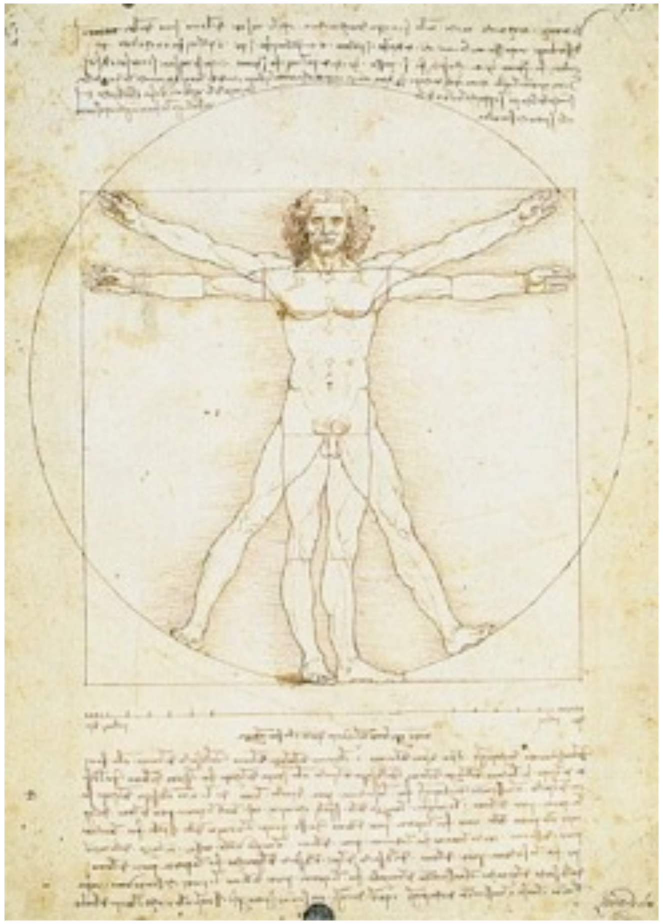

Sometimes,
quantum
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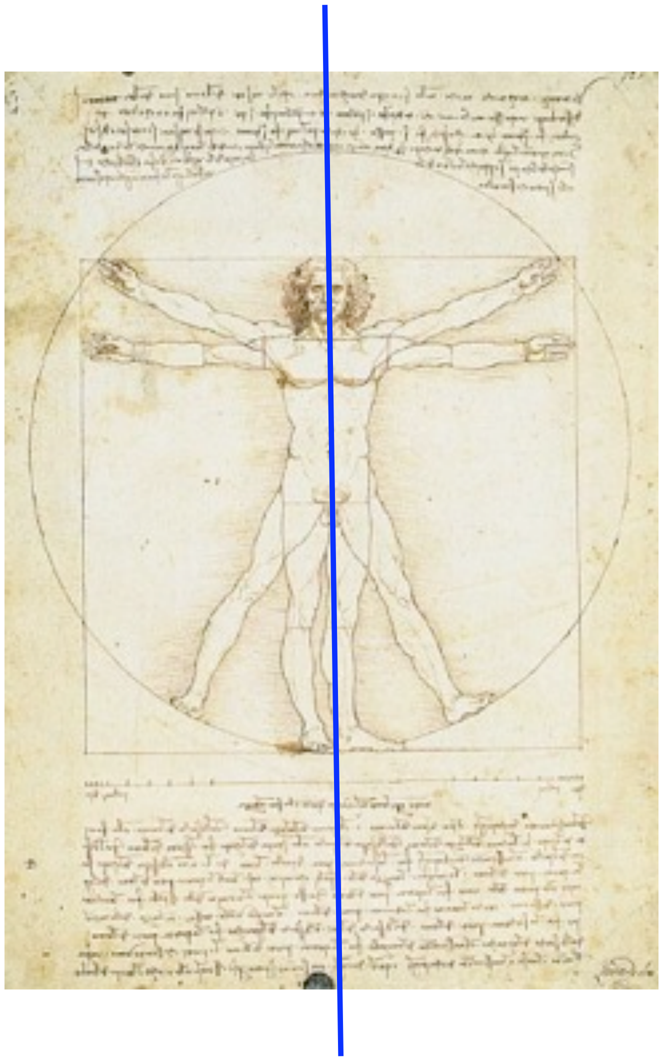
eliminates
the symmetries!

Anomalous Symmetry



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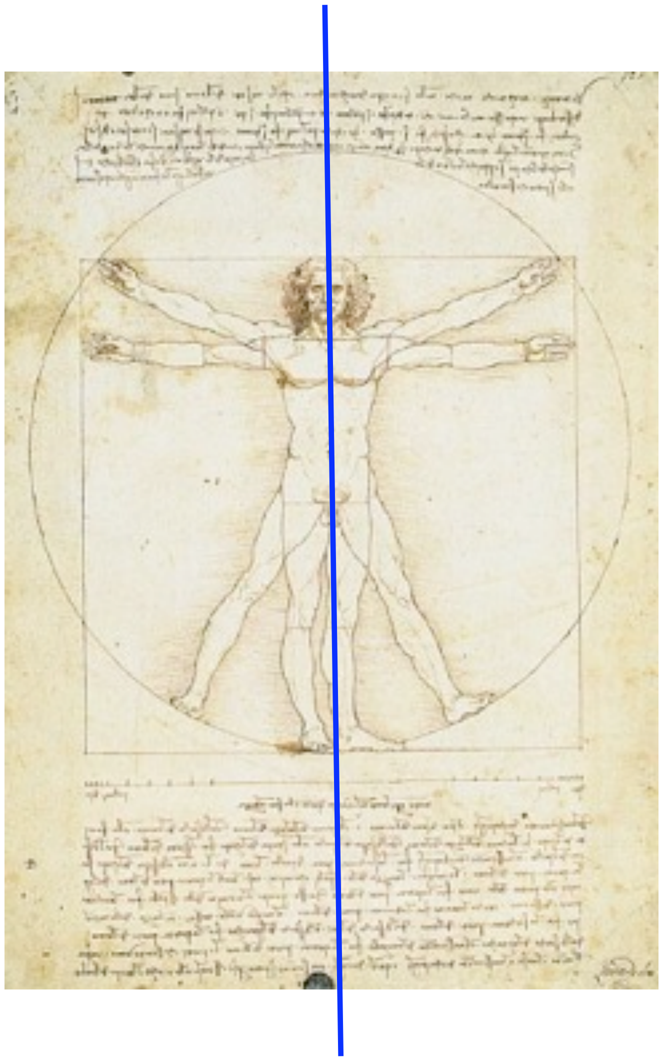

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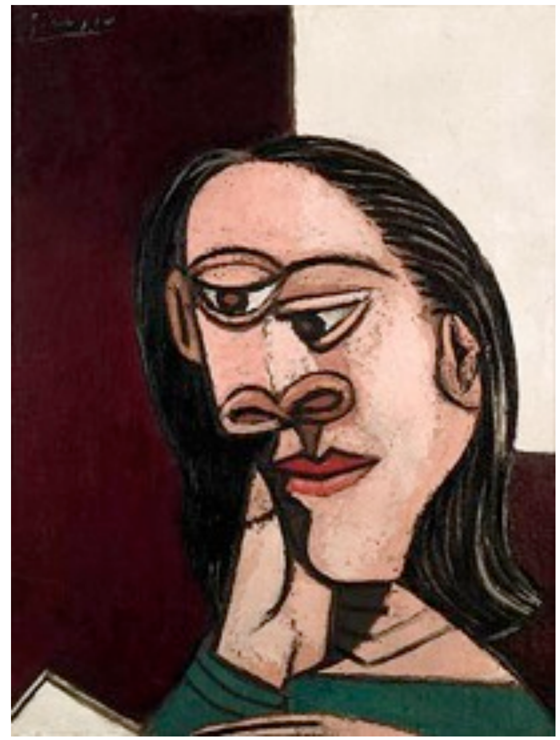

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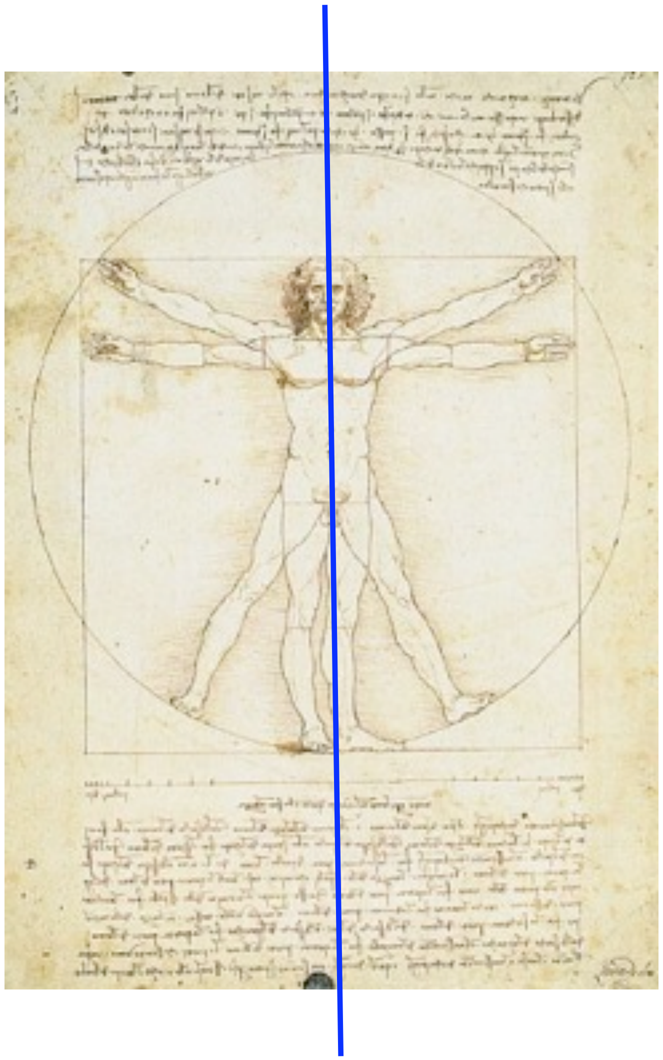

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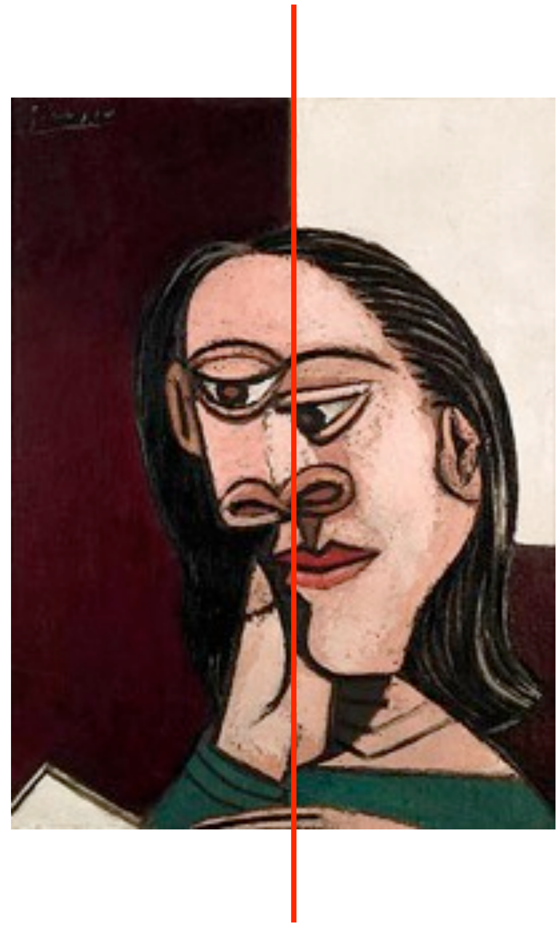


QCD is Still Important

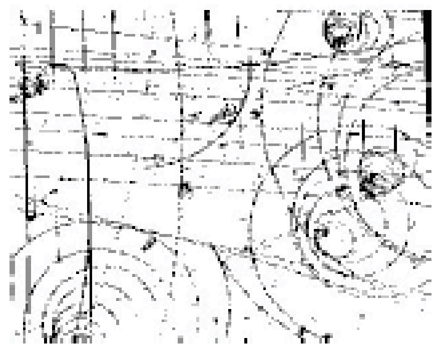
Yang-Mills and Mass Gap

The laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the "mass gap:" the quantum particles have positive masses, even though the classical waves travel at the speed of light. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of the Yang-Mills theory and a mass gap and will require the introduction of fundamental new ideas both in physics and in mathematics.

The Millennium Problems

Official Problem Description

Lecture by Lorenzo Sadun at University of Texas (video)



Especially at Tevatron/LHC...