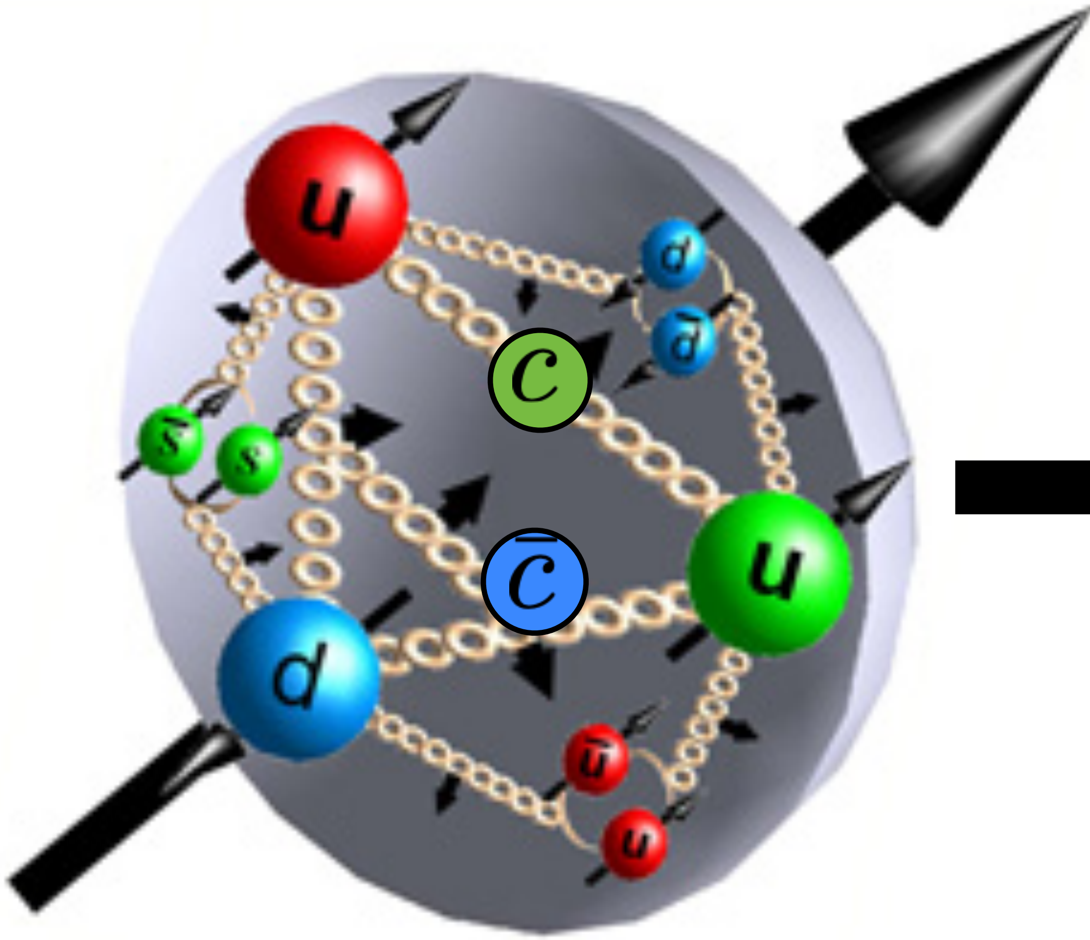
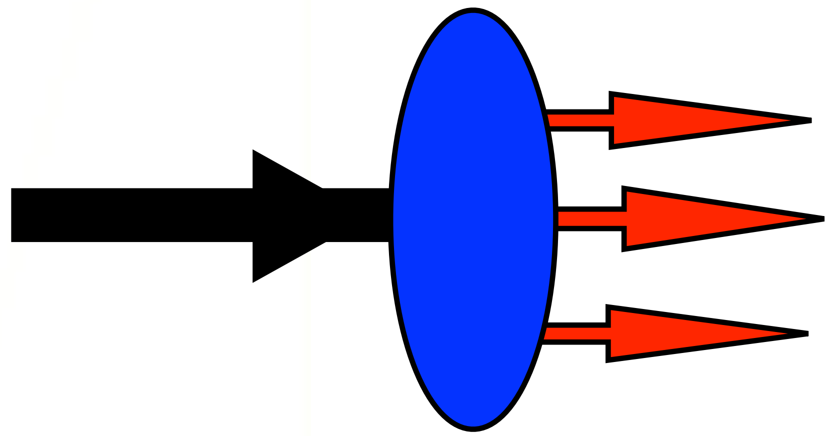
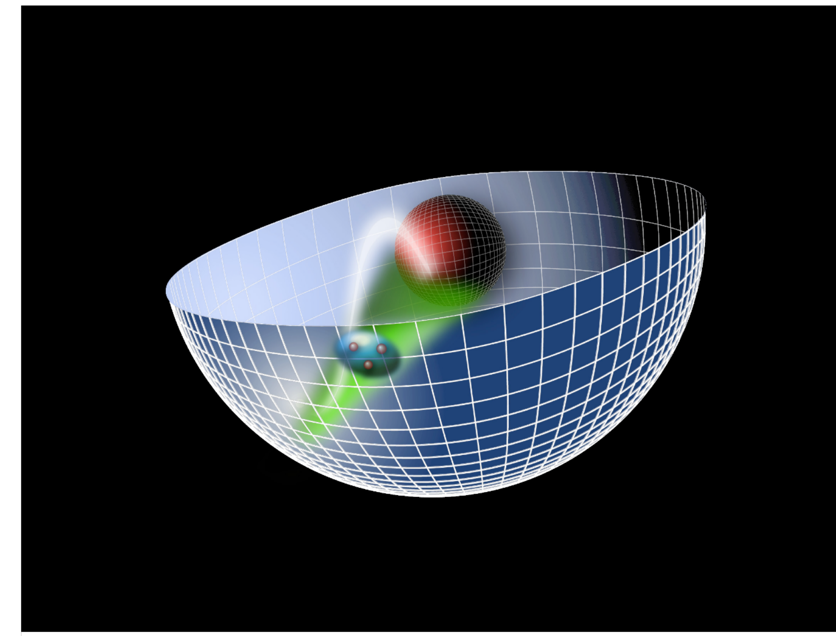
Supersymmetric Properties of Hadron Physics and Other Remarkable Features of Hadron Physics







Congratulations to Paul Mackenzie!

Fermilab

Colloquium
November 6, 2019

Stan Brodsky
SLAC NATIONAL ACCELERATOR LABORATORY

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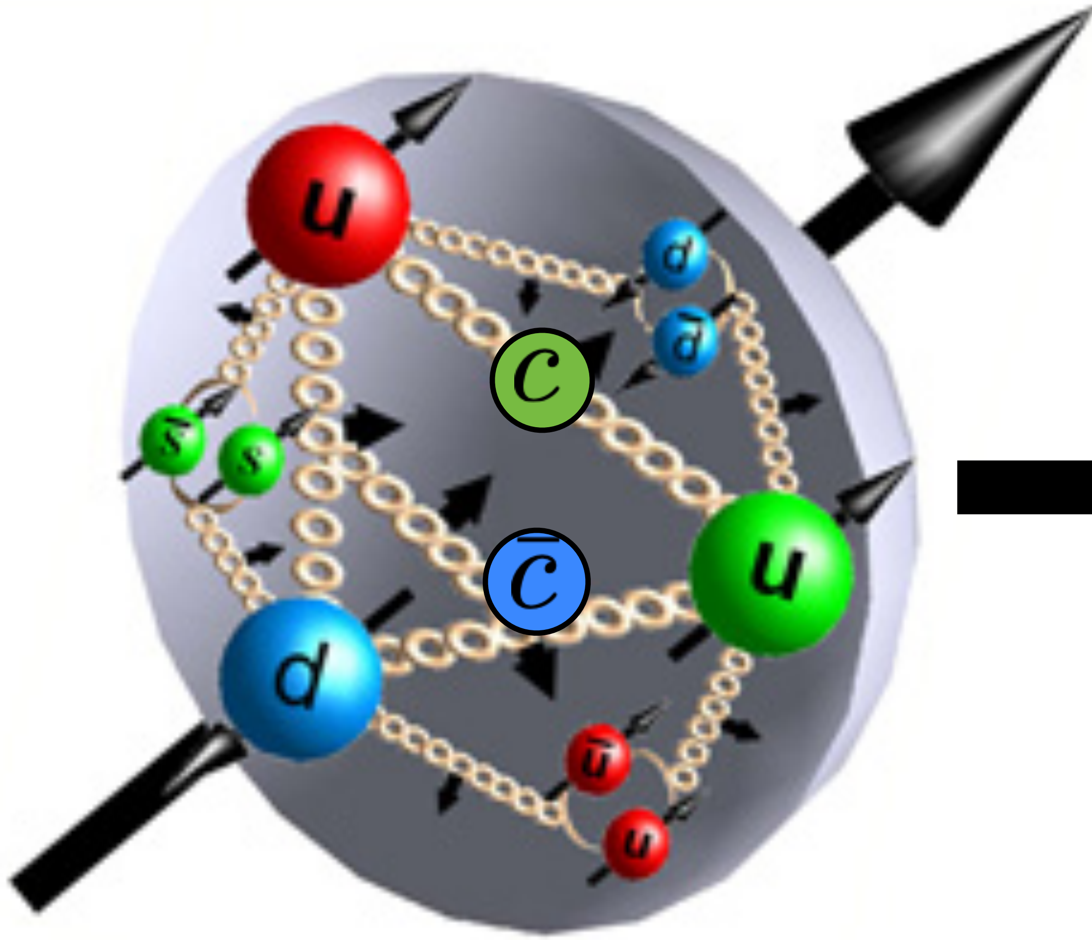

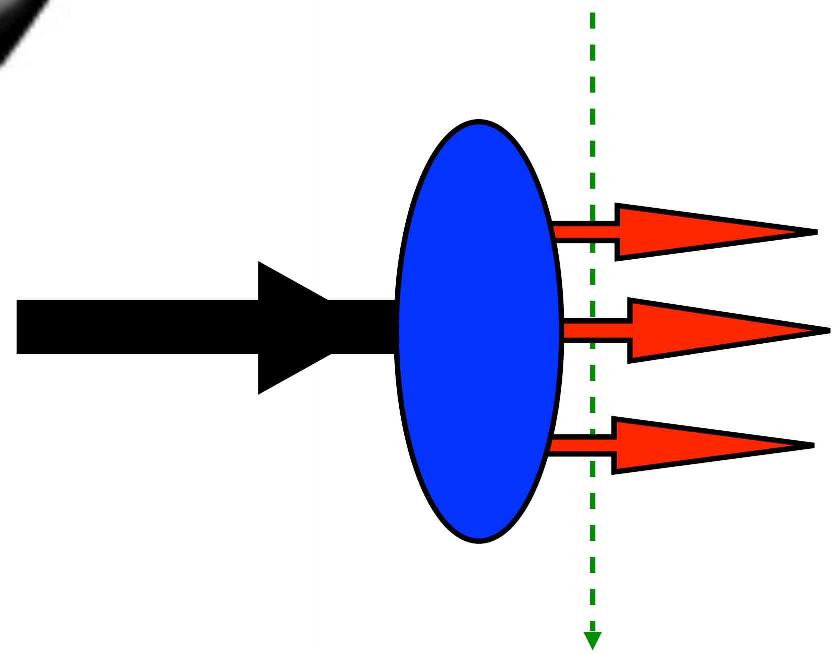

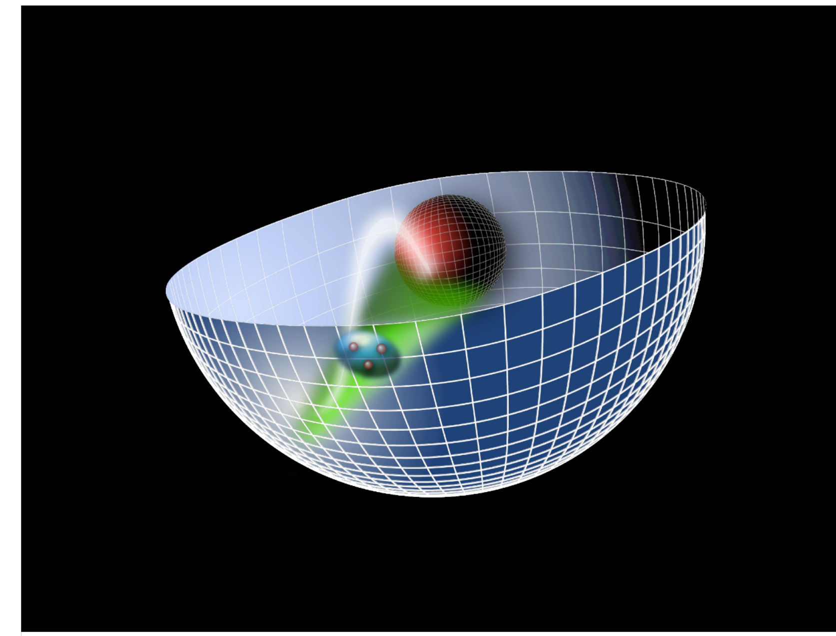

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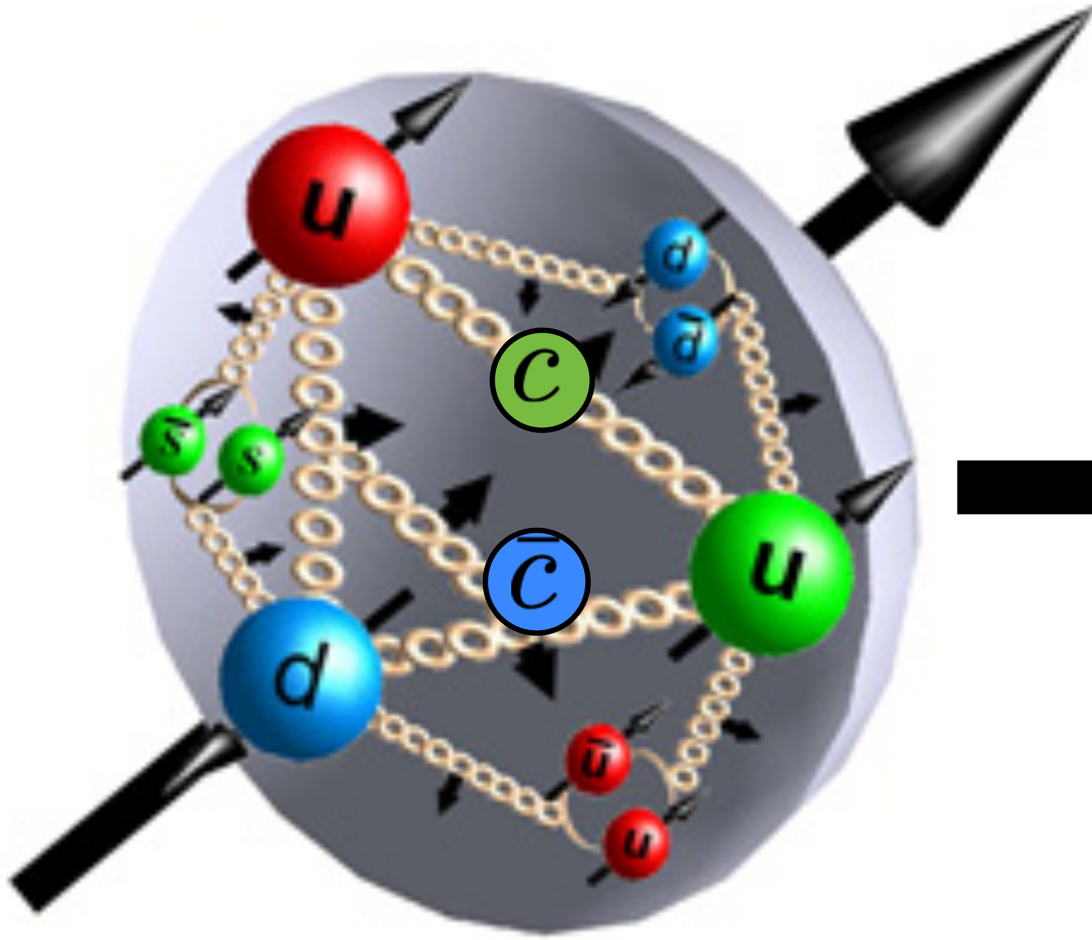
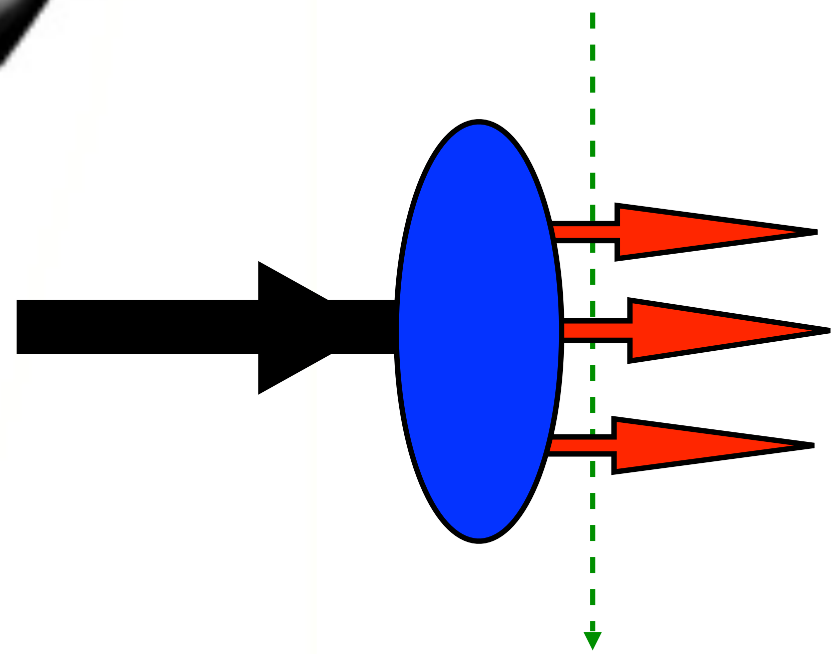
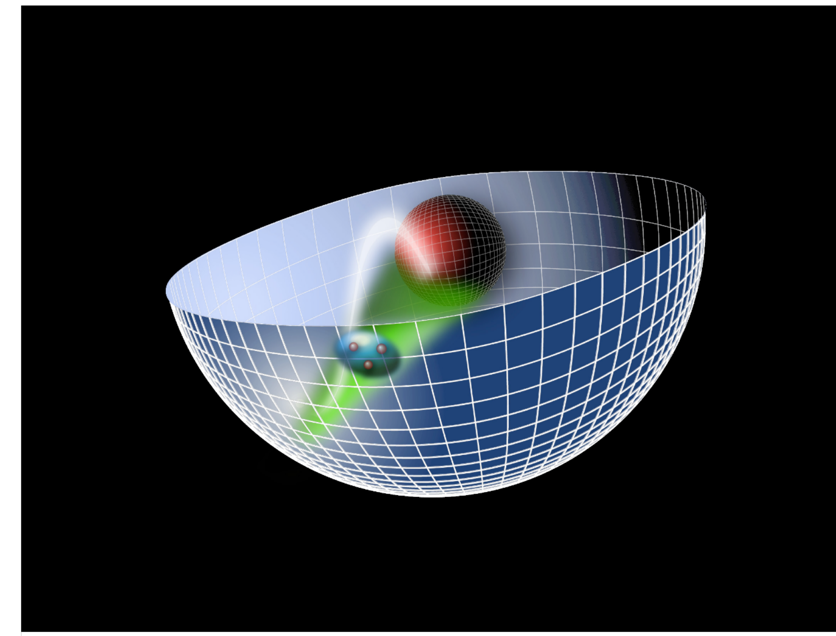


Fixed $\tau = t + z/c$





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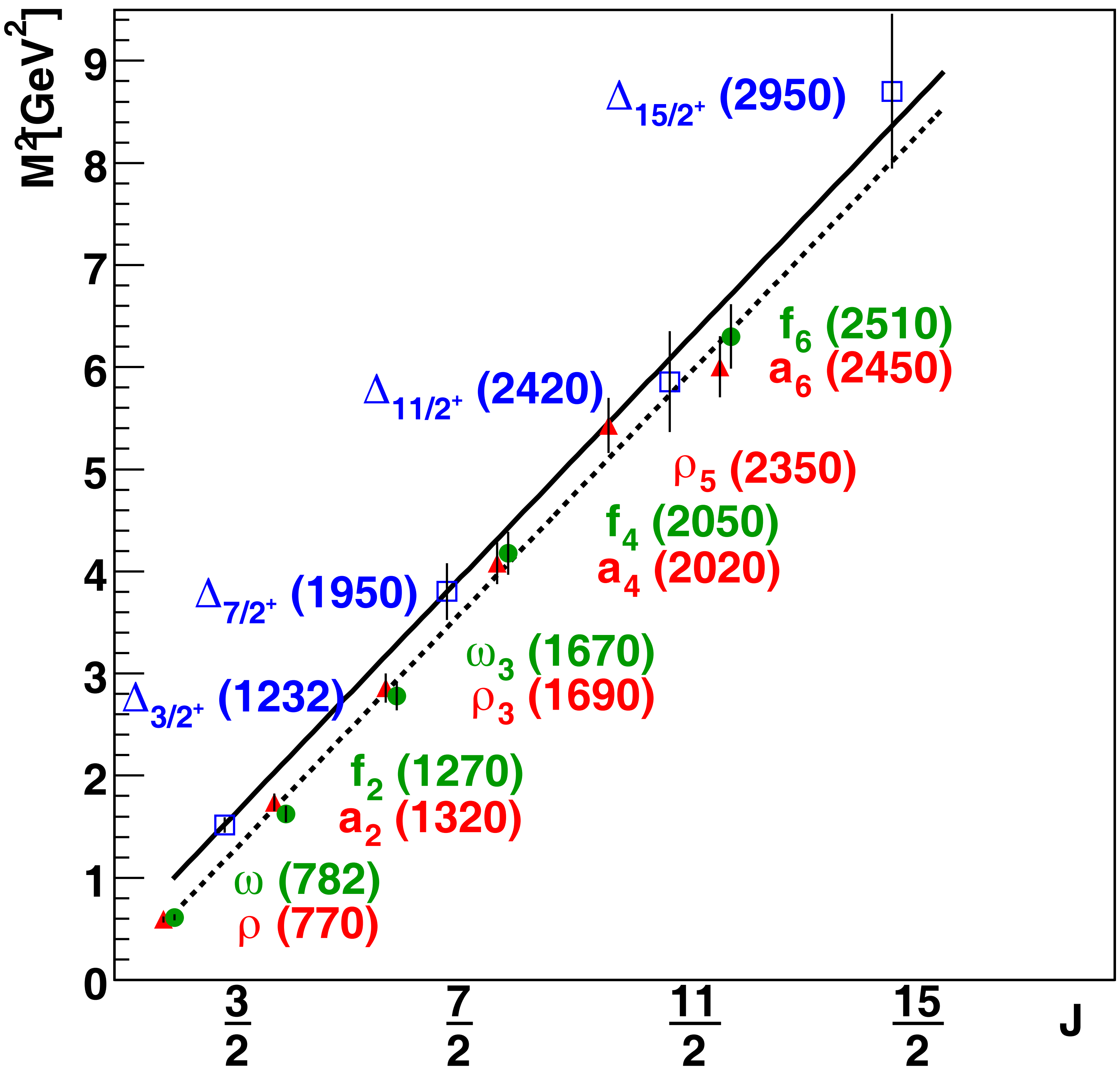
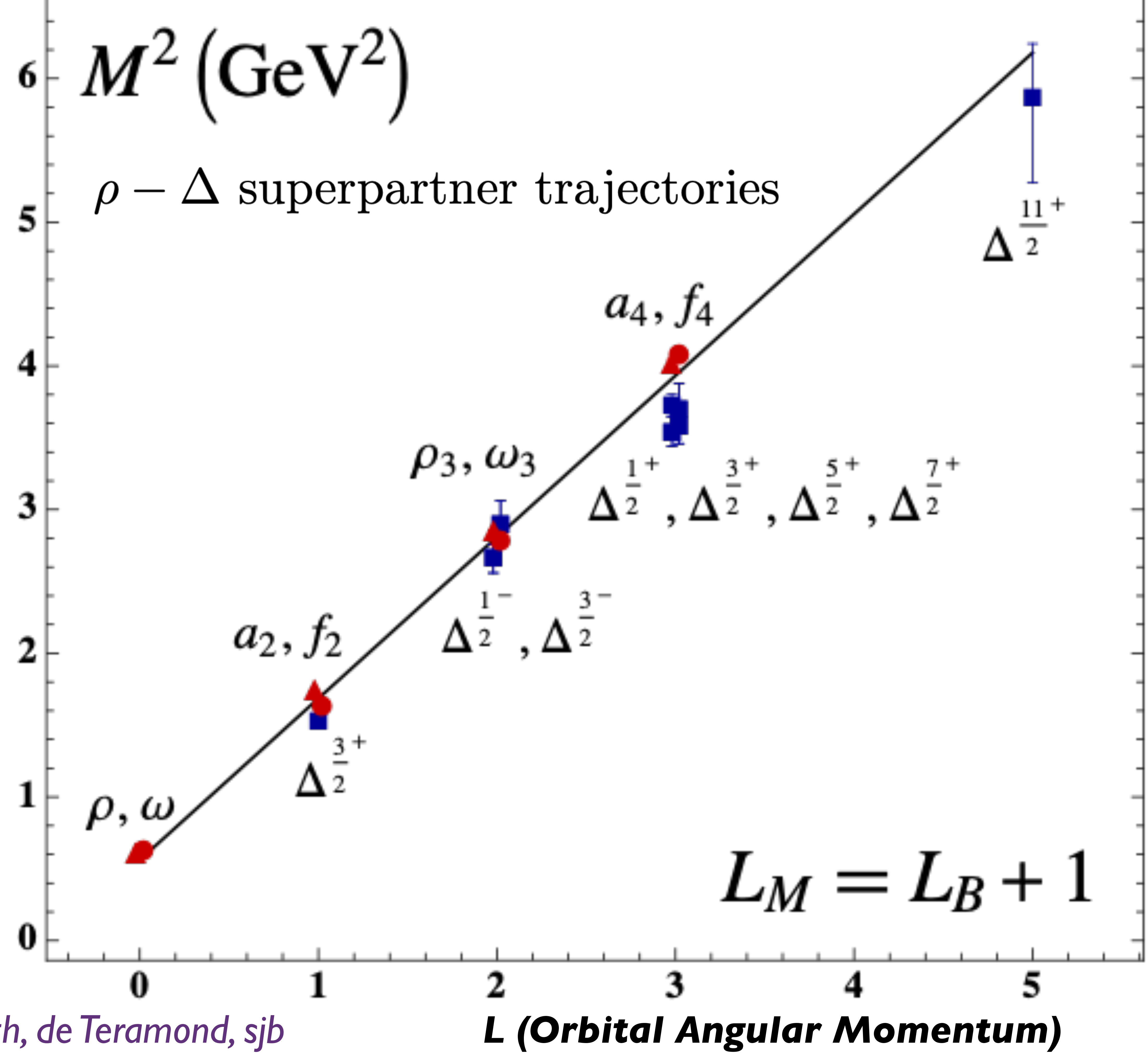
Mesons and Baryons: Same Regge Slope $M^2 \propto J$!

M²[GeV²]

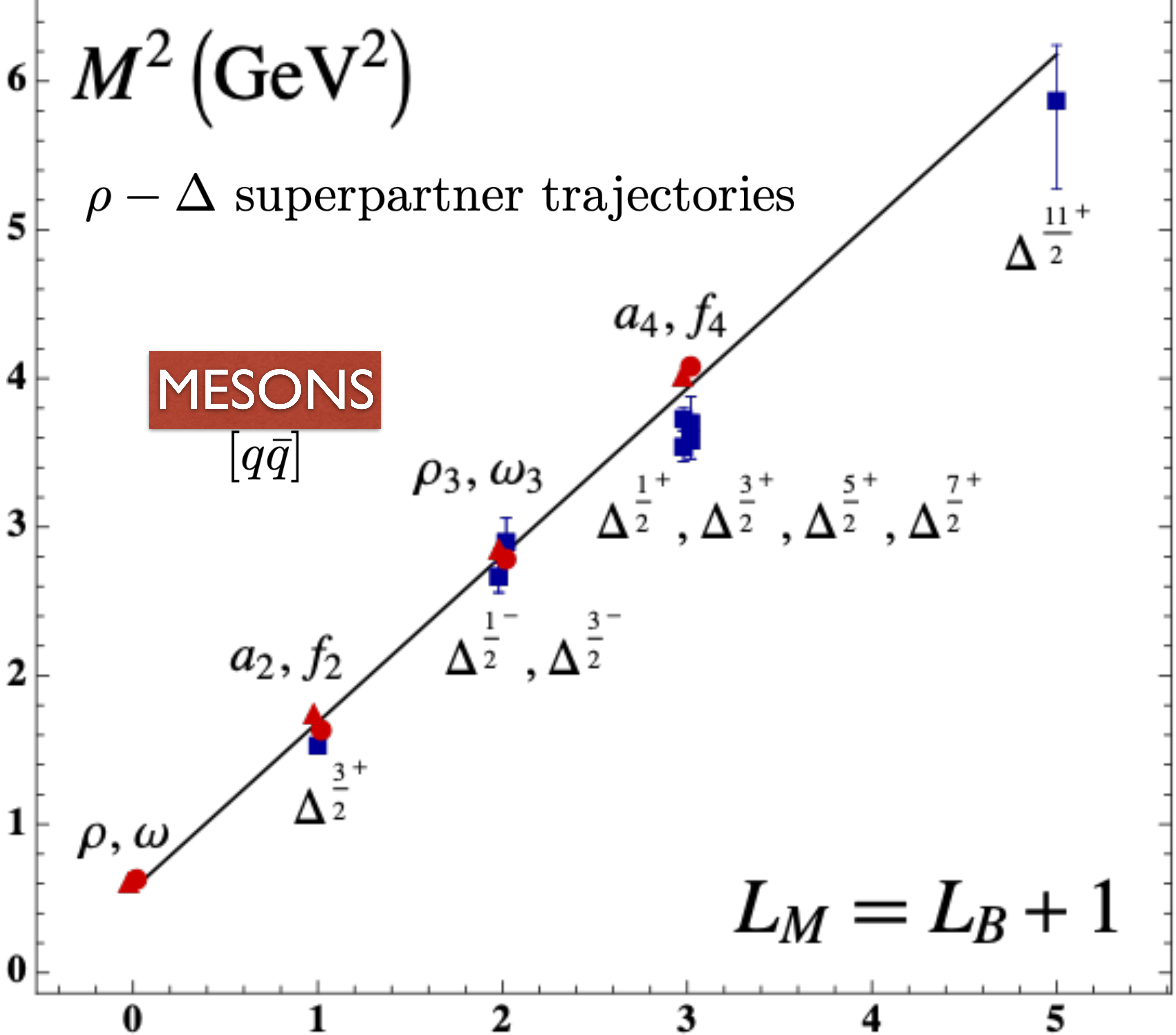


The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with J = L+S.

E. Klempt and B. Ch. Metsch

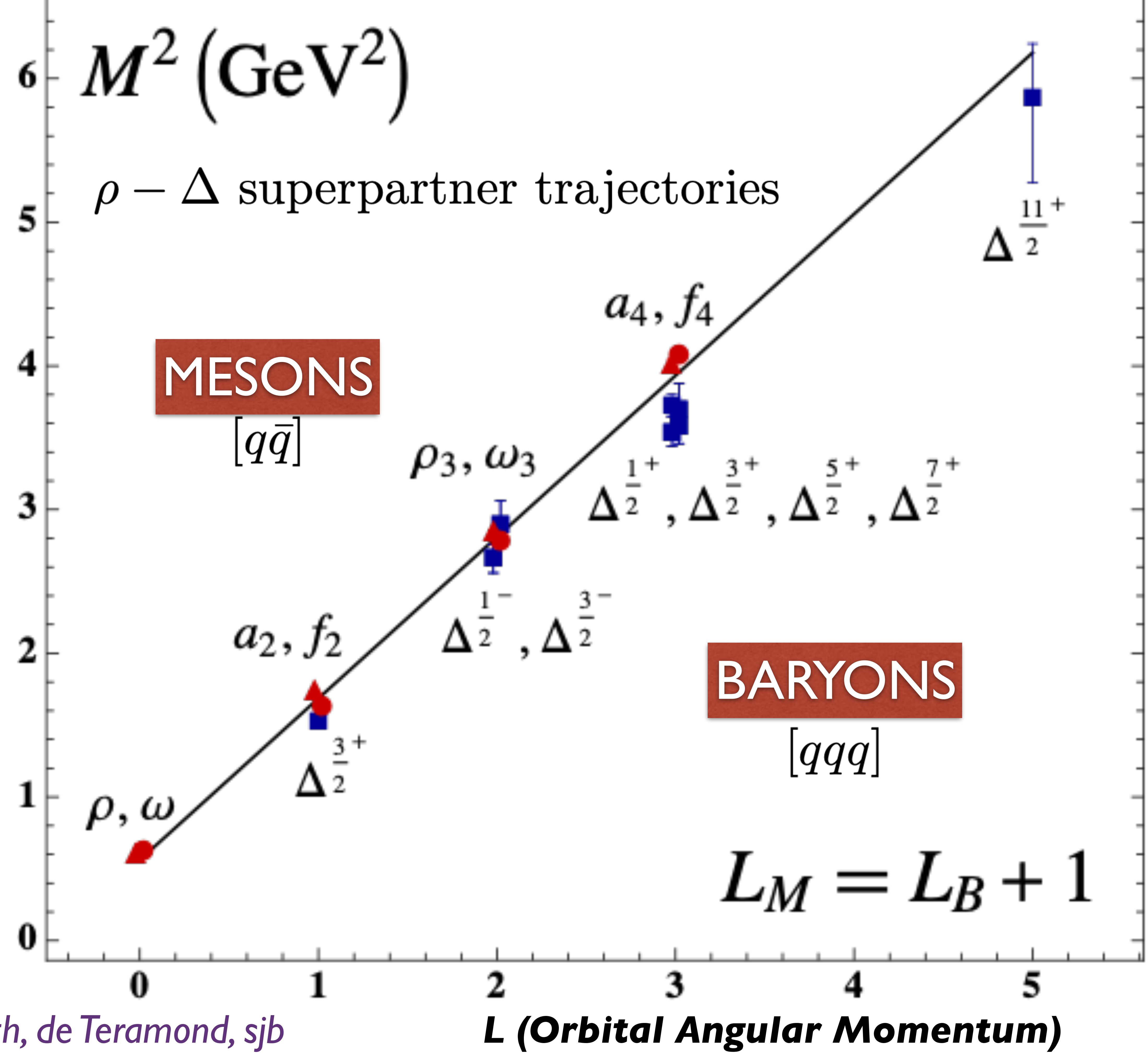


Dosch, de Teramond, sjb

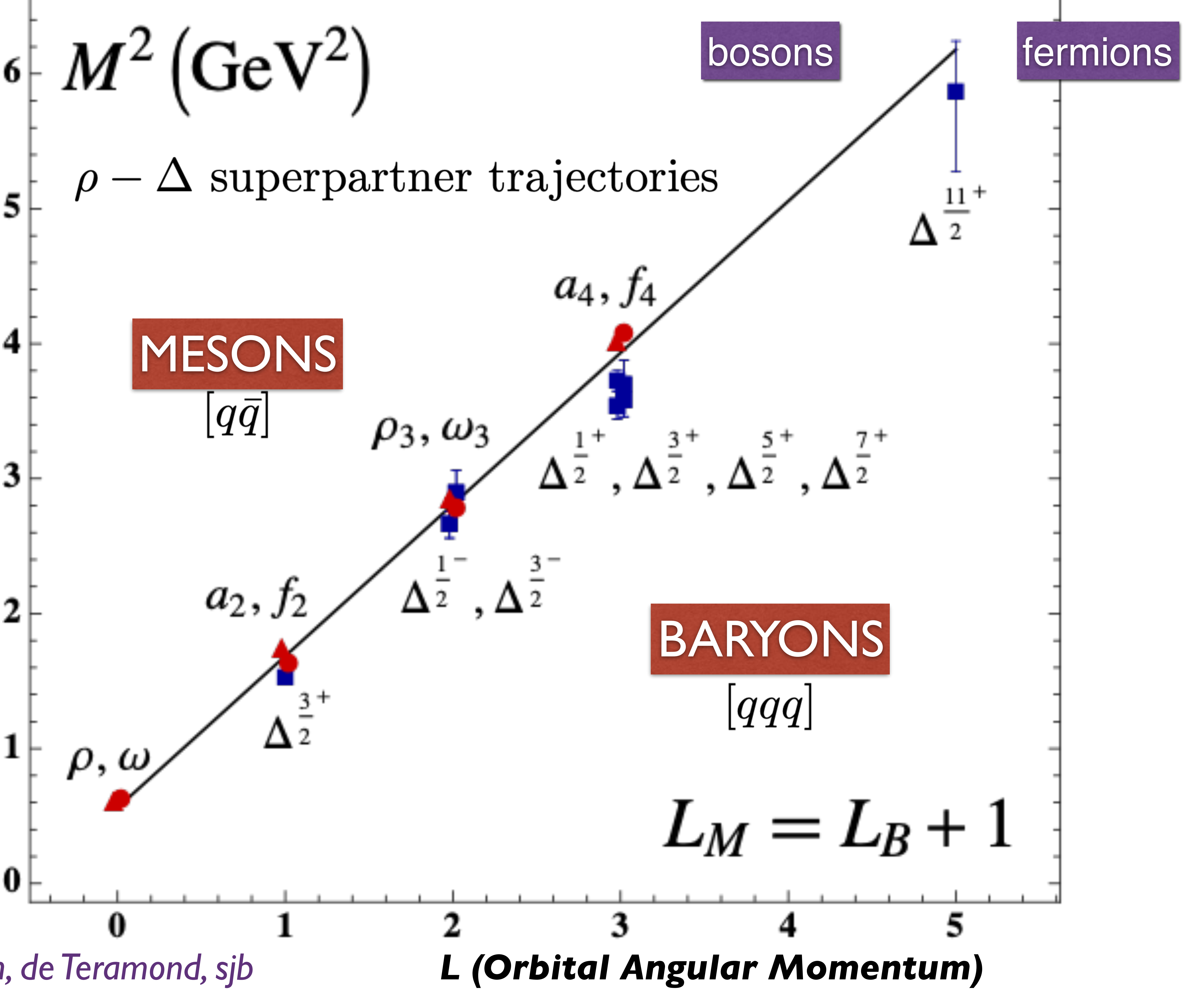


Dosch, de Teramond, sjb

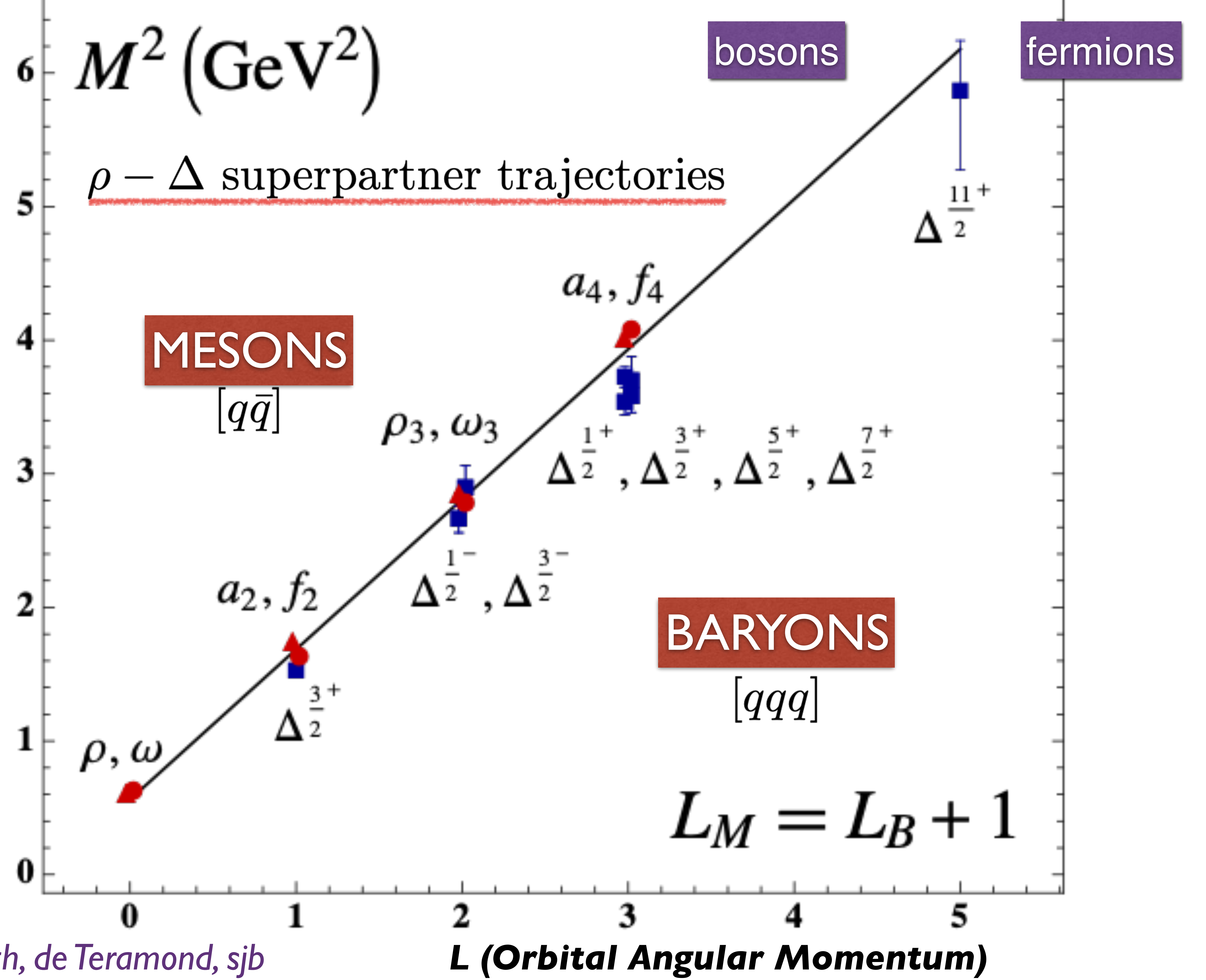
L (Orbital Angular Momentum)



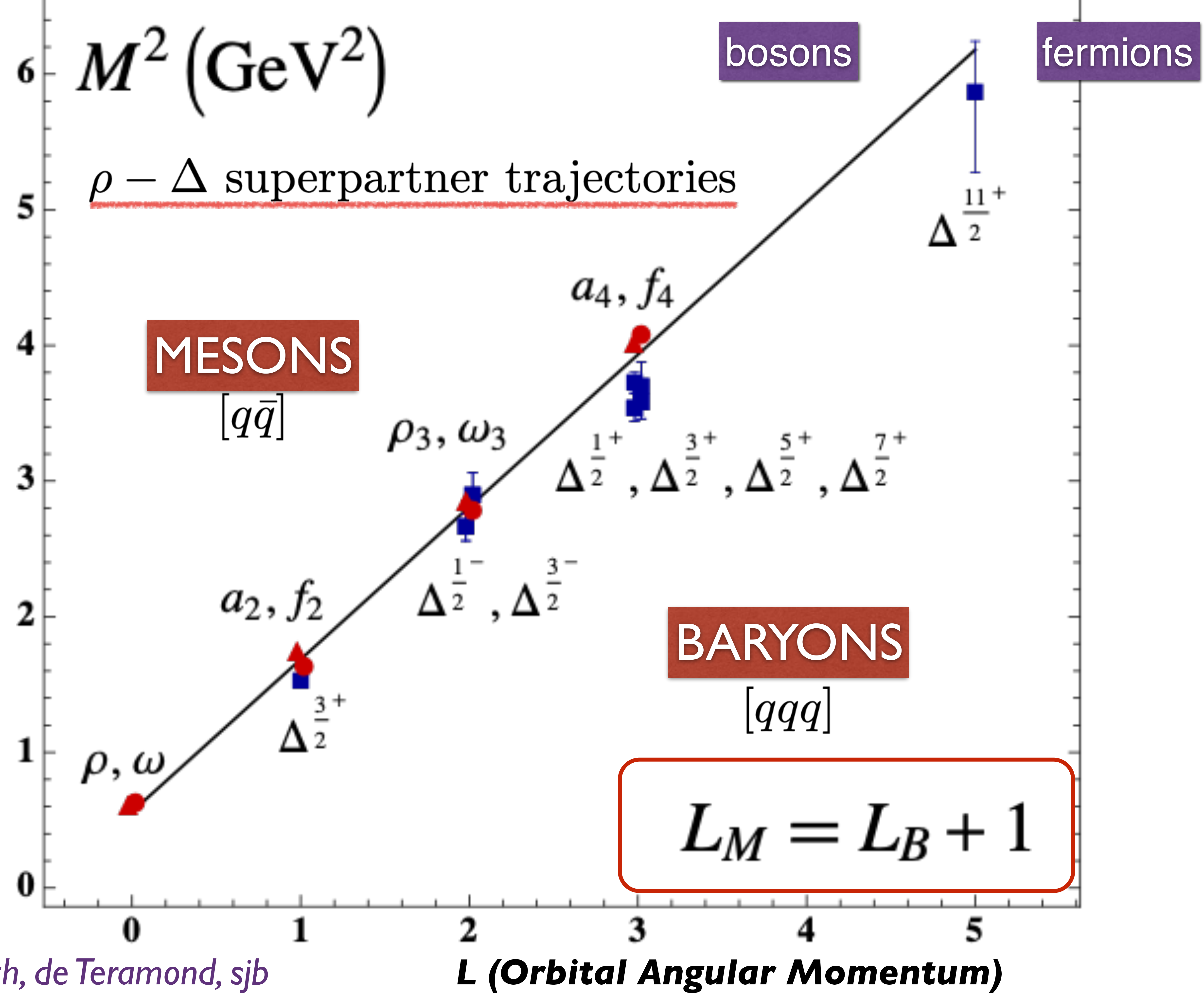
Dosch, de Teramond, sjb



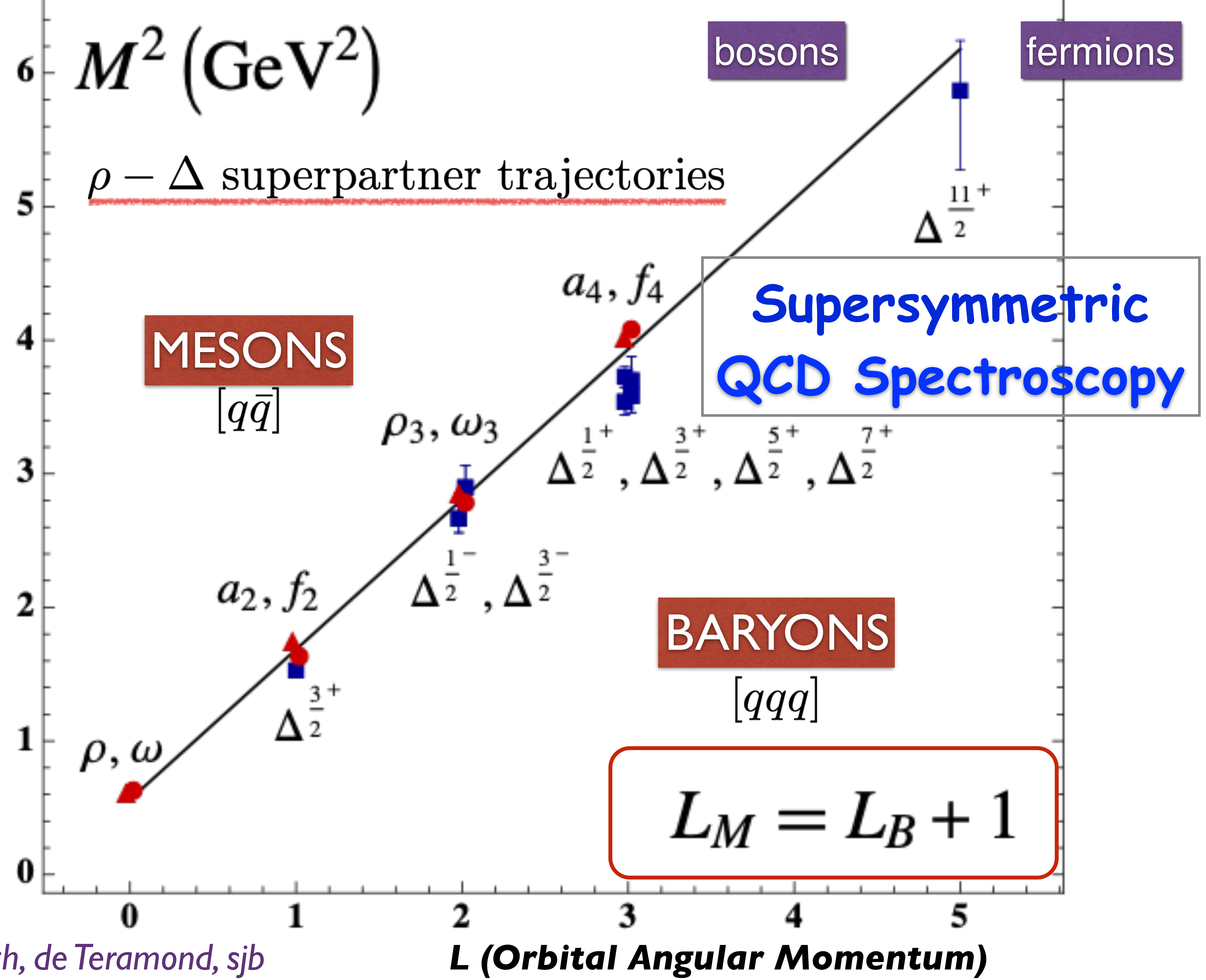
Dosch, de Teramond, sjb



Dosch, de Teramond, sjb



Dosch, de Teramond, sjb



Dosch, de Teramond, sjb

de Tèramond, Dosch, Lorce', sjb

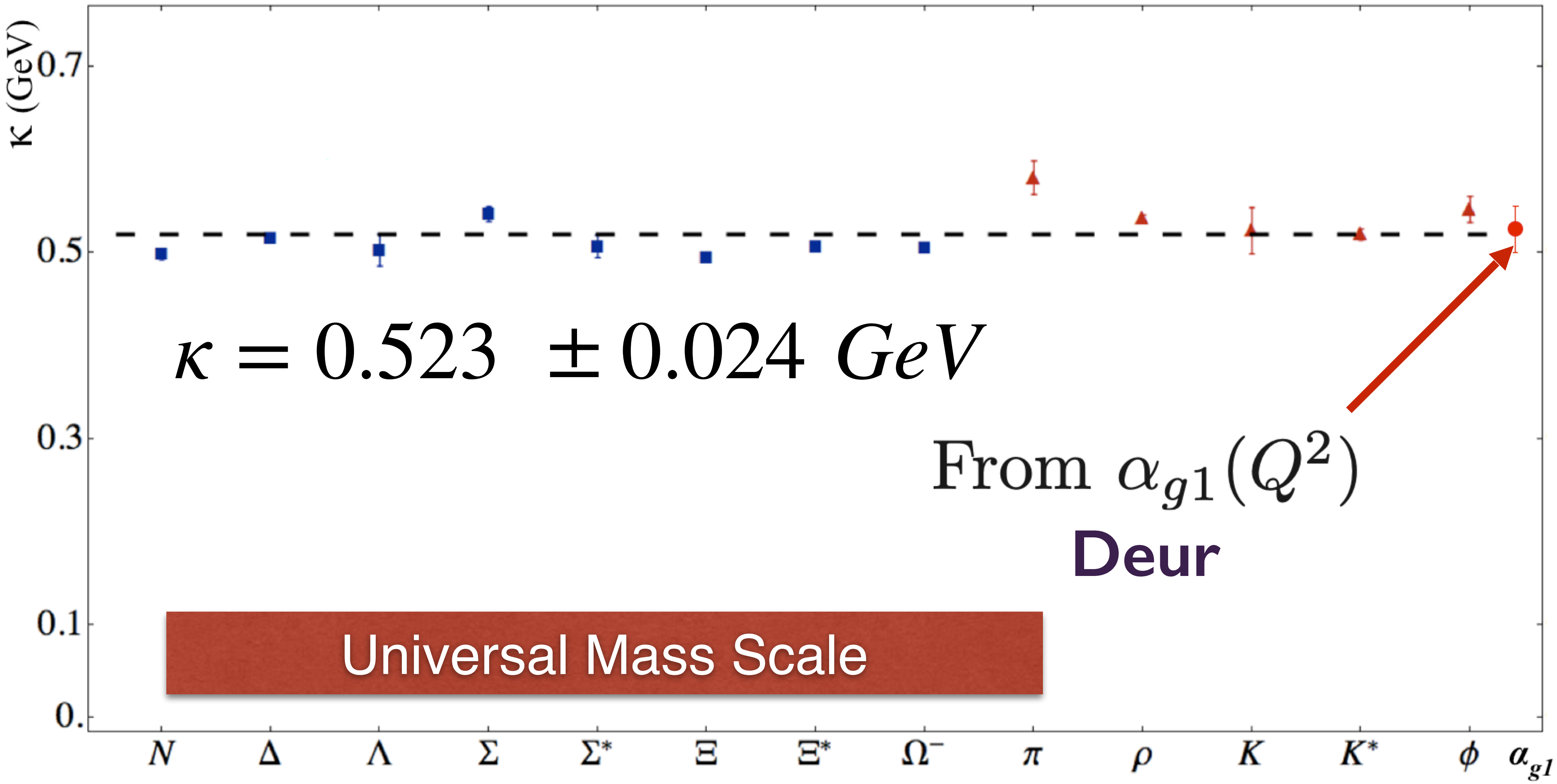
Mesons

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Baryons

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

κ



$$\kappa = 0.523 \ \pm 0.024 \ GeV$$

From $\alpha_{g1}(Q^2)$

Deur

Universal Mass Scale

Fit to the slope of Regge trajectories, including radial excitations

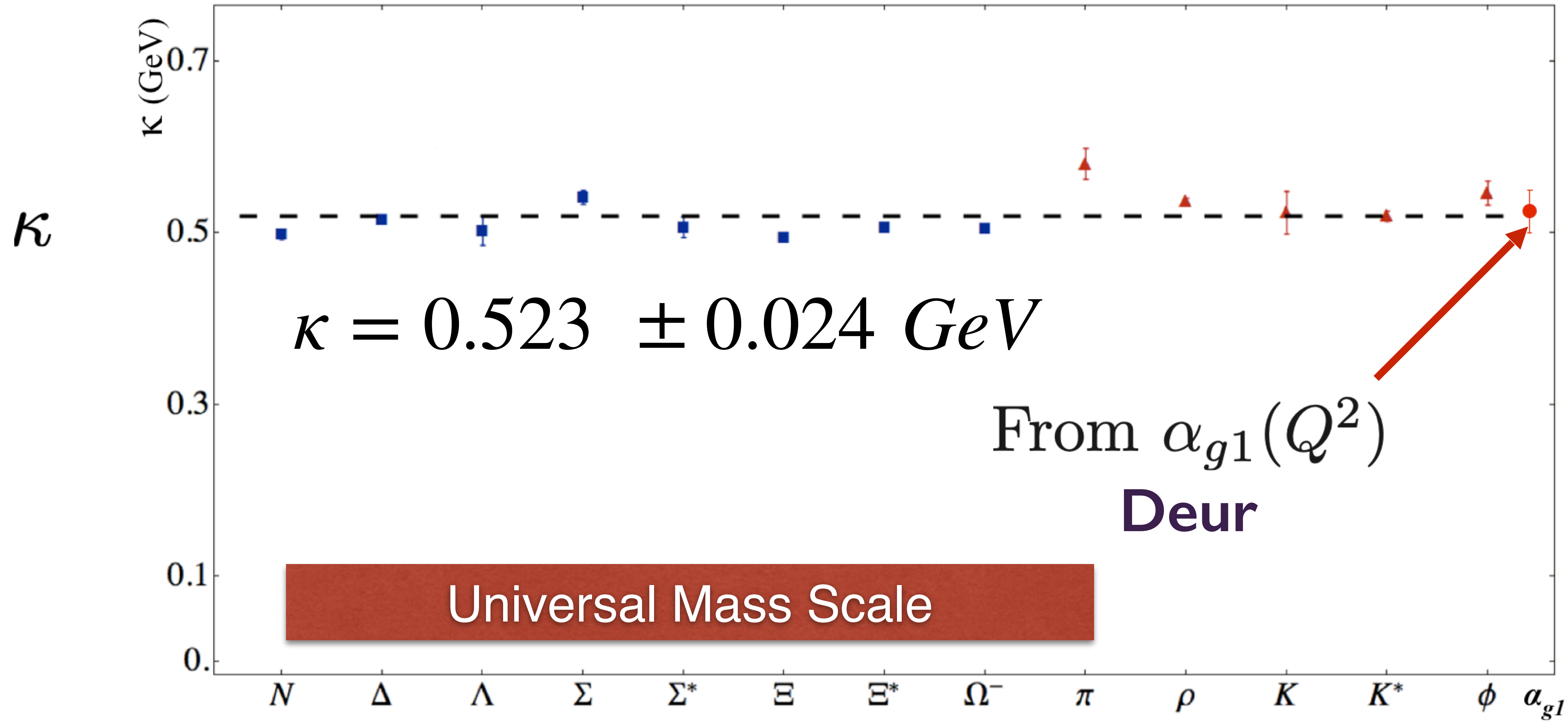
de Tèramond, Dosch, Lorce', sjb

Mesons

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Baryons

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$



Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons in n and L: Supersymmetric feature of hadron physics

Profound Questions for Hadron Physics

- *Color Confinement*
- *Origin of the QCD Mass Scale*
- *Meson and Baryon Spectroscopy and Structure*
- *Universal Regge Slopes: n, L, Mesons and Baryons: SUSY!*
- *Exotic States: Tetraquarks, Pentaquarks, Gluonium*
- *Massless Pion: Quark Anti-Quark Bound State*
- *Hadron Structure and Dynamics: QCD Coupling at all Scales*
- *Hadronization at the Amplitude Level*
- *Eliminate Scale Ambiguities*

BLM Renormalization Scale Setting

PHYSICAL REVIEW D VOLUME 28, NUMBER 1

On the elimination of scale ambiguities in perturbative quantum chromodynamics

Stanley J. Brodsky
Institute for Advanced Study, Princeton, New Jersey 08540
*and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305**

G. Peter Lepage
Institute for Advanced Study, Princeton, New Jersey 08540
*and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853**

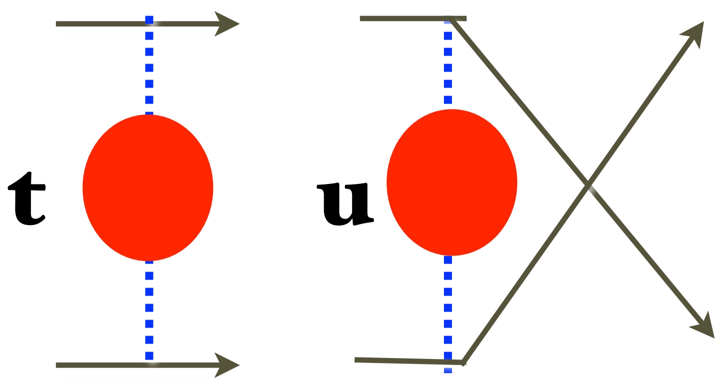
Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510
(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

Cited by 1182 records

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\ \alpha(t) + \frac{8\pi s}{u}\ \alpha(u)$$



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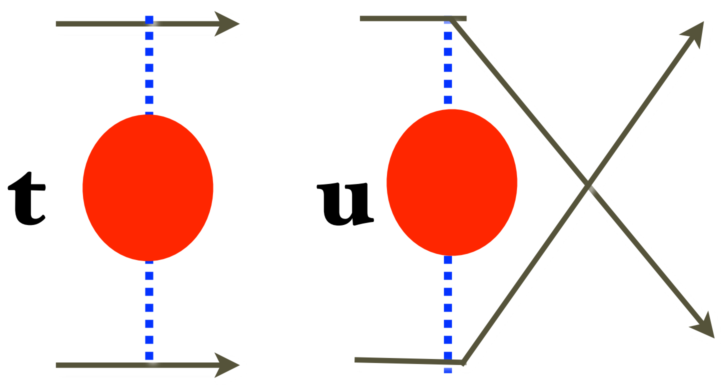


- No renormalization scale ambiguity in QED!
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, proper and improper
- All non-zero beta terms into running coupling. This is the purpose of the running coupling!
    - Two separate physical scales: t, u = photon virtuality
- If one chooses a different initial scale, one must sum an infinite number of graphs
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- No renormalization scale ambiguity!

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

- **Dressed Photon Propagator sums all β (vacuum polarization) contributions, proper and improper**

$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_o)}{1-\Pi(t_0)}$$

- **Initial Scale Choice t_o is Arbitrary!**

- **Any renormalization scheme can be used** $\alpha(t) \to \alpha_{\overline{MS}}(e^{-\frac{5}{3}}t)$

Features of BLM Scale-Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into QCD running coupling
- Resulting series identical to conformal series
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants
- $N_C \to 0: \ QCD \to QED$

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

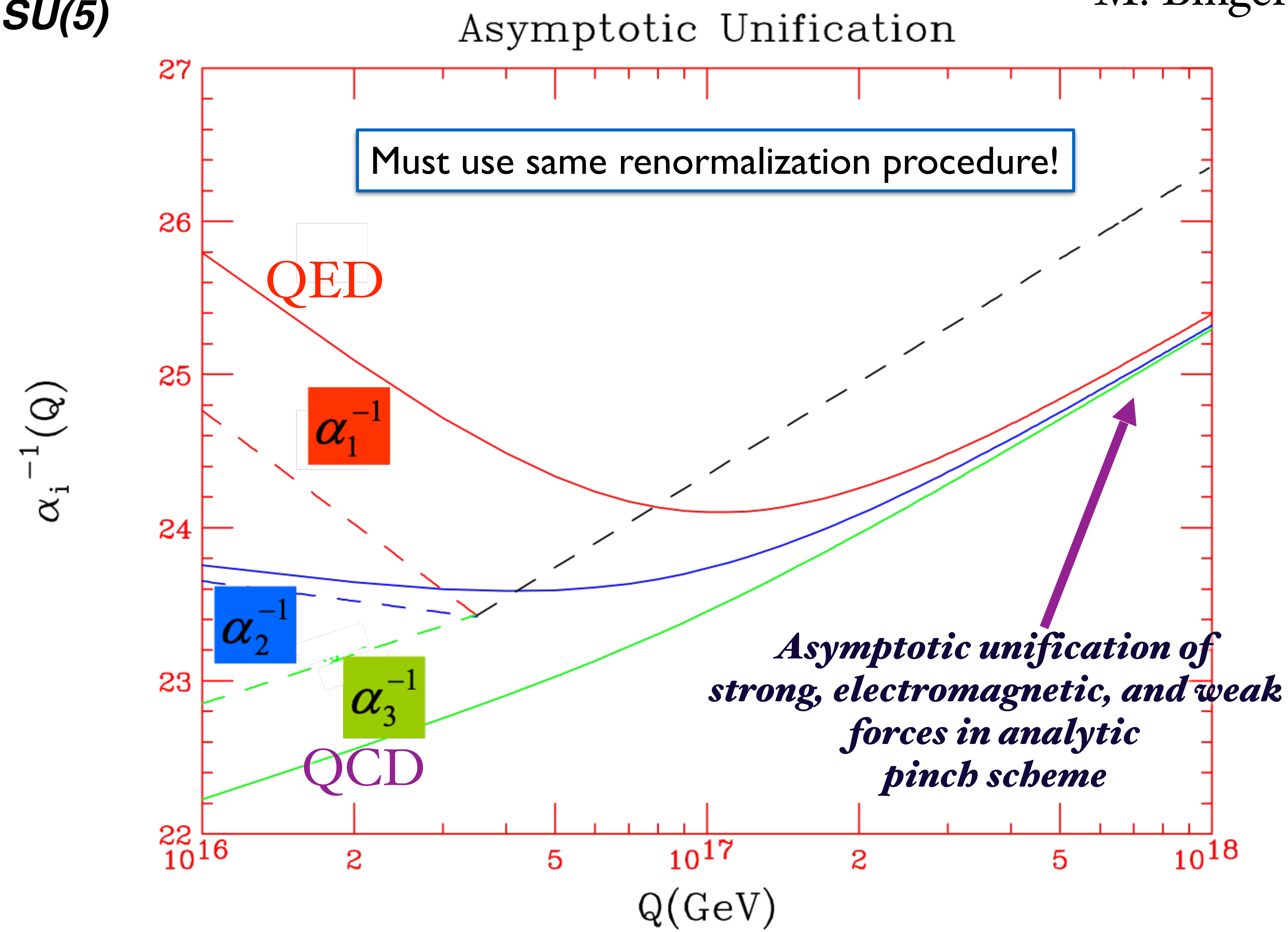
Analytic Feature of SU(Nc) Gauge Theory

Huet, sjb

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

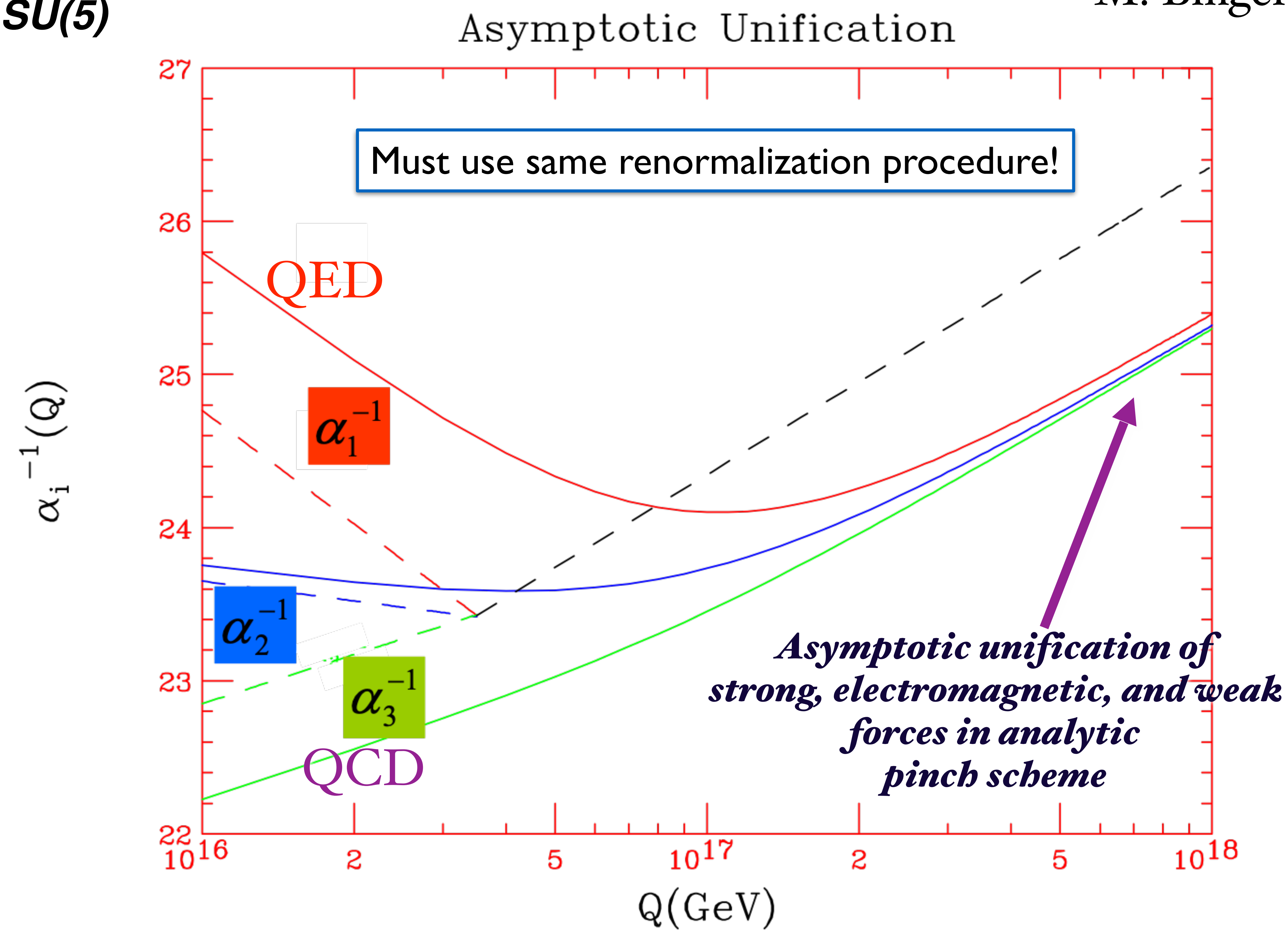
Supersymmetric SU(5)

M. Binger, sjb



Supersymmetric SU(5)

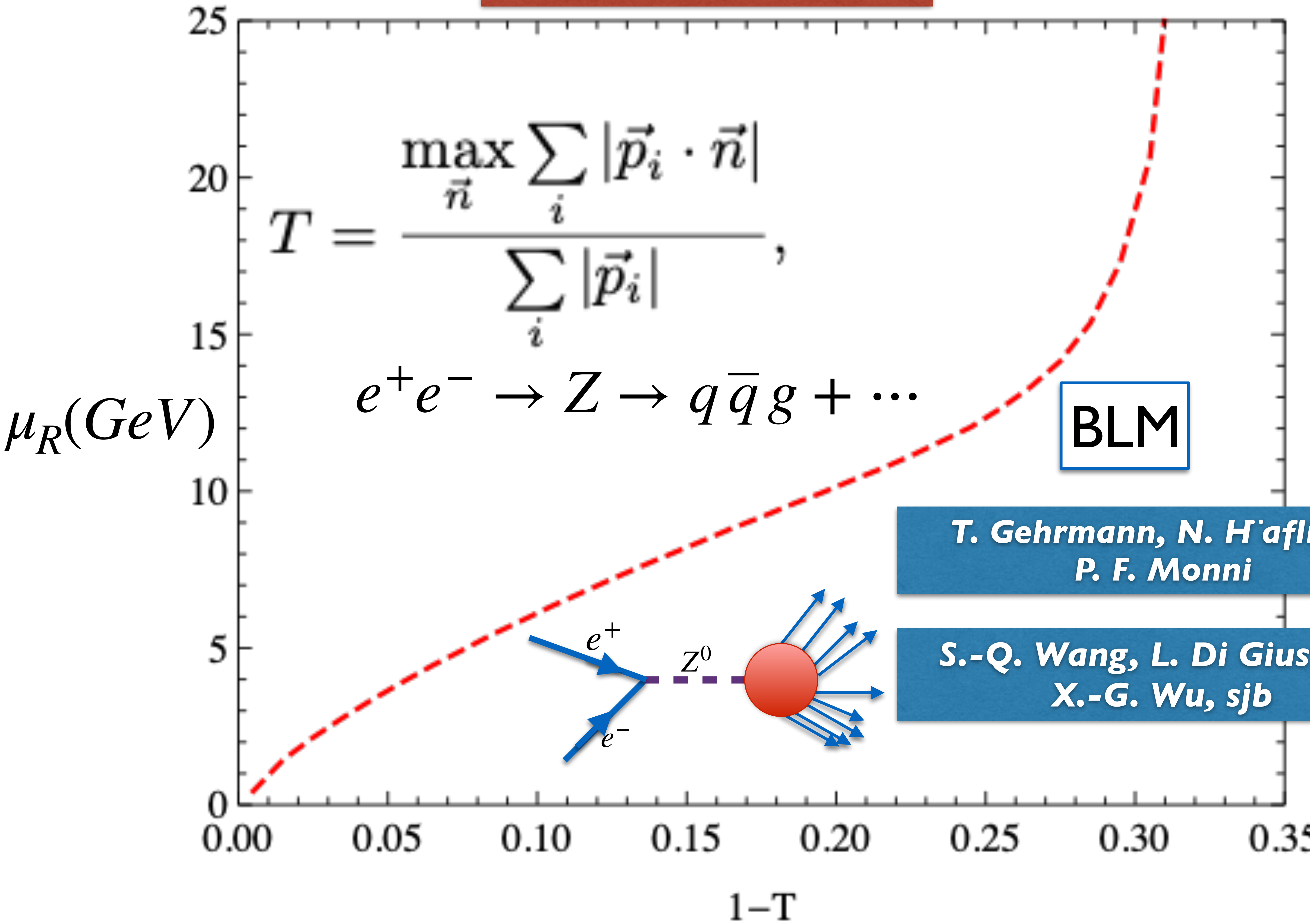
M. Binger, sjb



Must Use Same Scale-Setting Procedure! BLM/PMC

BLM Renormalization scale depends on the thrust

Not constant !



$$T = \frac{\max\limits_{\vec{n}} \sum\limits_i |\vec{p}_i \cdot \vec{n}|}{\sum\limits_i |\vec{p}_i|},$$

$$e^+e^- \to Z \to q\,\bar{q}\,g + \cdots$$

T. Gehrmann, N. H˙afliger, P. F. Monni

S.-Q. Wang, L. Di Giustino, X.-G. Wu, sjb

T. Gehrmann, N. H˙afliger, P. F. Monni

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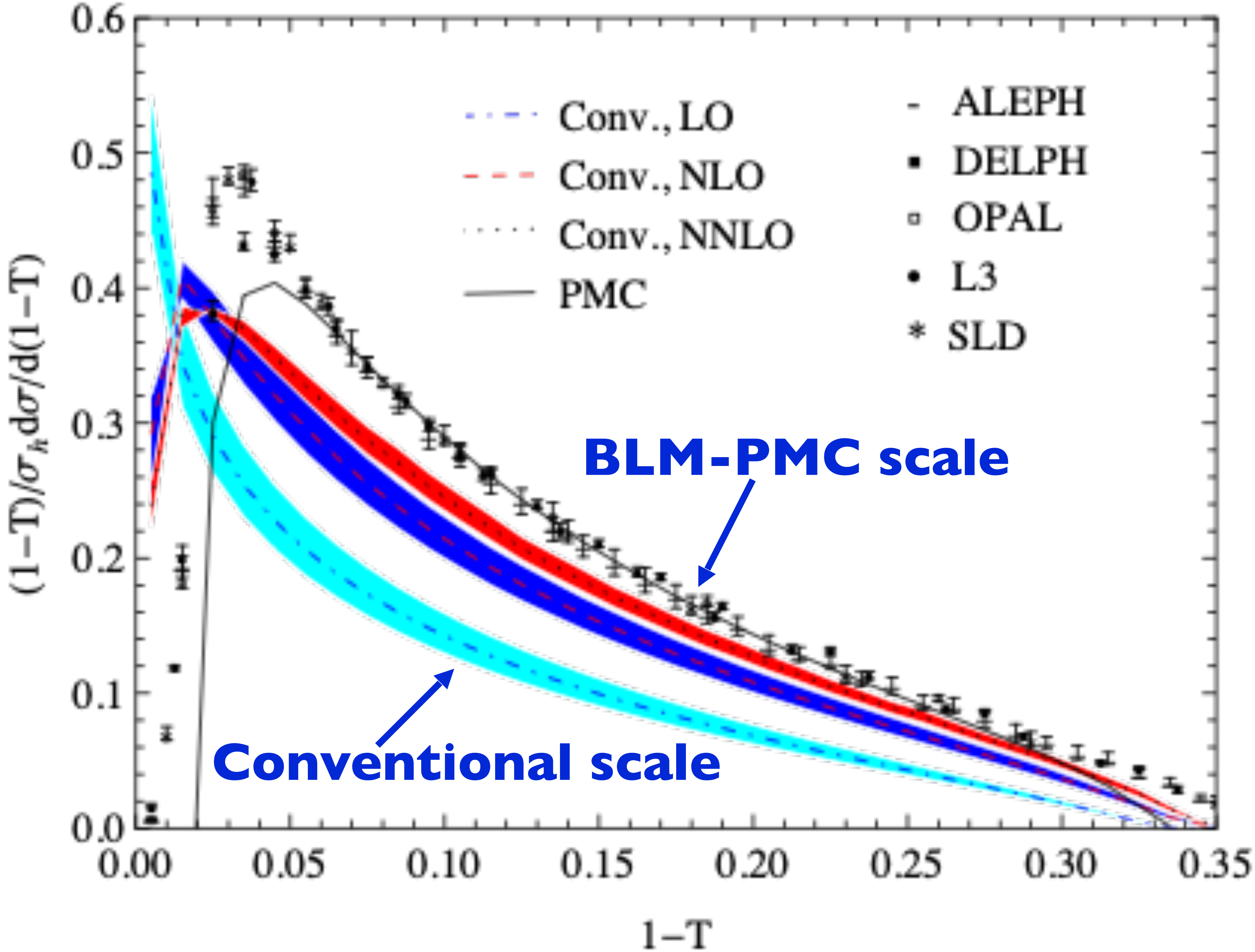
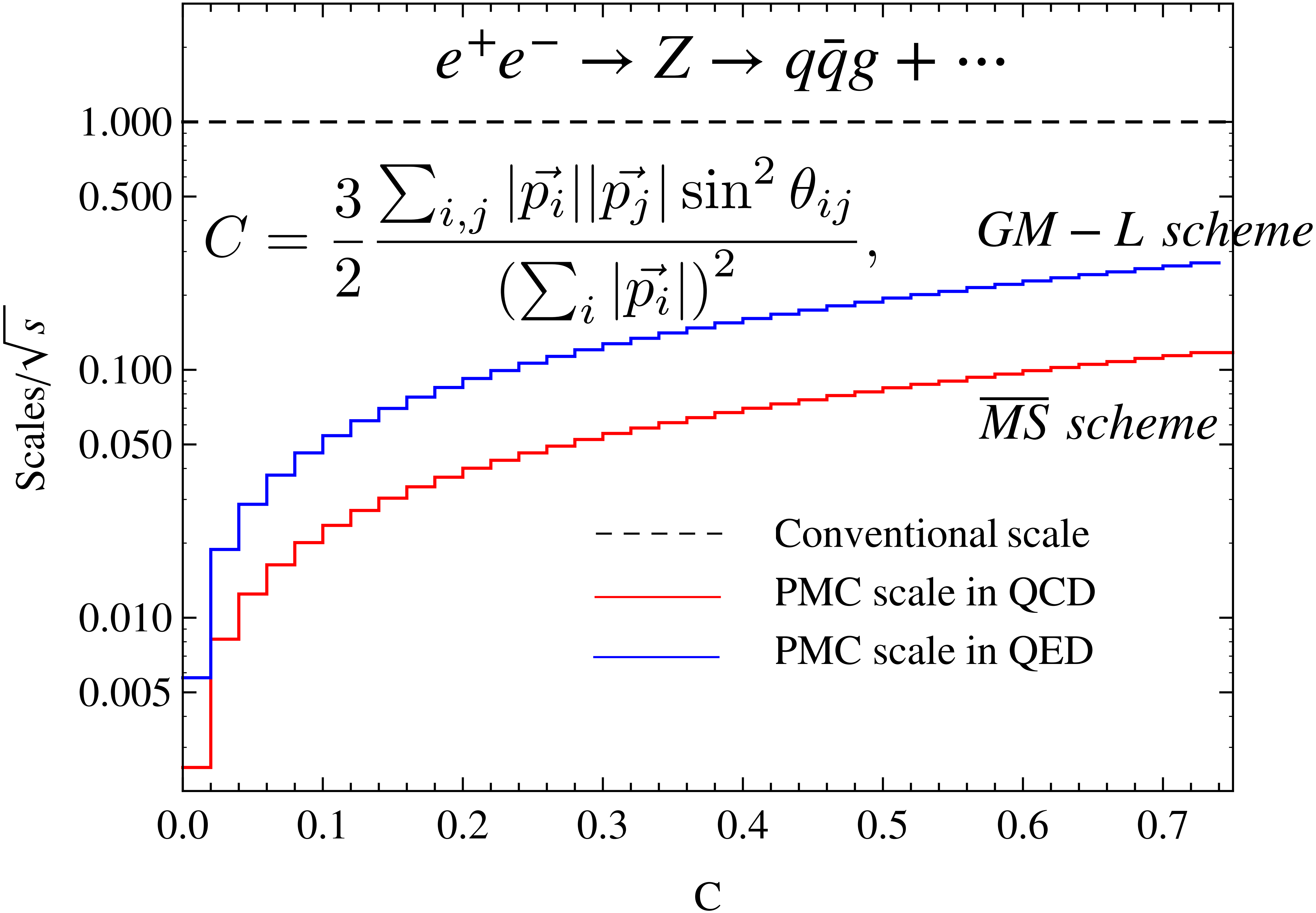
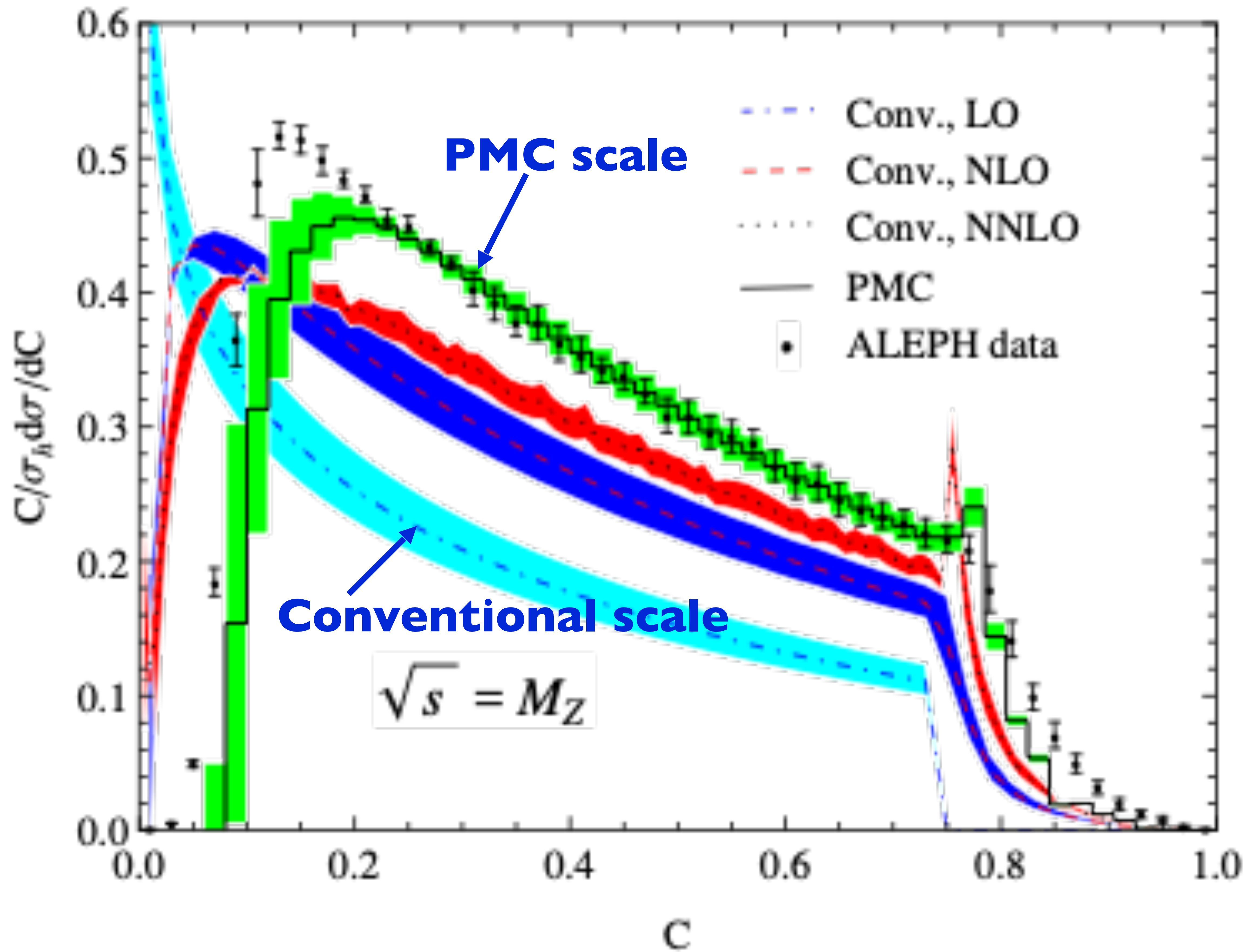


Principle of Maximum Conformality (PMC)

Renormalization scale depends on the C-parameter

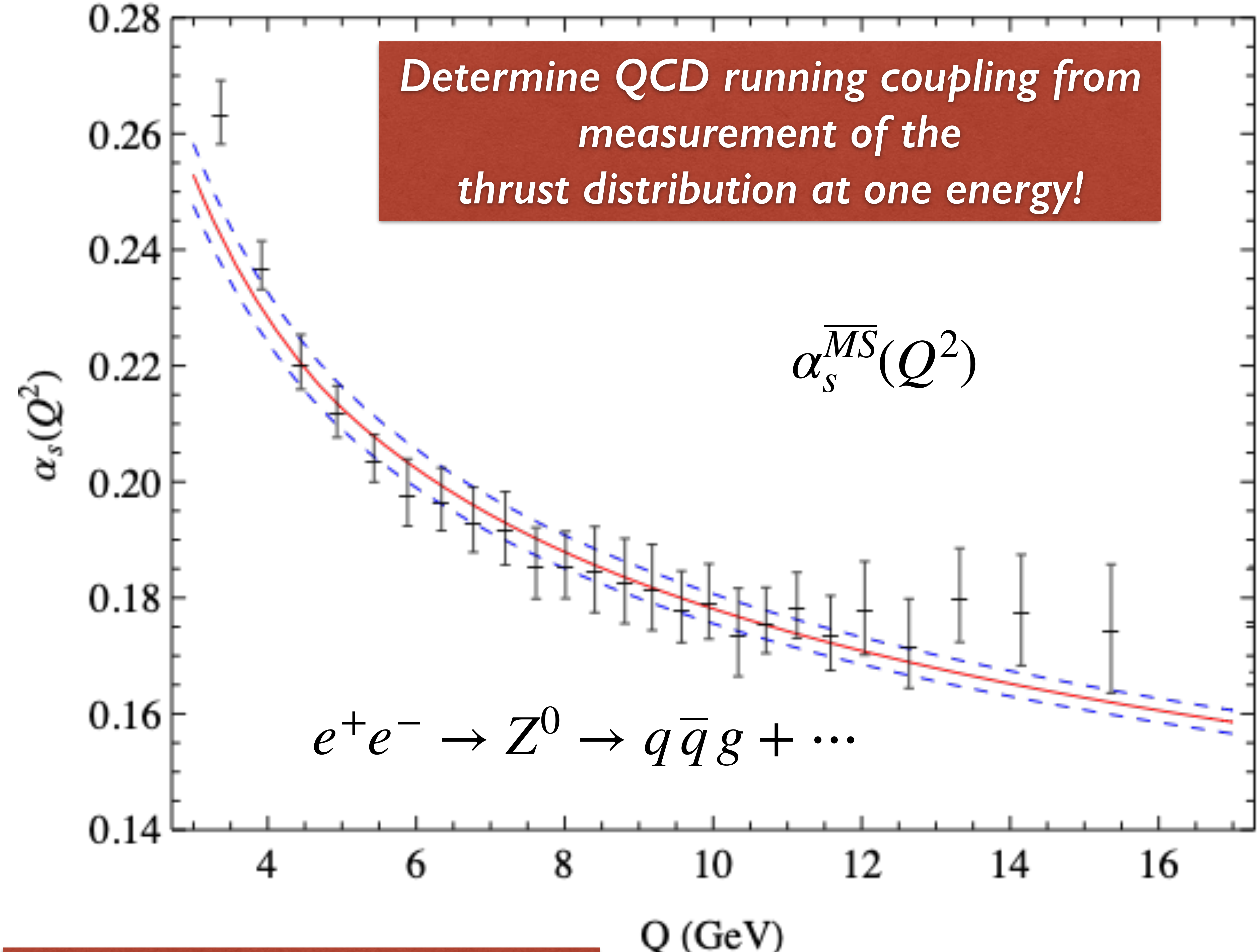


S.-Q. Wang, L. Di Giustino, X.-G. Wu, sjb



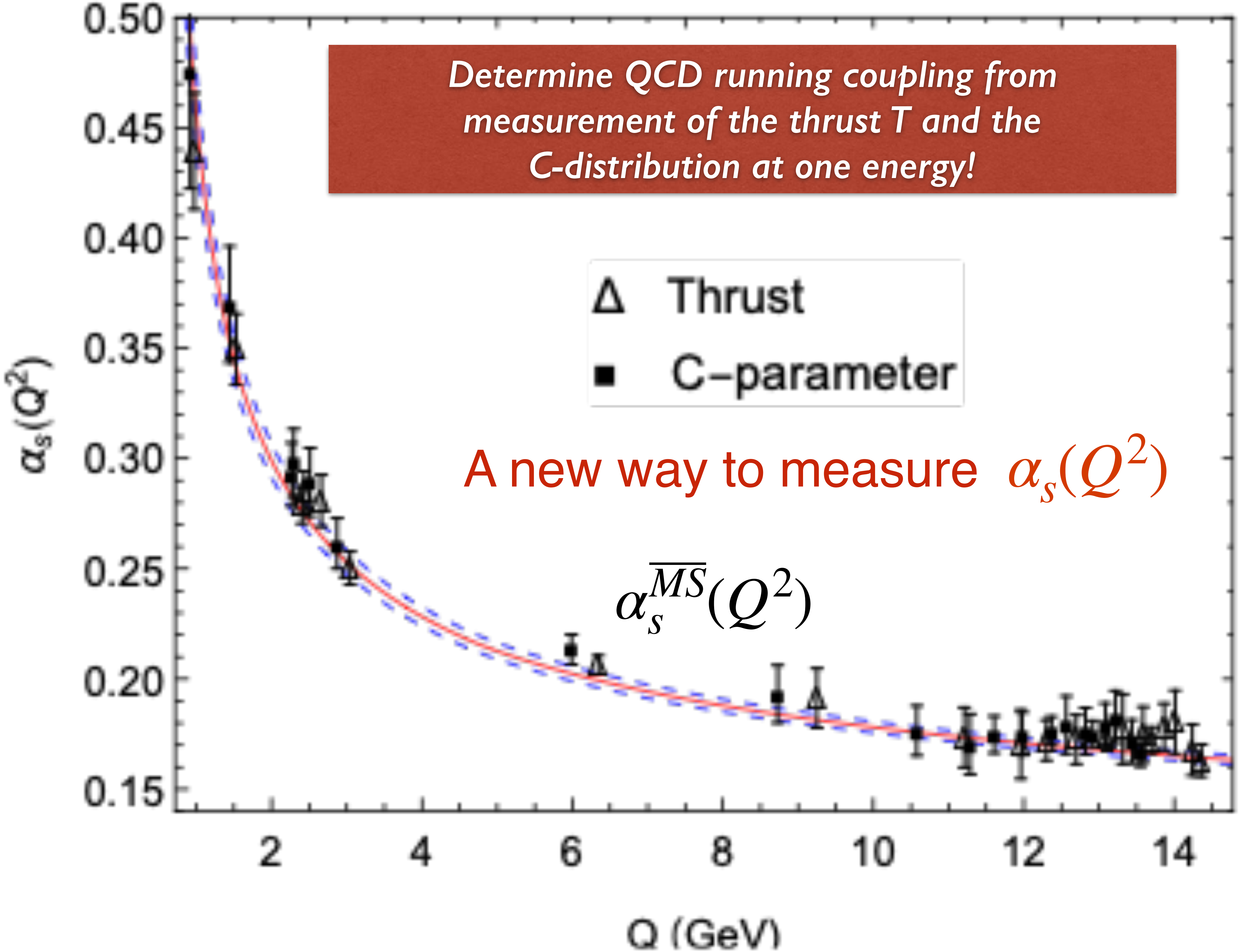
Principle of Maximum Conformality (PMC)

Determine QCD running coupling from measurement of the thrust distribution at one energy!



S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB

$$e^{+}e^{-} \rightarrow Z^{0} \rightarrow q\,\bar{q}\,g + \cdots$$



Determine QCD running coupling from measurement of the thrust T and the C-distribution at one energy!

A new way to measure $\alpha_s(Q^2)$

S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right]$$
$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2\right.$$
$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f$$
$$\left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right\}.$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right]$$
$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2\right.$$
$$\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648}f^2\right\}.$$

Apply BLM, Eliminate MSbar,
Find Amazing Simplification

$$\int_0^1 dx\,\left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)\right] \equiv \frac{1}{3}\left|\frac{g_A}{g_V}\right|\left[1 - \frac{\alpha_{g_1}(Q)}{\pi}\right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi}\right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi}\right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

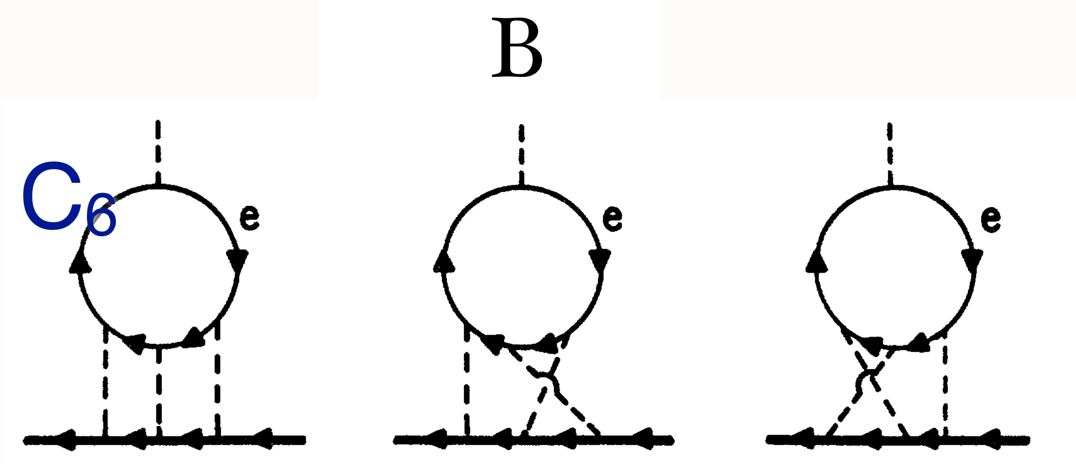
$$\sqrt{s^*} \simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

QED provides an asymptotic series relating g and α,

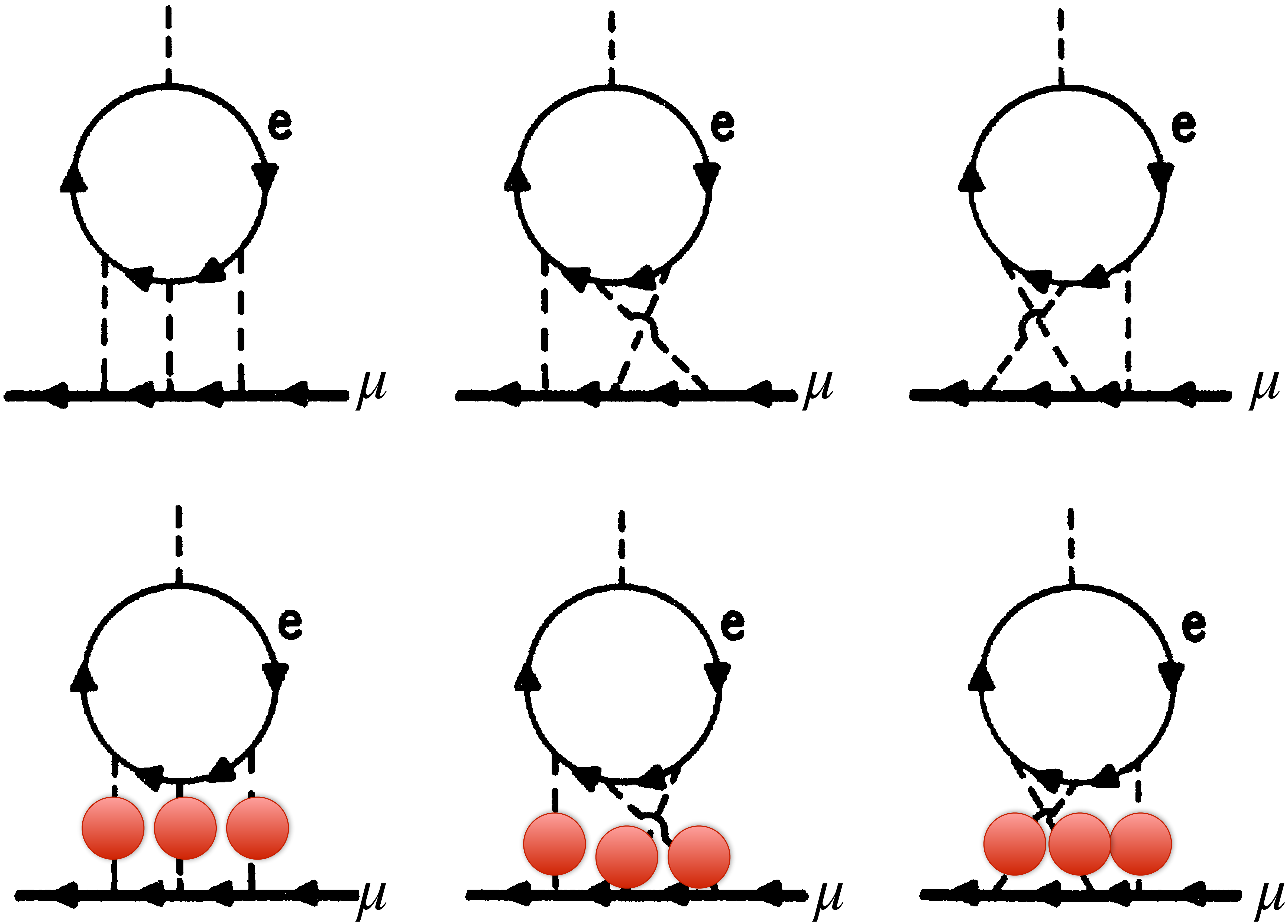
$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \dots$$
$$+ a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

Light-by-Light Scattering Contribution to C_6

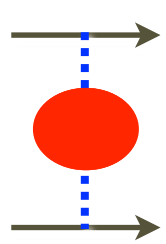


Aldins, Dufner, Kinoshita, sjb

G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, Phys. Rev. Lett. **97**, 030802 (2006).



$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$



Apply BLM : $\alpha(\hat{t})$:
Sums an infinite number of vacuum polarization insertions

Principle of Maximum Conformality (PMC)

PRL **110**, 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza[*]

CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Stanley J. Brodsky[†]

SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Xing-Gang Wu[‡]

Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China

(Received 13 January 2013; published 10 May 2013)

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

BLM/PMC

Principle of Maximum Conformality

$$\alpha_s(q^2) \text{ sums all } \beta \text{ terms}$$

- Eliminates renormalization scale ambiguities for pQCD and SM predictions
- Predictions are independent of scheme and initial scale choice
- Convergent conformal series: No "renormalons" $C_n \sim \alpha_s^n \beta_o^n n!$
- Consistent with Gell-Mann Low for QED $\alpha(t) = \dfrac{\alpha(t_o)}{1 - \Pi(t, t_0)}$
- Eliminates many outstanding conflicts of pQCD with experiment
- Maximizes sensitivity of LHeC measurements to new physics

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$$\textit{Choose renormalization scheme; e.g. } \alpha_s^R(\mu_R^{\rm init})$$

↓

$$\textit{Choose } \mu_R^{init}\textit{; arbitrary initial renormalization scale}$$

↓

$$\textit{Identify } \{\beta_i^R\} - \textit{terms using } n_f - \textit{terms}$$

$$\textit{through the } PMC - BLM \textit{ correspondence principle}$$

or R_δ scheme dependence

↓

$$\textit{Shift scale of } \alpha_s \textit{ to } \mu_R^{\rm PMC} \textit{ to eliminate } \{\beta_i^R\} - \textit{terms}$$

↓

$$\textit{Conformal Series}$$

$$\textit{Result is independent of } \mu_R^{\rm init} \textit{ and scheme at fixed order}$$

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at $N_C=0$

Eliminates unnecessary systematic uncertainty

Scale fixed at each order

R_δ-Scheme automatically identifies β-terms!

Principle of Maximum Conformality

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB

Predictions for the Top-Quark Forward-Backward Asymmetry at High Invariant Pair Mass Using the Principle of Maximum Conformality

Sheng-Quan Wang[1,2,*] Xing-Gang Wu[1,†] Zong-Guo Si[3,‡] and Stanley J. Brodsky[4§]

Top-quark pair asymmetries $A_{\rm FB}(M_{t\bar{t}} > M_{\rm cut})$										
$M_{\rm cut}$(GeV)	350	400	450	500	550	600	650	700	750	800
$A_{\rm FB}(M_{t\bar{t}} > M_{\rm cut})$|Conv.	8.9%	10.9%	12.9%	14.7%	16.4%	17.8%	19.3%	20.5%	21.9%	23.2%
$A_{\rm FB}(M_{t\bar{t}} > M_{\rm cut})$|PMC	9.6%	17.1%	29.9%	43.5%	45.1%	37.8%	33.5%	31.4%	30.5%	30.1%
$\overline{\alpha}_s(\overline{\mu}_r^{\rm PMC})$	0.123	0.131	0.146	0.157	0.153	0.138	0.129	0.123	0.120	0.117
$\overline{\mu}_r^{\rm PMC}$(GeV)	71	48	26	18	20	35	53	69	83	94

TABLE III: Top-quark pair asymmetries $A_{\rm FB}(M_{t\bar{t}} > M_{\rm cut})$ using conventional (Conv.) and PMC scale-setting procedures, respectively. The Conv. predictions are for the NLO pQCD predictions with $\mathcal{O}(\alpha_s^2\alpha)$ and the $\mathcal{O}(\alpha^2)$ electroweak contributions and the PMC predictions are calculated by Eq.(5). The predictions are shown for typical values of $M_{\rm cut}$. The last two lines give the values of the effective couplings $\overline{\alpha}_s(\overline{\mu}_r^{\rm PMC})$ and the underlying effective scale $\overline{\mu}_r^{\rm PMC}$, respectively. The initial scale is taken as $\mu_r = m_t$.

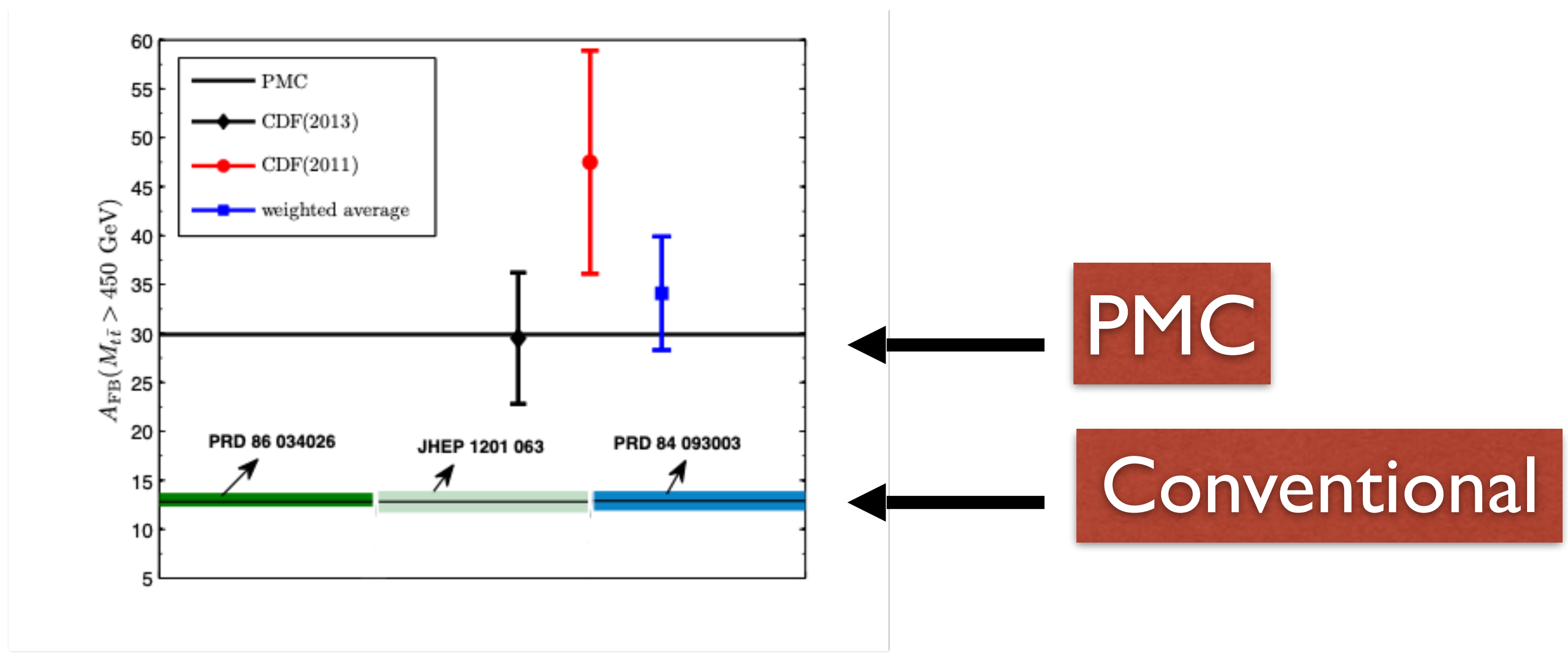
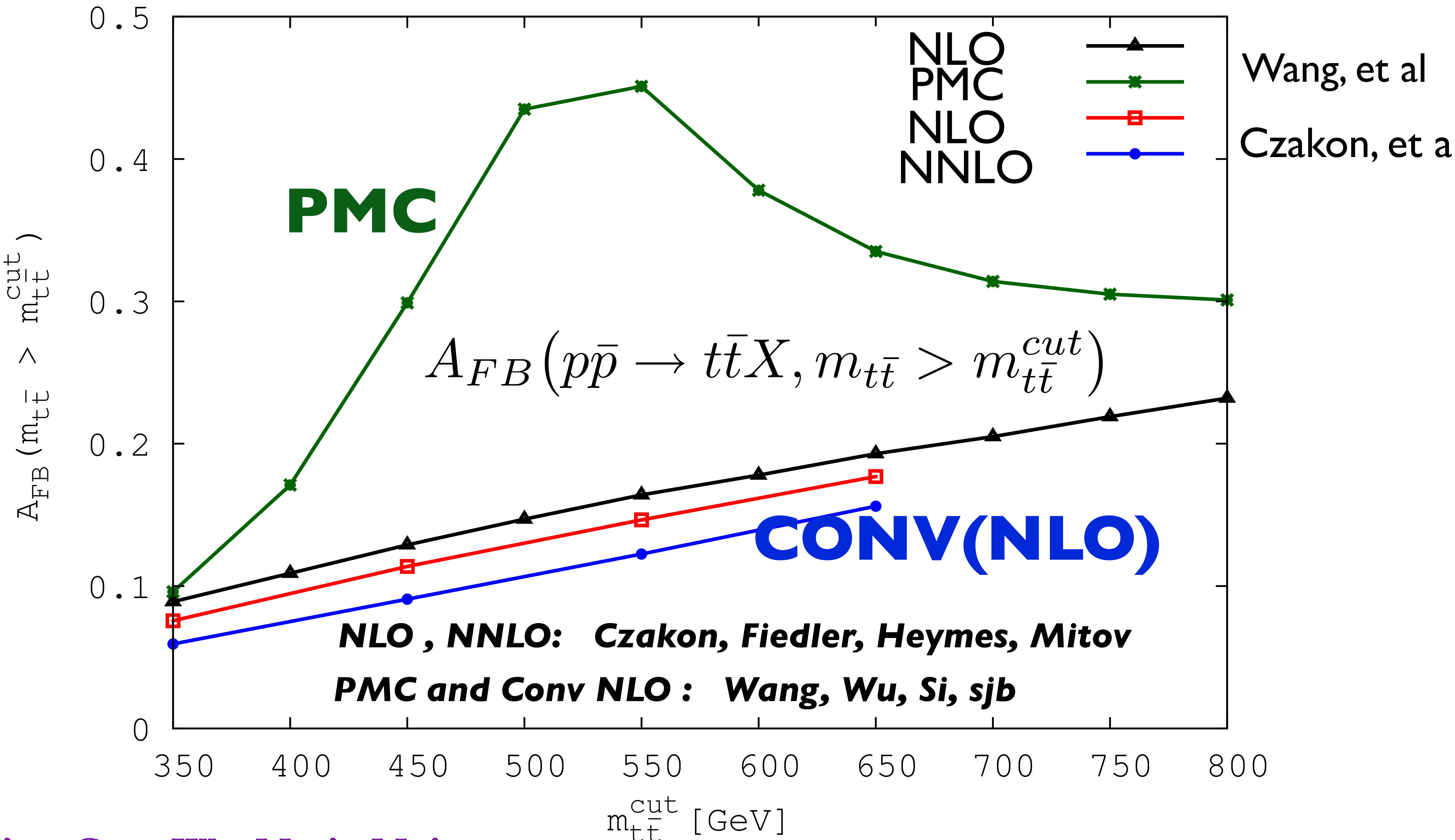


FIG. 1: Comparison of the PMC prediction for the top-pair asymmetry $A_{\rm FB}(M_{t\bar{t}} > 450$ GeV) with the CDF measurement [5, 6]. The NLO results predicted by Refs. [10–12] under conventional scale-setting are presented as a comparison, which are shown by shaded bands.

NNLO QCD predictions for fully-differential top-quark pair production at the Tevatron arXiv:1601.05375

Michał Czakon,[a] Paul Fiedler,[a] David Heymes[b] and Alexander Mitov[b]

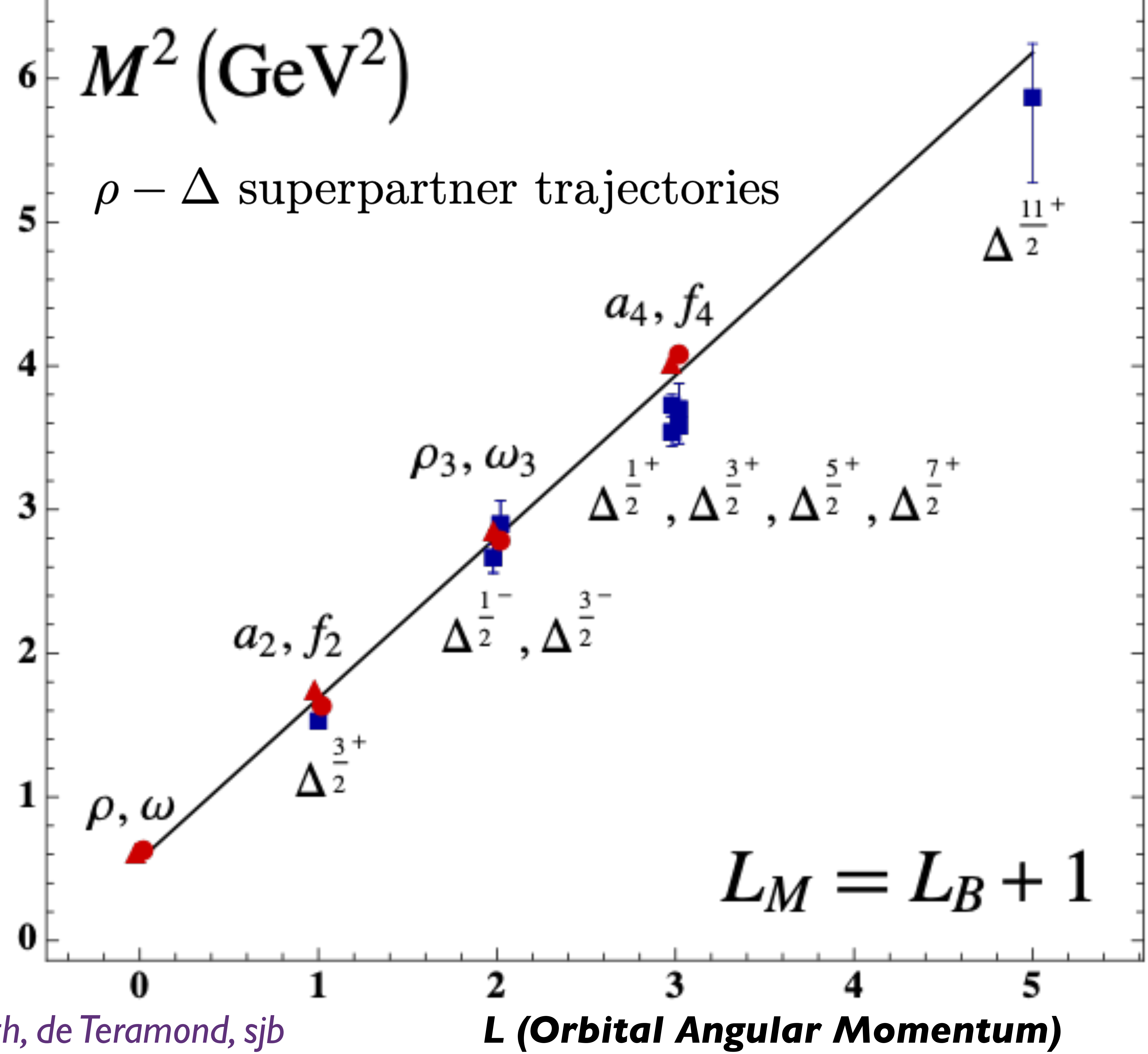


Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

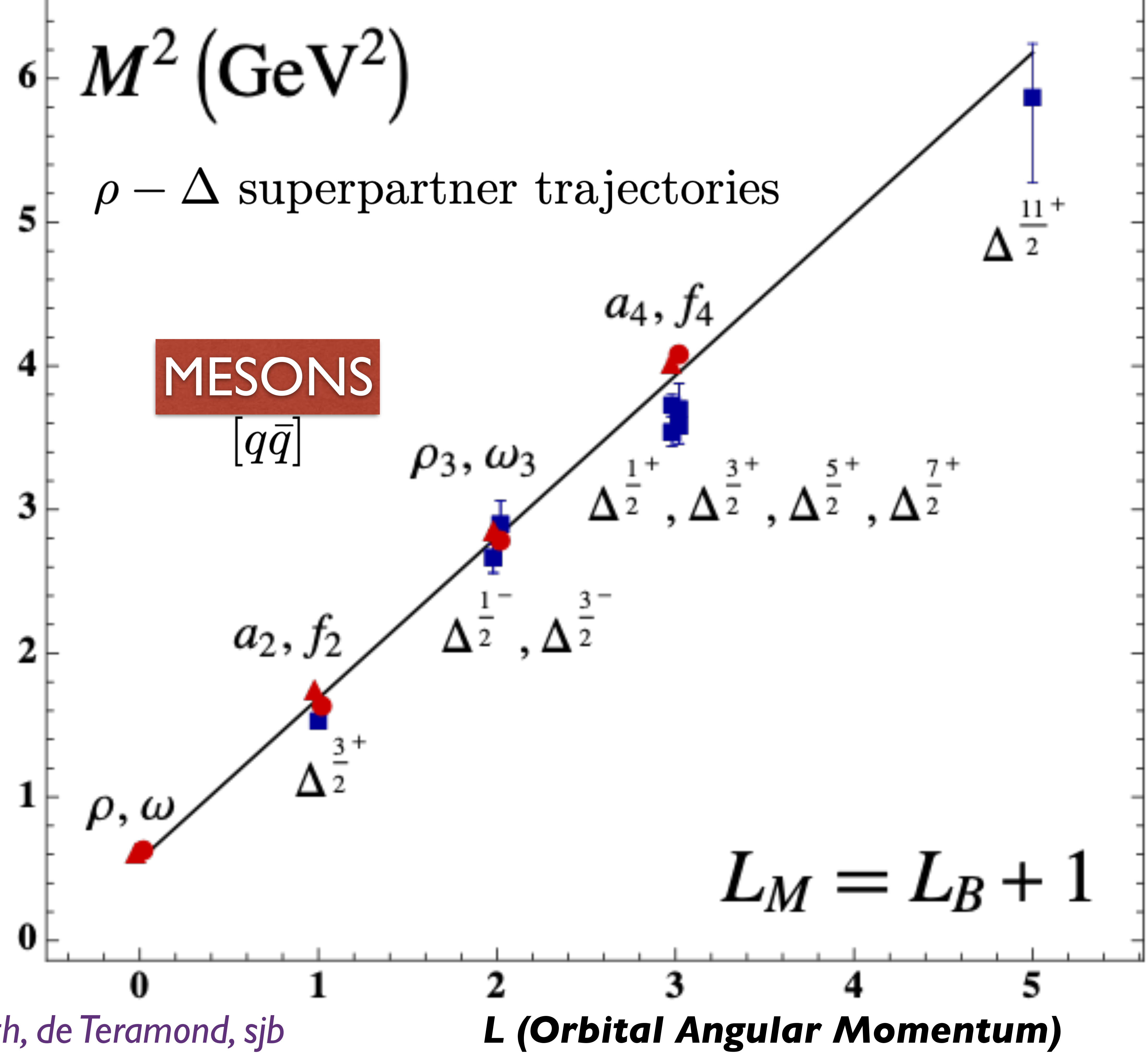
Predictions for the cumulative front-back asymmetry.

Features of BLM/PMC

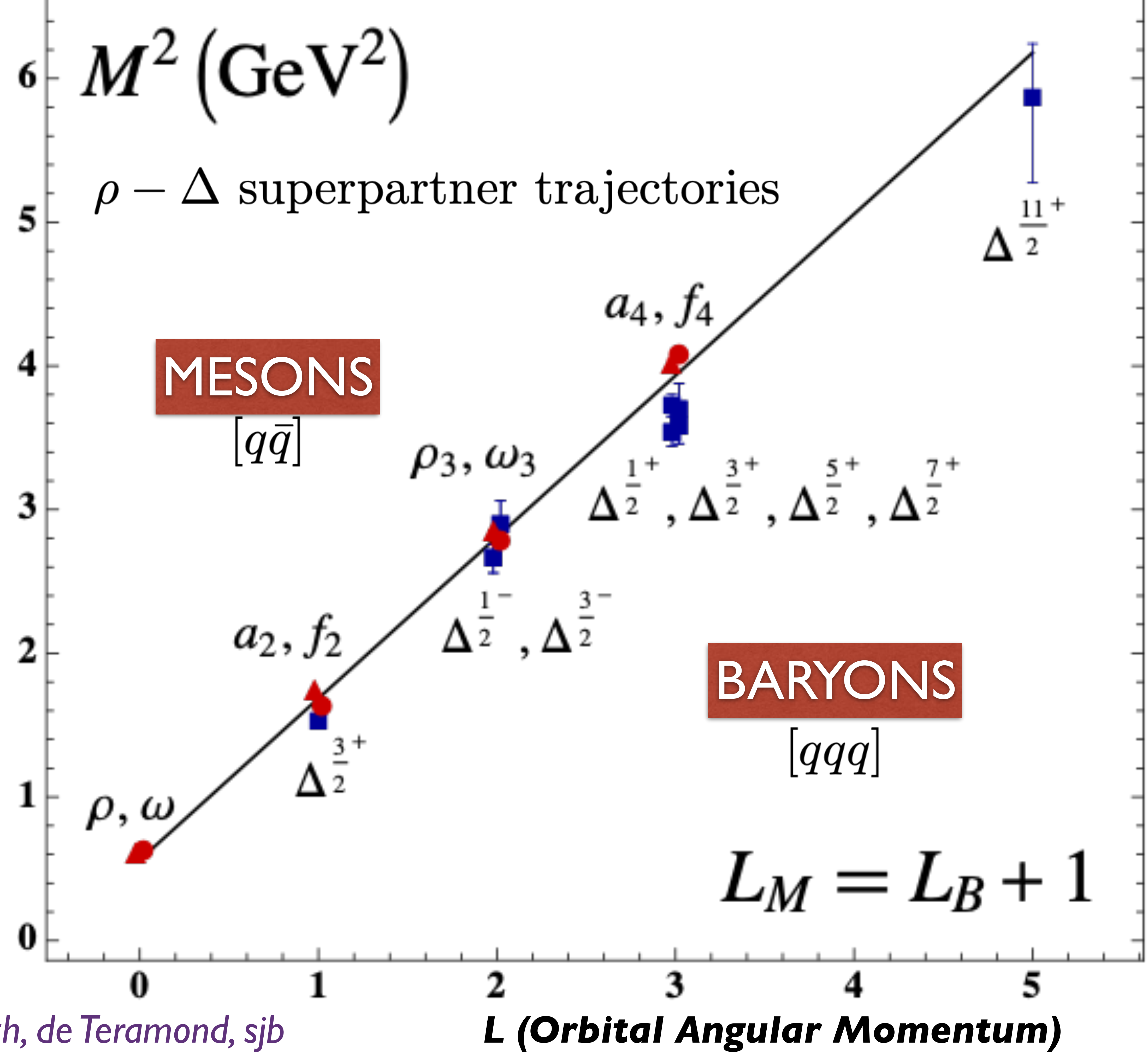
- ***Predictions are scheme-independent at every order***
- ***Matches conformal series***
- ***No n! Renormalon growth of pQCD series***
- ***New scale appears at each order; n_F determined at each order - matches virtuality of quark loops***
- ***Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Realistic Estimate of Higher-Order Terms***
- ***Reduces to standard QED scale*** $N_C \to 0$
- ***GUT: Must use the same scale setting procedure for QED, QCD***
- ***Eliminates unnecessary theory error***
- ***Maximal sensitivity to new physics***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)***
- ***PMC Reduces to BLM at NLO: Example: BFKL intercept*** **(Fadin, Kim, Lipatov, Pivovarov, sjb)**



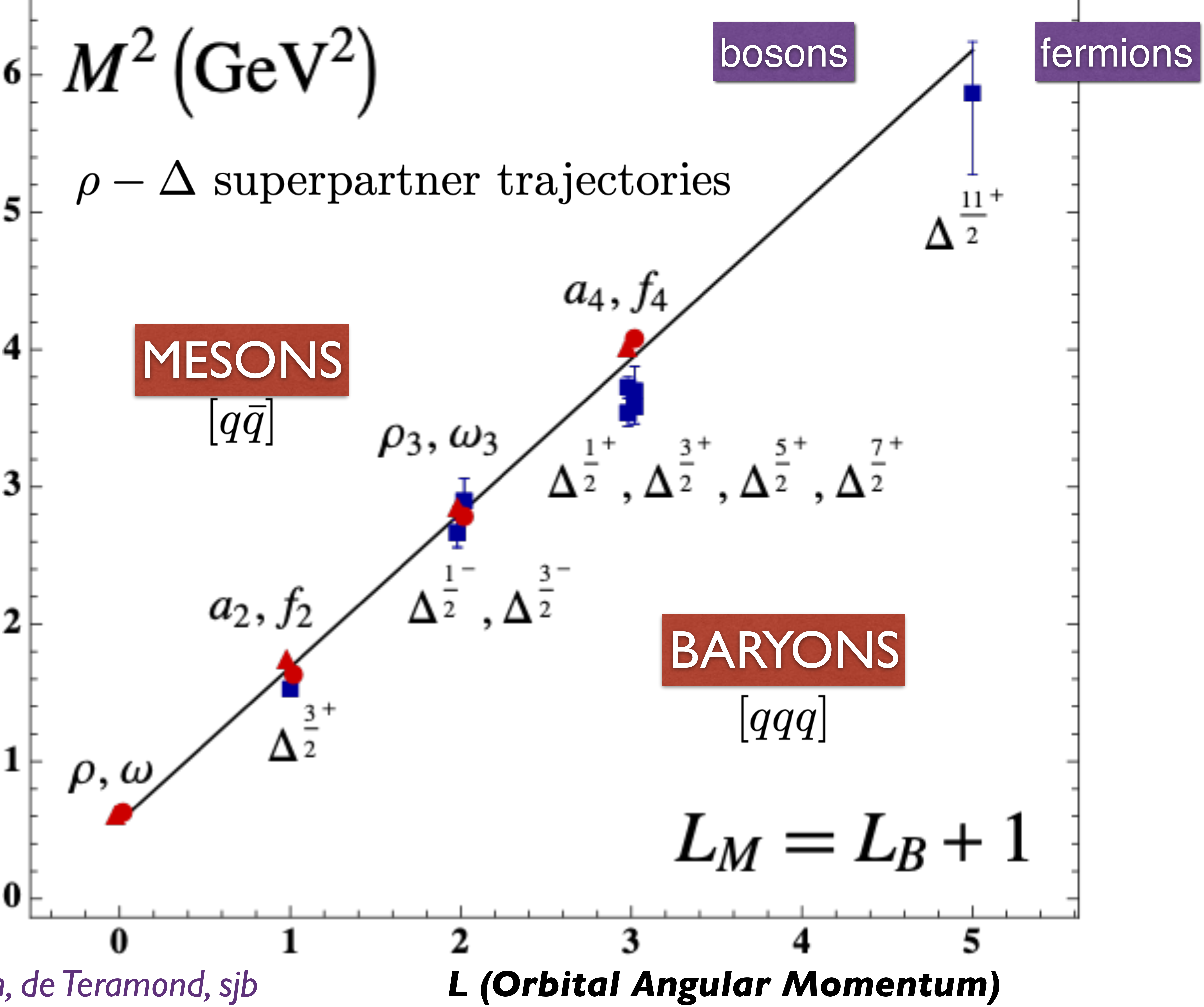
Dosch, de Teramond, sjb



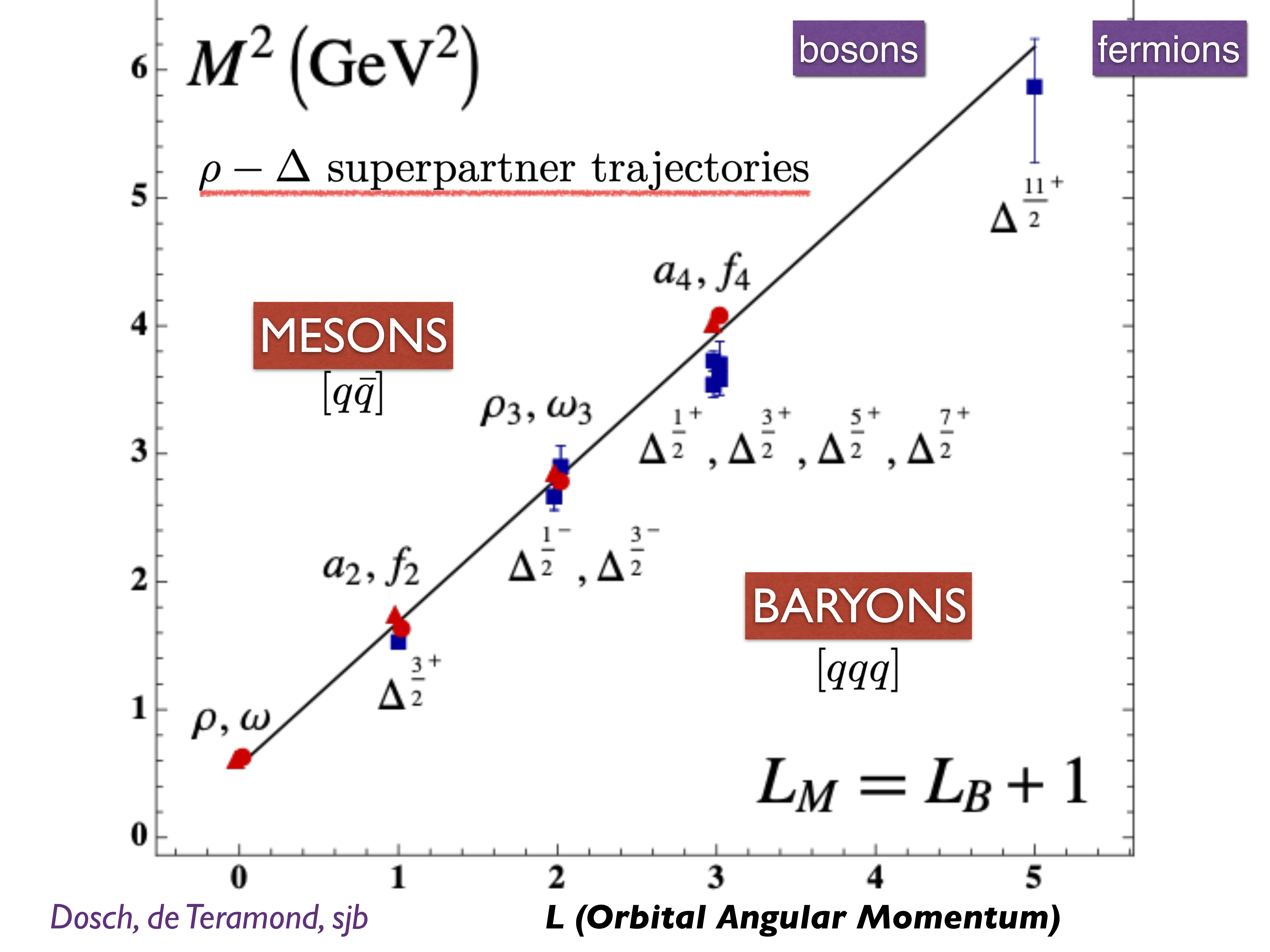
Dosch, de Teramond, sjb



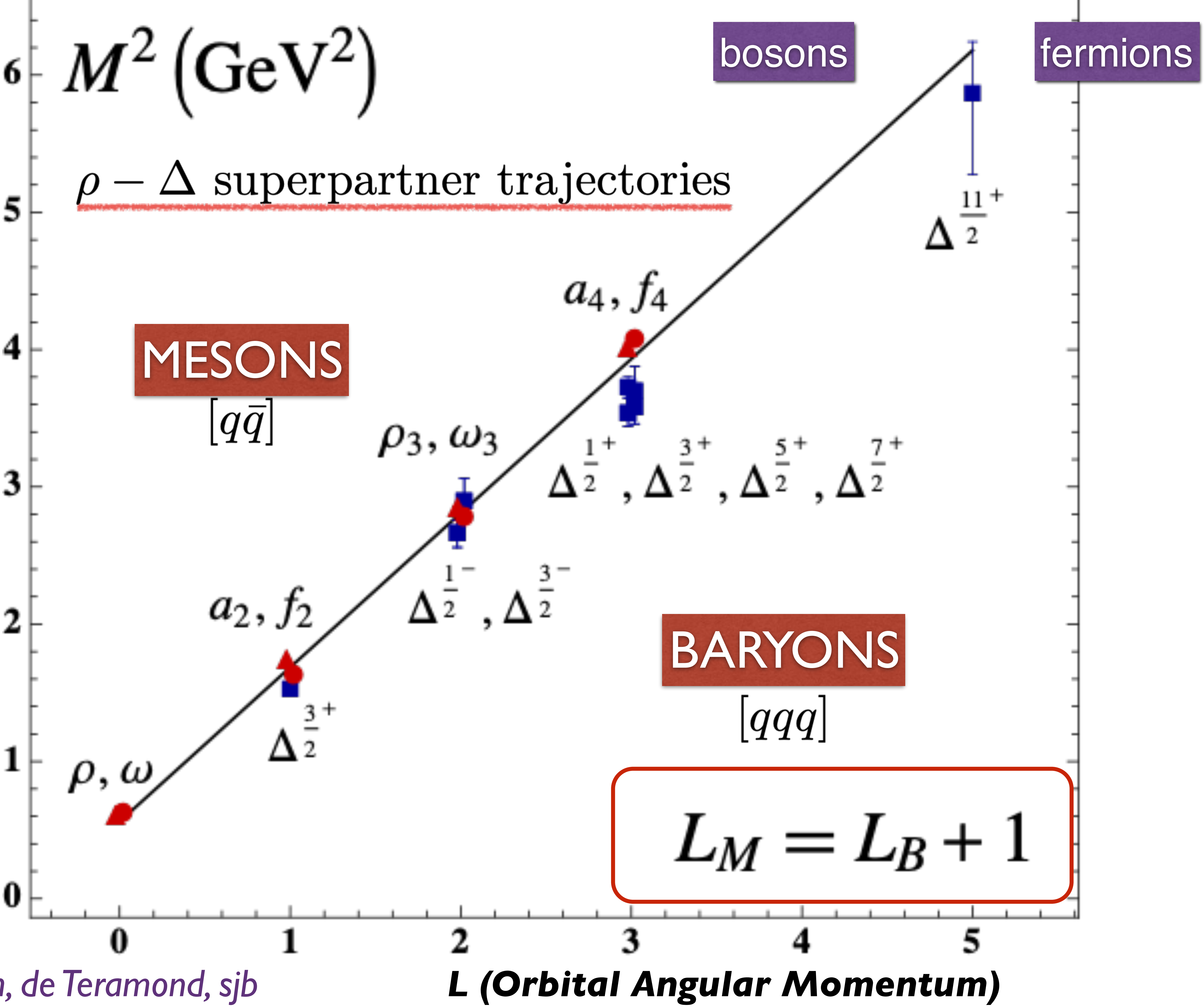
Dosch, de Teramond, sjb



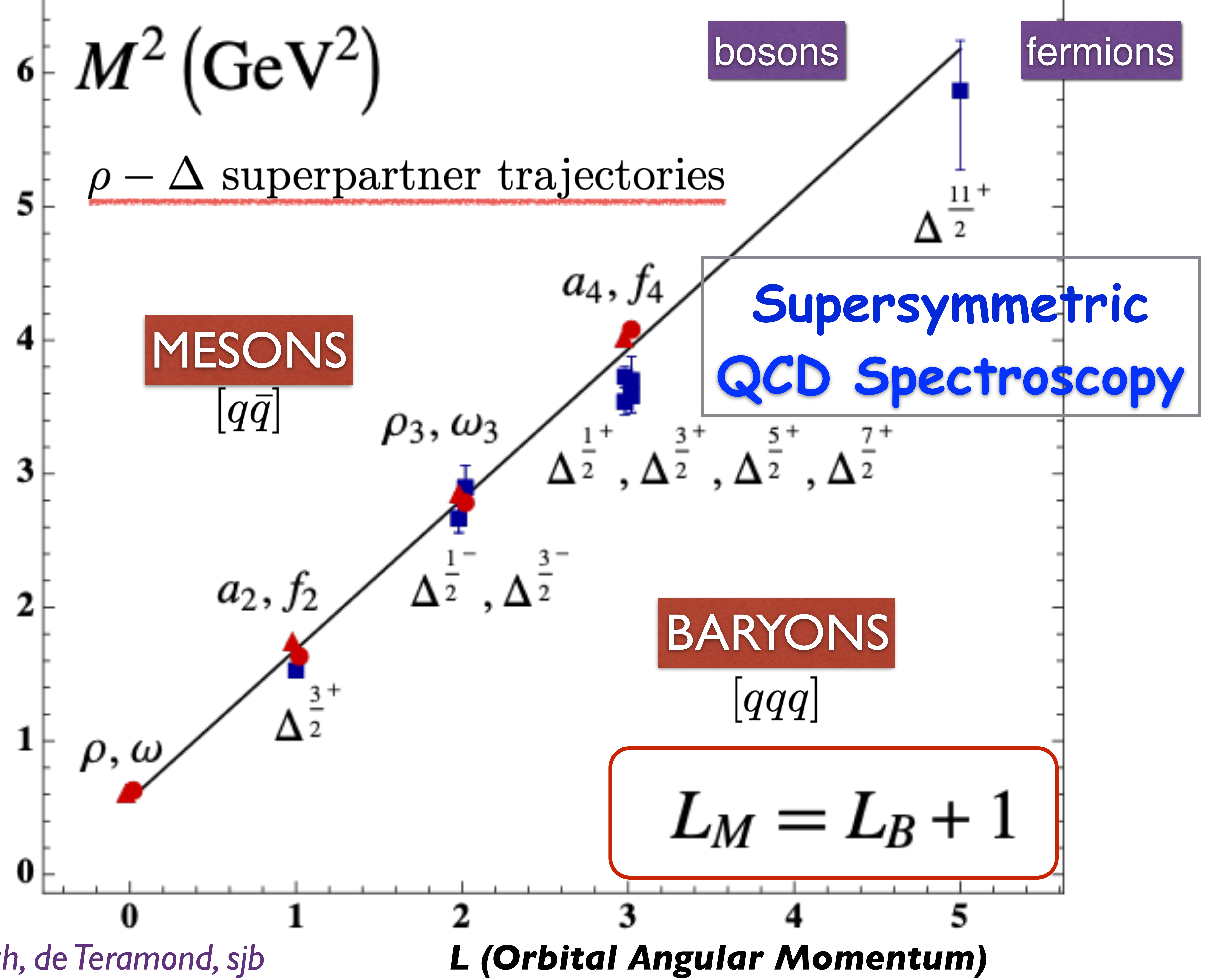
Dosch, de Teramond, sjb



$M^2\ (\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta^{\frac{3}{2}^+}$
$\Delta^{\frac{1}{2}^-}, \Delta^{\frac{3}{2}^-}$
$\Delta^{\frac{1}{2}^+}, \Delta^{\frac{3}{2}^+}, \Delta^{\frac{5}{2}^+}, \Delta^{\frac{7}{2}^+}$
$\Delta^{\frac{11}{2}^+}$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb



Dosch, de Teramond, sjb



Dosch, de Teramond, sjb

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

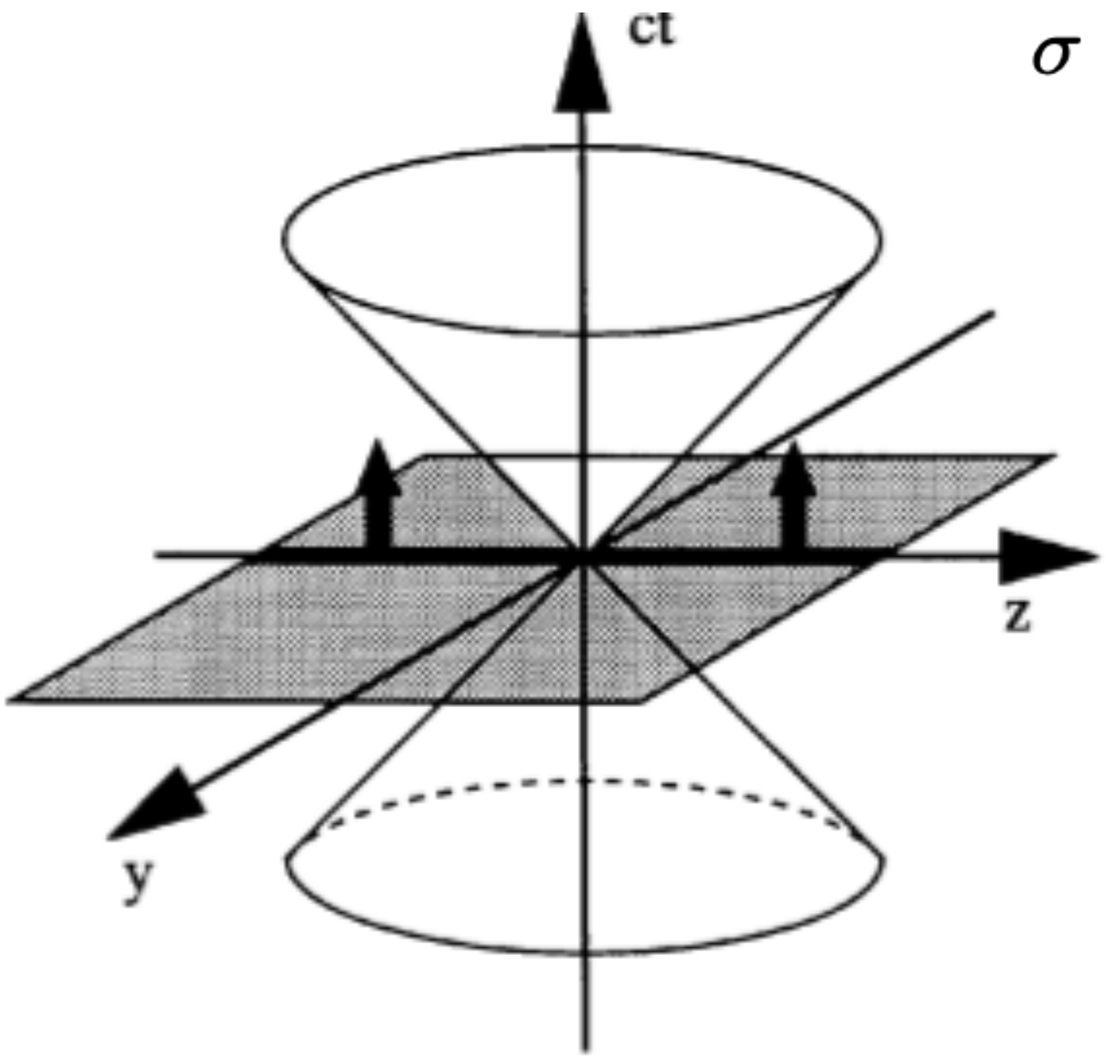
AdS/QCD
Light-Front Holography
Superconformal Algebra

No parameters except for quark masses!

Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

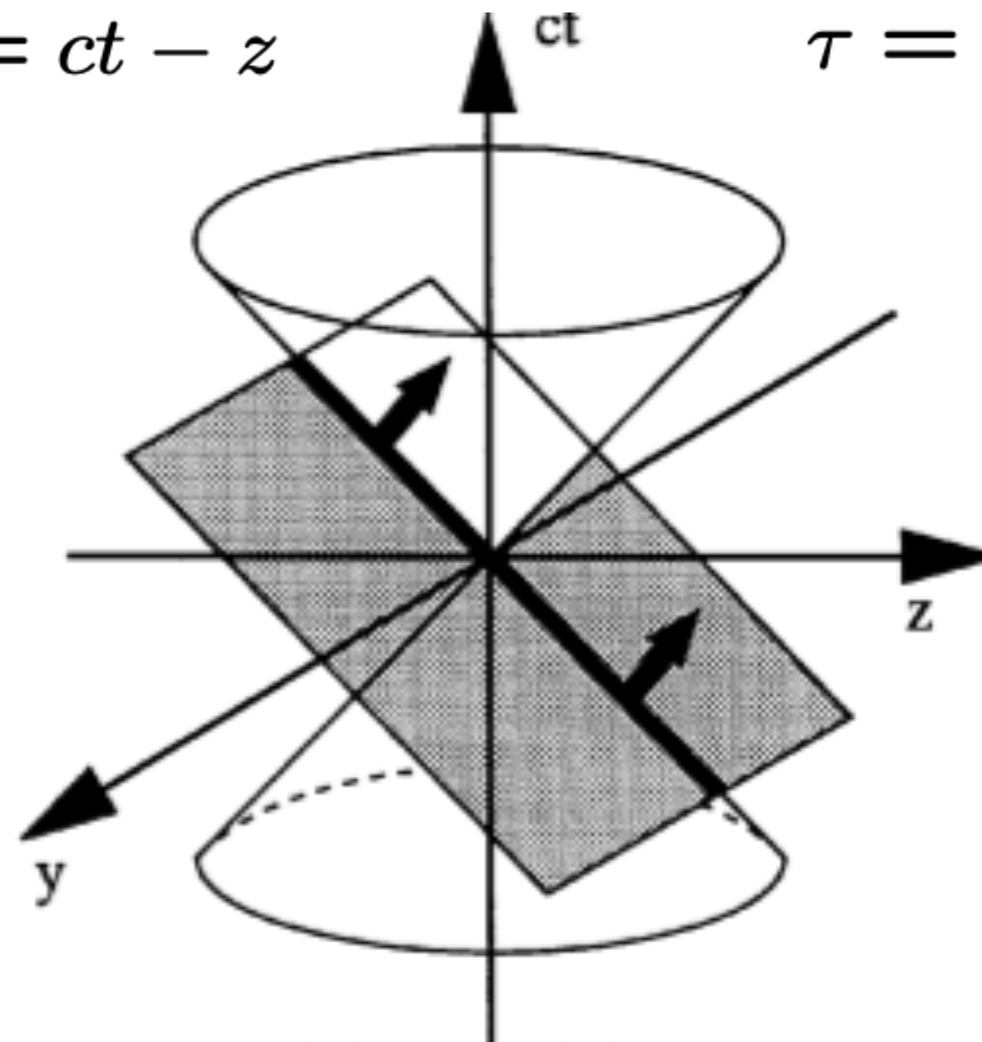
Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- *Color Confinement*
- *Origin of the QCD Mass Scale*
- *Meson and Baryon Spectroscopy*
- *Exotic States: Tetraquarks, Pentaquarks, Gluonium,*
- *Universal Regge Slopes: n, L, Mesons and Baryons*
- *Massless Pion! (Quark Anti-Quark Bound State)*
- *QCD Coupling at all Scales* $\alpha_s(Q^2)$
- *Eliminate Scale Uncertainties and Scheme Dependence*
- *Heavy Quark Distributions*



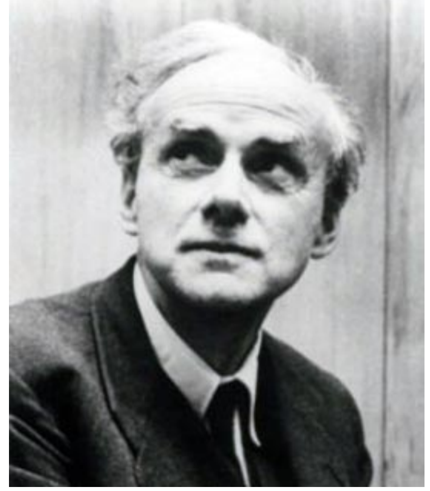
Instant Form

$$\sigma = ct - z$$



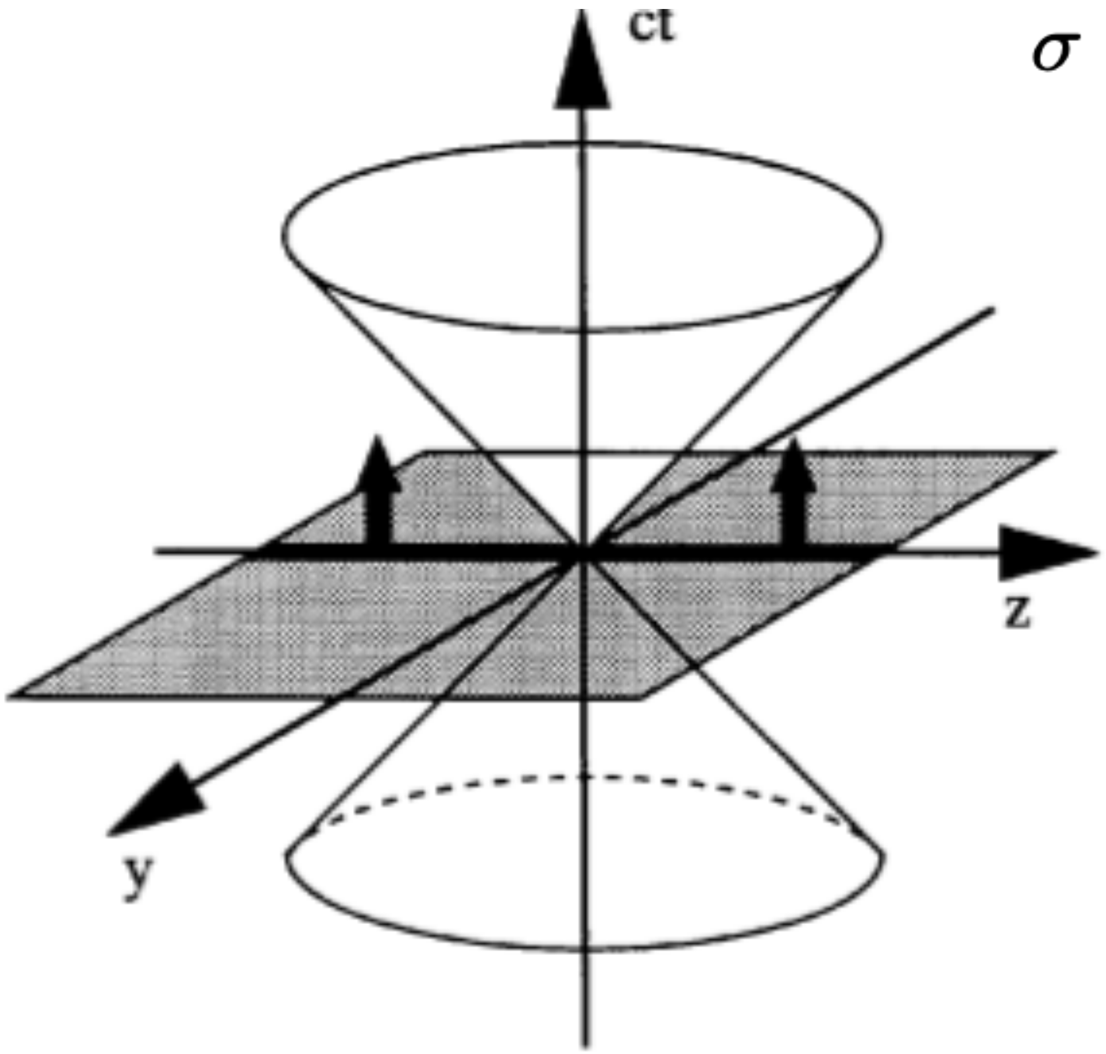
$$\tau = t + z/c$$

Front Form

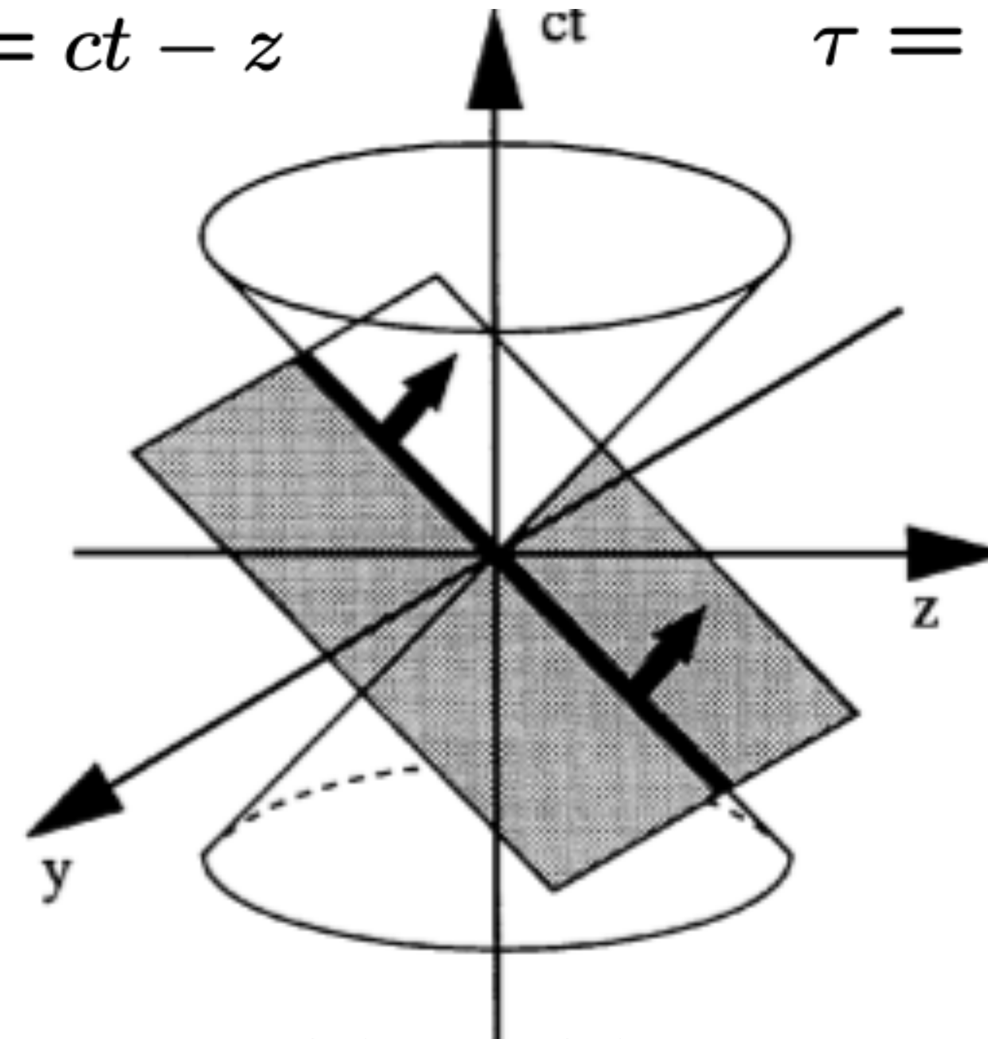


P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

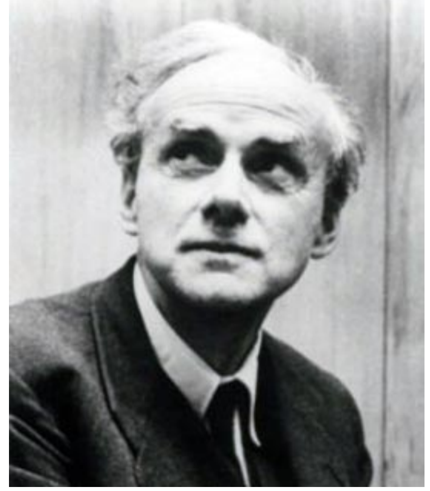
$$\sigma = ct - z \qquad \tau = t + z/c$$



Instant Form



Front Form



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Dirac's Amazing Idea:
The "Front Form"

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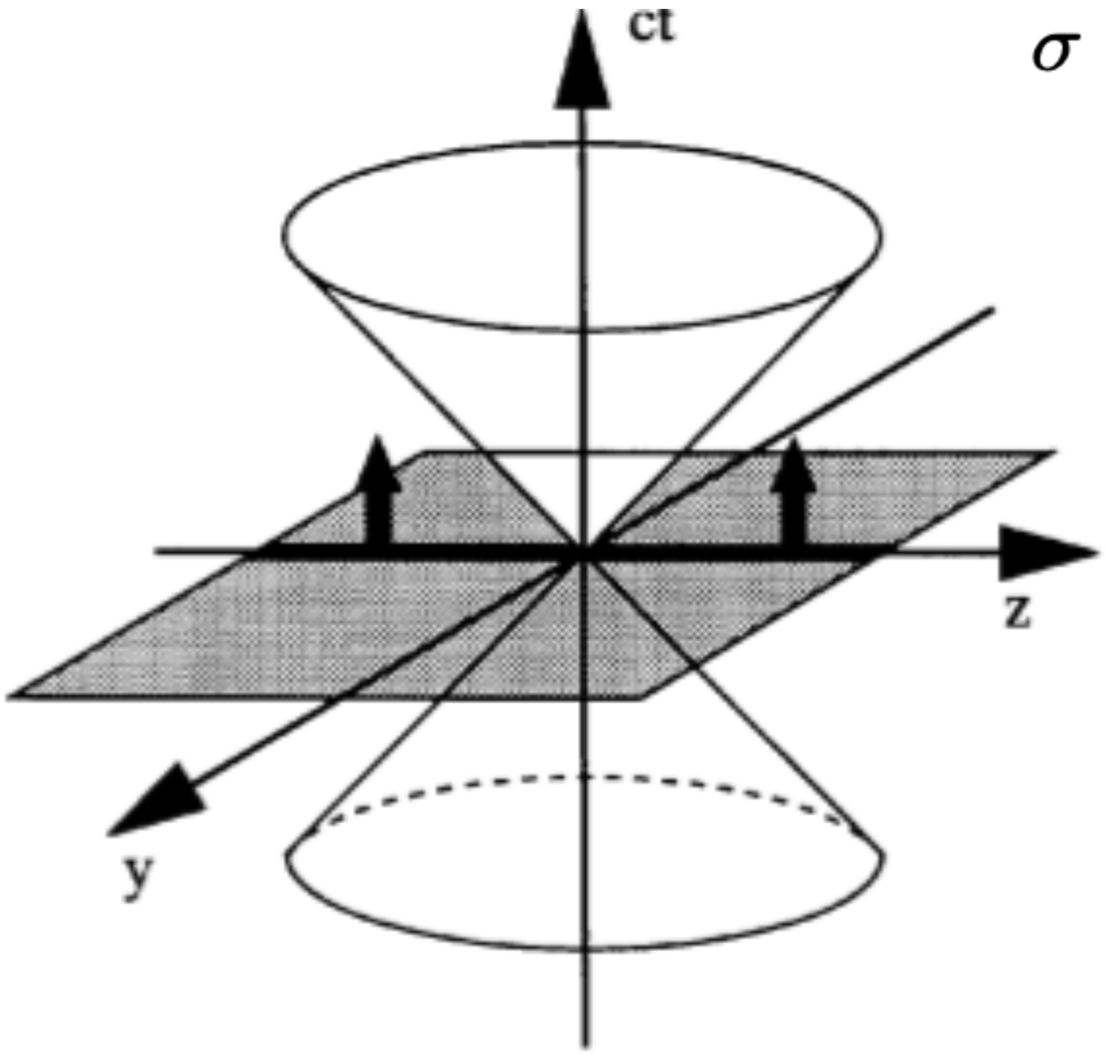
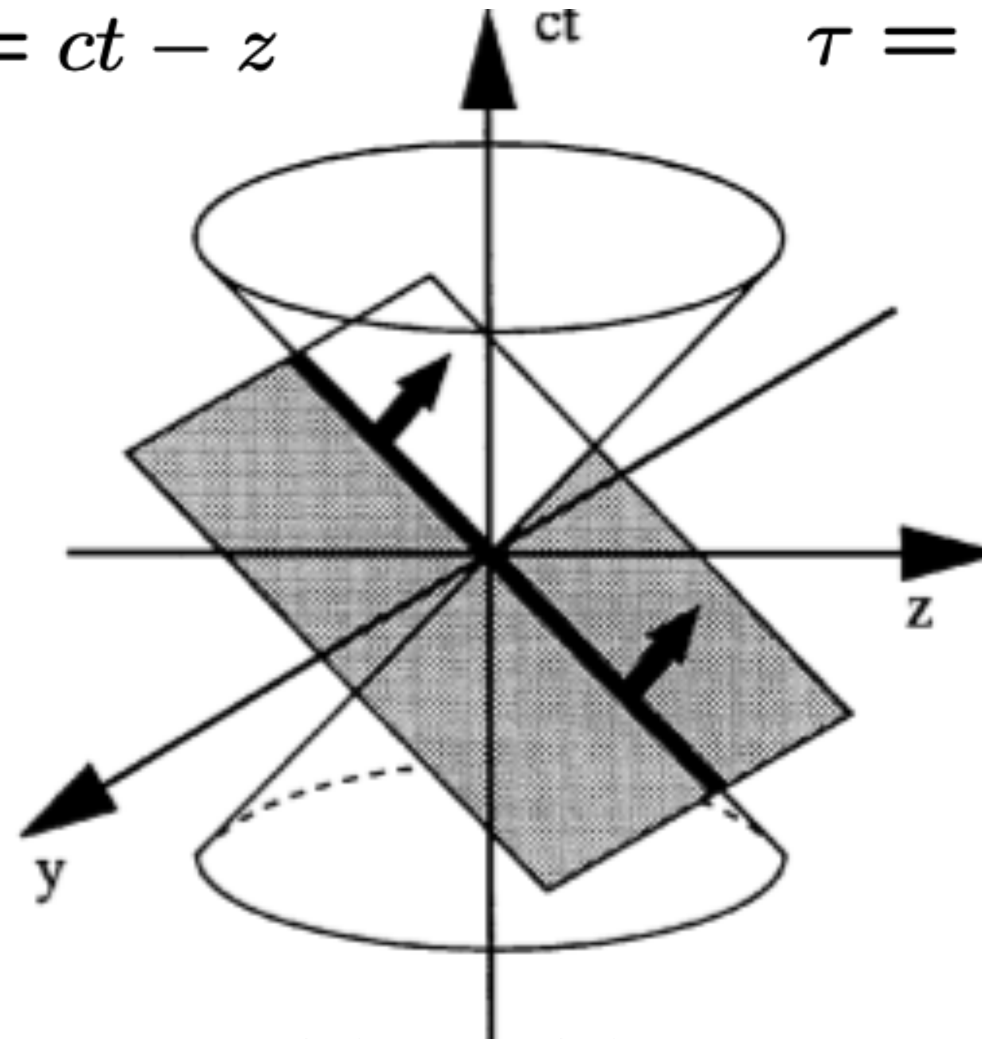
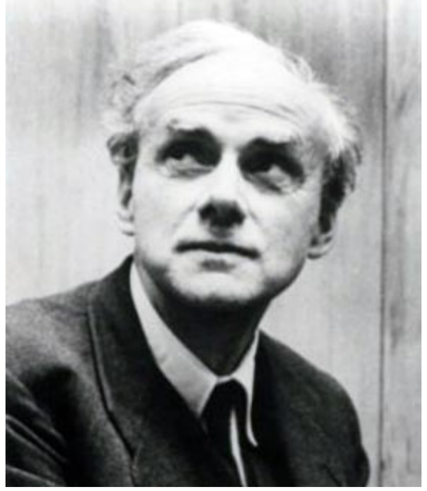


Instant Form



Front Form

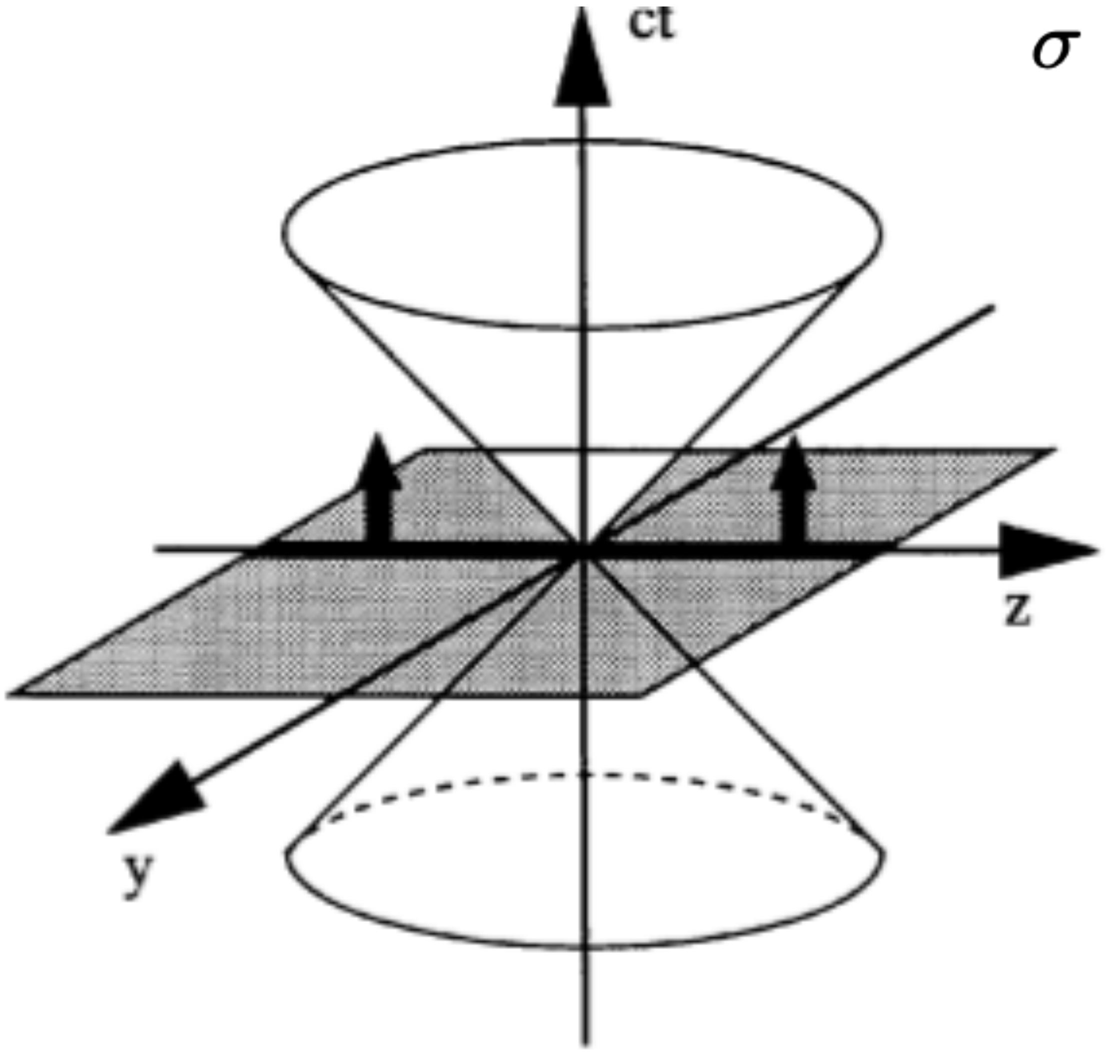


P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

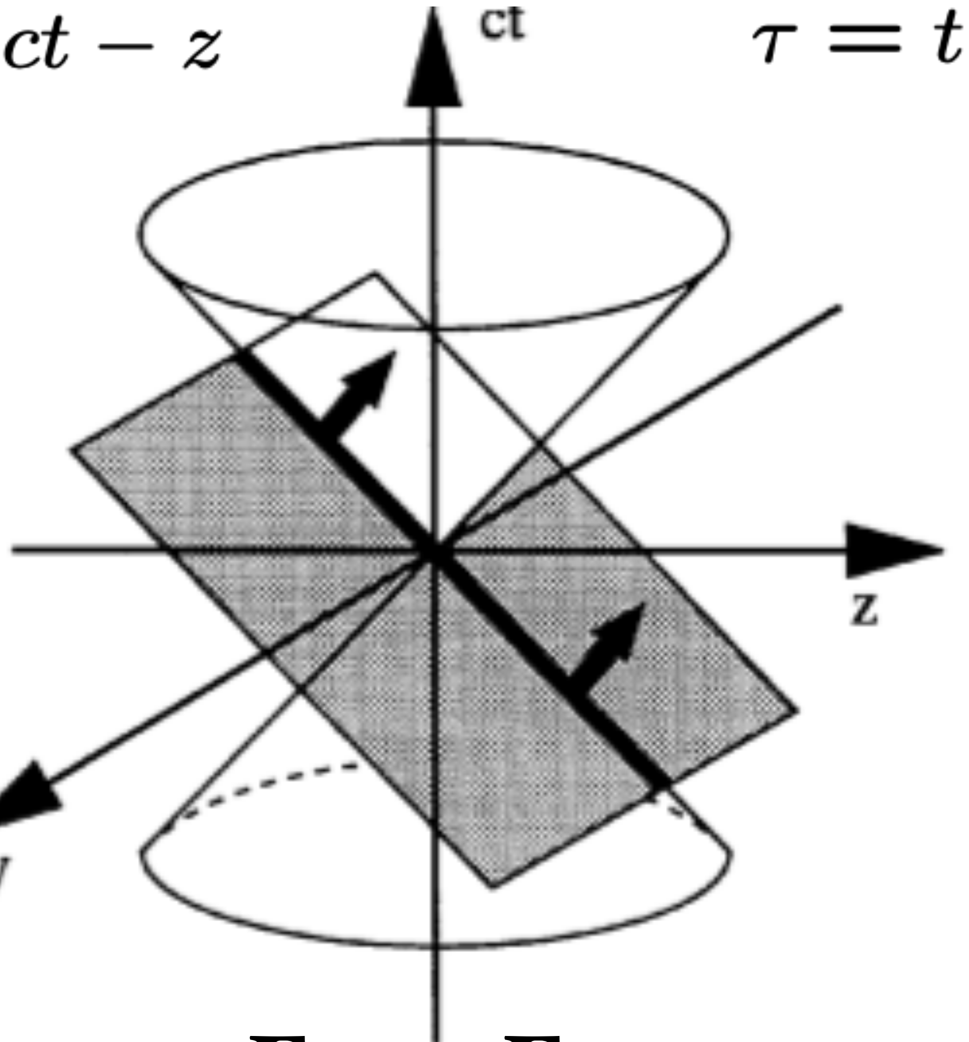
Dirac's Amazing Idea:
The "Front Form"

Evolve in ordinary time

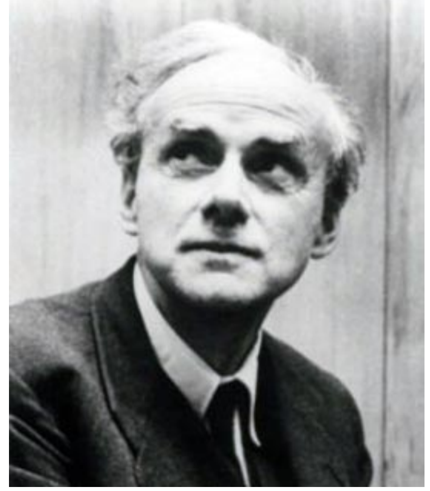
$\sigma = ct - z$ $\quad\quad$ $\tau = t + z/c$



Instant Form



Front Form



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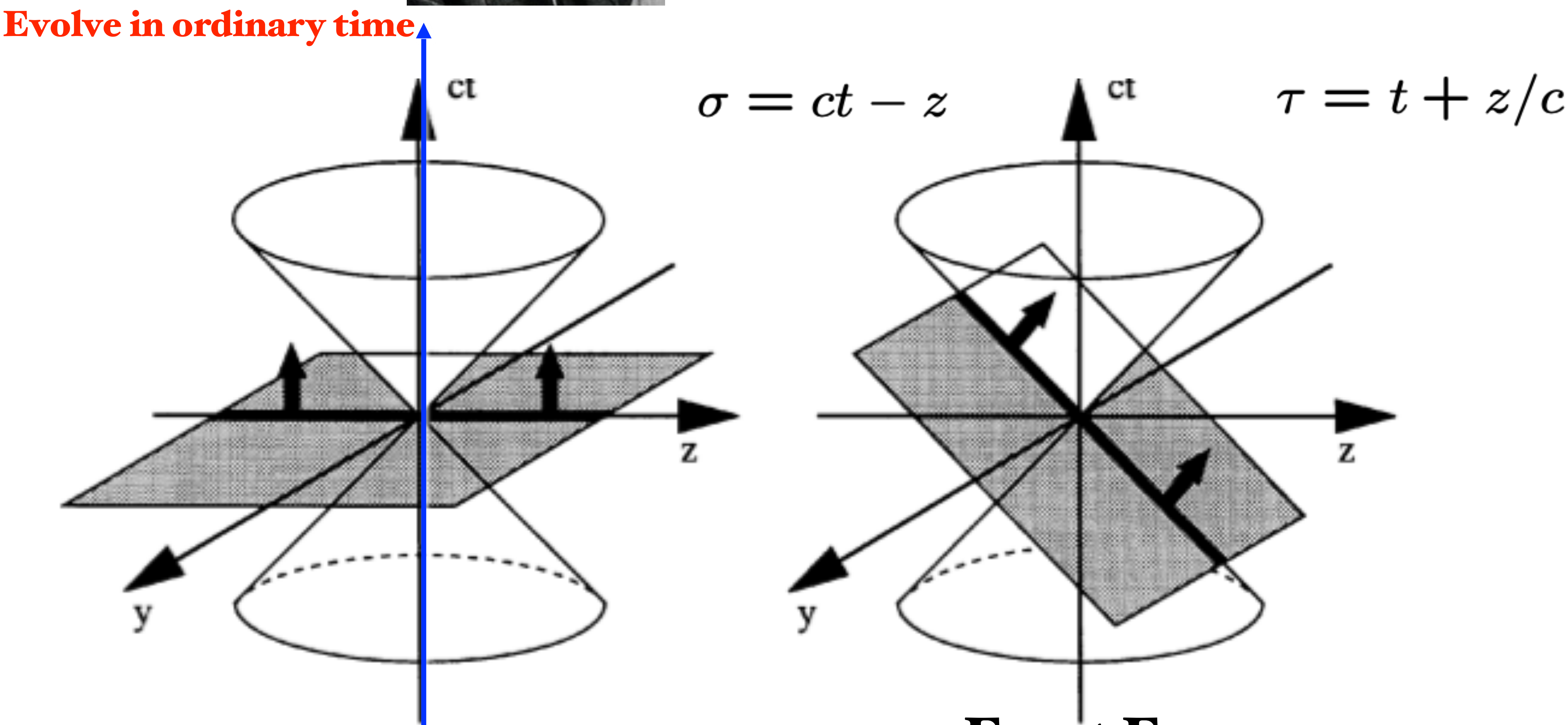
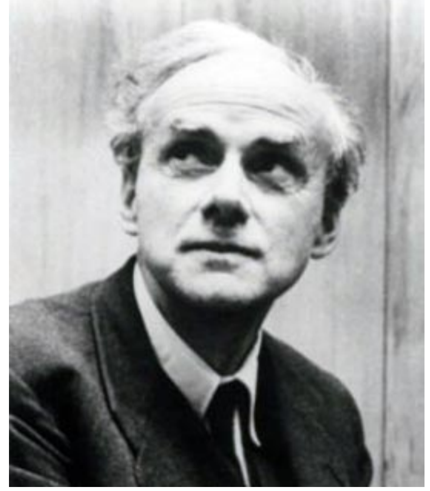


Instant Form

Front Form



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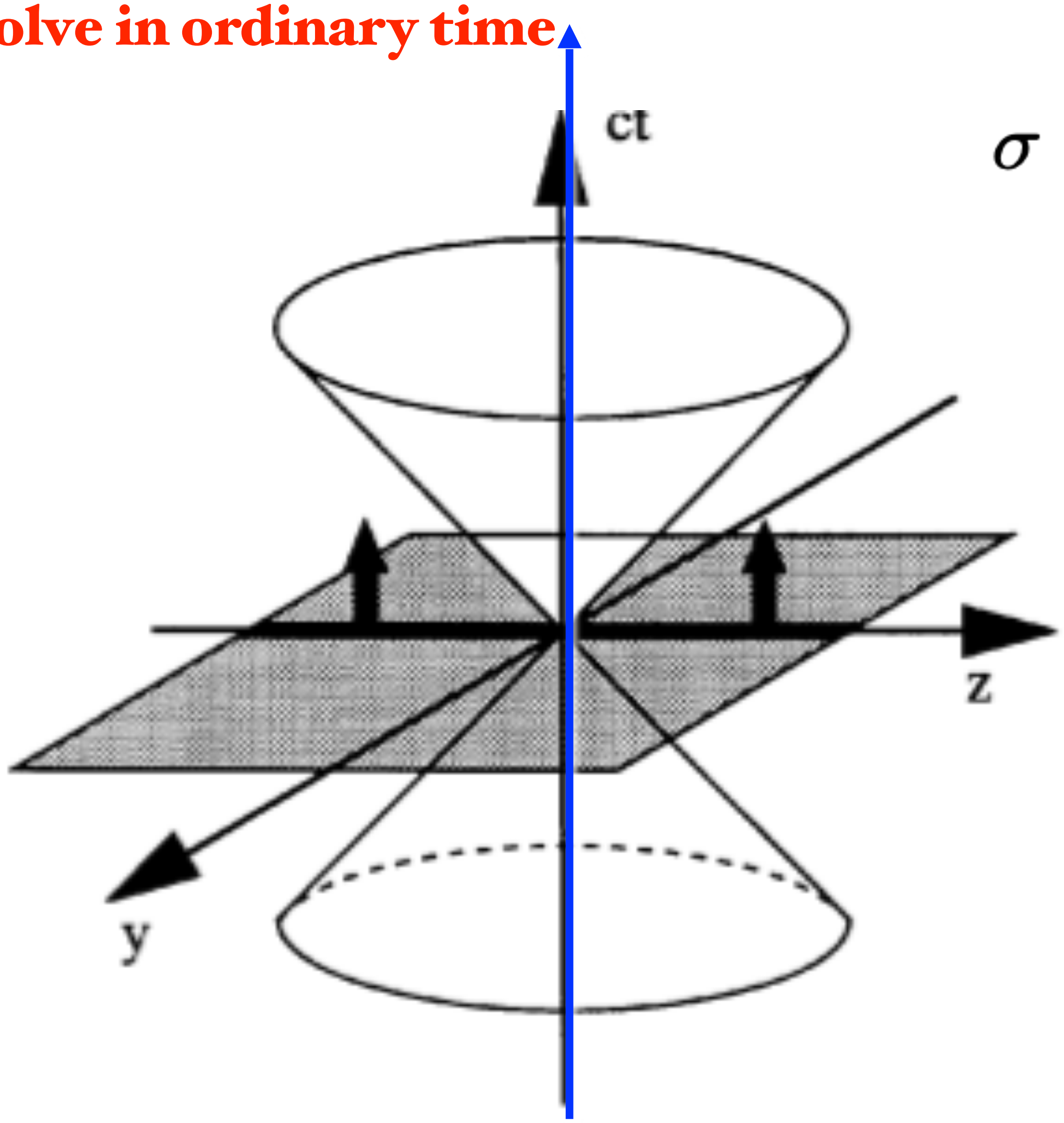
The "Front Form"

Evolve in ordinary time

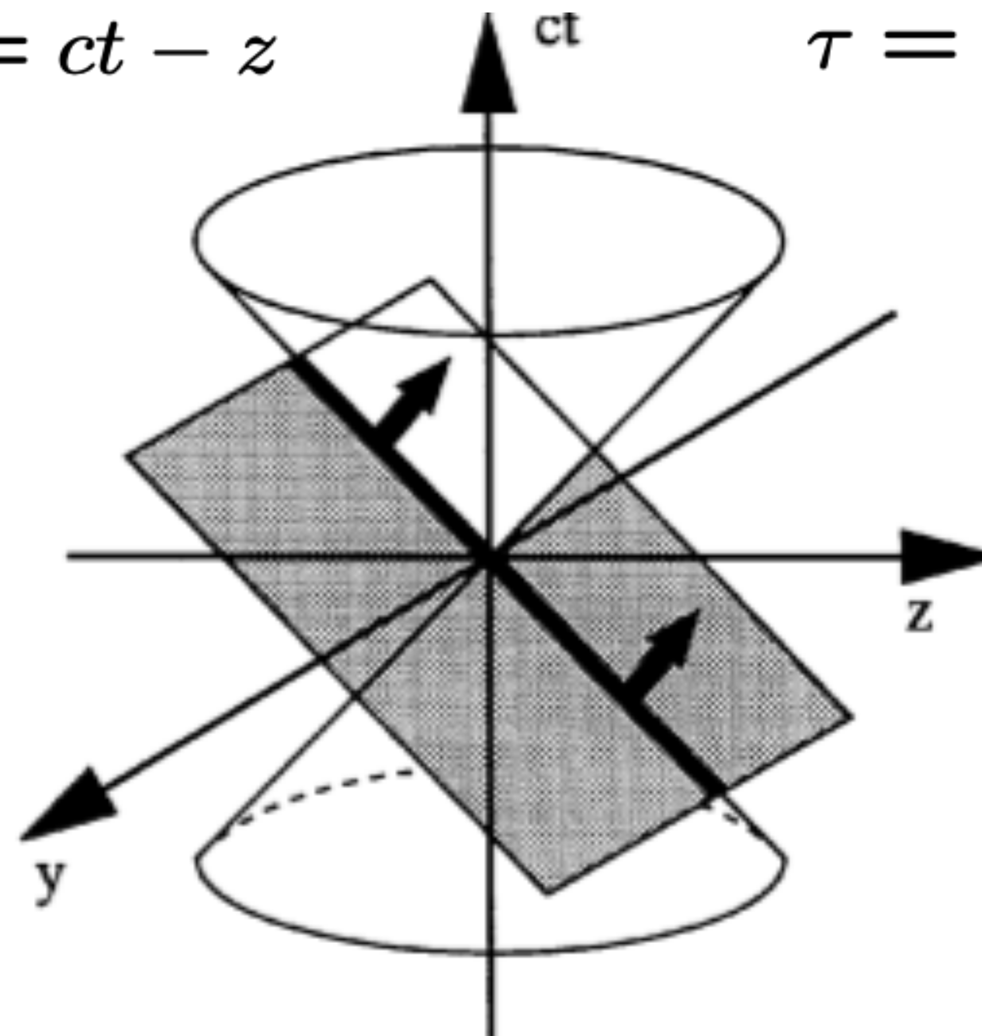
Evolve in light-front time!

$$\sigma = ct - z$$

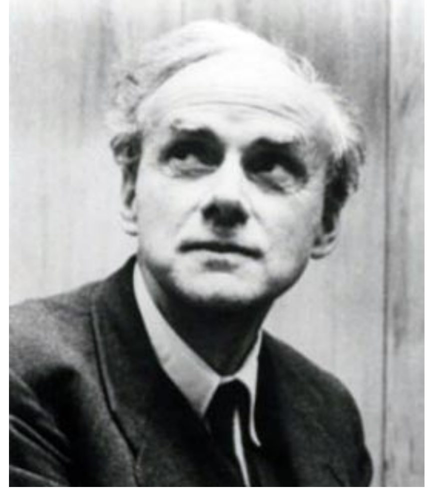
$$\tau = t + z/c$$



Instant Form



Front Form



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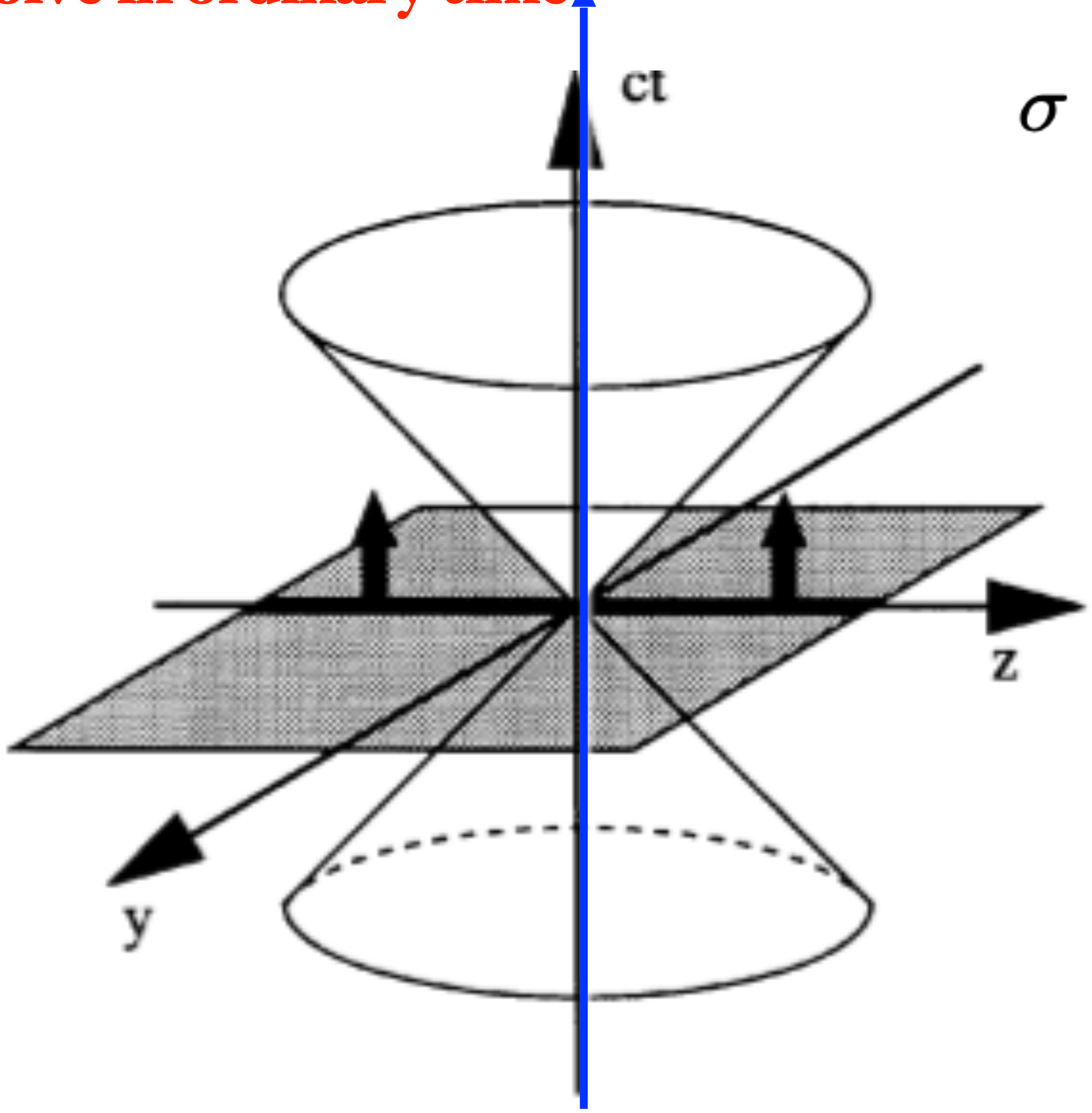

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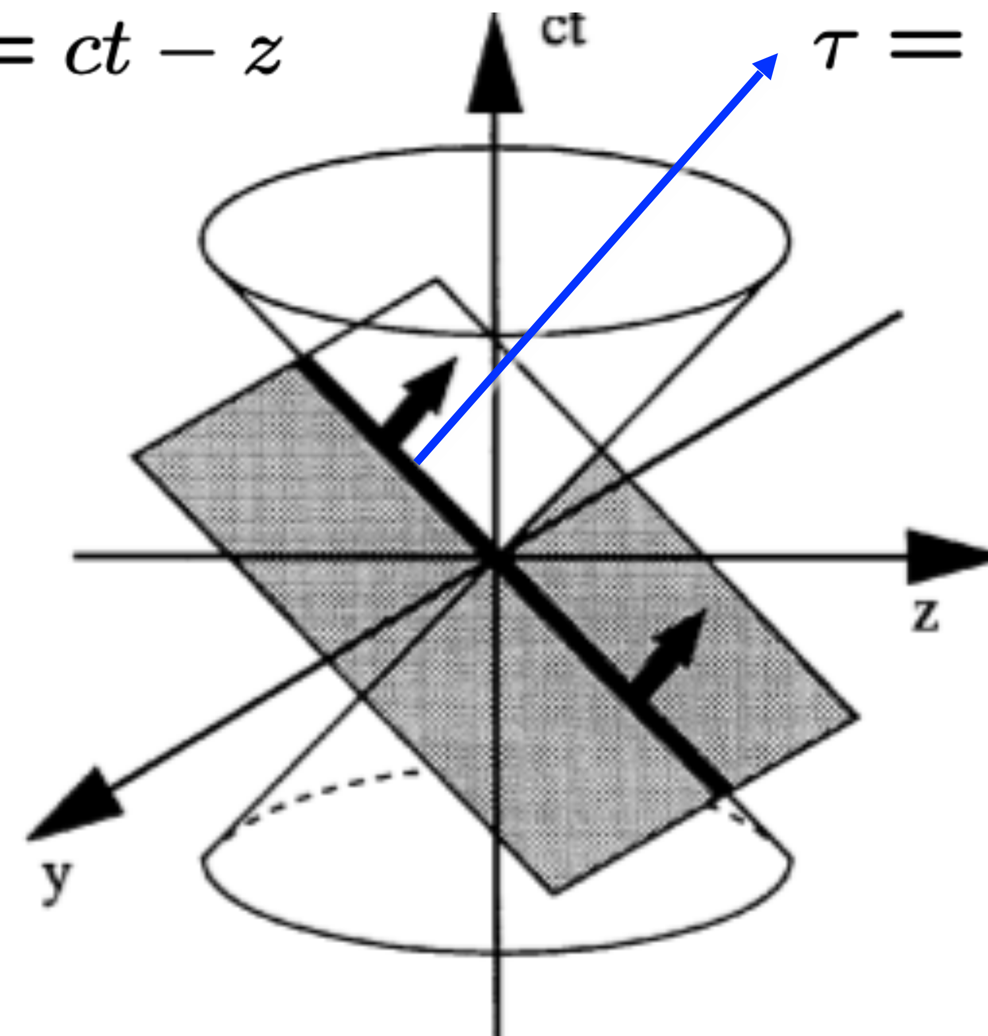

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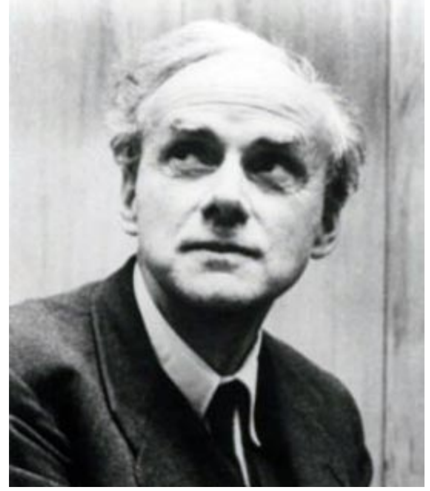

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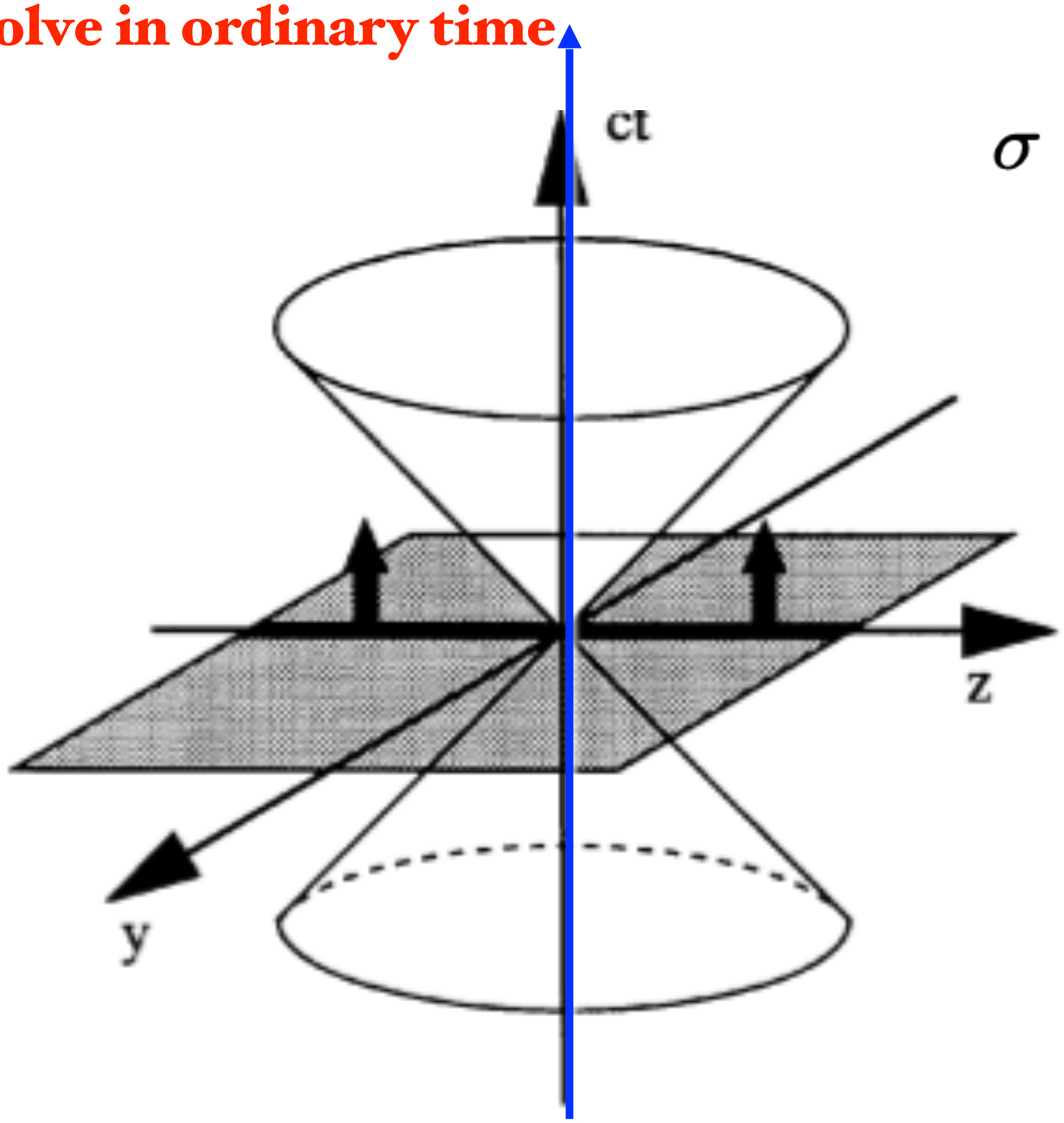
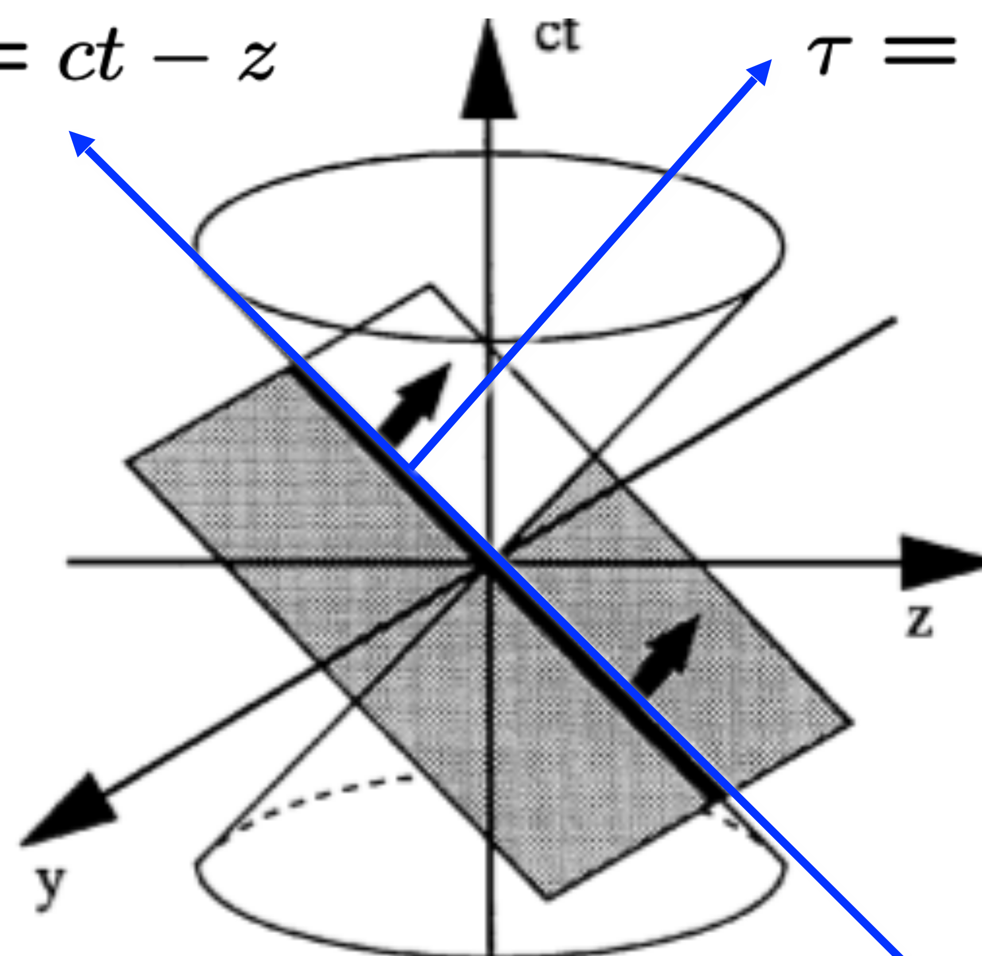


Instant Form

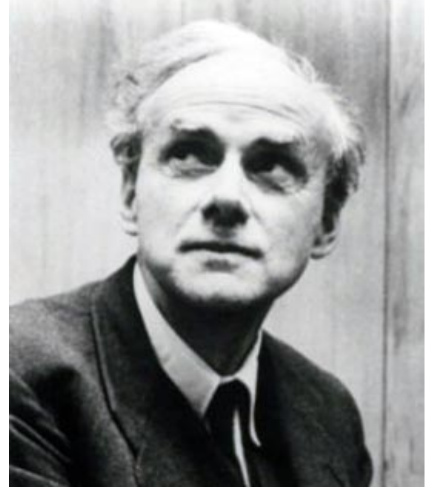
Evolve in light-front time!

$\sigma = ct - z$

$\tau = t + z/c$



Front Form



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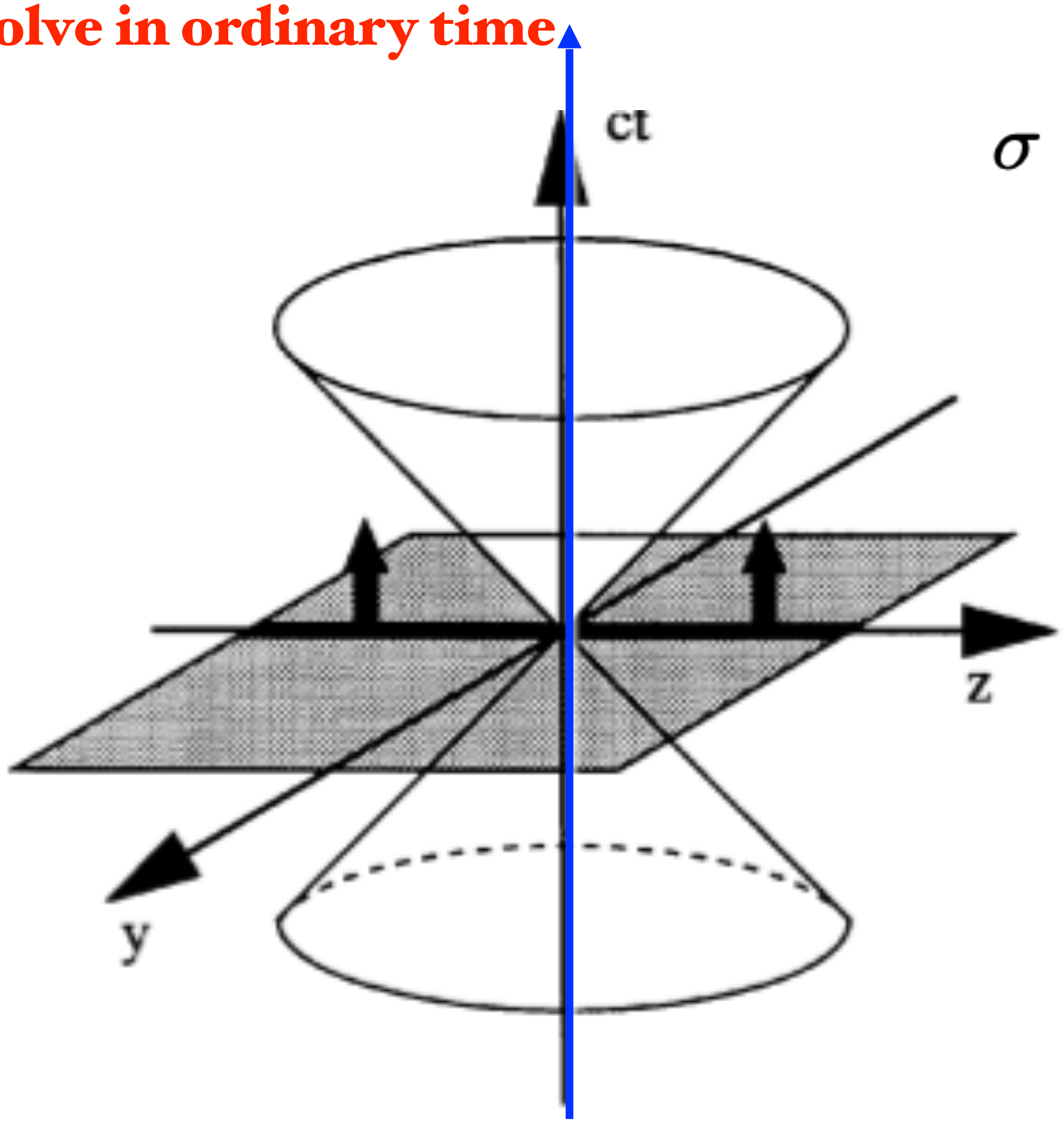
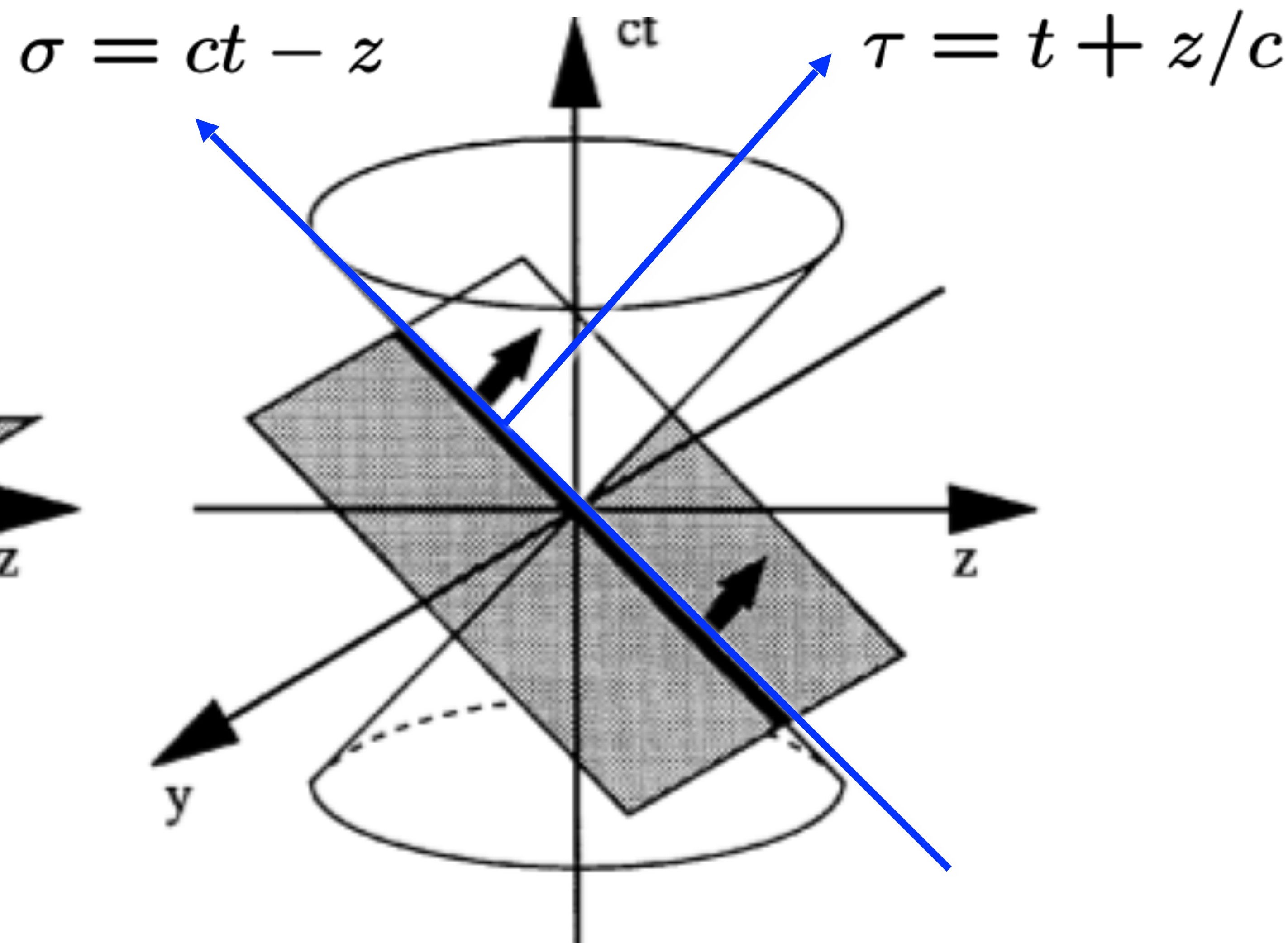


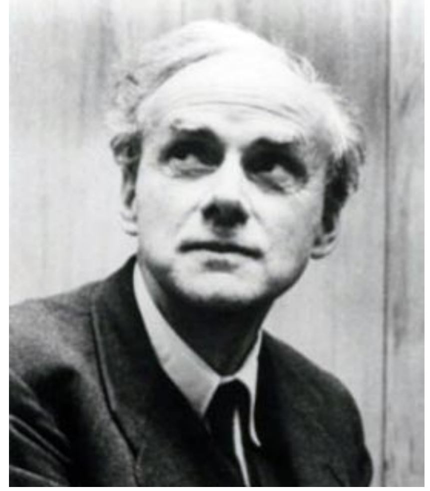
Instant Form

Evolve in light-front time!



Front Form

Casual, Boost Invariant!



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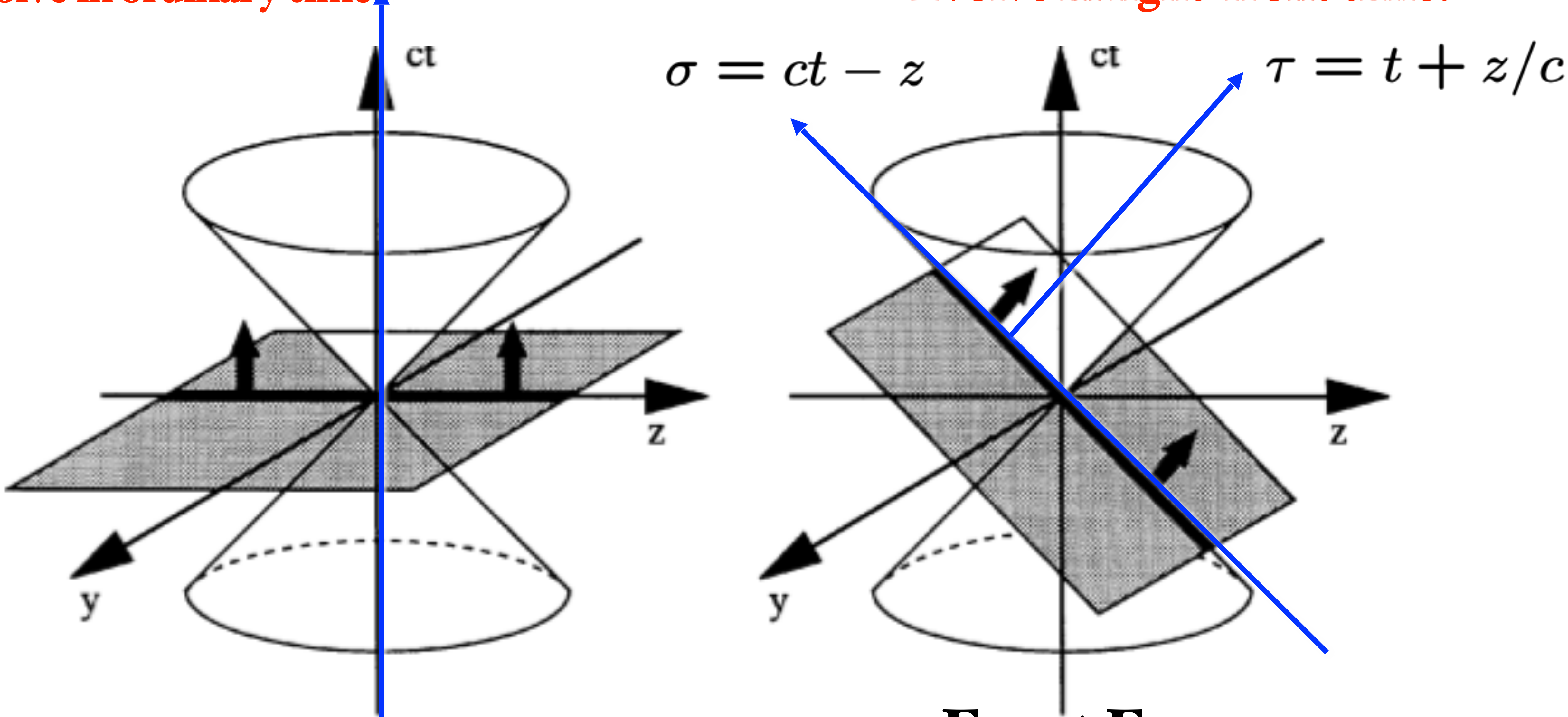
Evolve in light-front time!

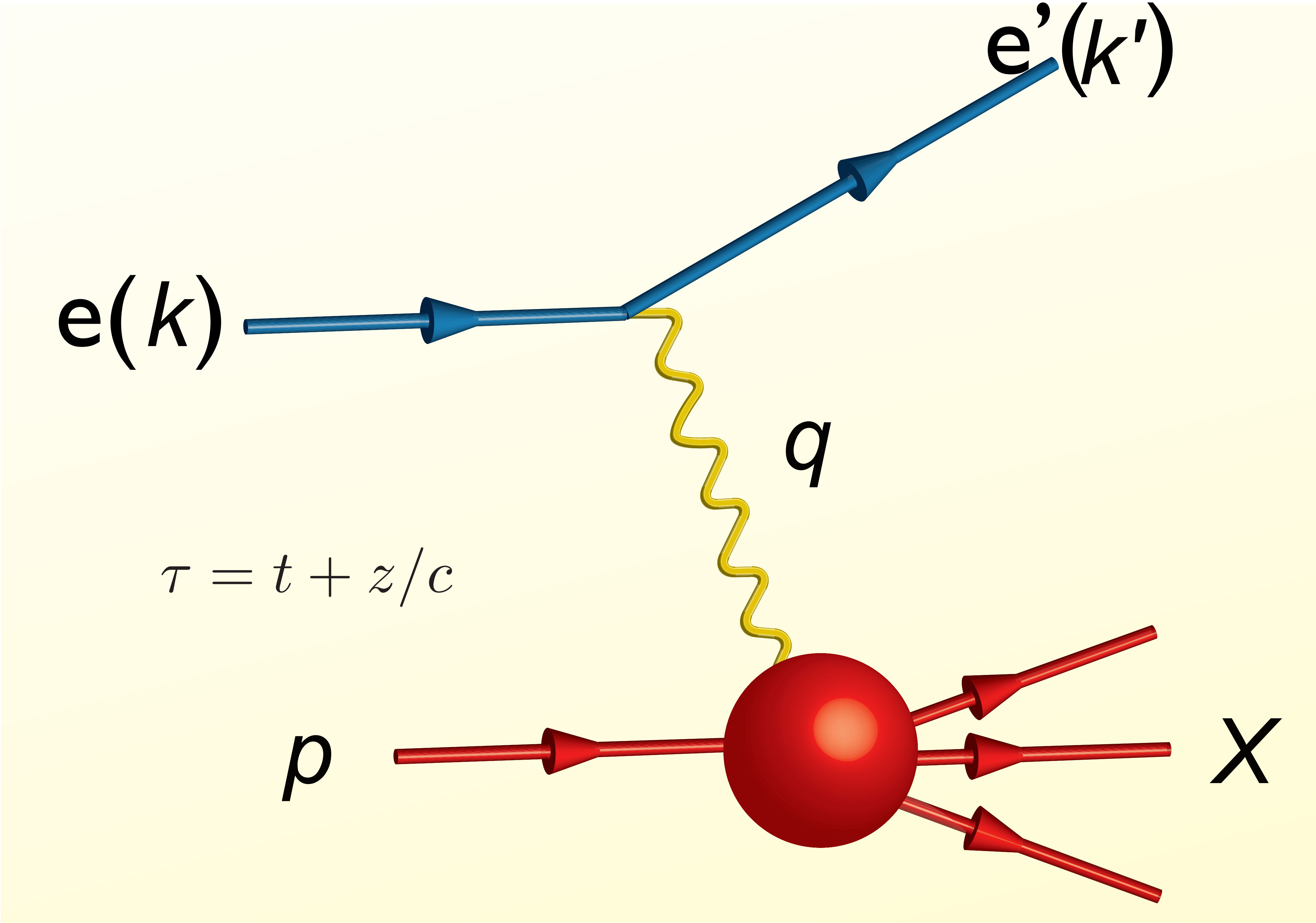


Instant Form

Front Form

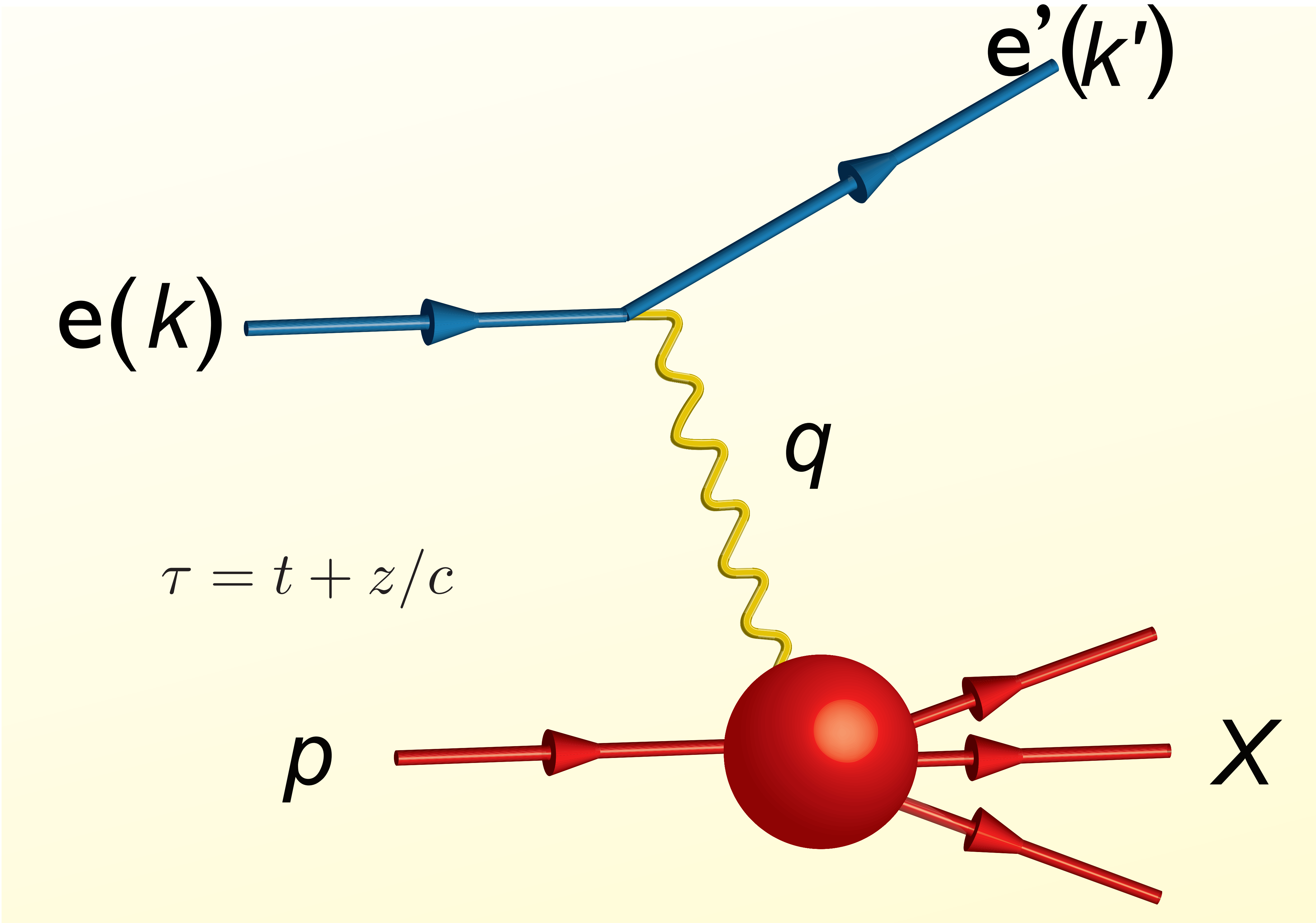
Casual, Boost Invariant!

- ***Trivial LF Vacuum (up to zero modes)***



The scattered electron measures the proton's structure at the speed of light — like a flash photograph

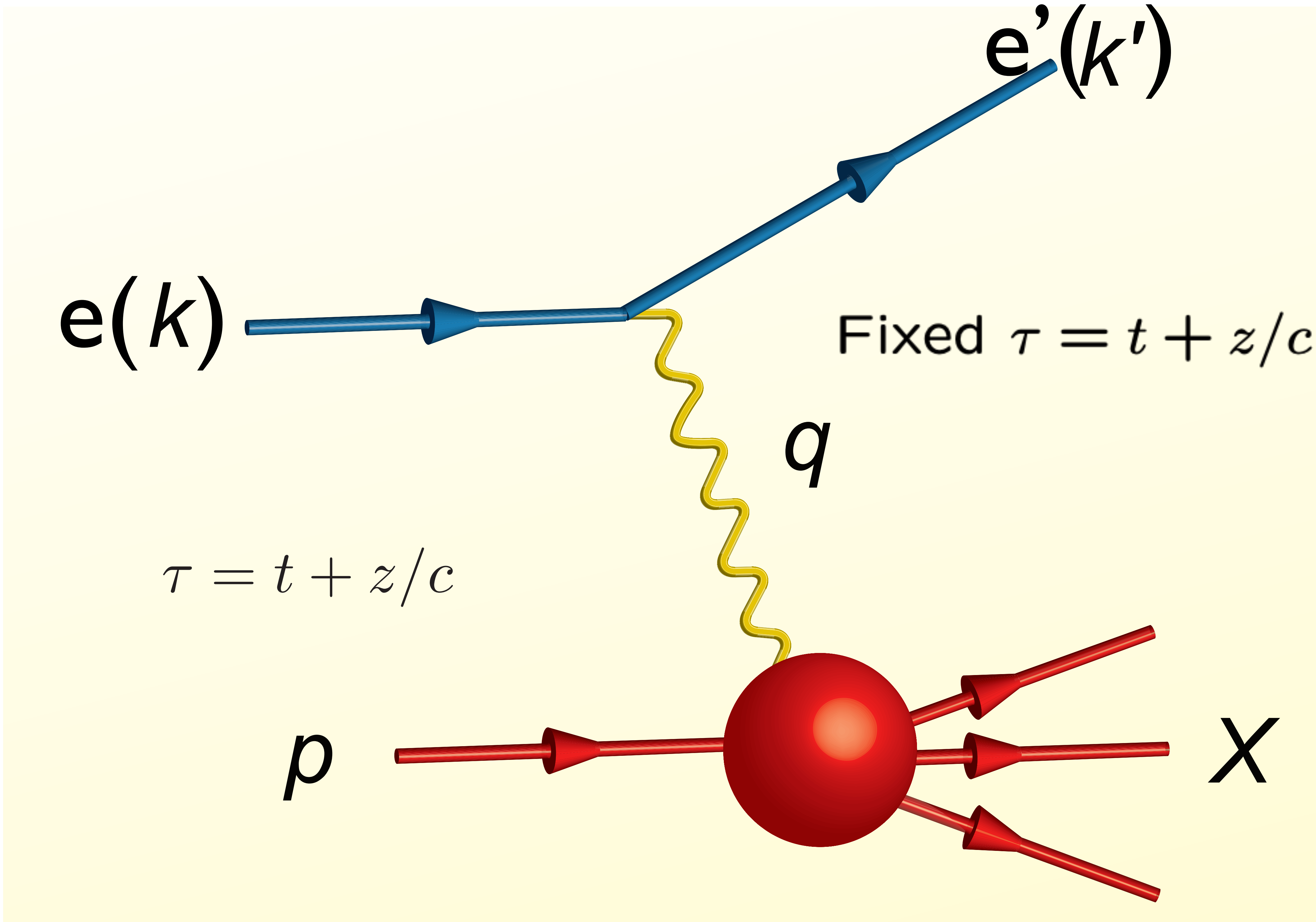
Causality: Information and correlations constrained by speed of light



$$\tau = t + z/c$$

The scattered electron measures the proton's structure at the speed of light — like a flash photograph

Causality: Information and correlations constrained by speed of light

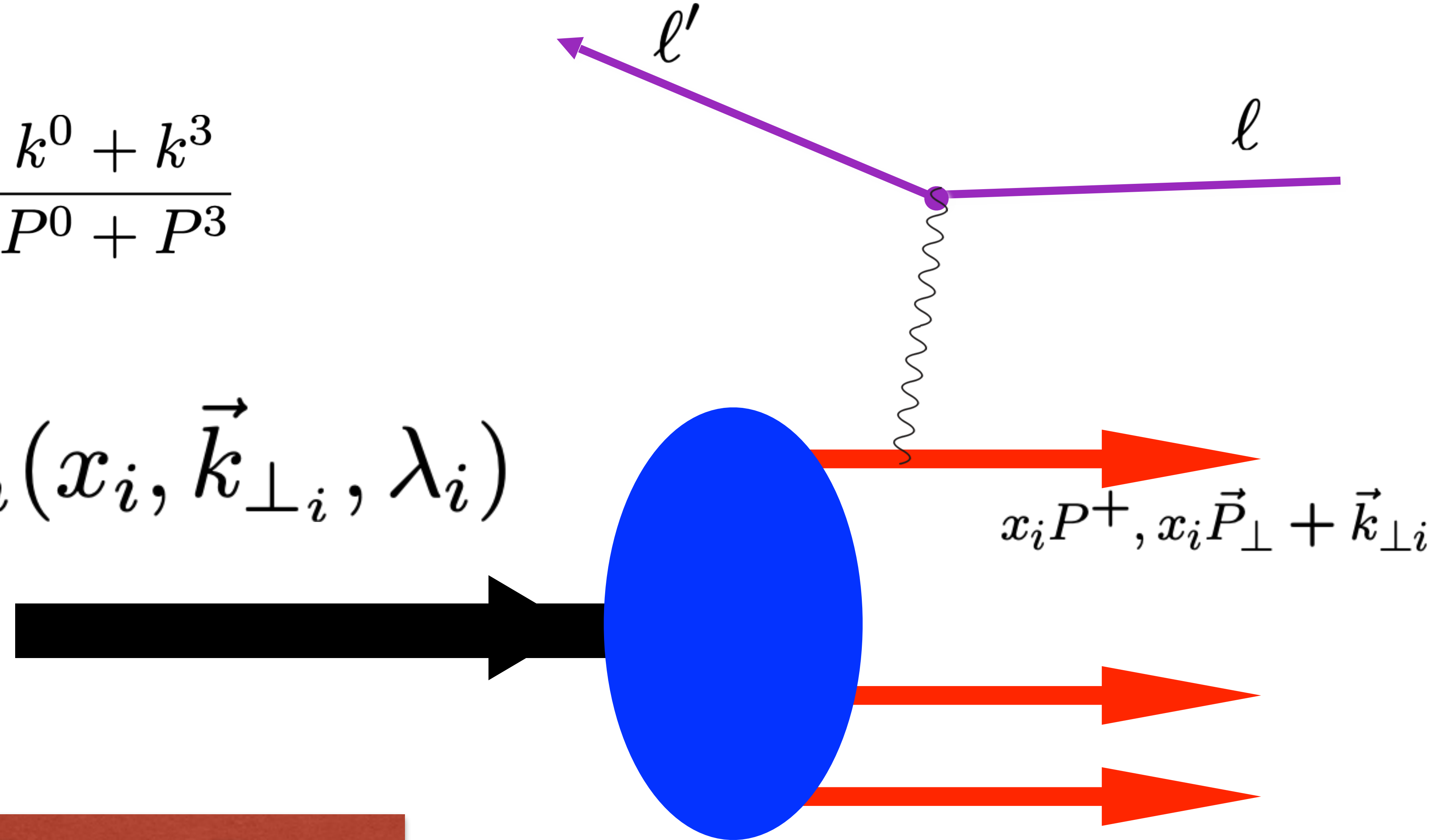


The scattered electron measures the proton's structure at the speed of light — like a flash photograph

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$P^+, \vec{P}_\perp$



Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

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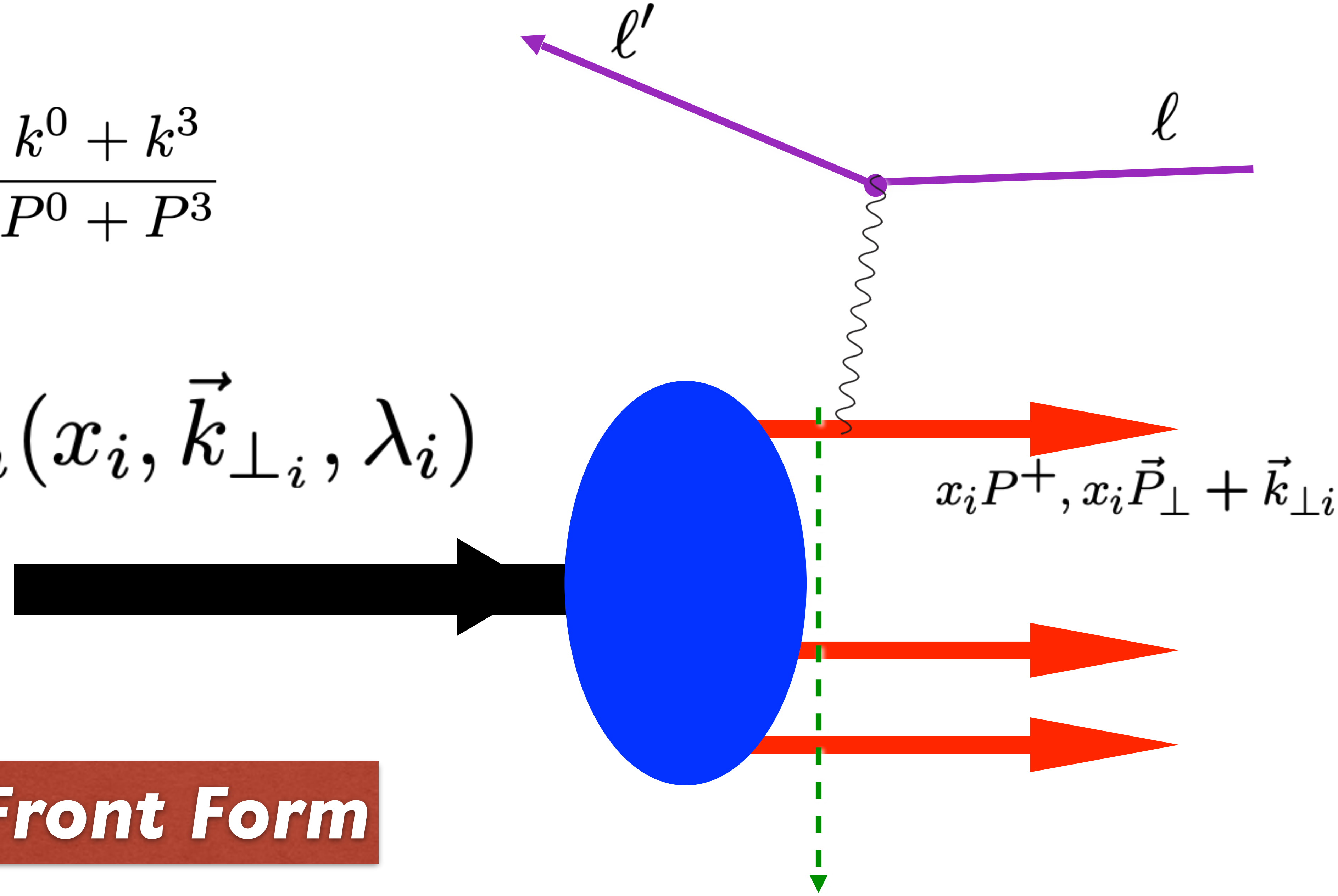

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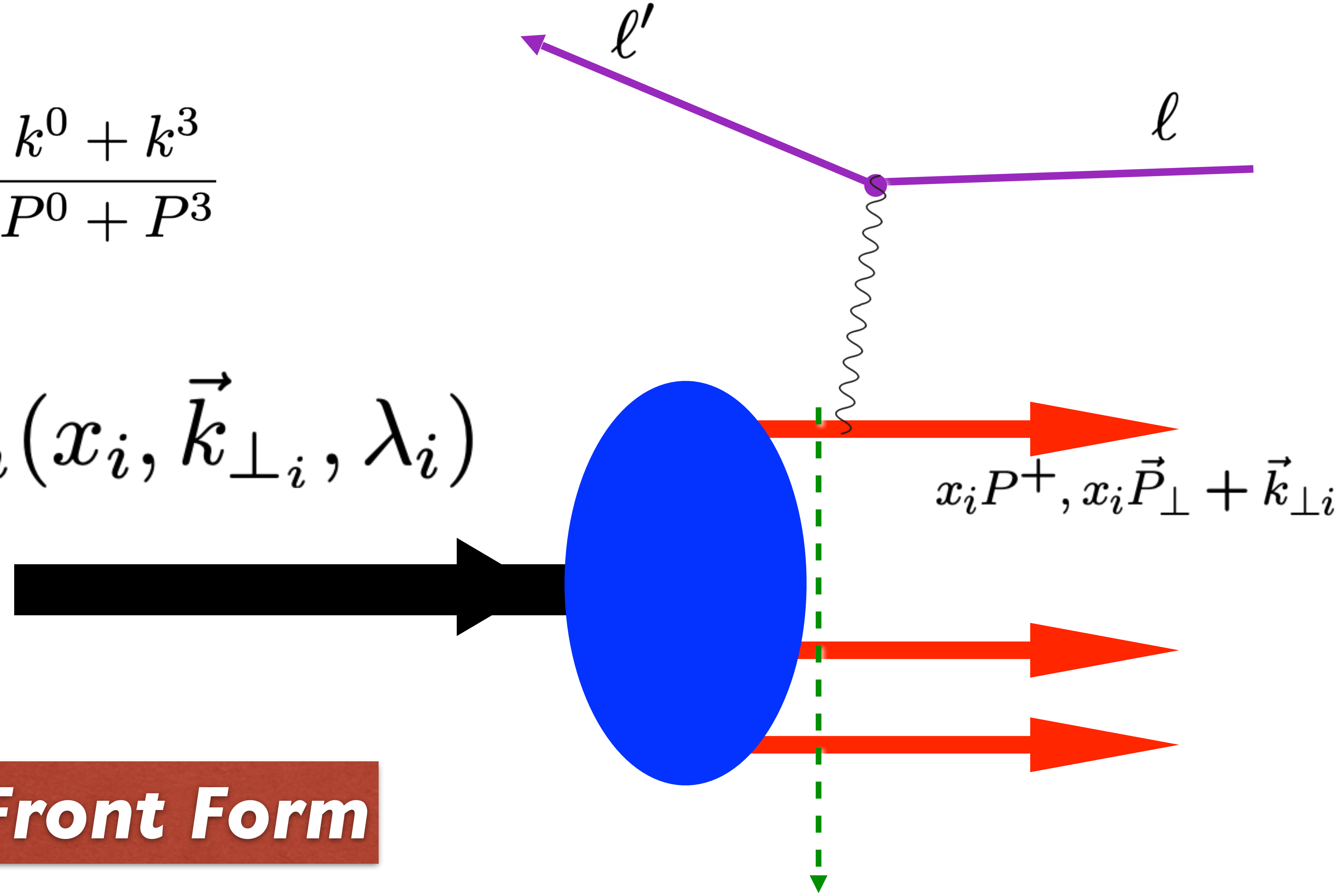

Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

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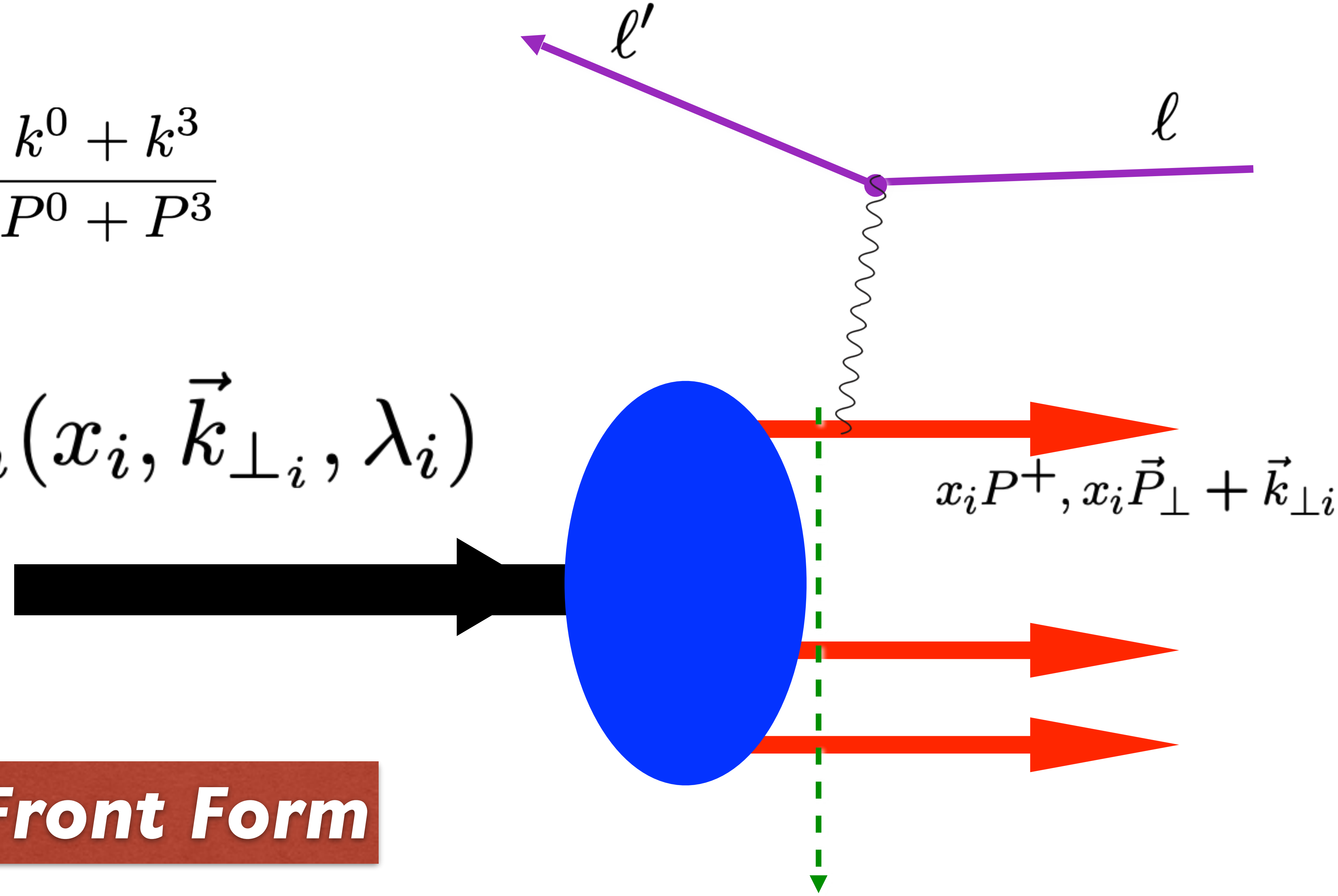

Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

H E L E N B R A D L E Y - P H O T O G R A P H Y

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

$$P^\pm = P^0 \pm P^z$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

$$P^\pm = P^0 \pm P^z$$

Eigenstate -- independent of τ

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

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Causal, frame-independent

Evolve in LF time

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$$P^\pm = P^0 \pm P^z$$

Eigenstate -- independent of τ

$$\text{Eigenvalue } P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

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HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

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Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

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Eigenstate -- independent of τ

Eigenvalue $P^- = \frac{\mathcal{M}^2 + \vec{P}^2_\perp}{P^+}$

$$H_{LF} = P^\mu P_\mu = P^+ P^- - \vec{P}^2_\perp$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

$$P^\pm = P^0 \pm P^z$$

Eigenstate -- independent of τ

Eigenvalue $P^- = \frac{\mathcal{M}^2 + \vec{P}^2_\perp}{P^+}$

$$H_{LF} = P^\mu P_\mu = P^+ P^- - \vec{P}^2_\perp$$

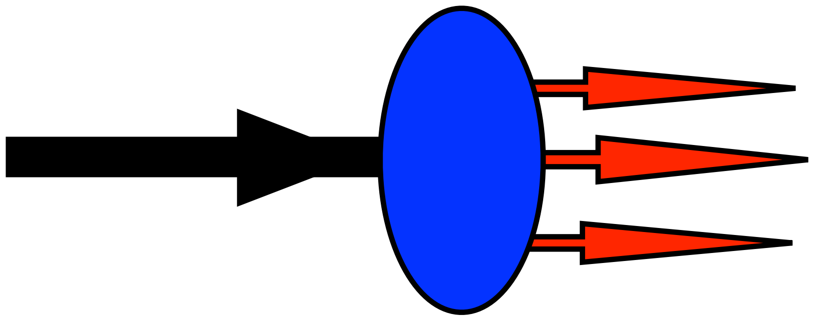
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

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$$\boxed{H^{QCD}_{LF} |\psi> = M^2 |\psi>}$$

Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

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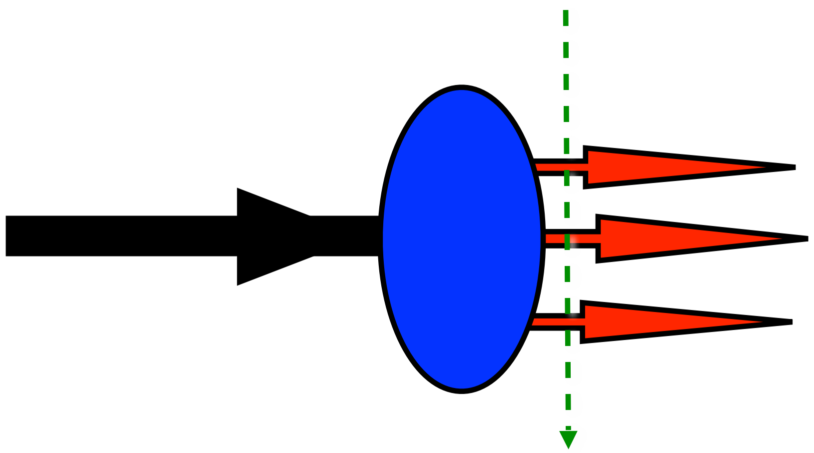
Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



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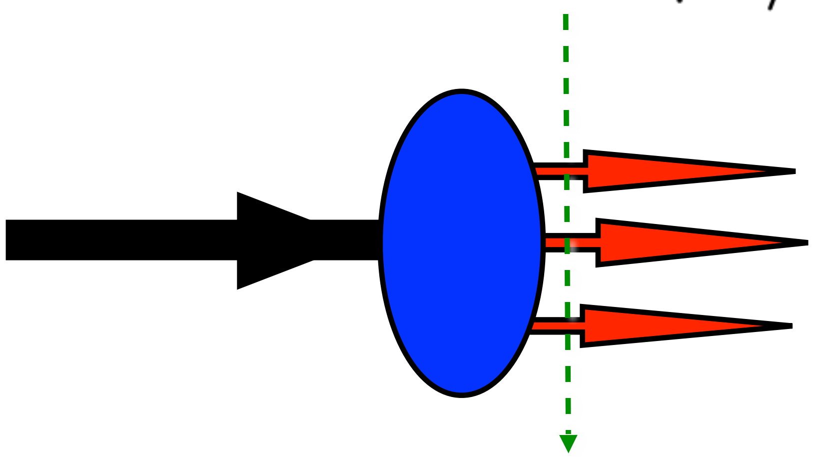

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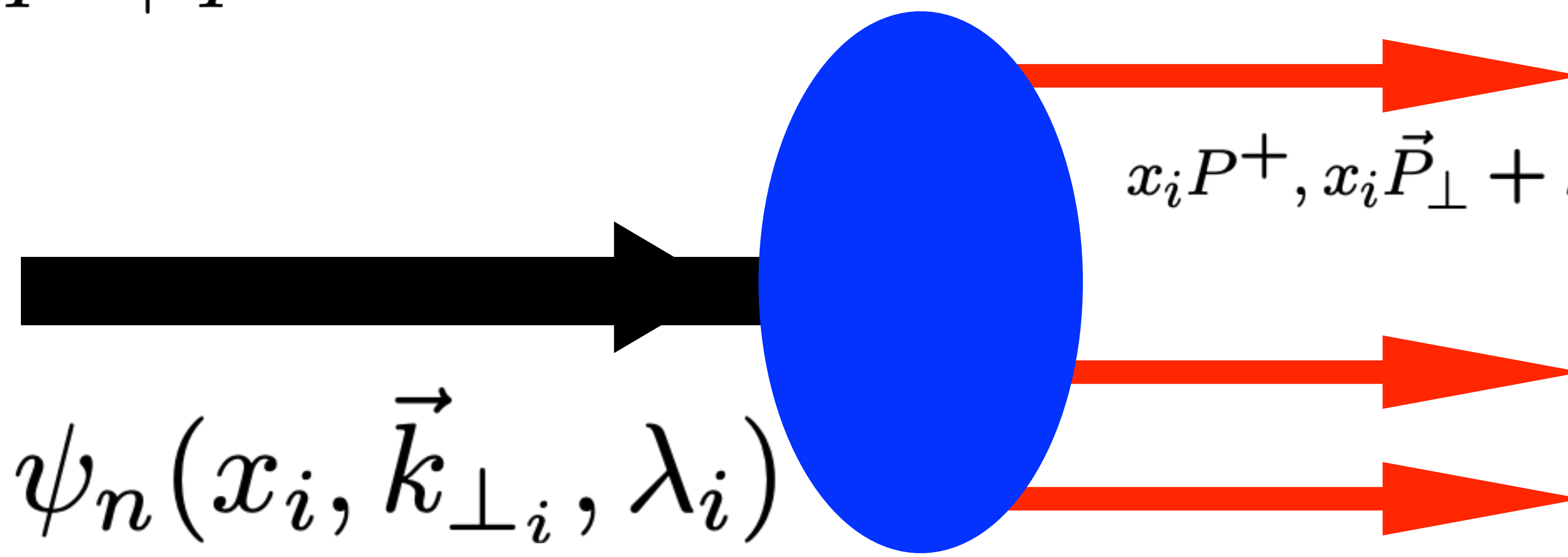

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Eigenstate of LF Hamiltonian

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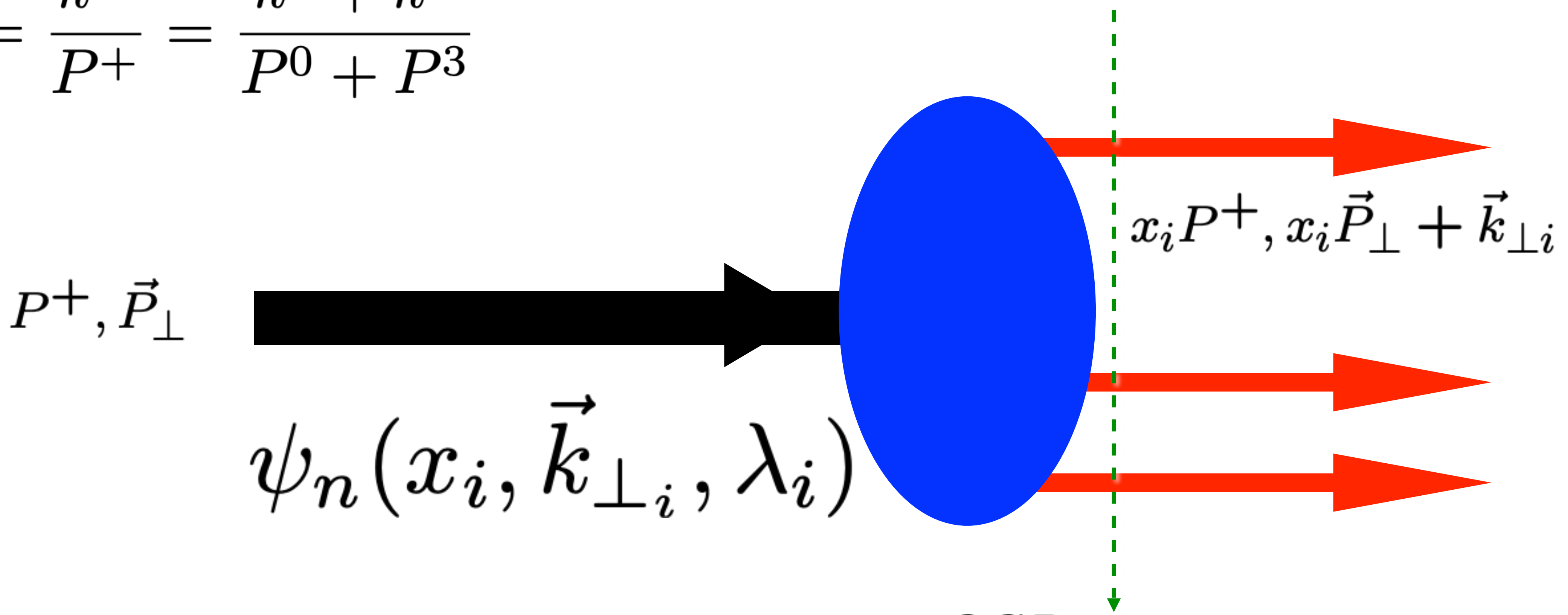

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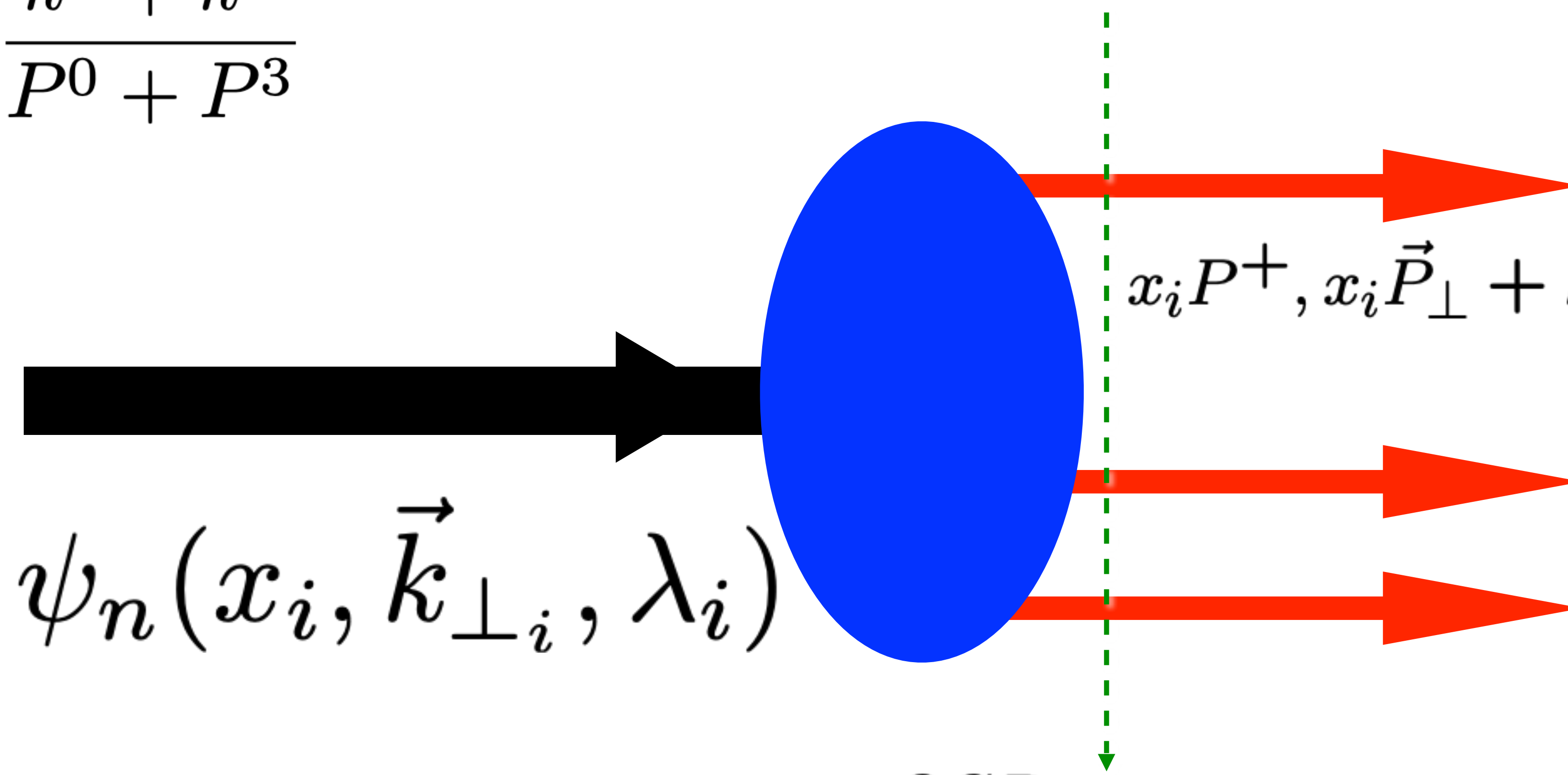

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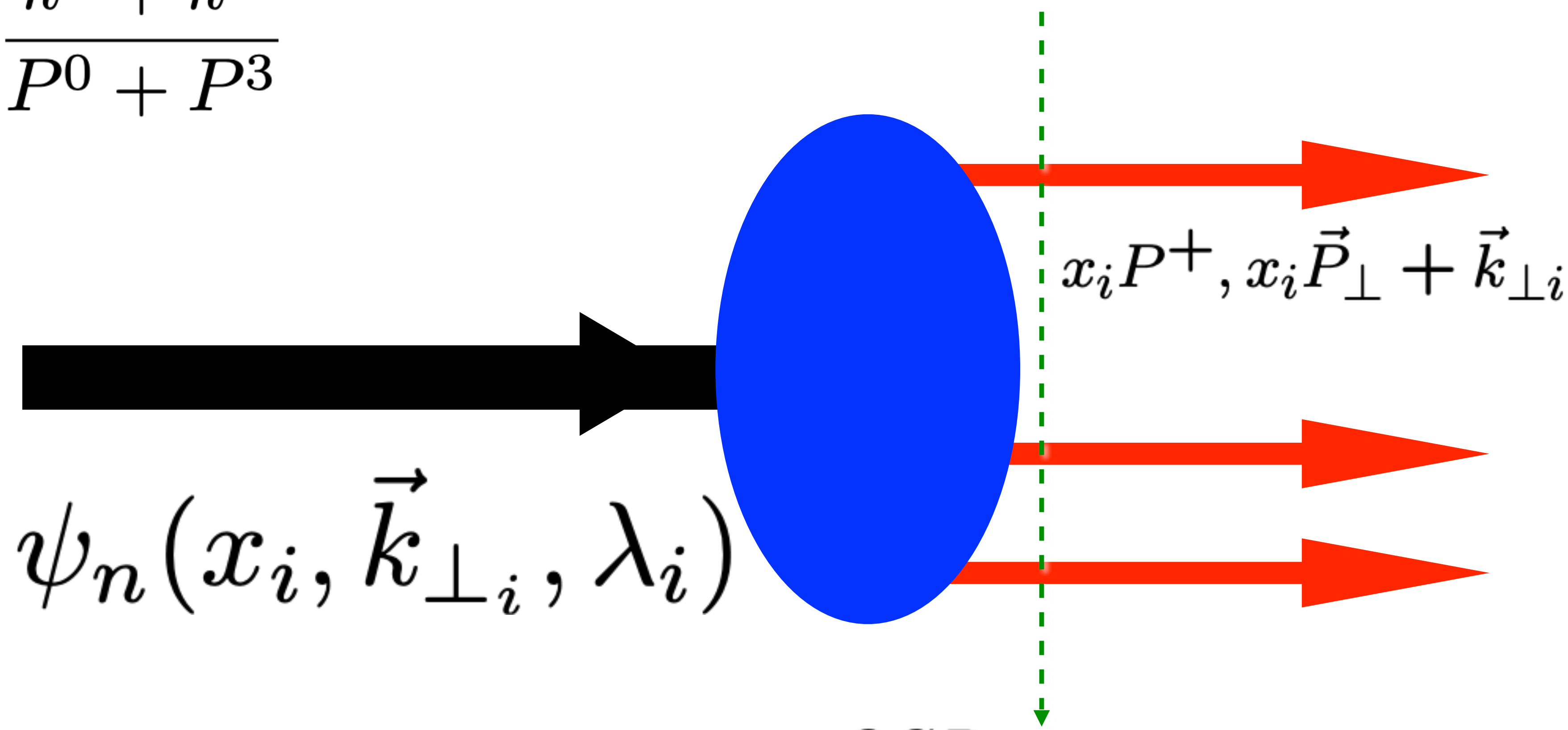

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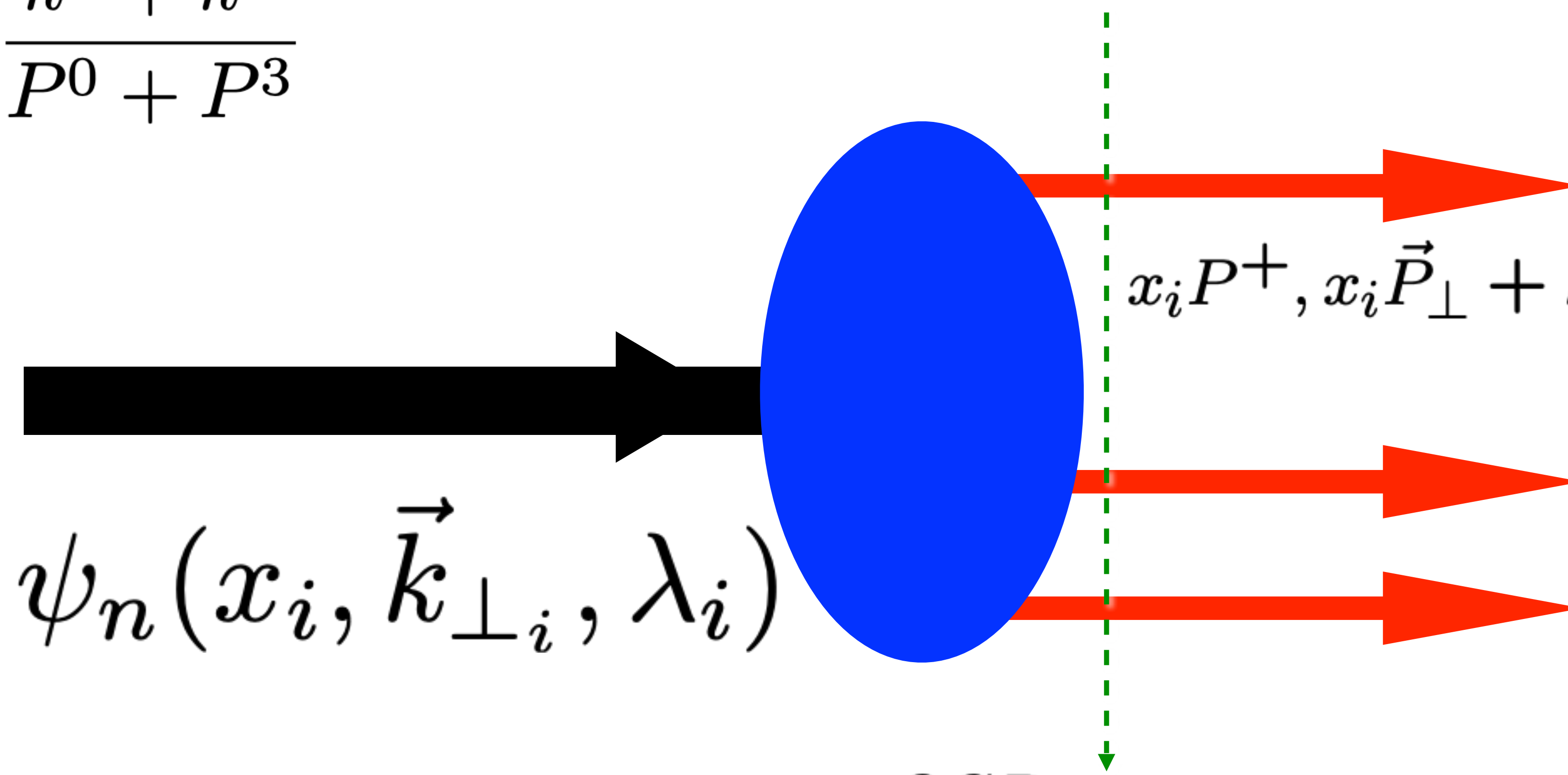

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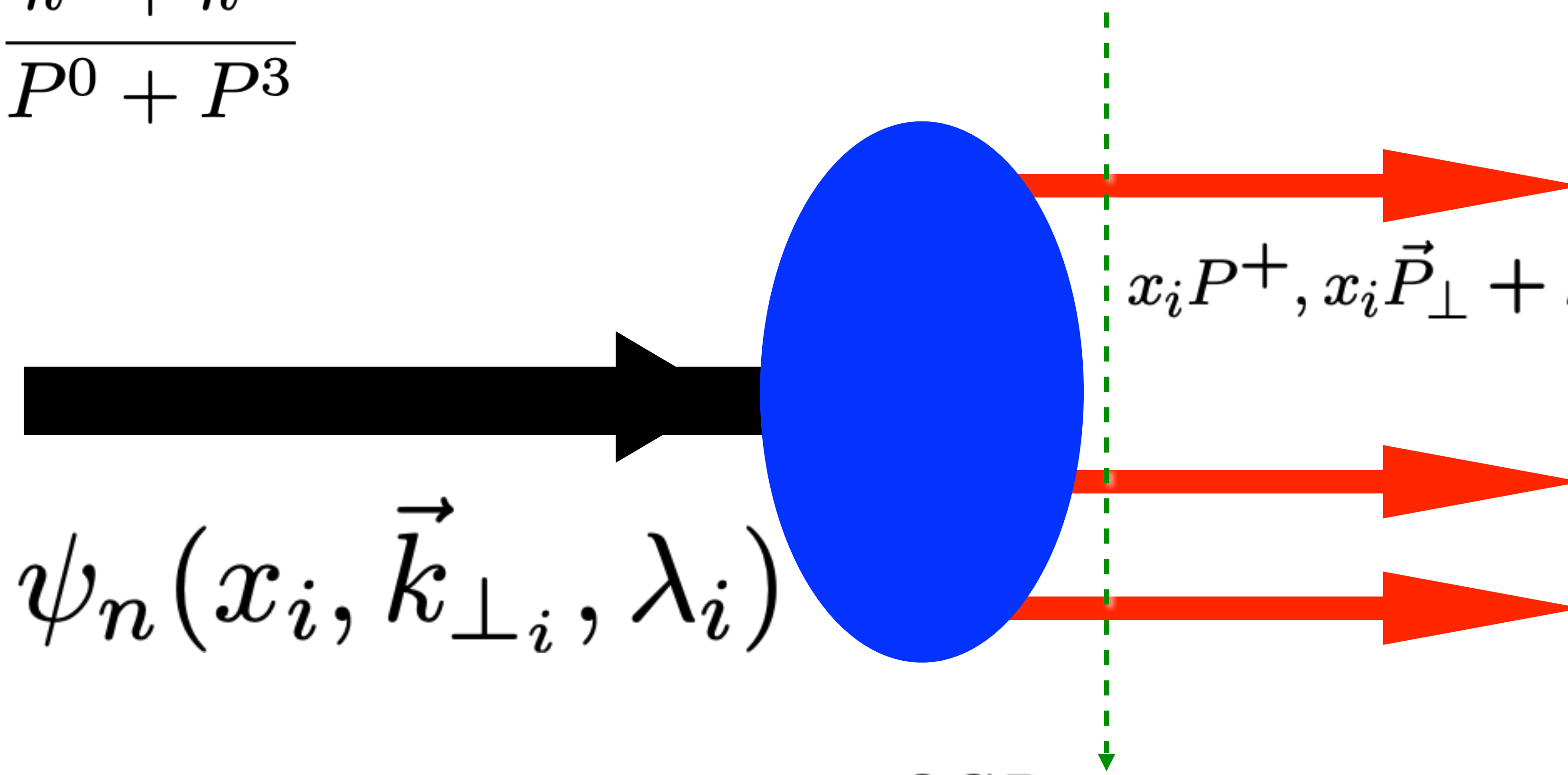

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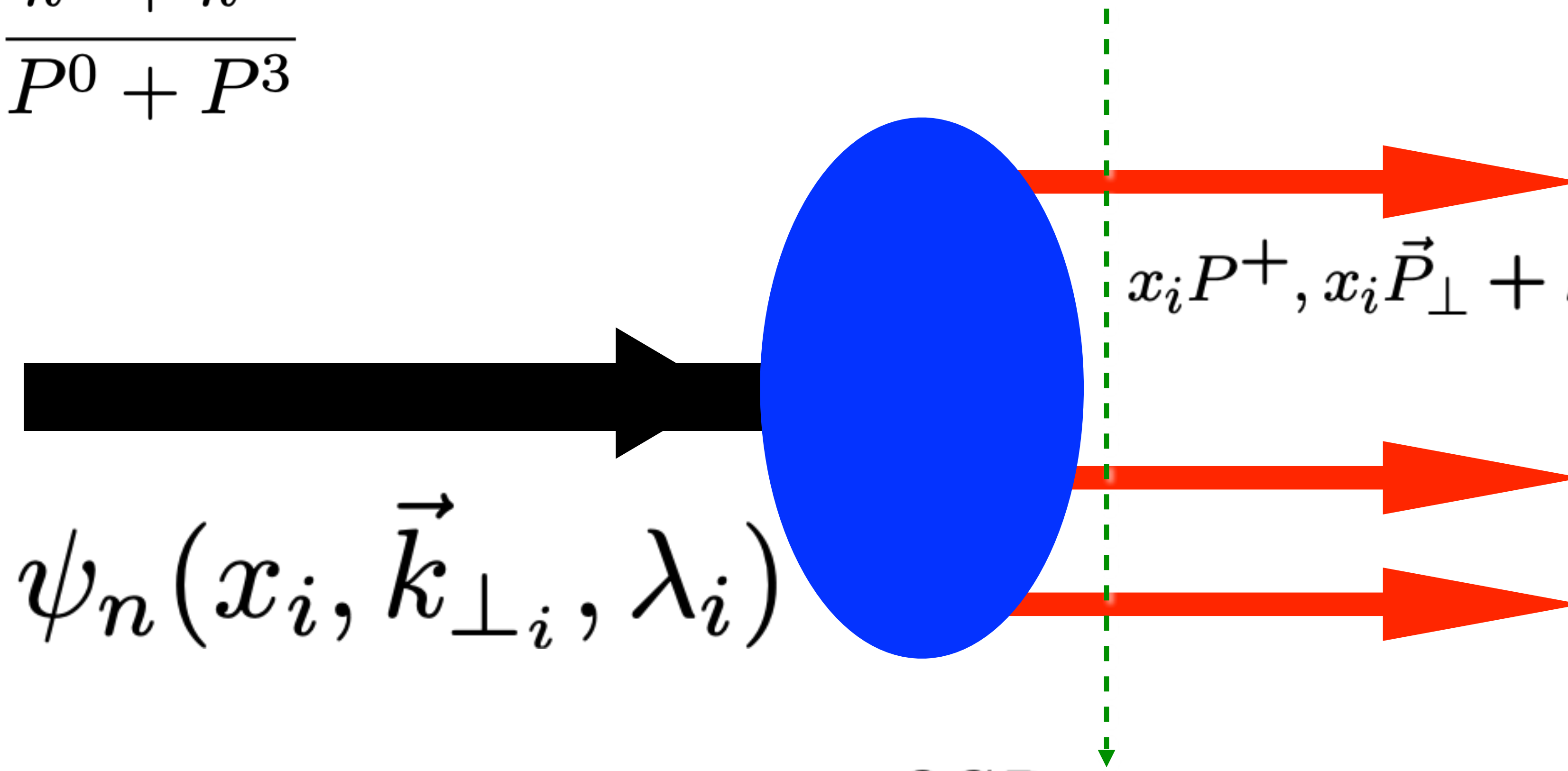

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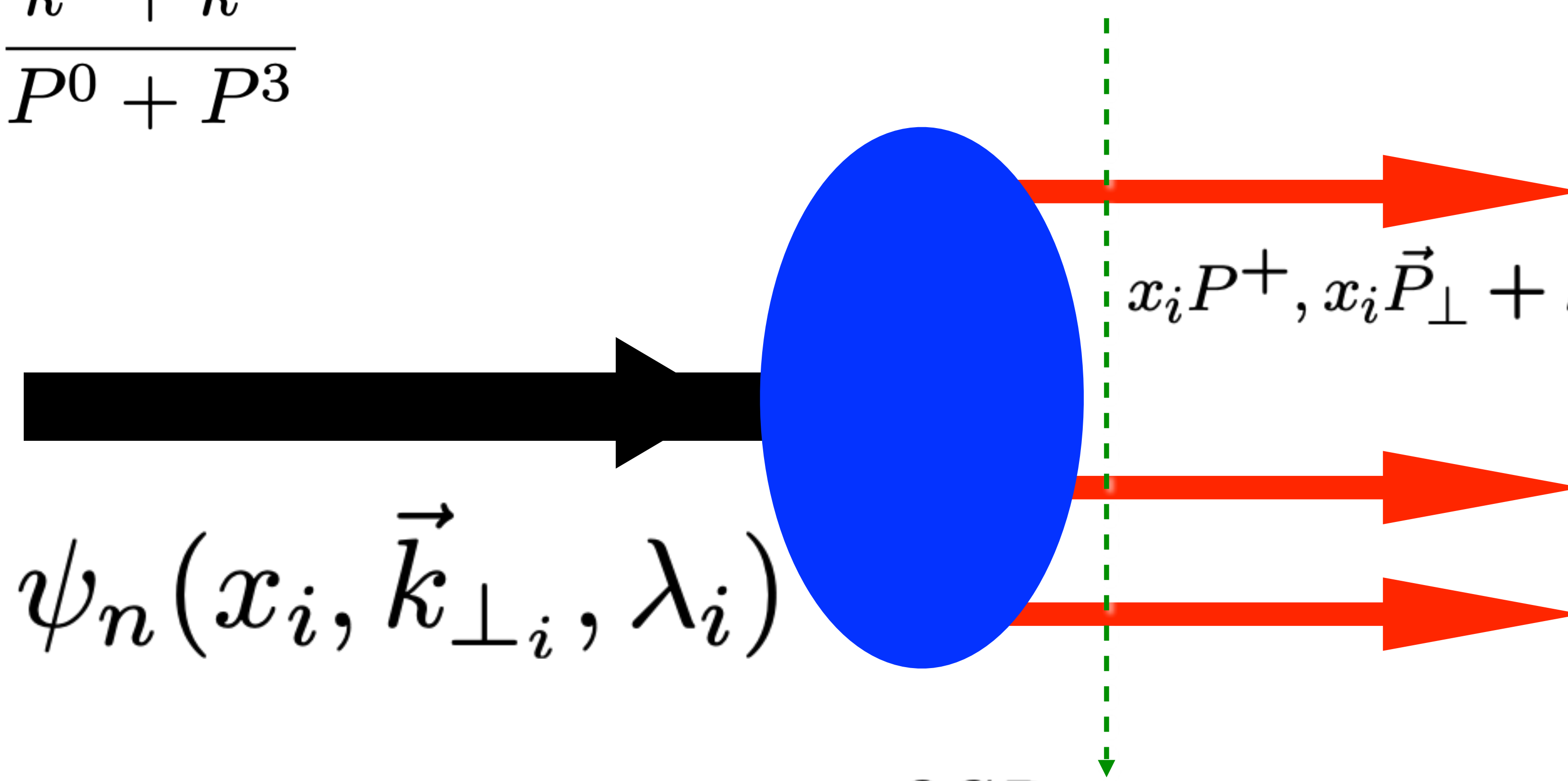

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Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

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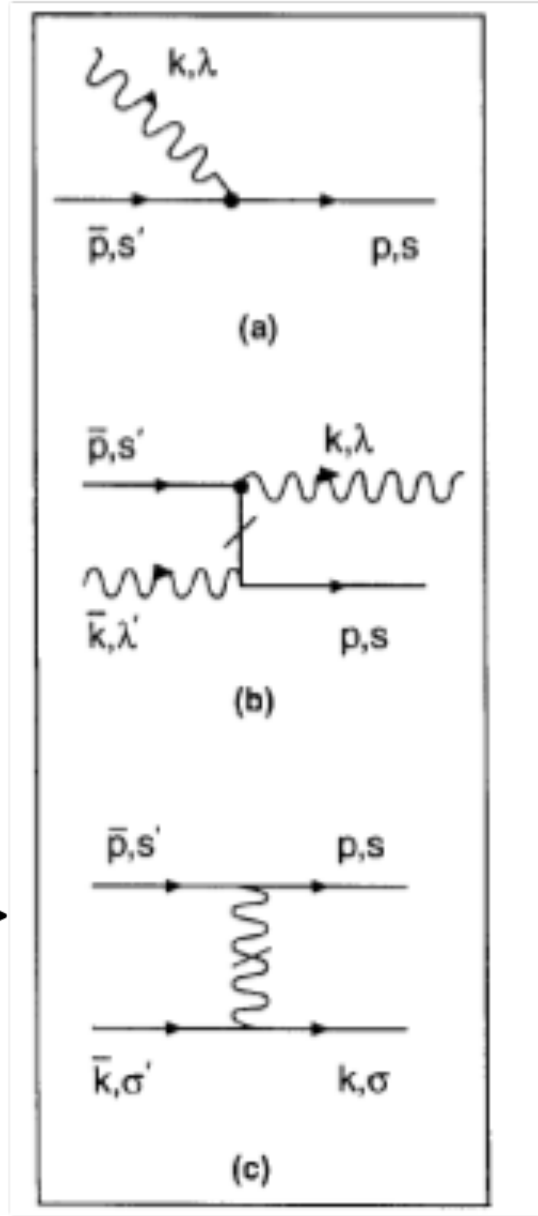

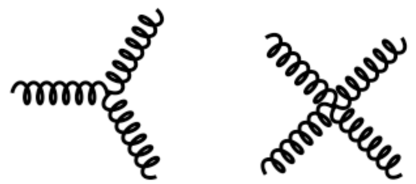

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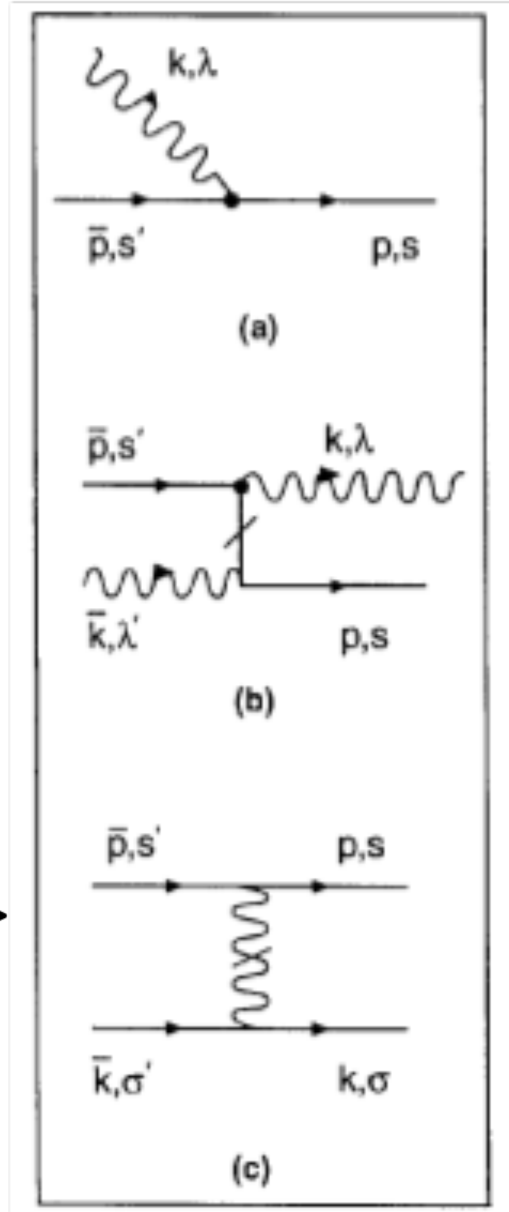

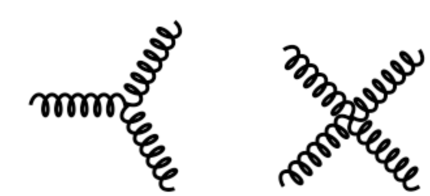

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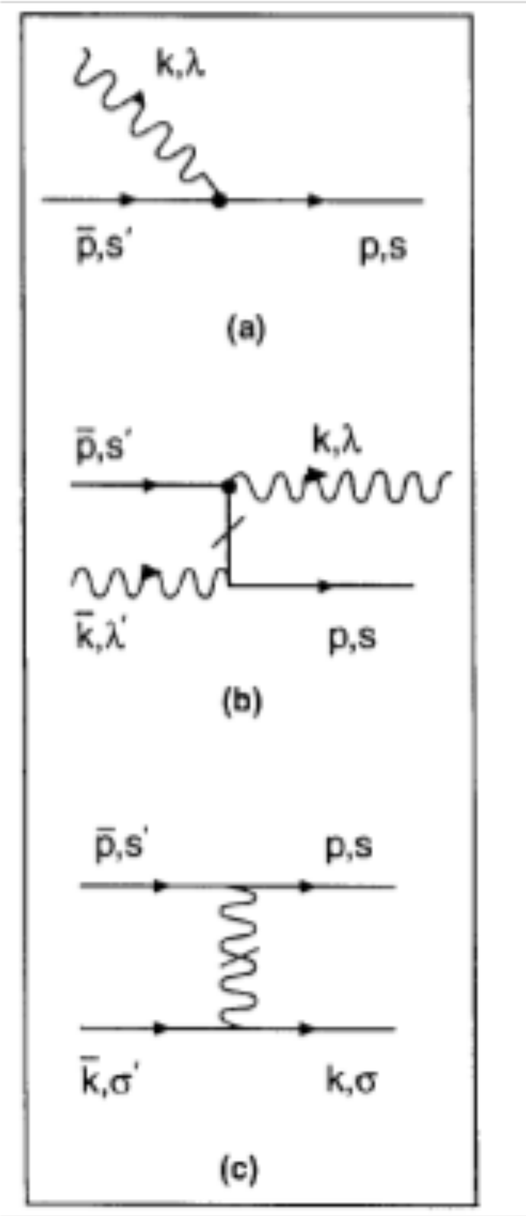
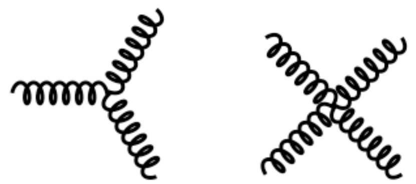
Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions





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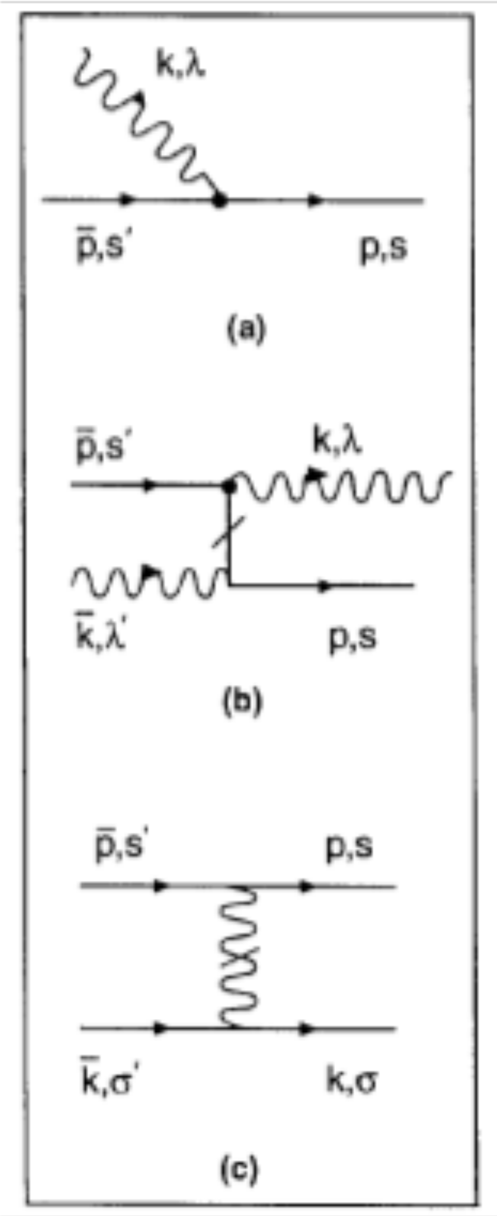

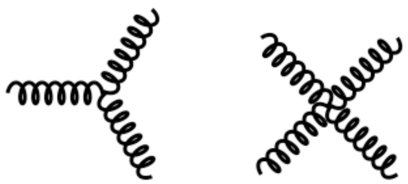

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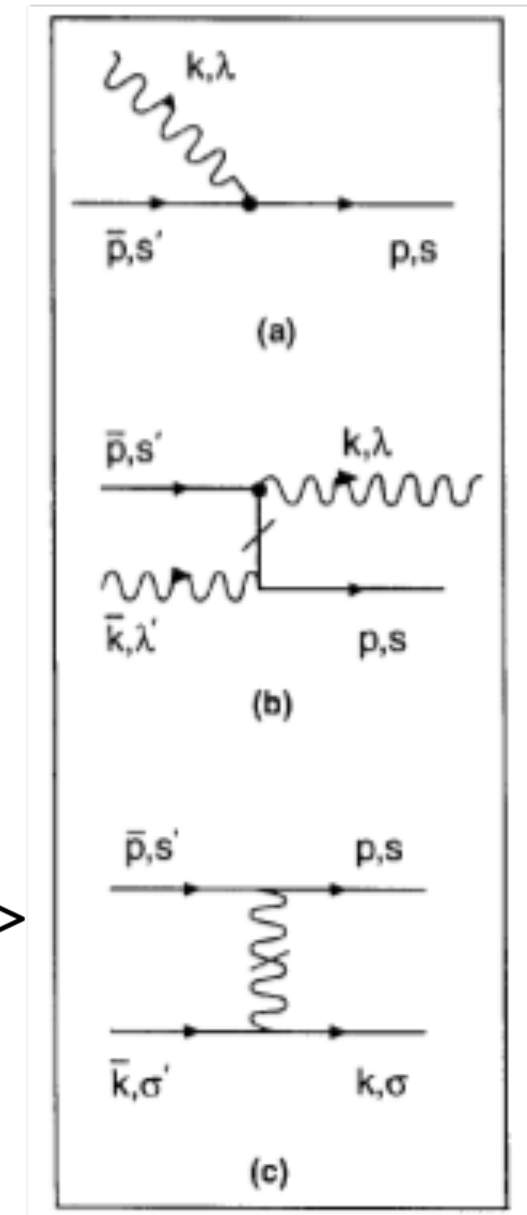

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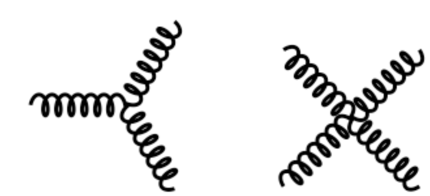
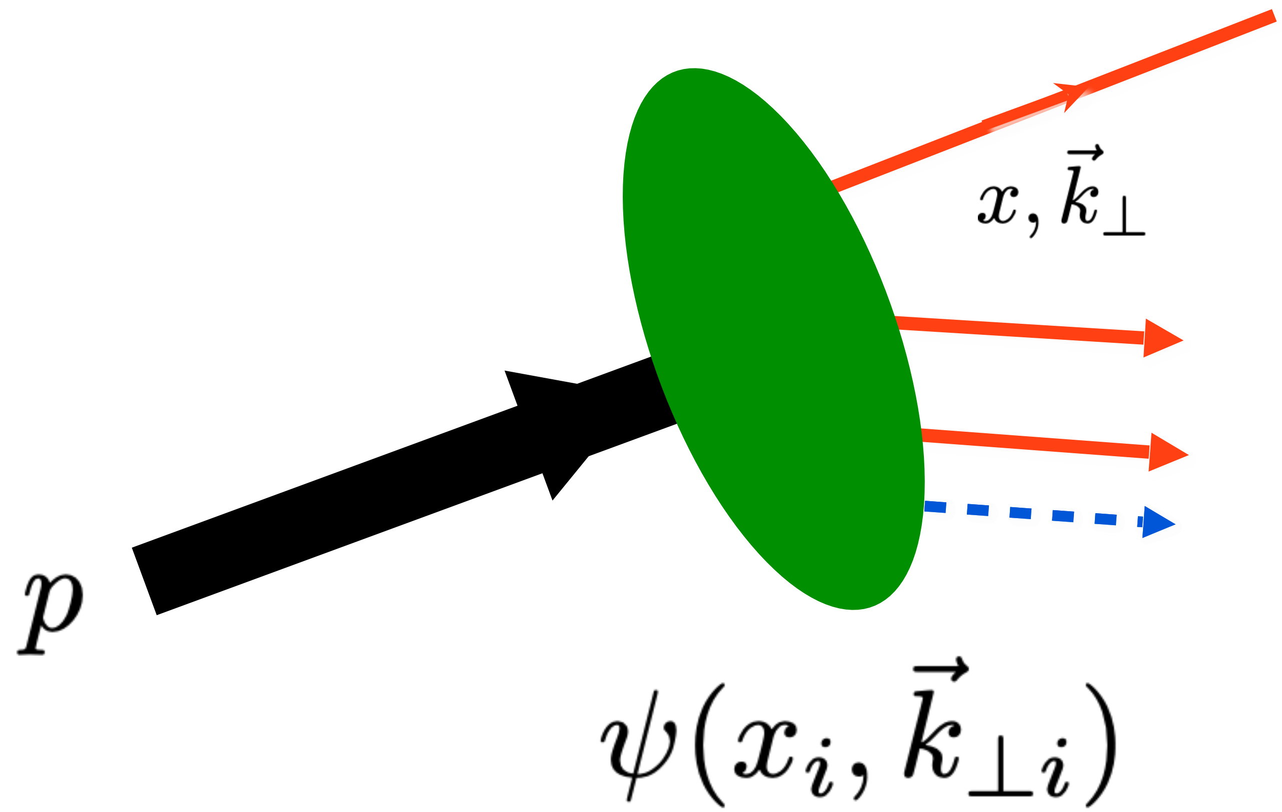
LFWFs: Off-shell in P- and invariant mass



$$H^{int}_{LF}$$

Front Form

Interaction picture



Drell &Yan, West
Exact LF formula!

Drell, sjb

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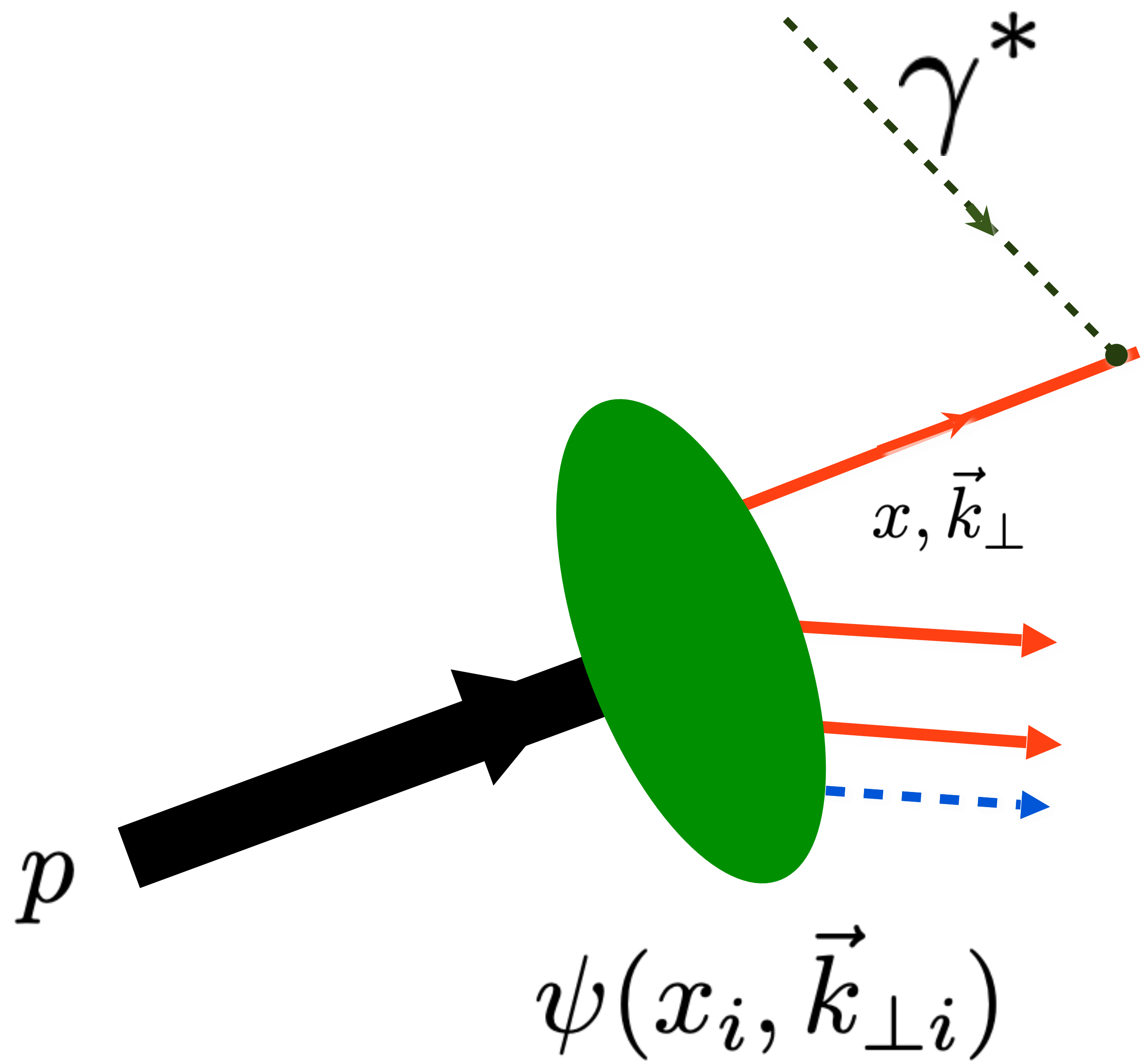

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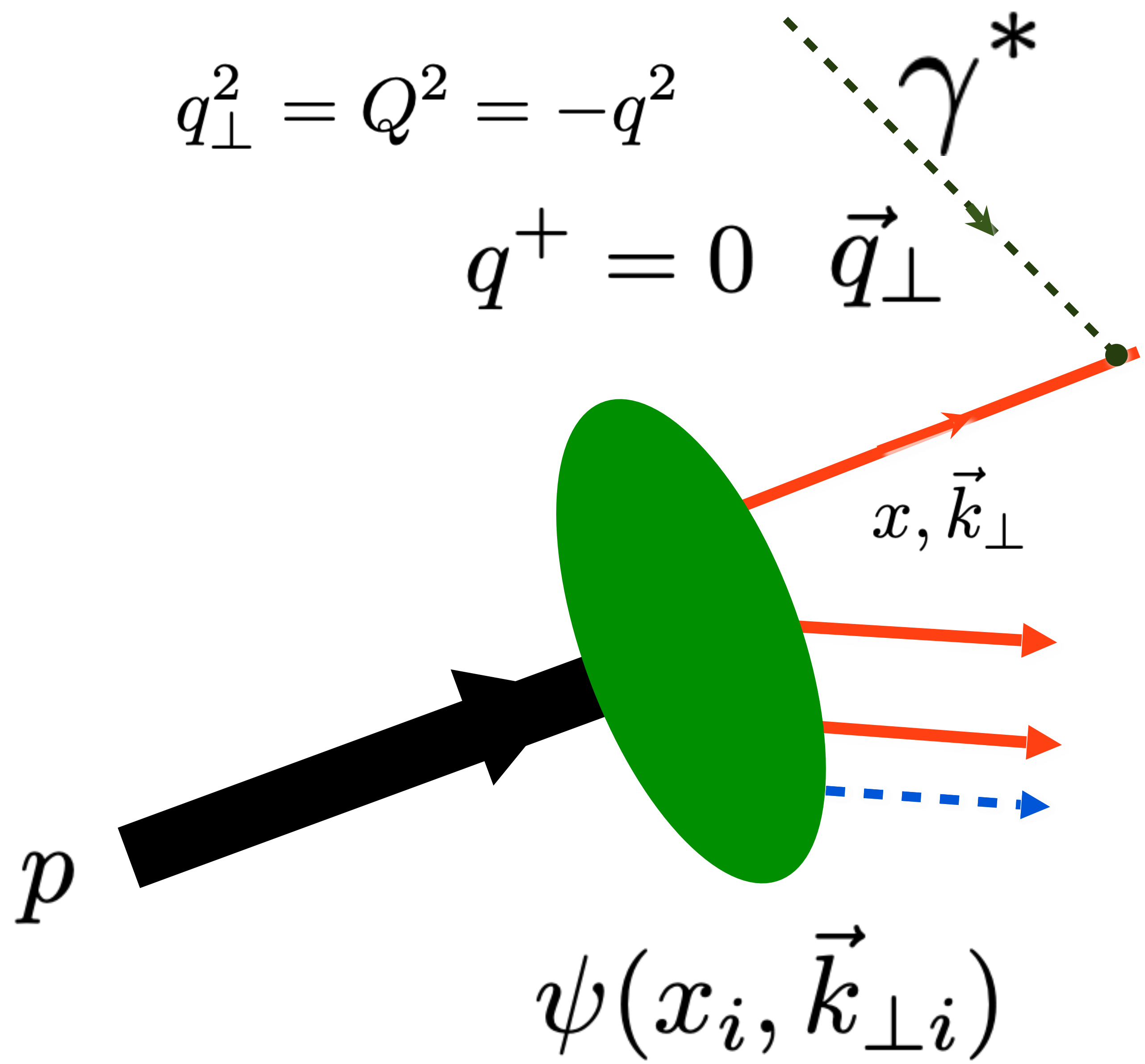

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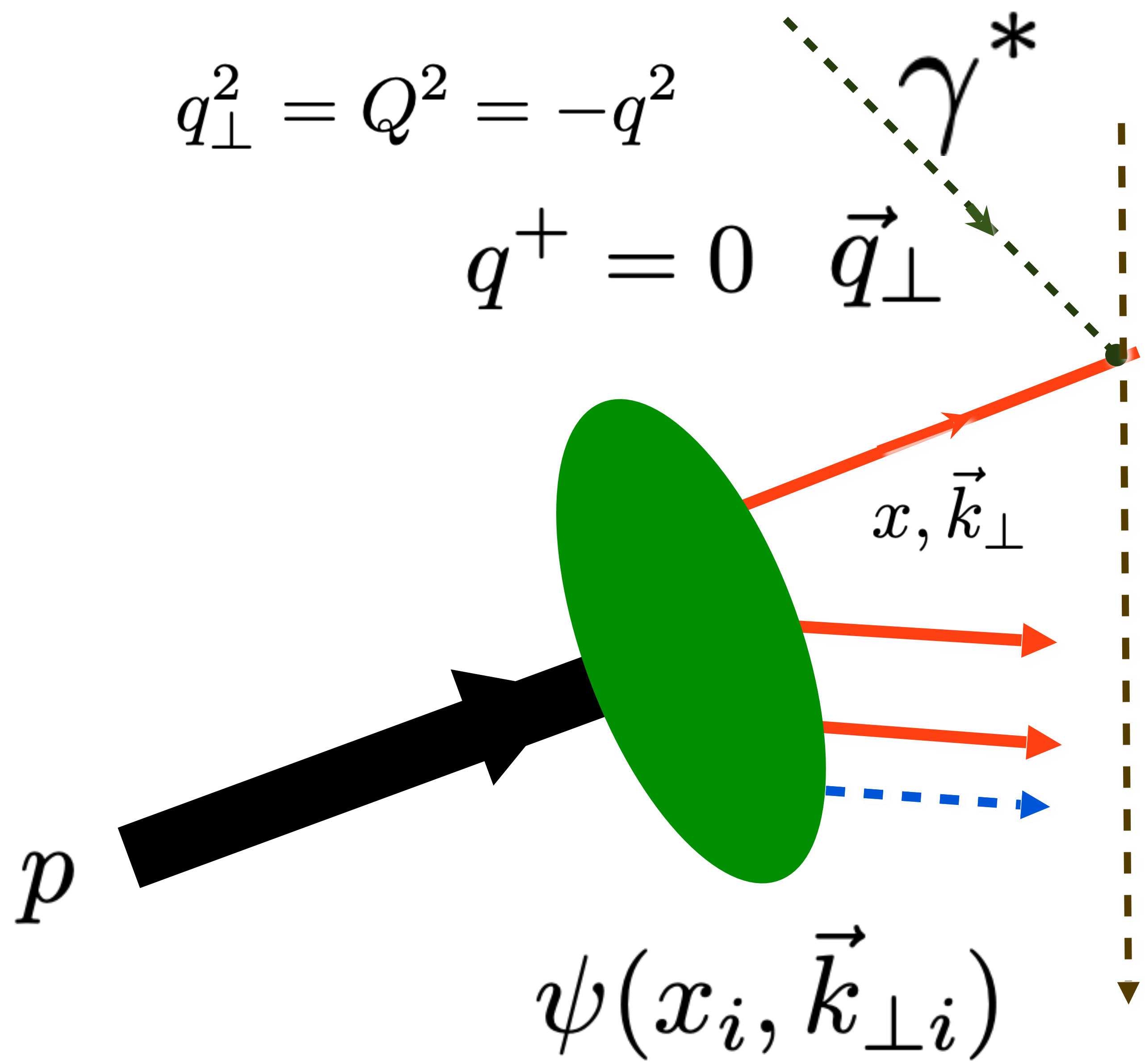

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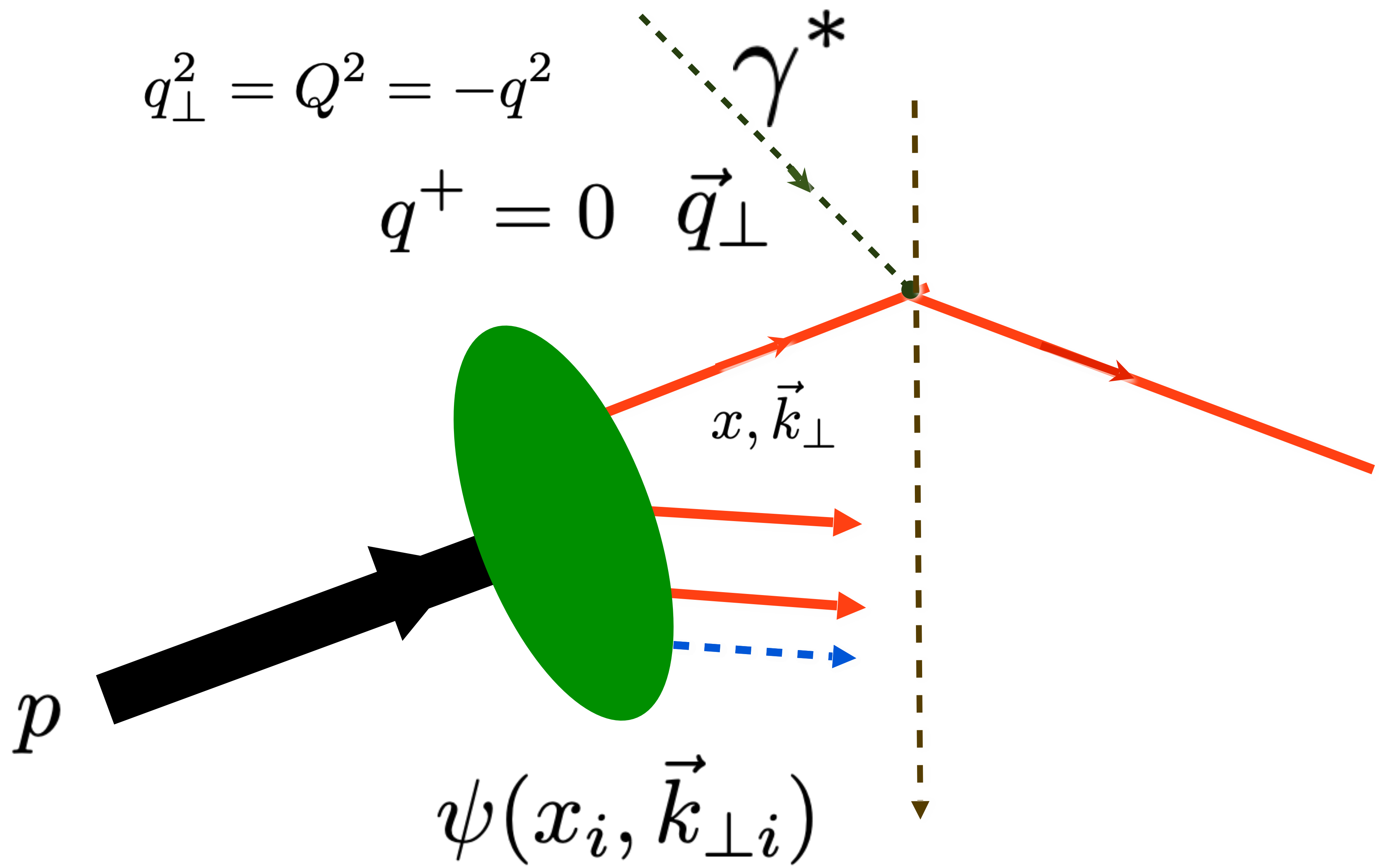

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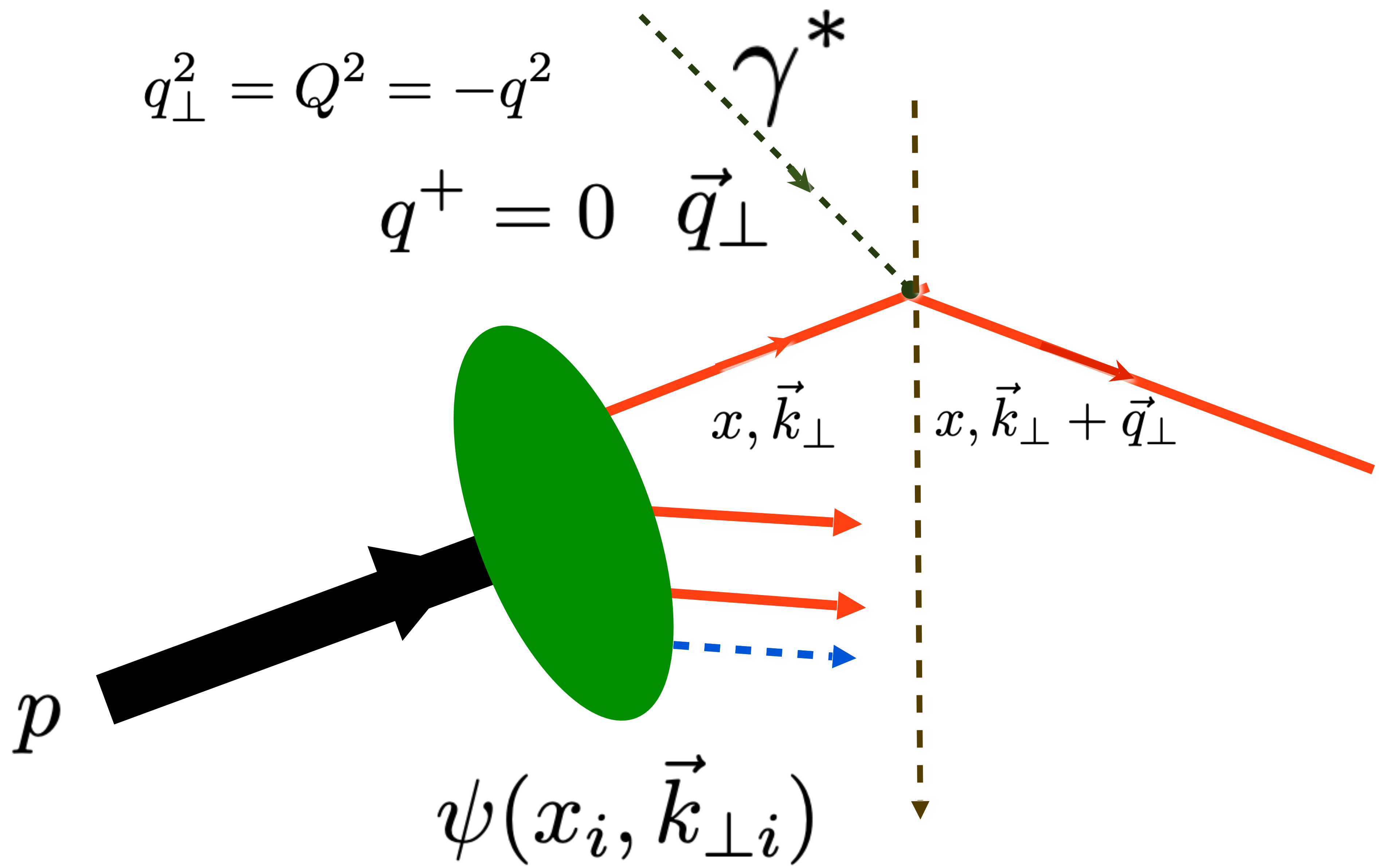

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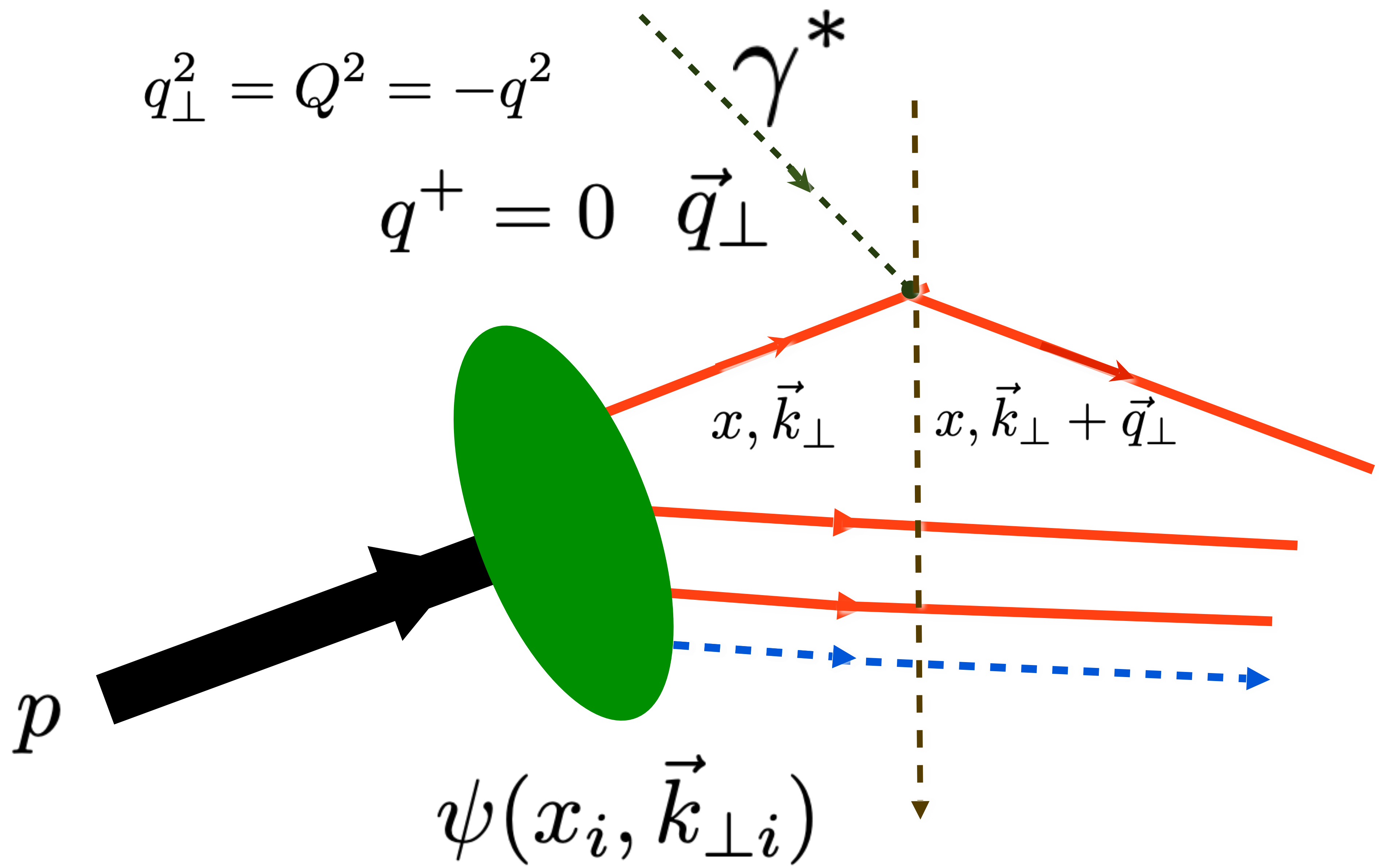

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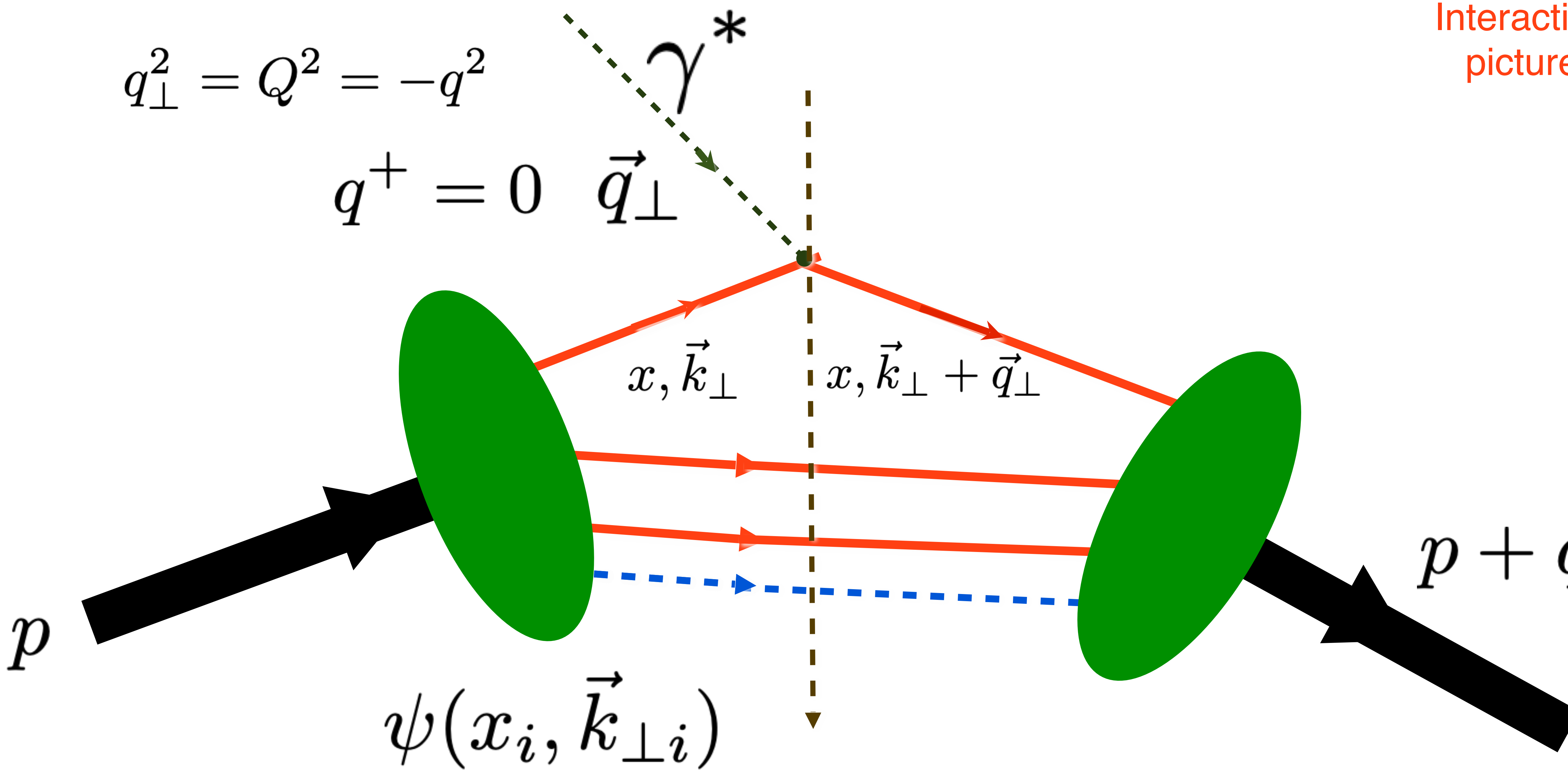

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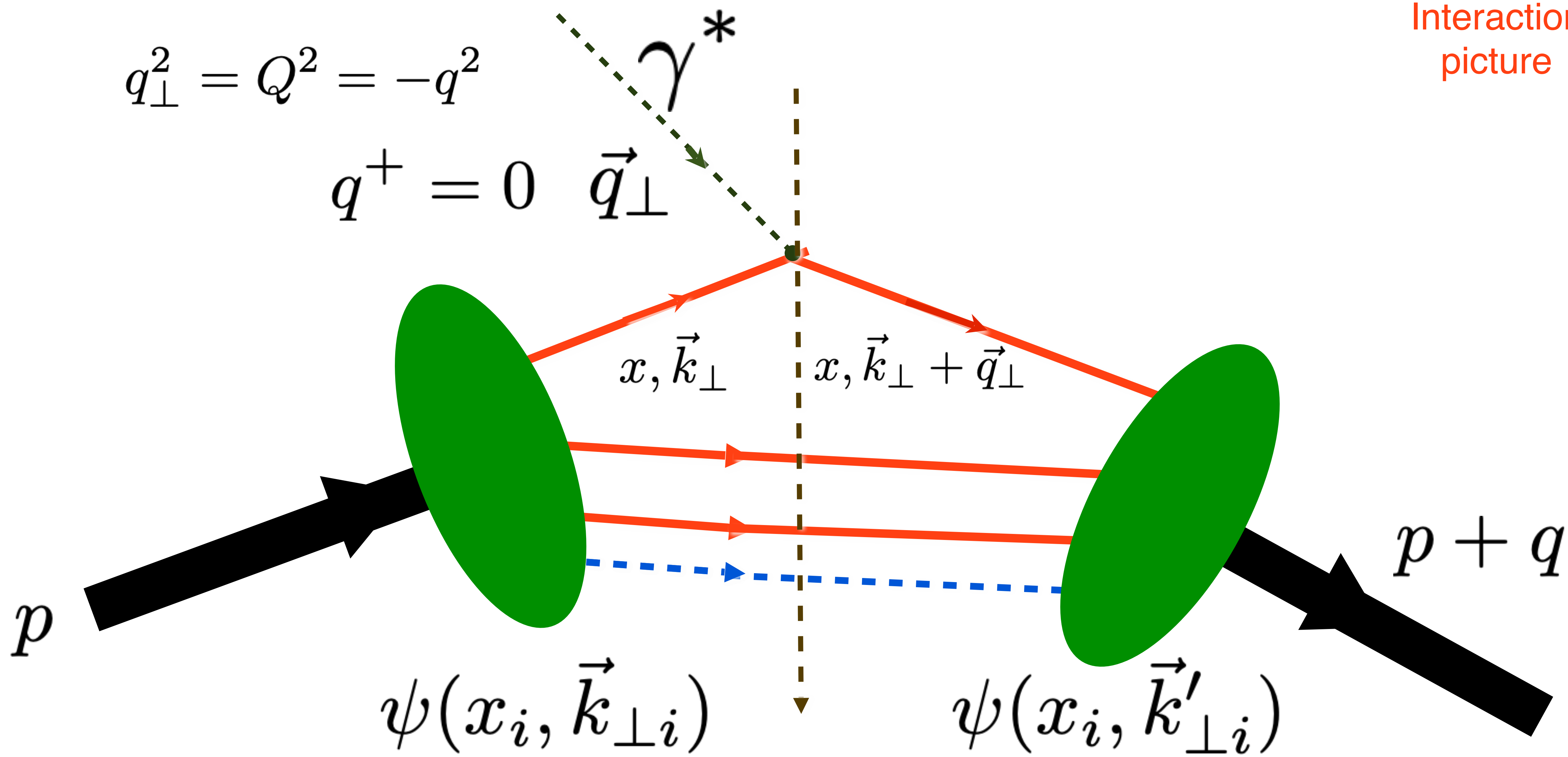

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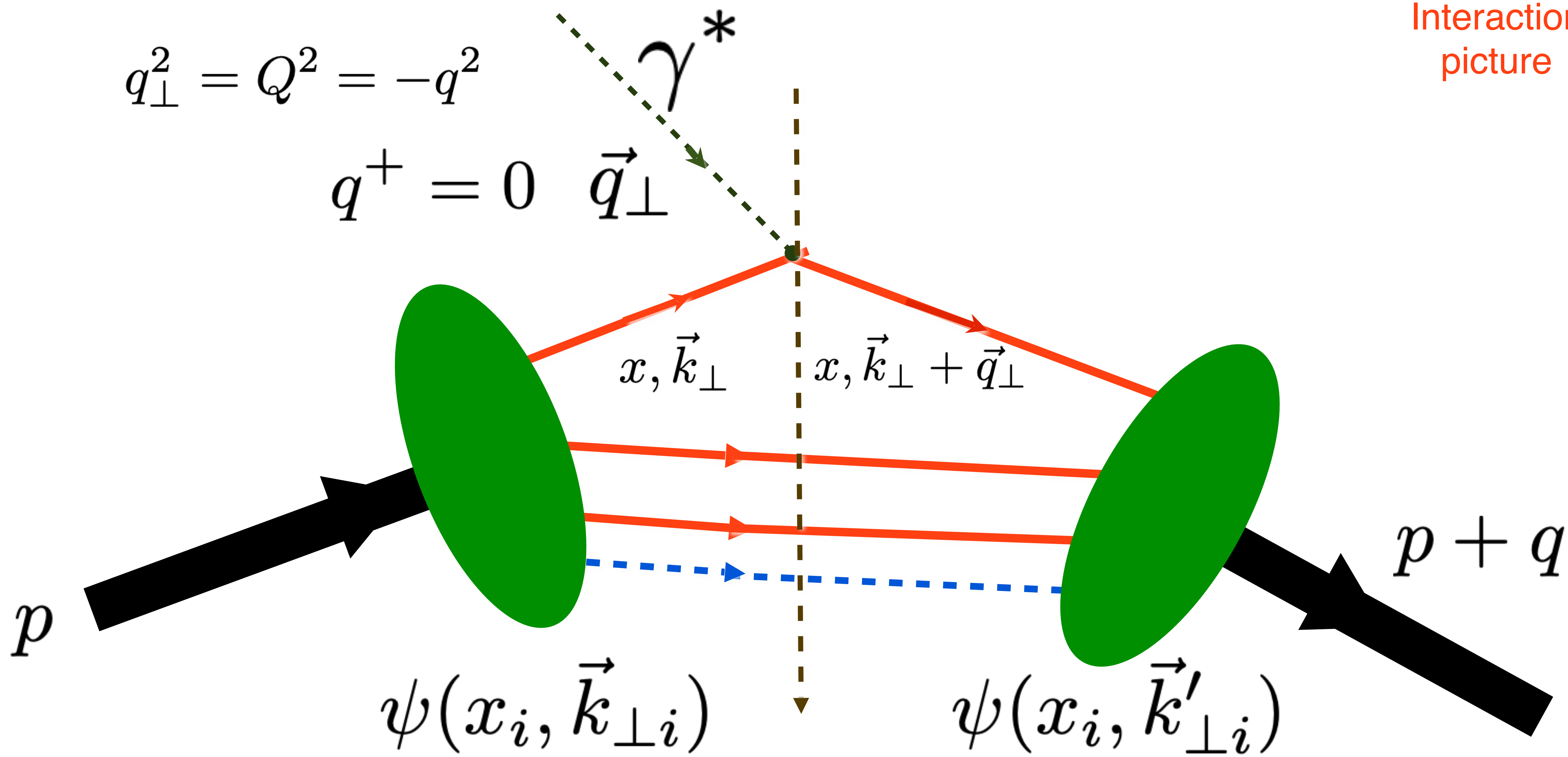

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Drell, sjb

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

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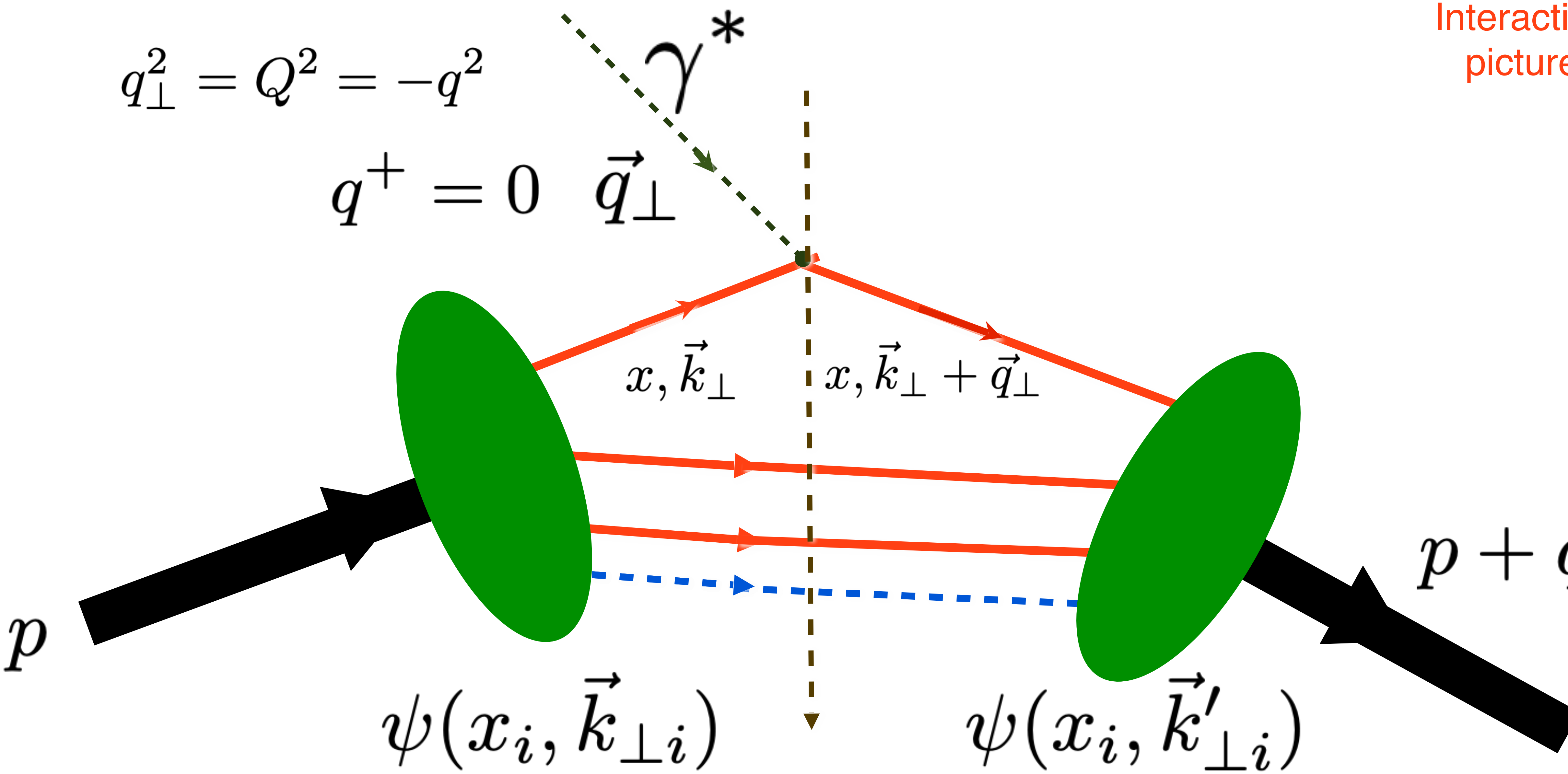

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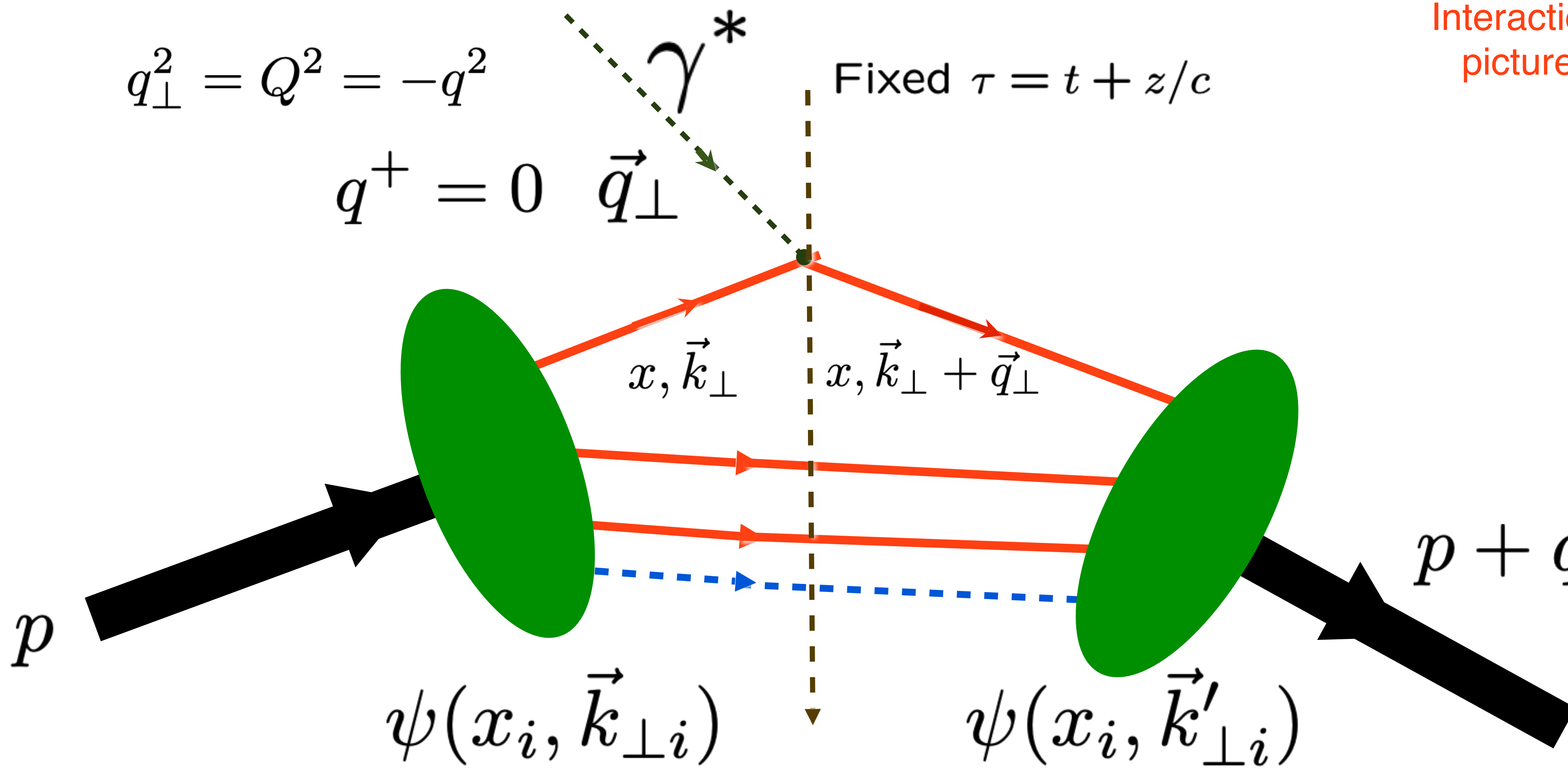

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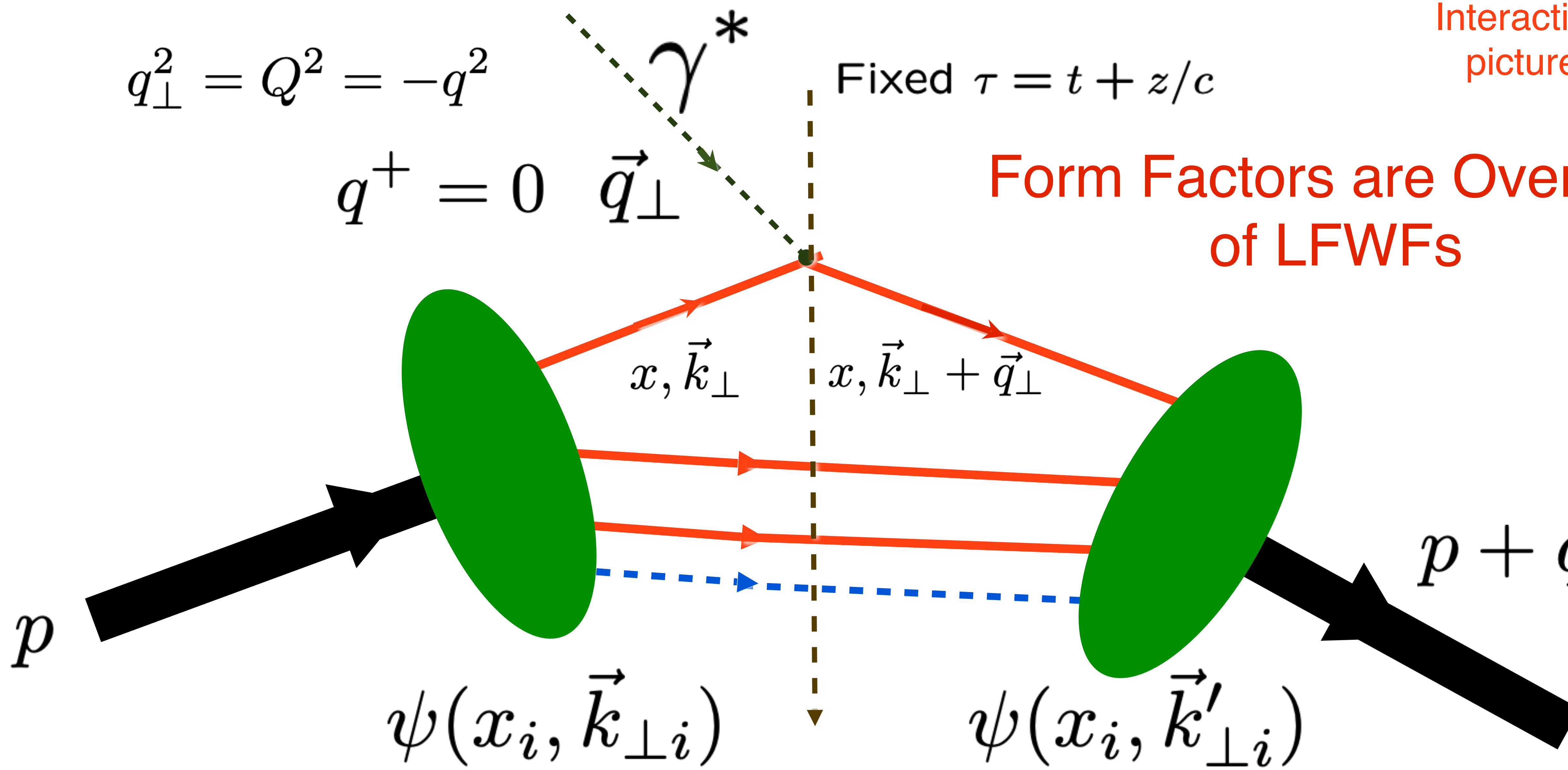
$$q_\perp^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_\perp$$

Fixed $\tau = t + z/c$

Form Factors are Overlaps of LFWFs



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Drell &Yan, West
Exact LF formula!

Drell, sjb

Exact LF Formula for Pauli Form Factor

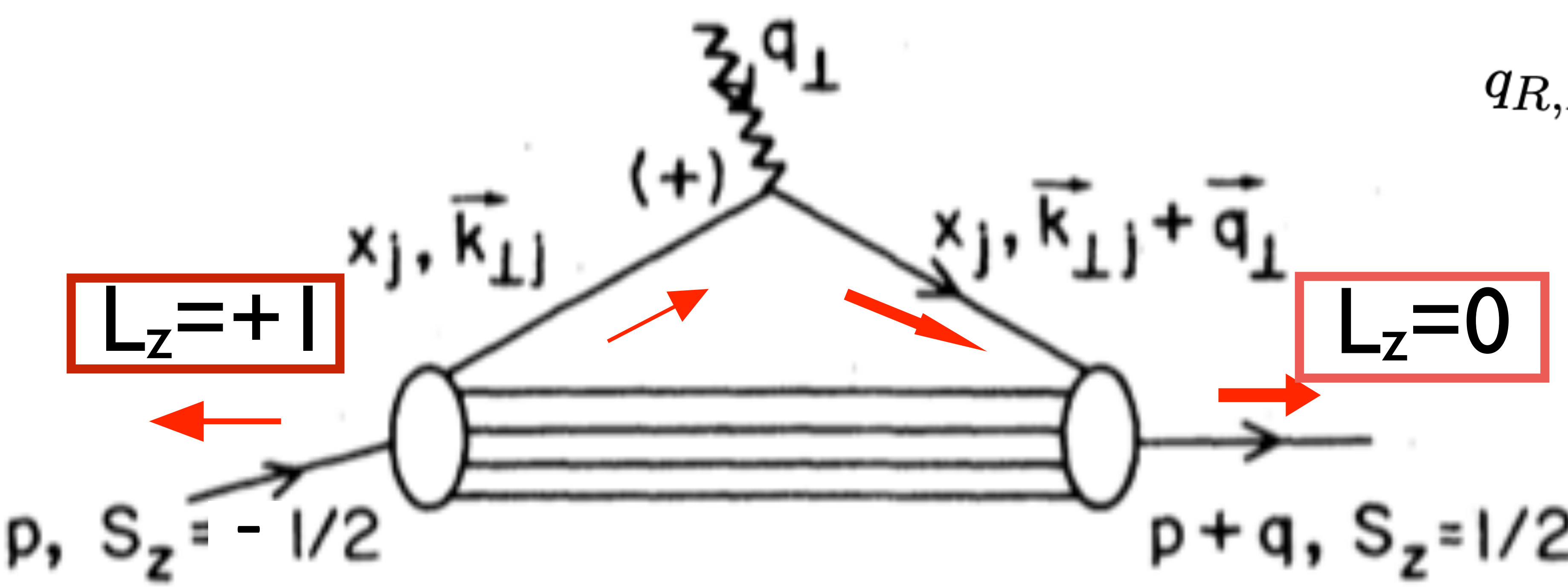
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



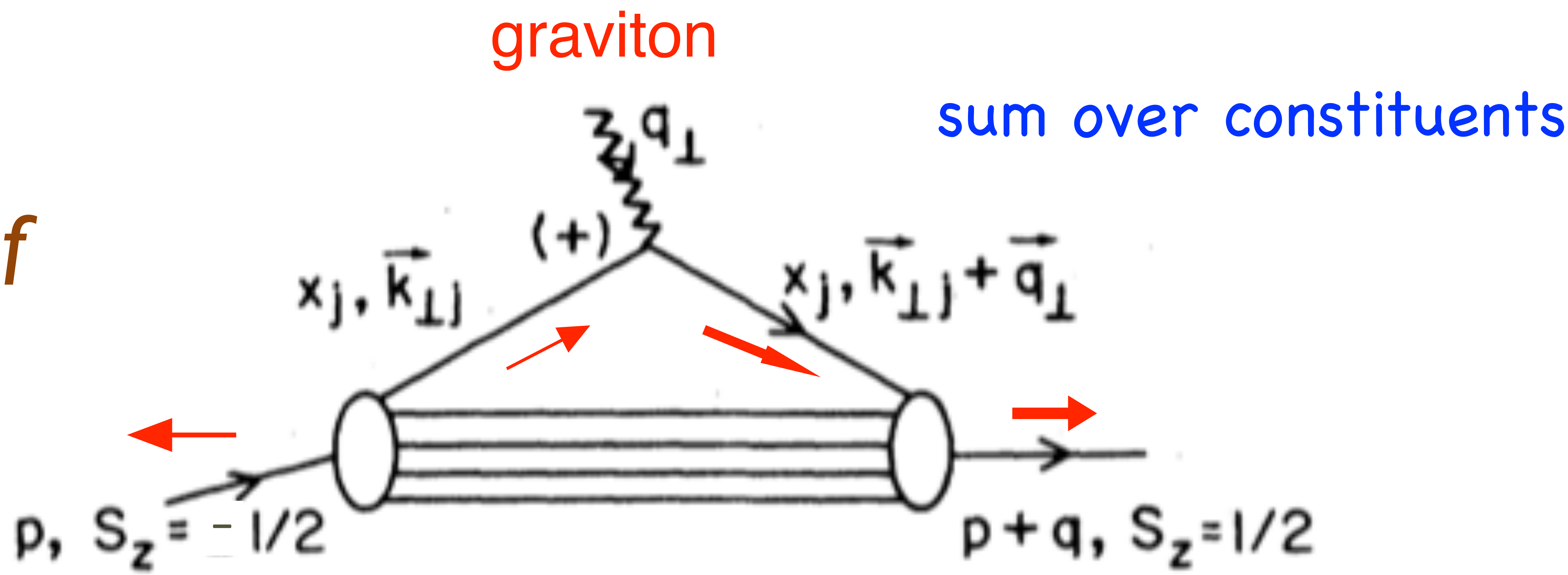
Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Dae Sung Hwang, Bo-Qiang Ma, Ivan Schmidt, sjb

Terayev, Okun: B(0) Must vanish because of Equivalence Theorem

LF Proof



$\boxed{B(0) = 0}$

Each Fock State

Vanishing Anomalous gravitomagnetic moment B(0)

Advantages of the Dirac's Front Form for Hadron Physics
Poincare' Invariant

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!

Penrose, Terrell, Weisskopf

- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial up to zero modes
- Implications for Cosmological Constant

Roberts, Shrock, Tandy, sjb

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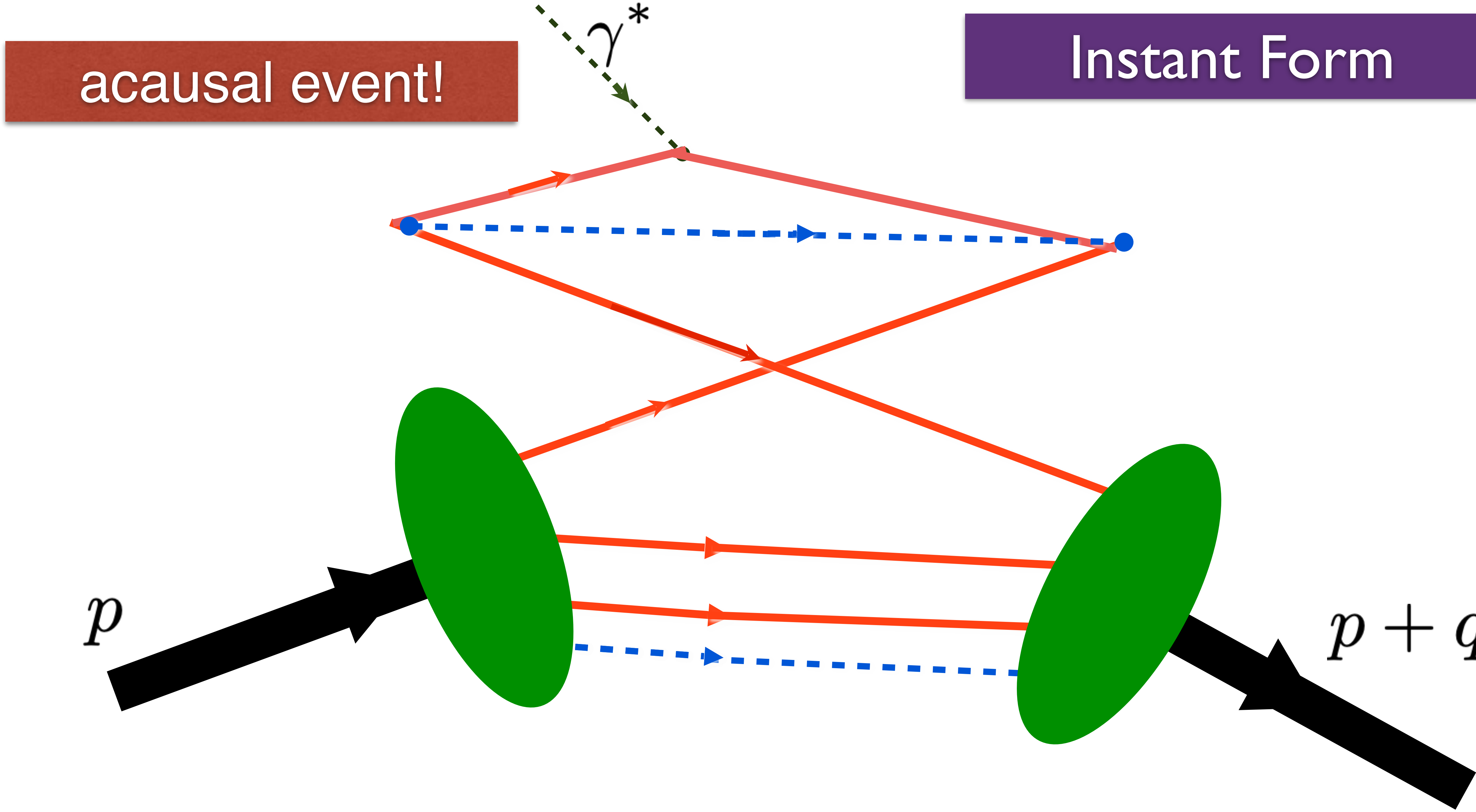
Roberts, Shrock, Tandy, sjb

Instant Form



Must include vacuum-induced currents to compute form factors and other current matrix elements in instant form

Boost are dynamical in instant form

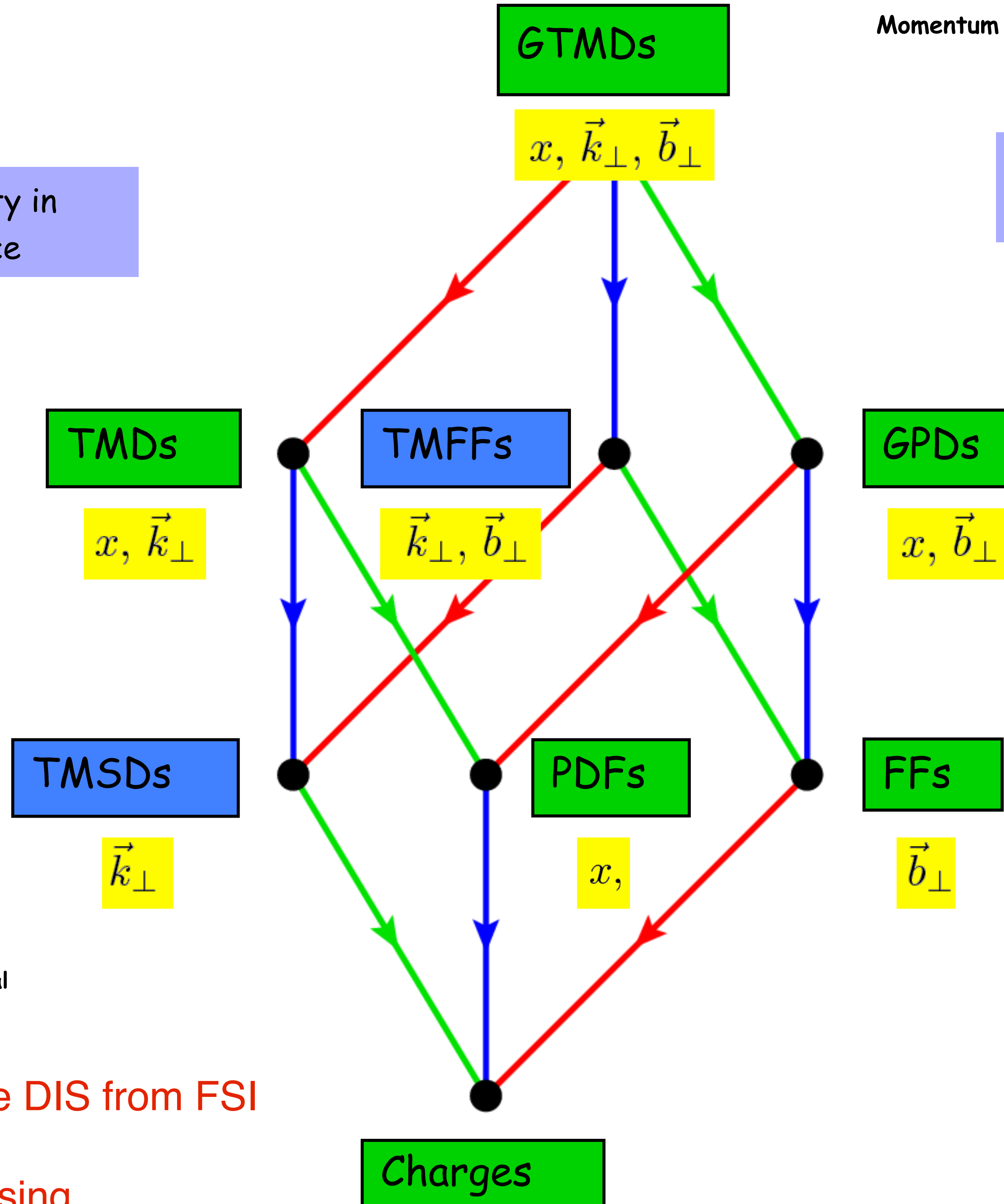
Lorce, Pasquini

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

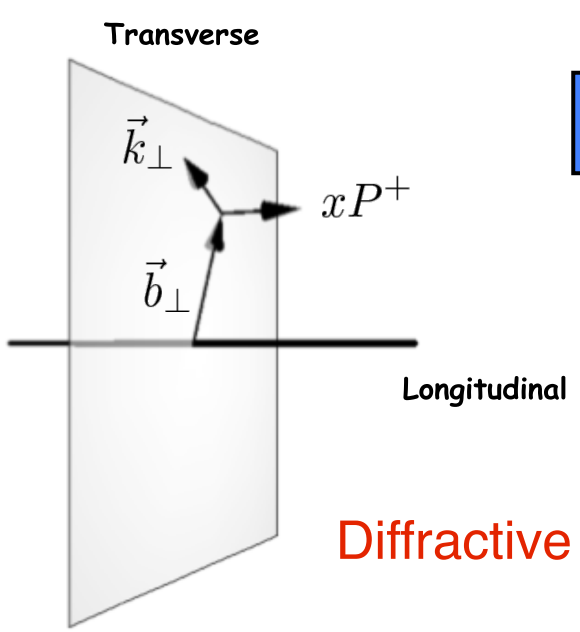
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

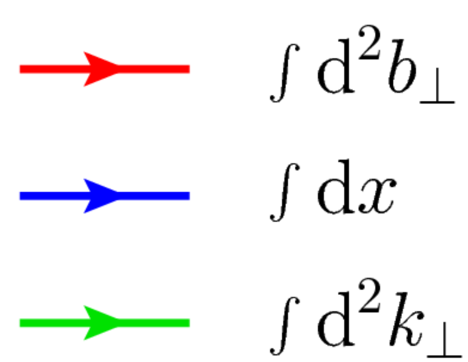
Transverse density in momentum space

Transverse density in position space



DGLAP, ERBL Evolution
Factorization Theorems

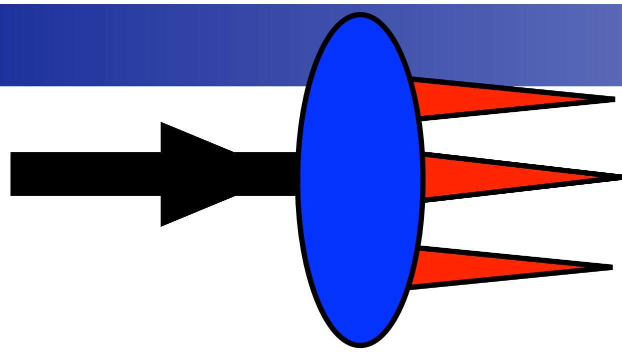




Diffractive DIS from FSI

Sivers, T-odd from lensing

Lorce, Pasquini



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

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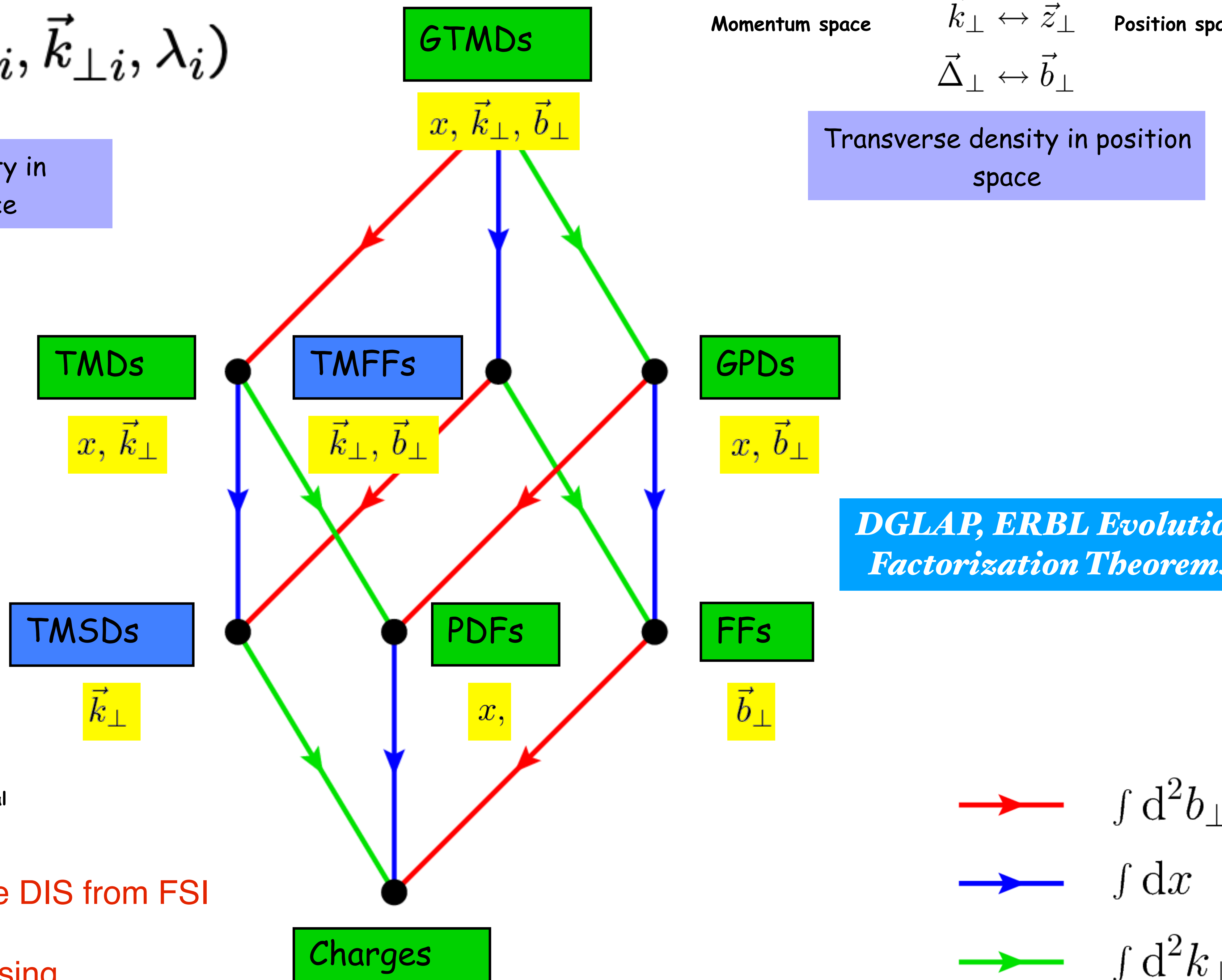

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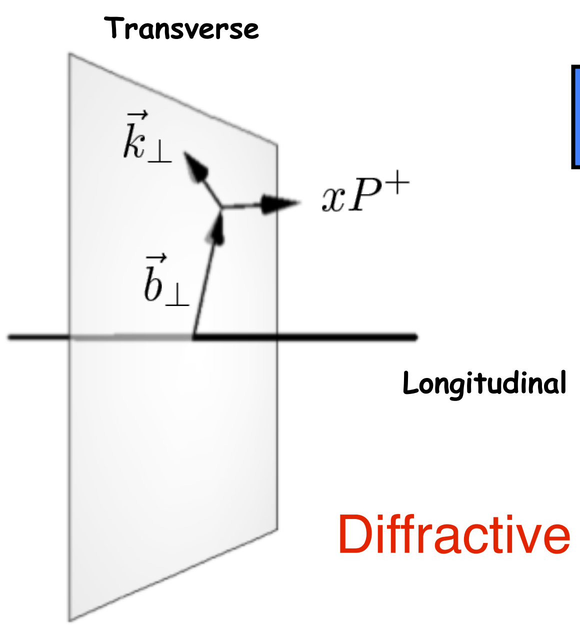

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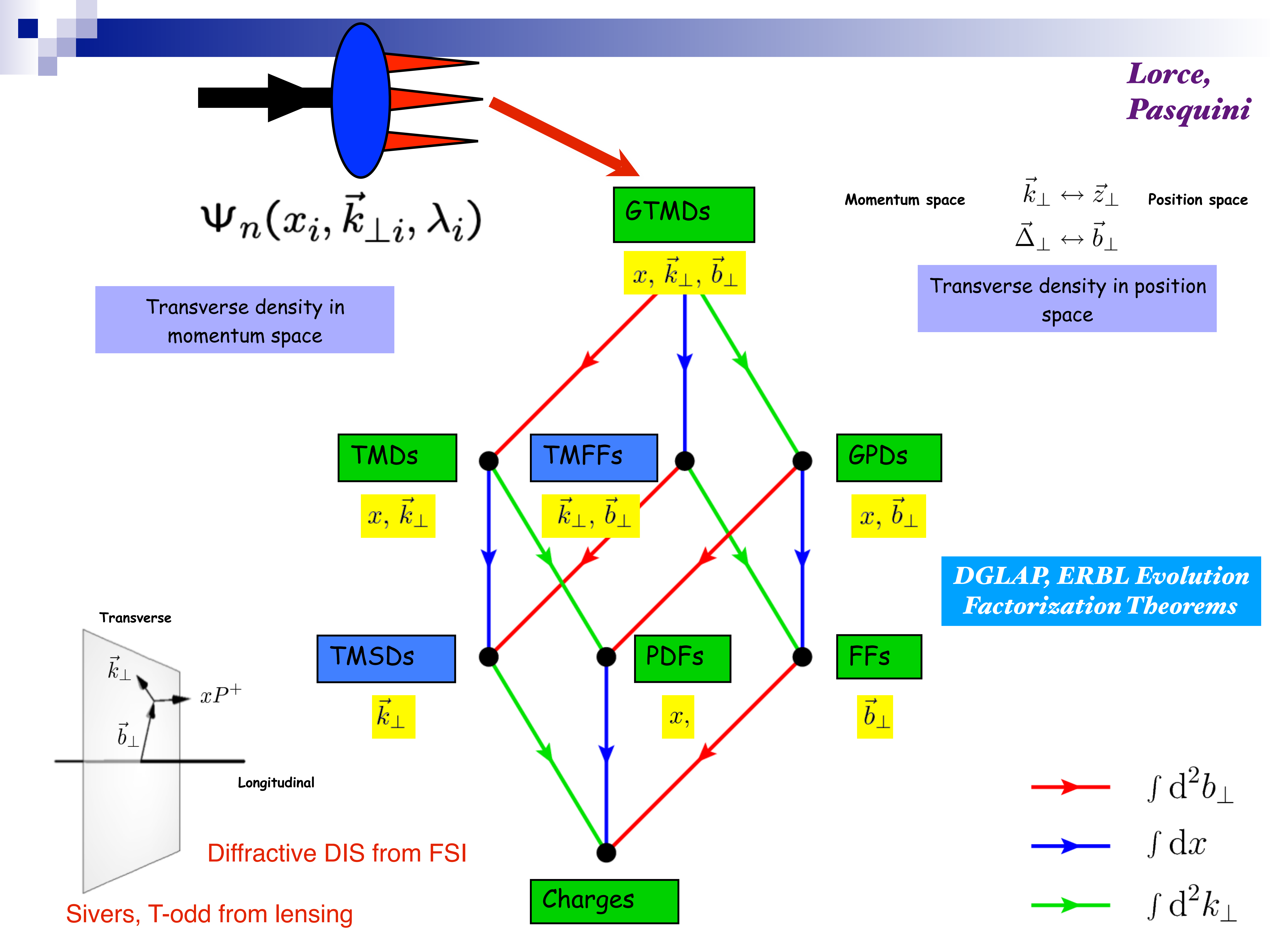
Lorce, Pasquini
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Transverse density in momentum space
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GTMDs
$x, \vec{k}_\perp, \vec{b}_\perp$
TMDs
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DGLAP, ERBL Evolution
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xP^+
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$\int d^2 b_\perp$
$\int dx$
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Diffractive DIS from FSI
Charges
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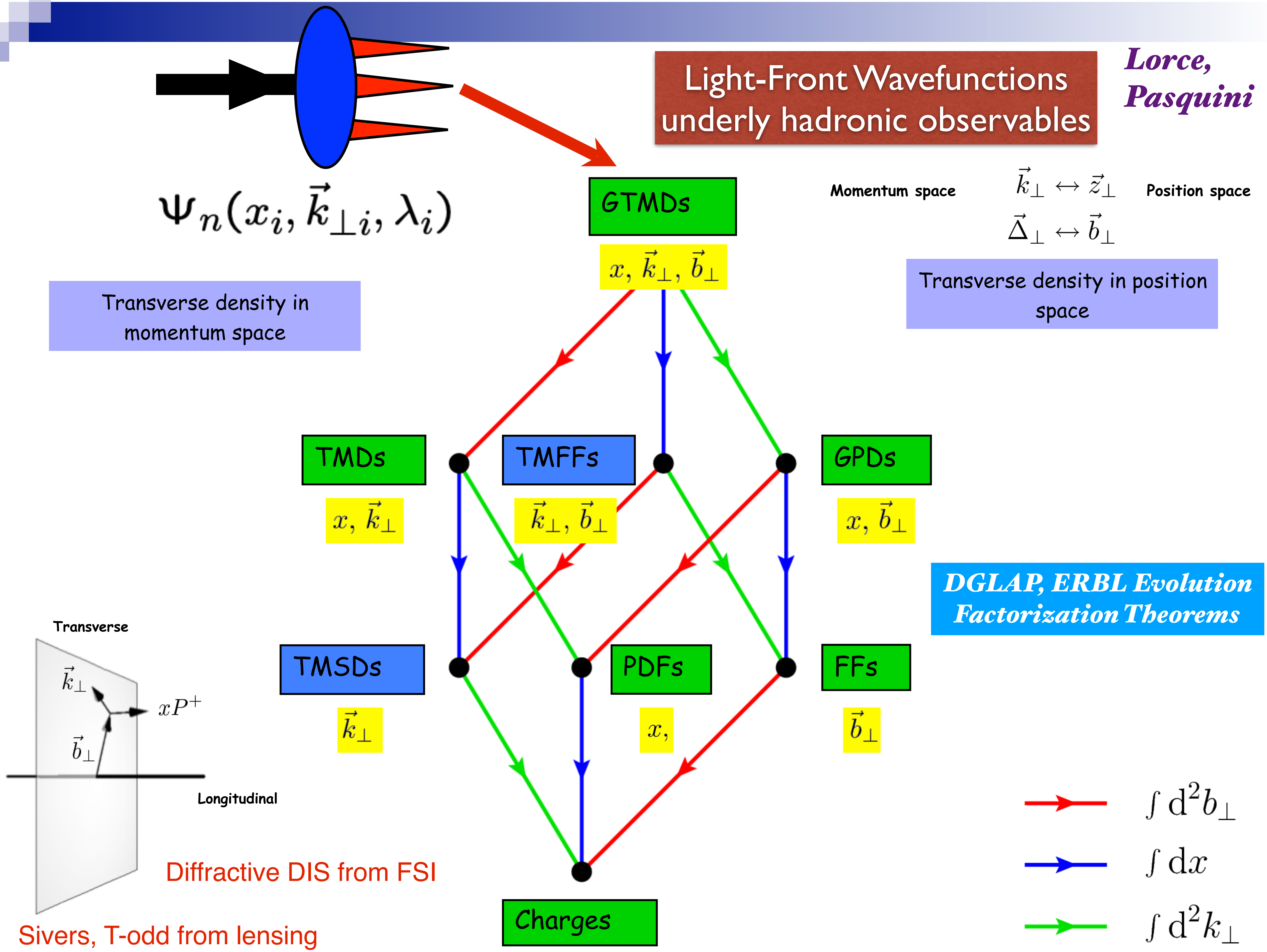

Light-Front Wavefunctions underly hadronic observables
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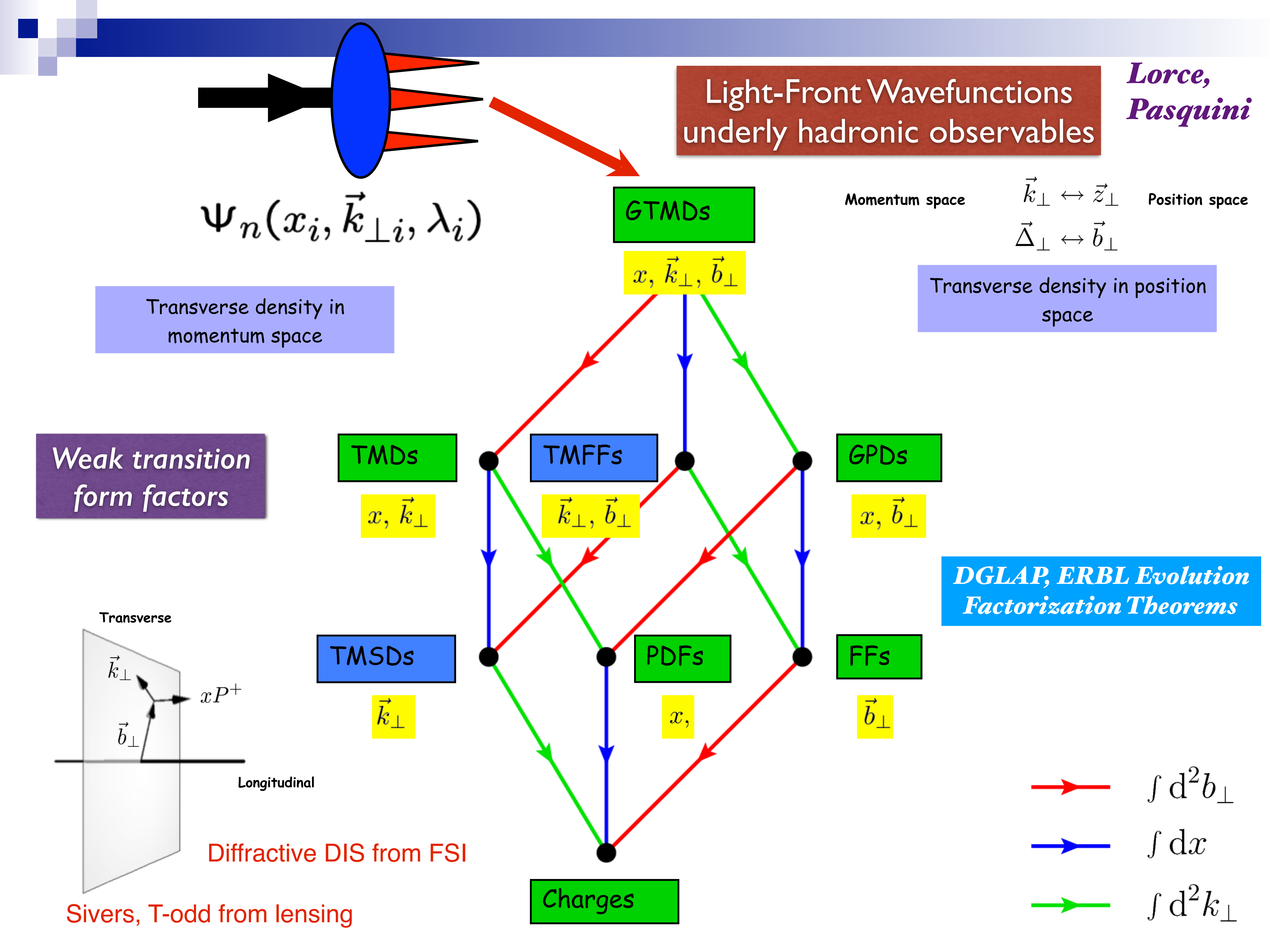

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Weak transition form factors
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$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi> = E\ |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\boxed{[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)}$$

Spherical Basis r, θ, ϕ

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$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$\boxed{H_{QED}}$$

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Bohr Spectrum

Schrödinger Eq.

Light-Front QCD

$$\mathcal{L}_{QCD} \longrightarrow \boxed{H^{LF}_{QCD}}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

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$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \ \psi_{LF}(x, \vec{k}_\perp)$$

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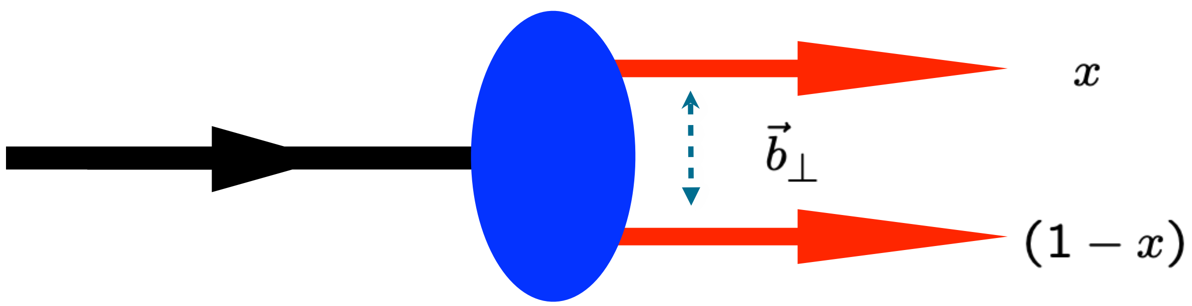
Azimuthal Basis ζ, ϕ

Single variable Equation

$m_q = 0$

Light-Front QCD

Fixed $\tau = t + z/c$



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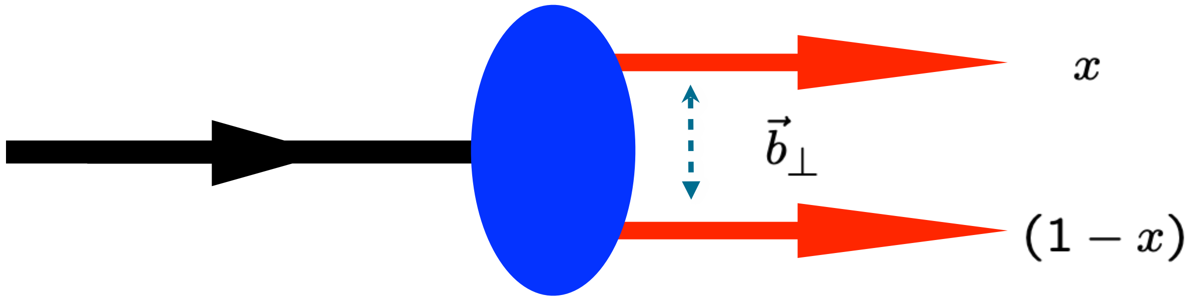

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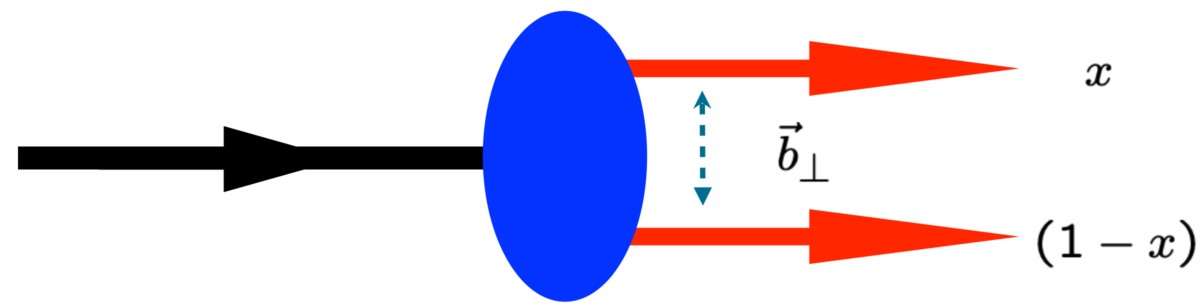

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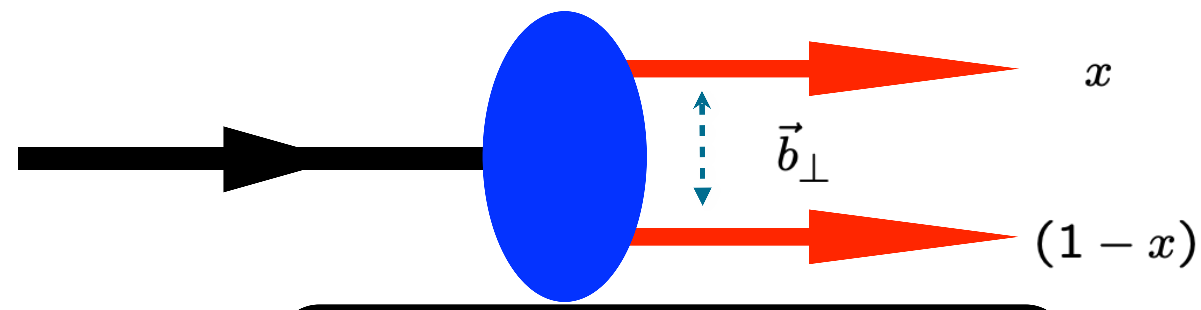

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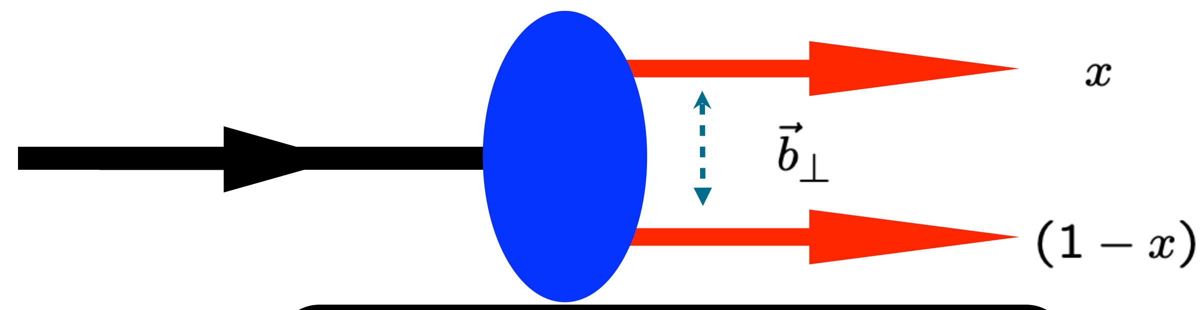

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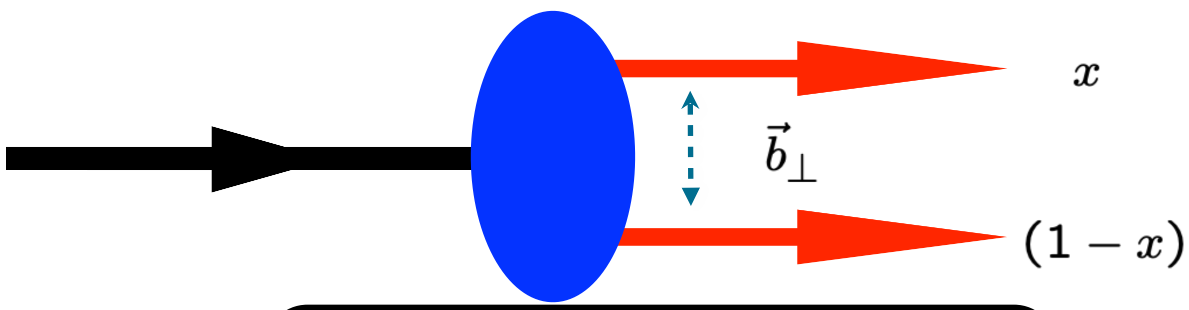

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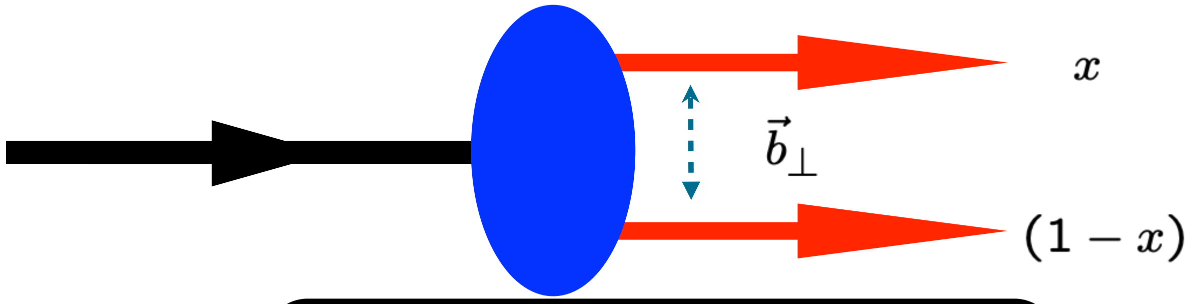

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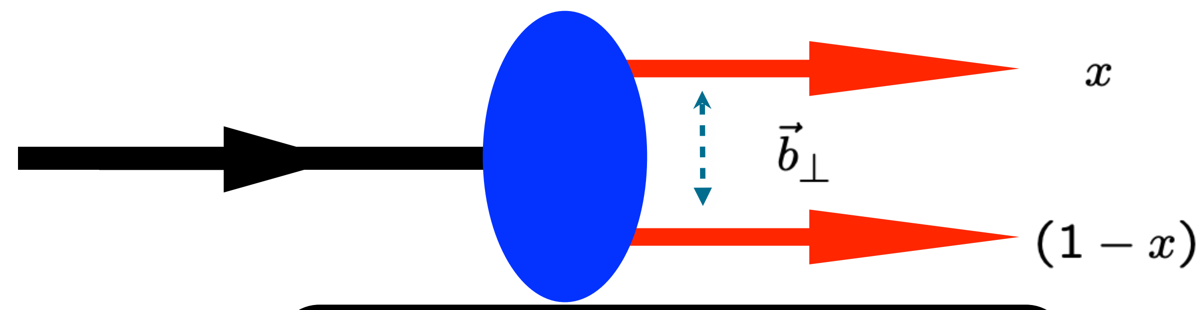

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Semiclassical first approximation to QCD

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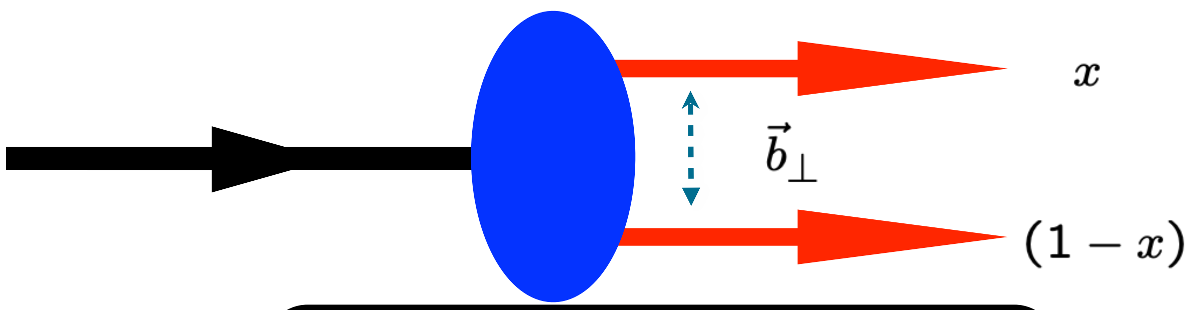

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Confining AdS/QCD potential!

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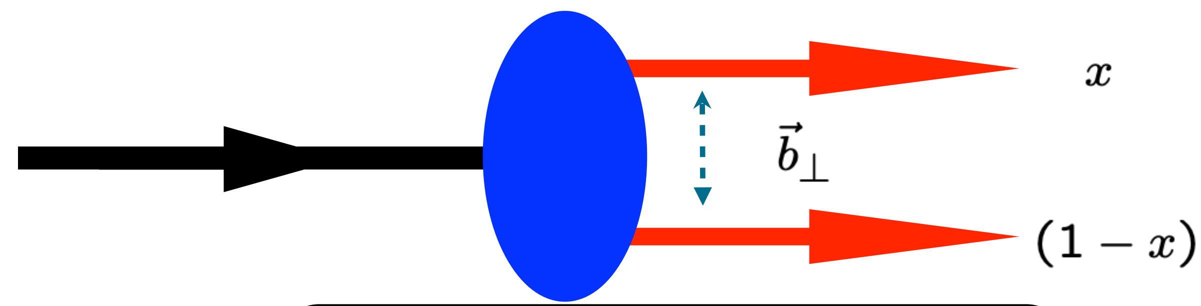

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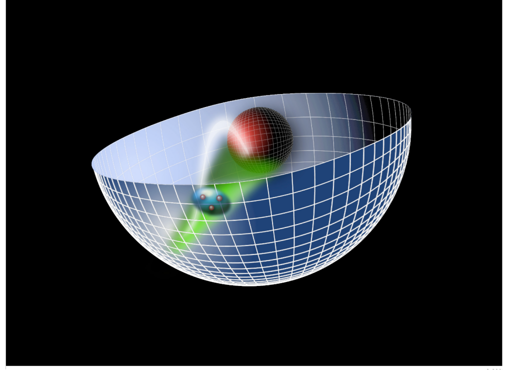
Semiclassical first approximation to QCD

Sums an infinite # diagrams

Maldacena

AdS$_5$



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

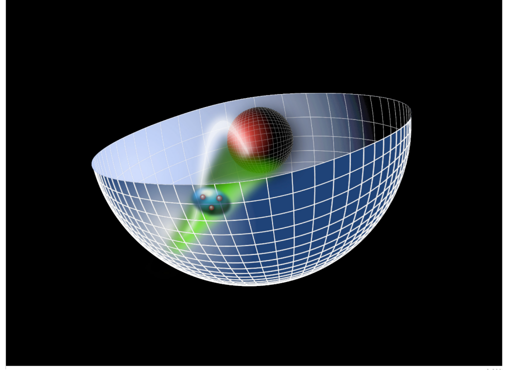
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Maldacena

AdS$_5$



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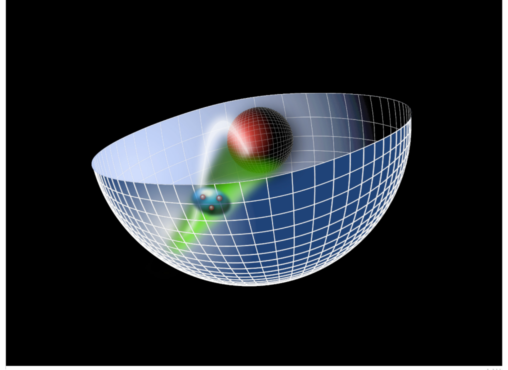

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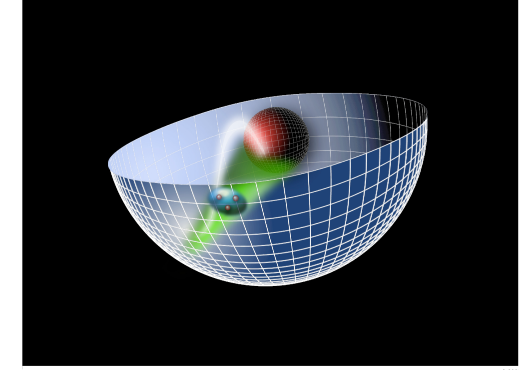

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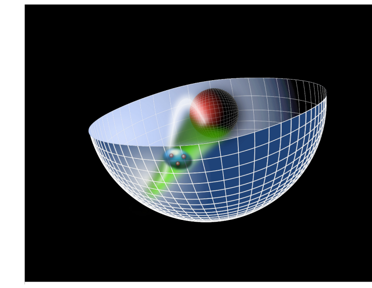
AdS/CFT

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- ***Soft-wall dilaton profile breaks conformal invariance*** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- ***Color Confinement in z***
- ***Introduces confinement scale*** κ
- ***Uses AdS_5 as template for conformal theory***

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

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Positive-sign dilaton

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Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

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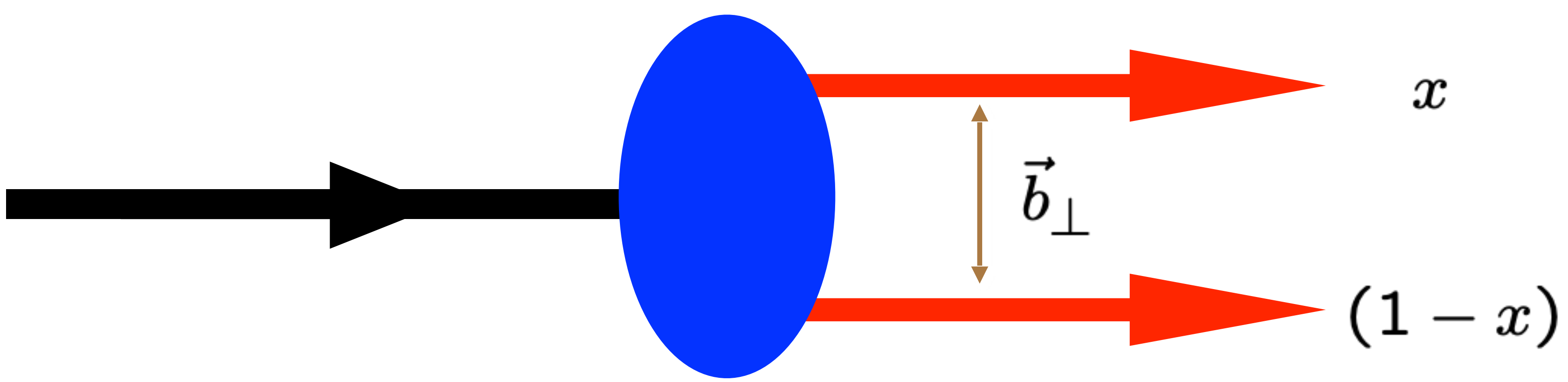
$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

LF(3+1) ⟷ AdS_5

de Teramond, sjb

Light-Front Holographic Dictionary



$$(\mu R)^2 = L^2 - (J-2)^2$$

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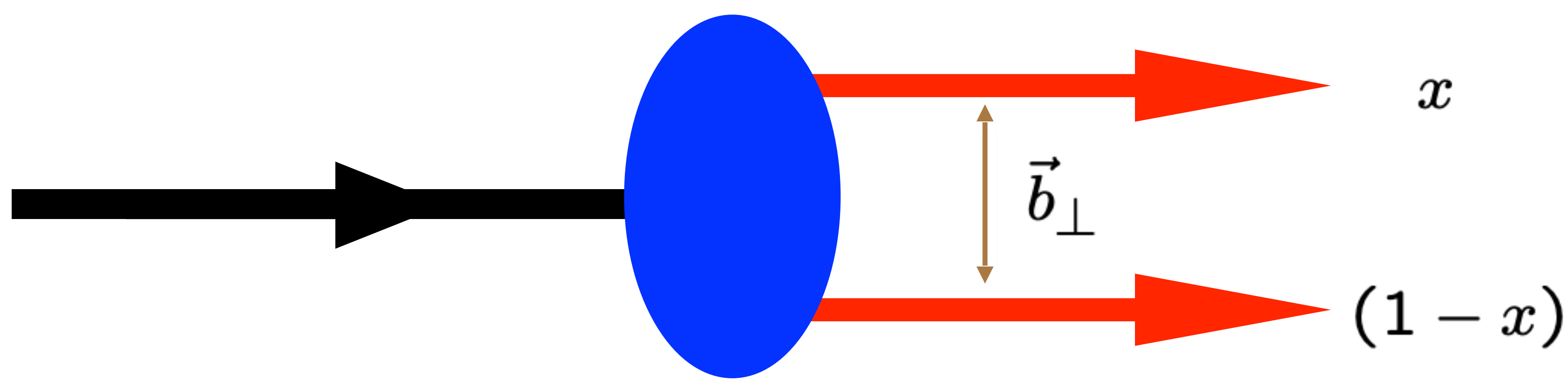
Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp)$$



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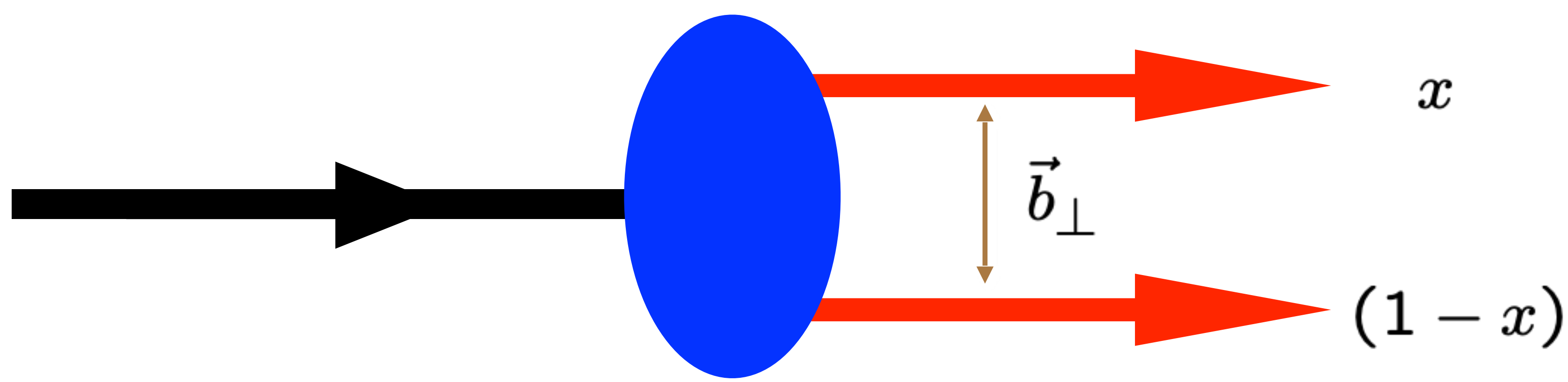

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LF(3+1) AdS5

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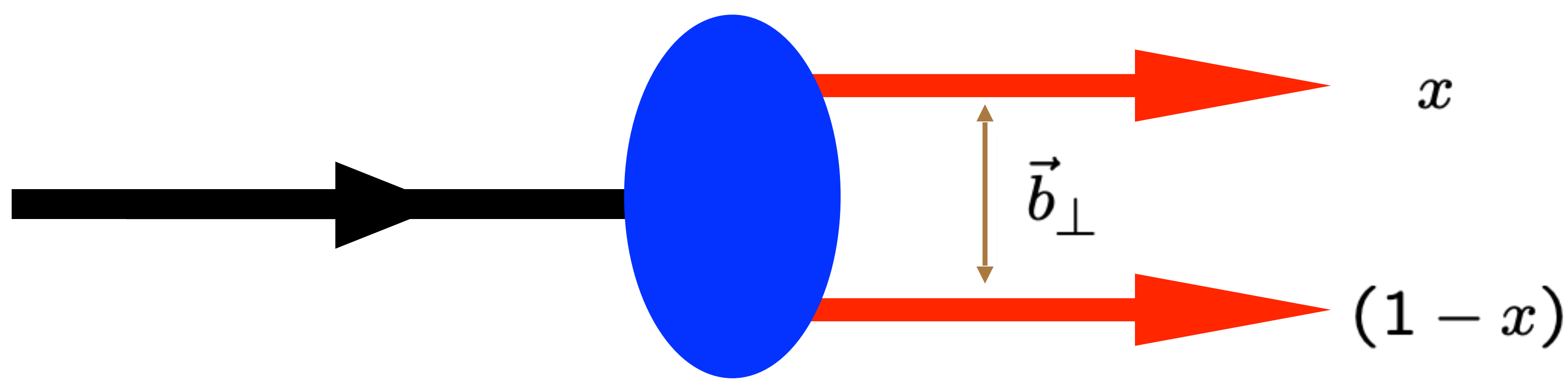

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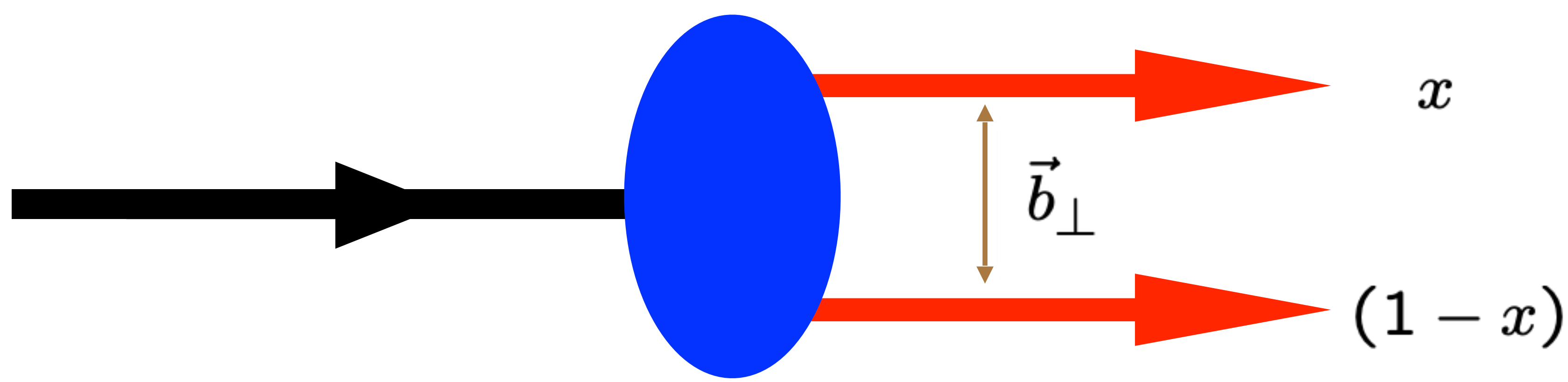

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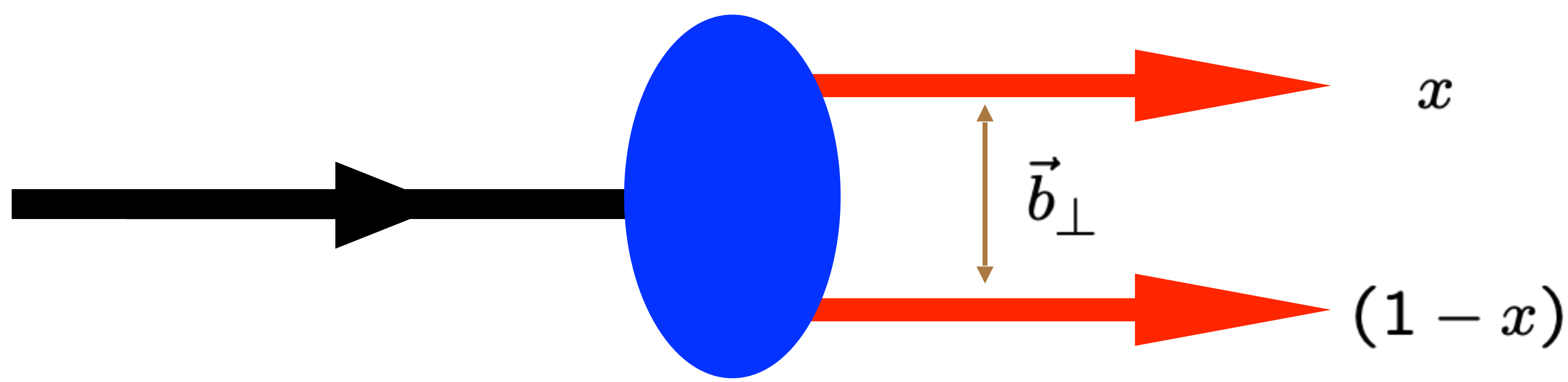

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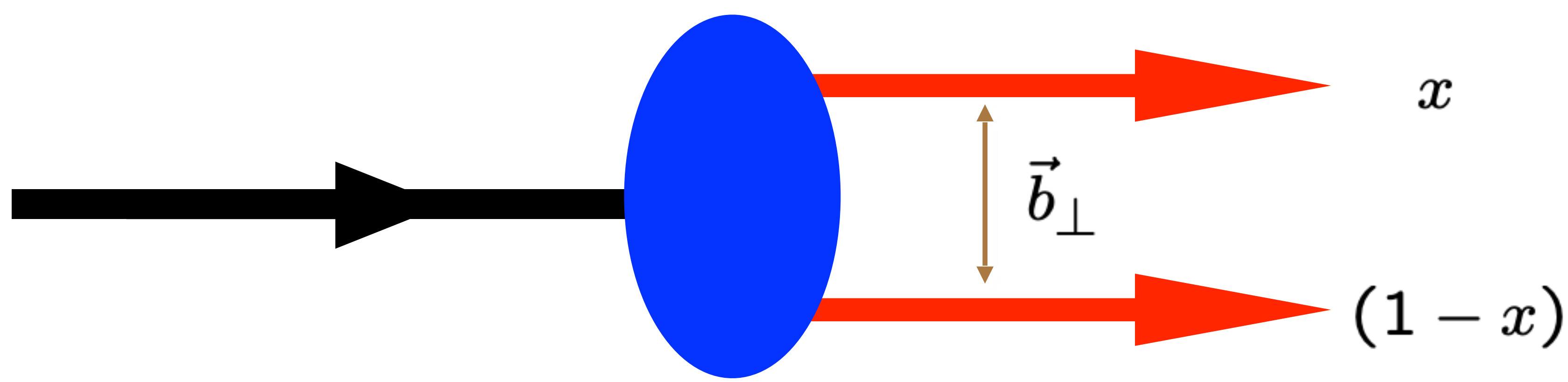

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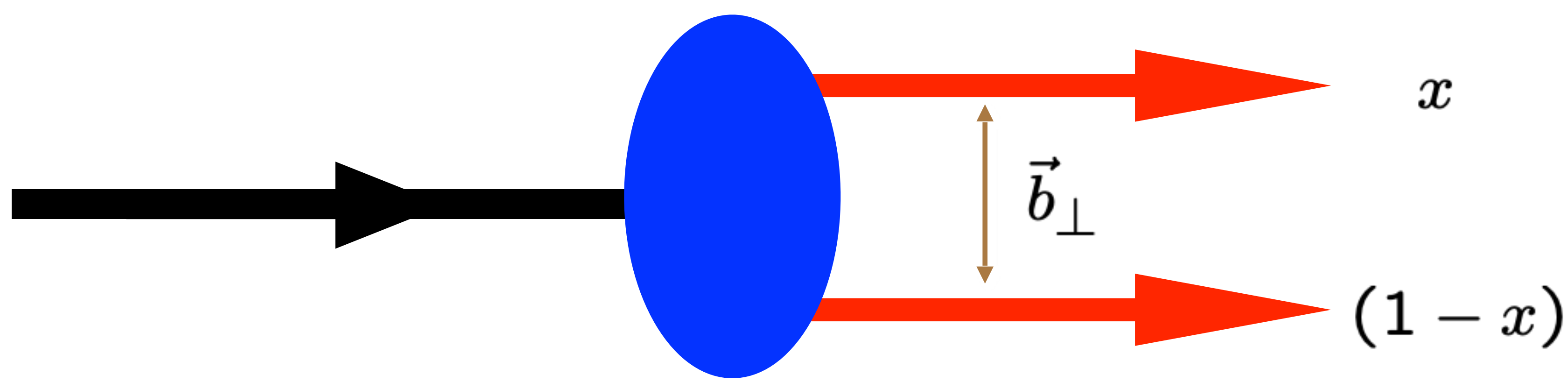

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Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

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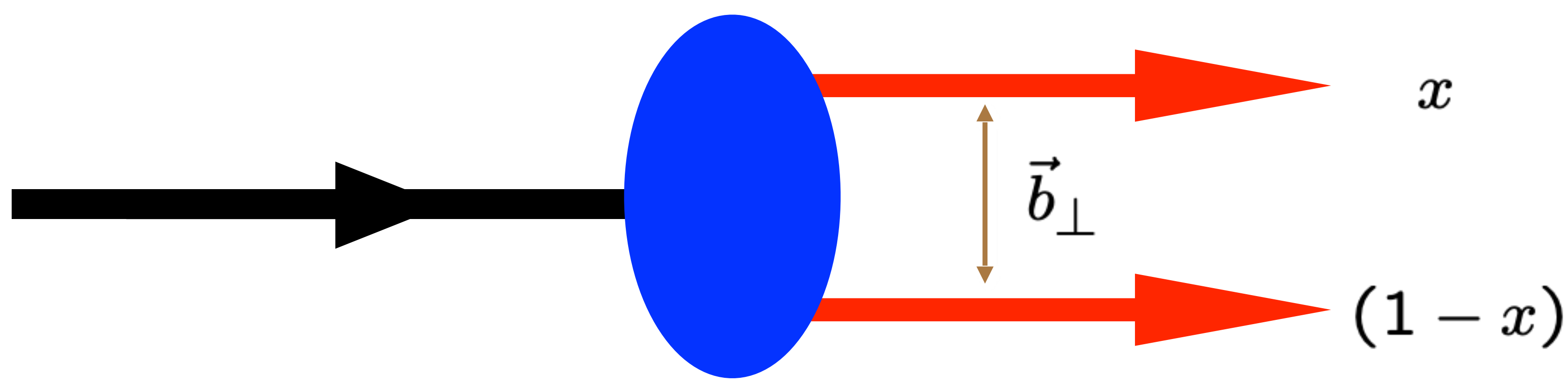

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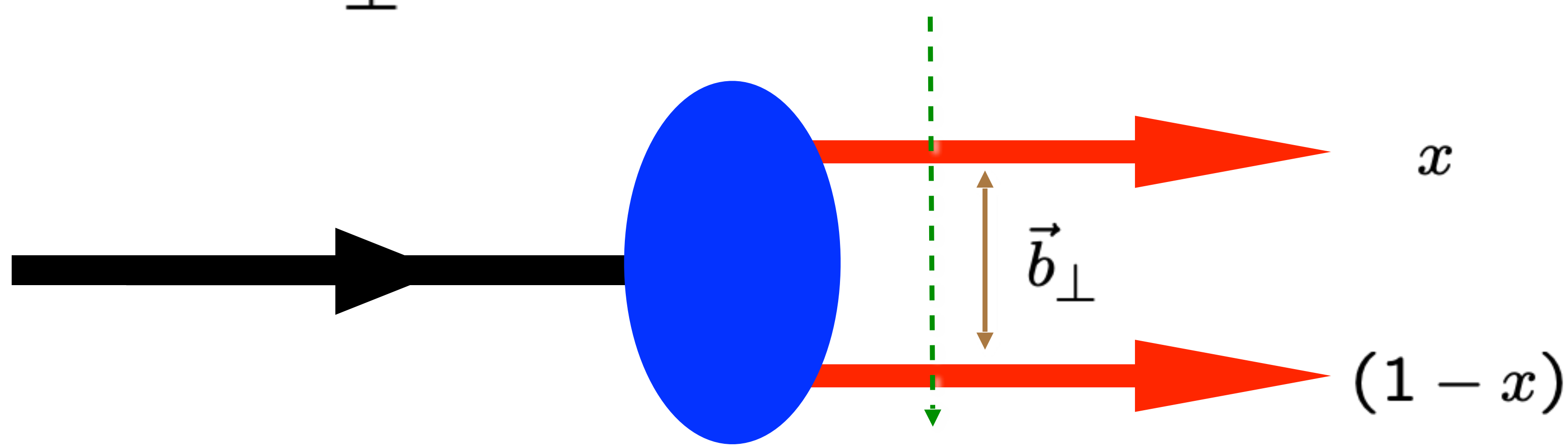

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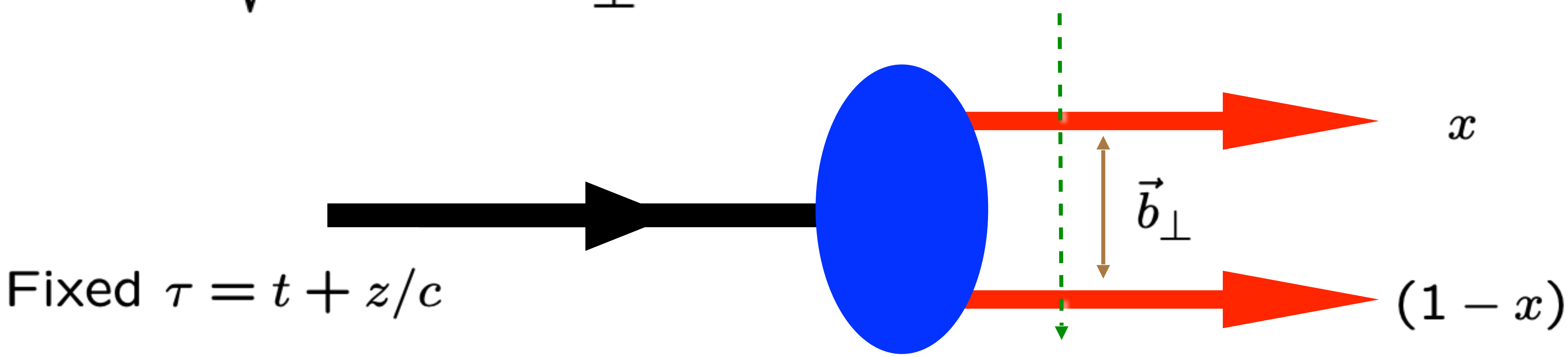

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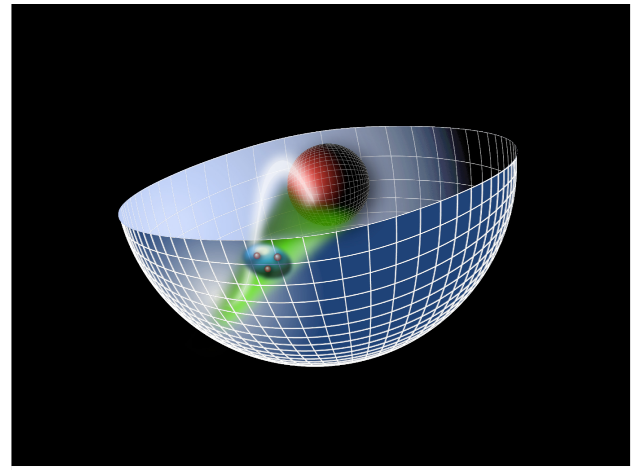
AdS/QCD
Soft-Wall Model



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Light-Front Schrödinger Equation

Single variable ζ

Confinement scale: $\kappa \simeq 0.5\ GeV$

de Tèramond, Dosch, sjb

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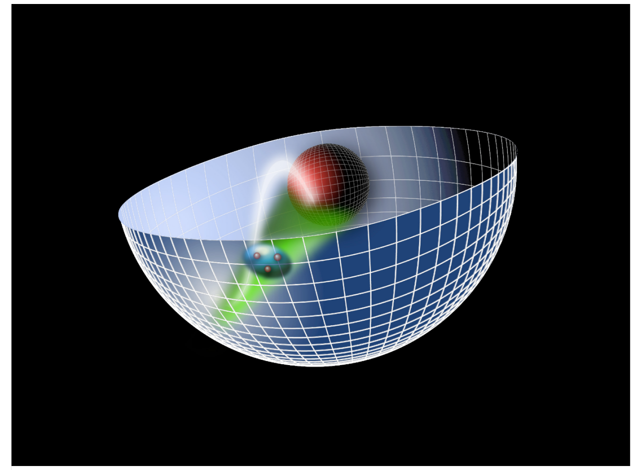

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de Tèramond, Dosch, sjb

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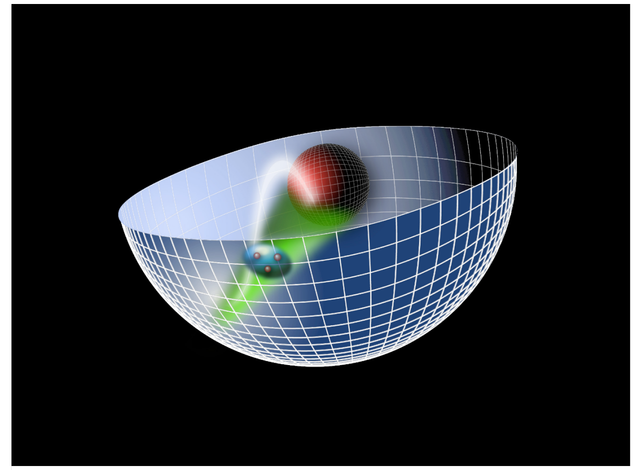

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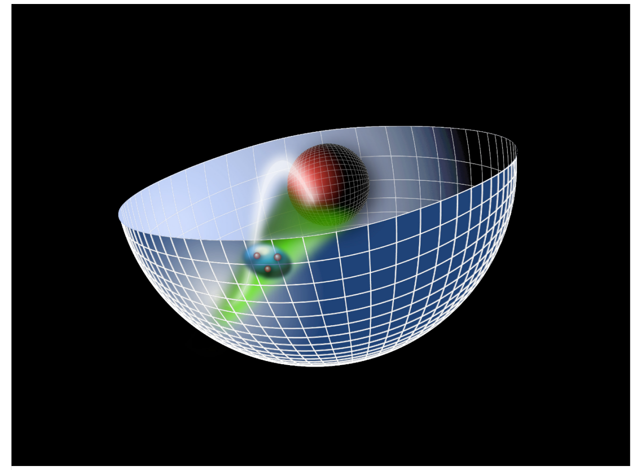

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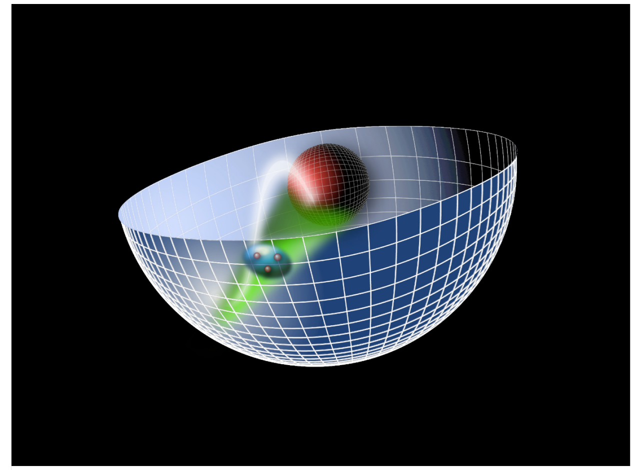

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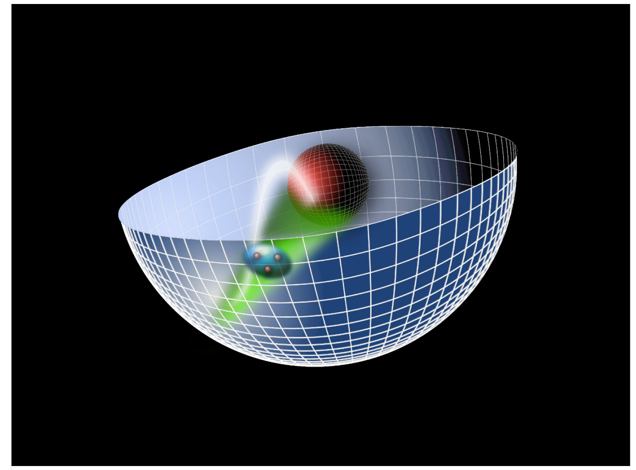

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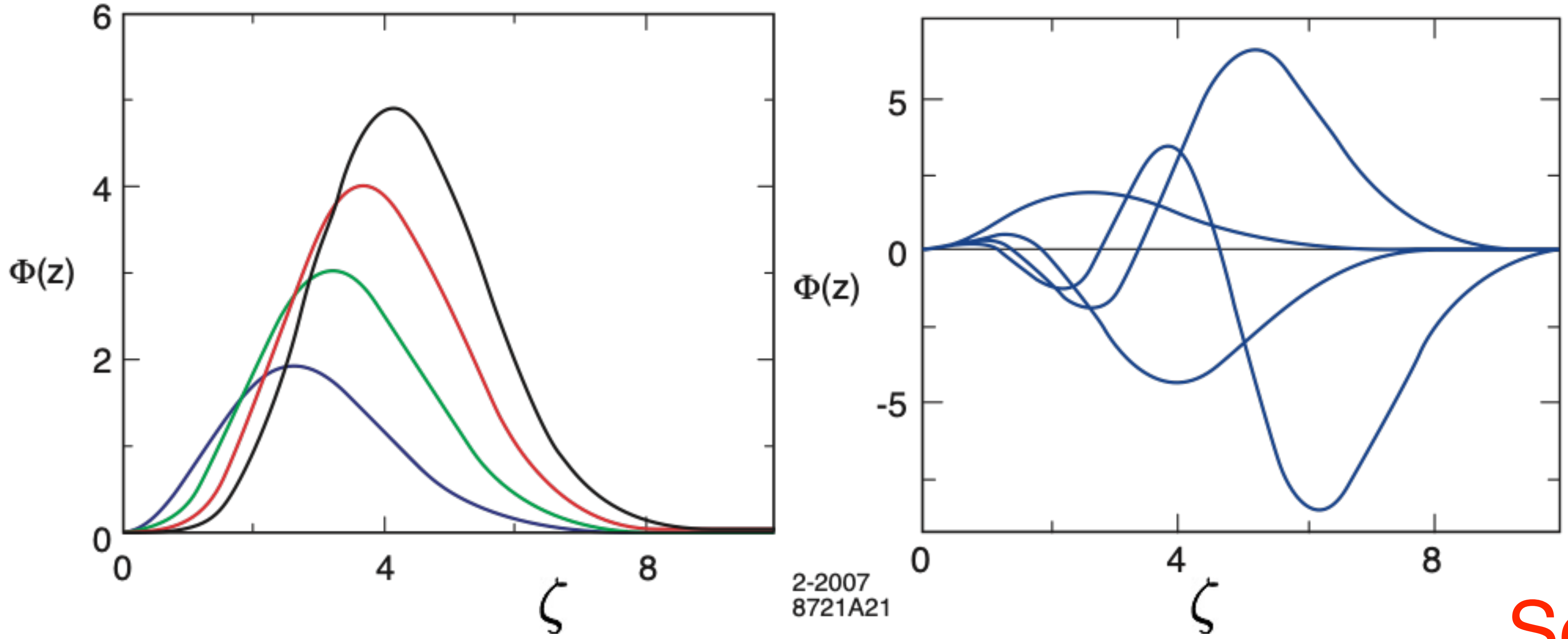
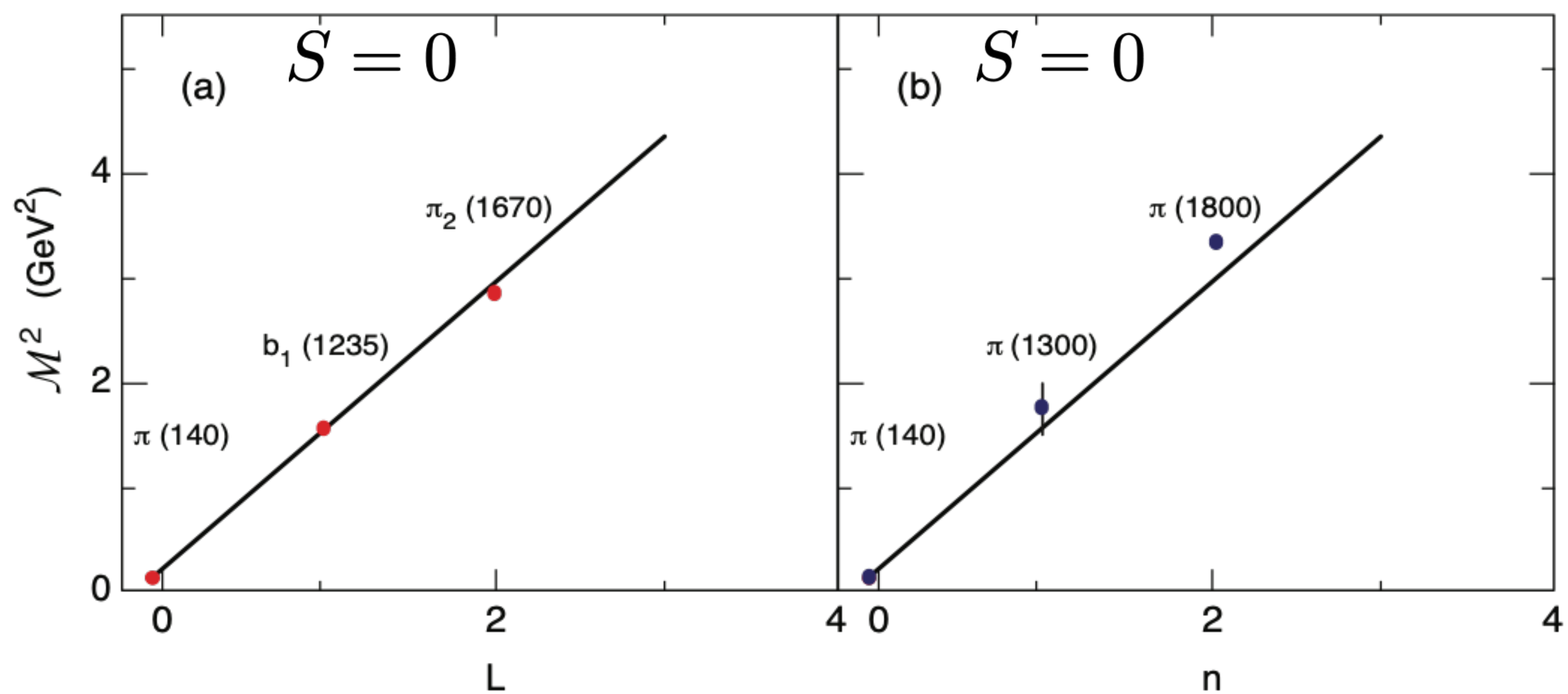
GeV units external to QCD: Only Ratios of Masses Determined



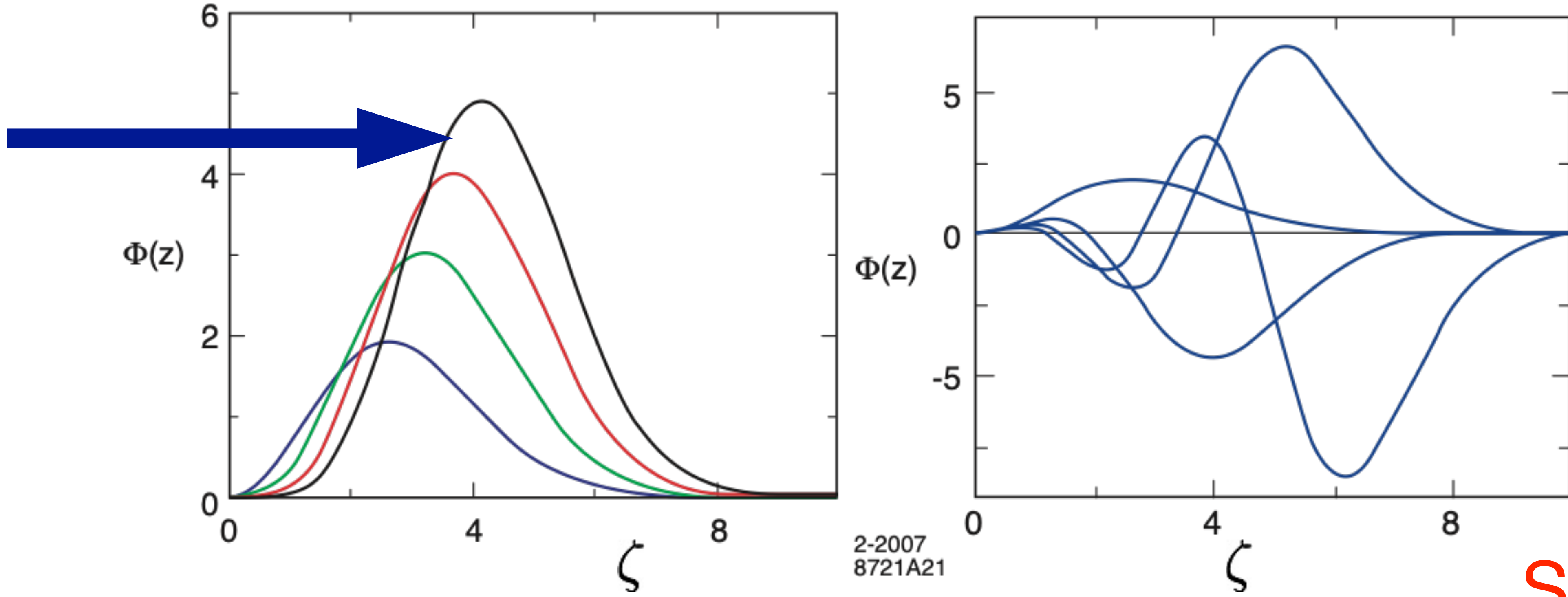
Soft Wall Mode

Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .



$m_q = 0$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.



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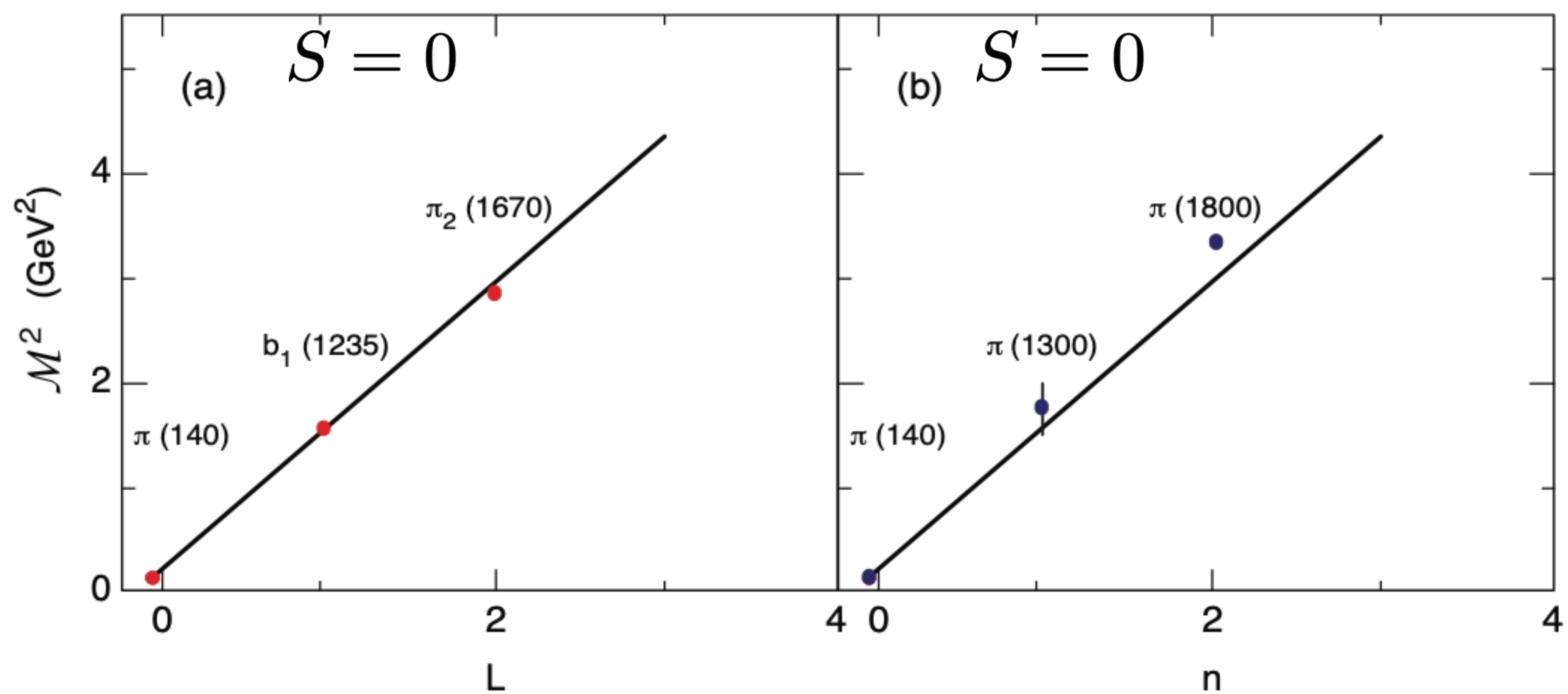

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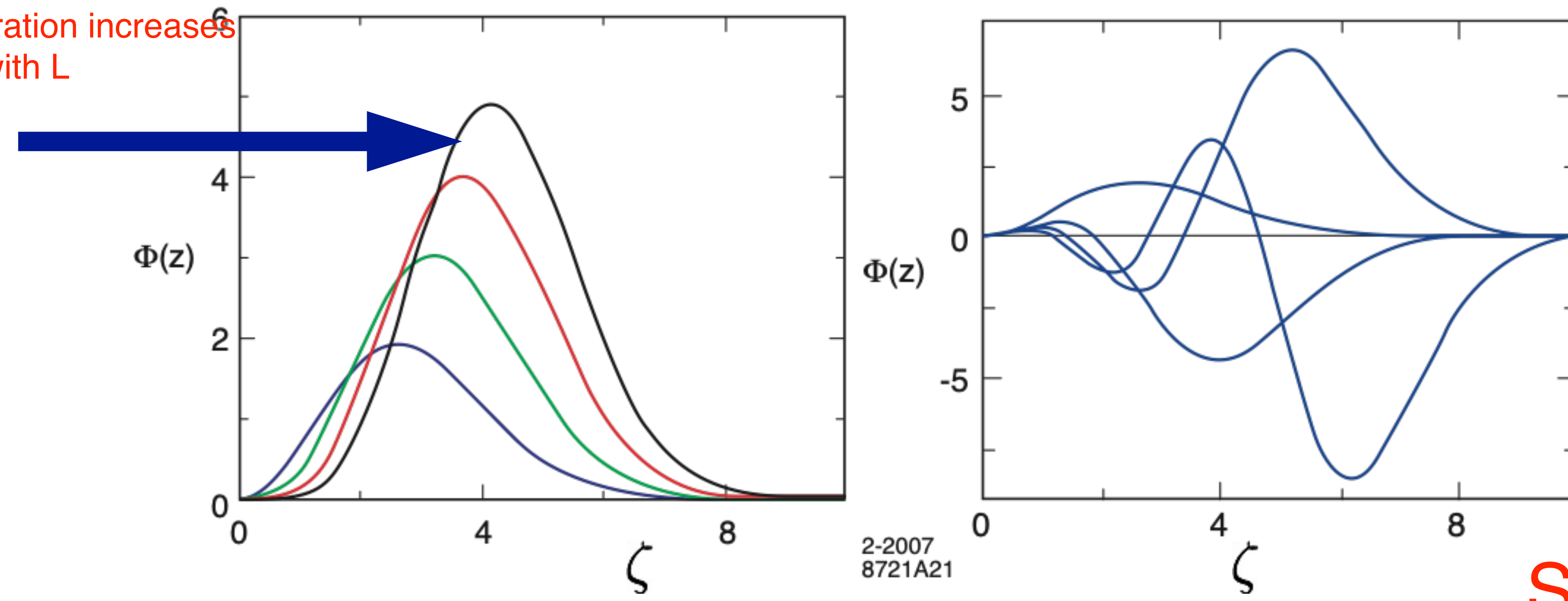
Quark separation increases with L



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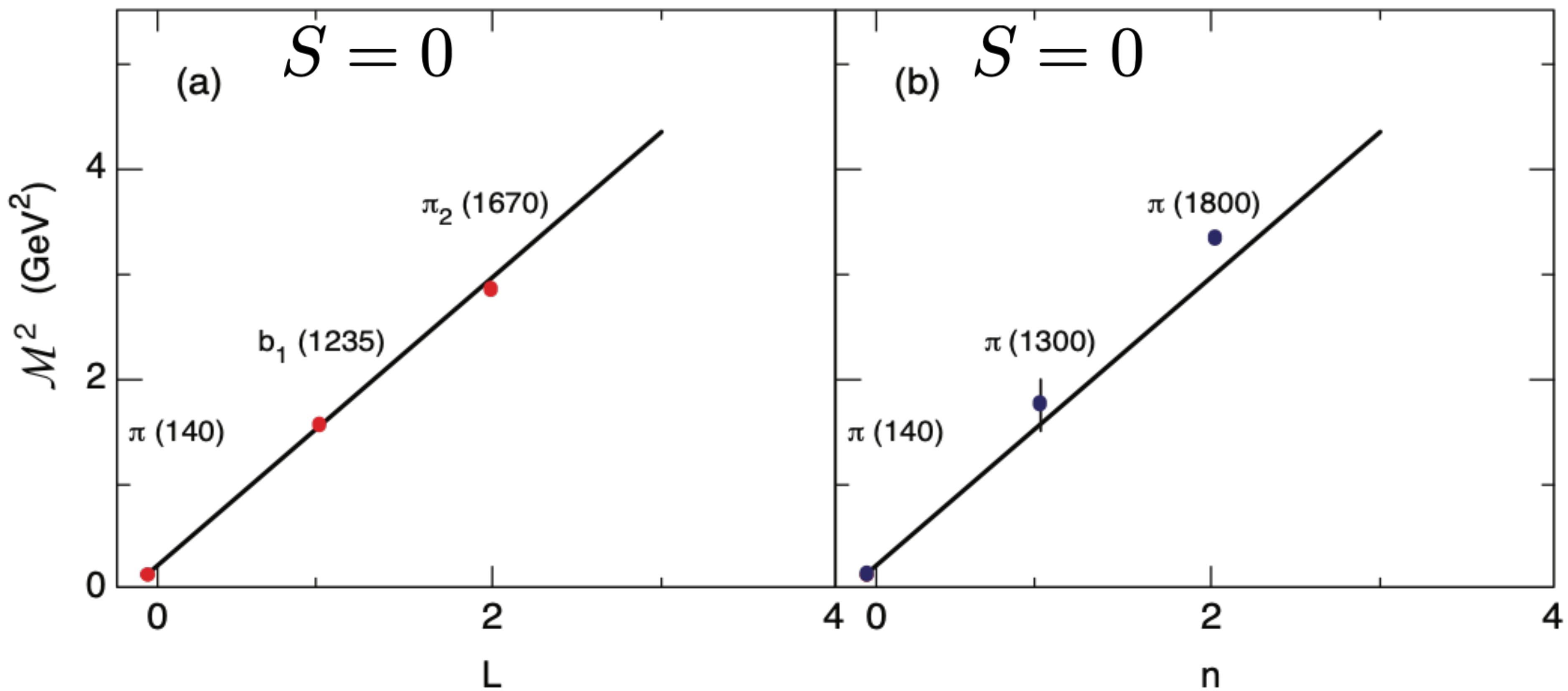

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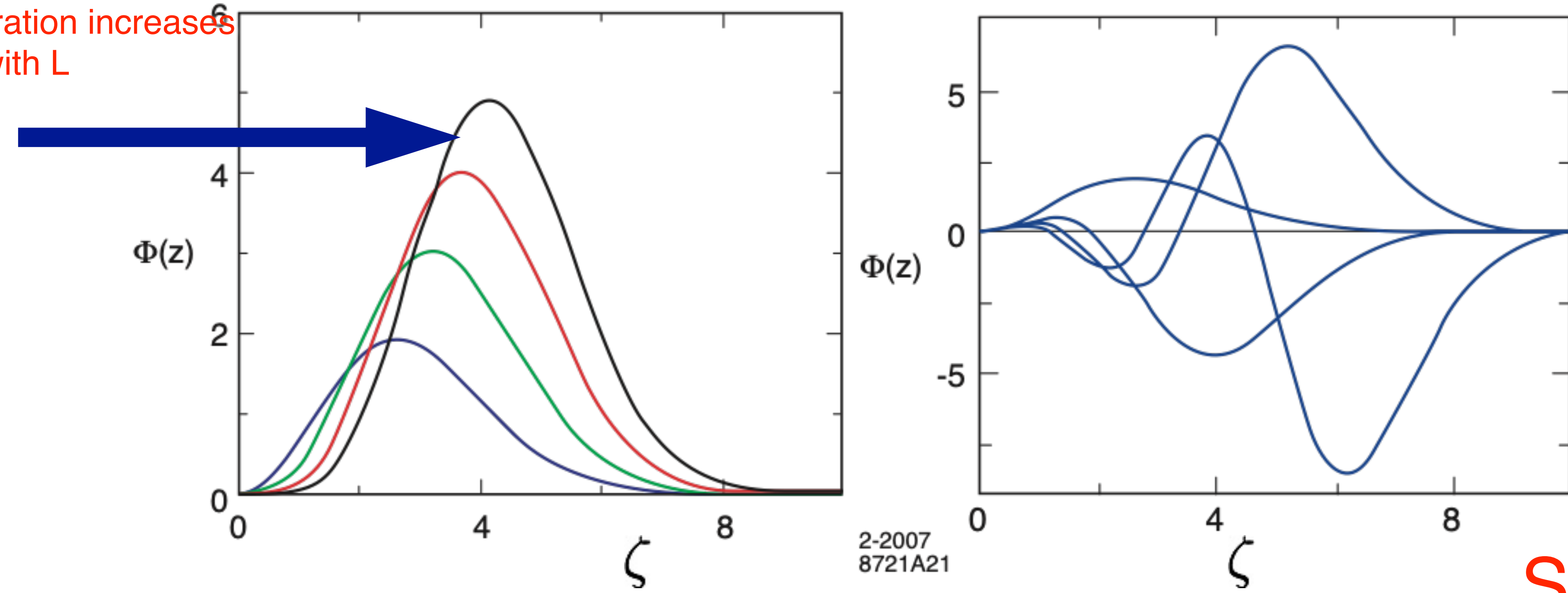

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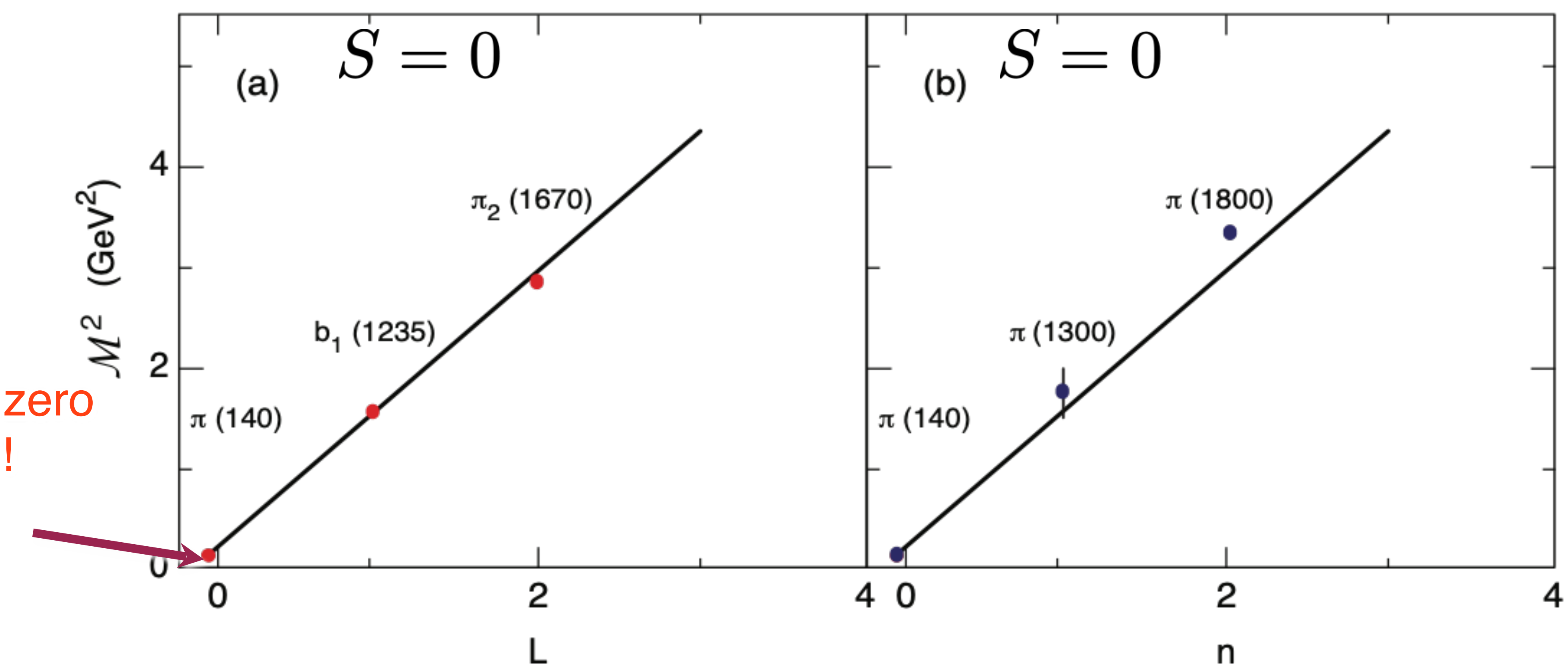

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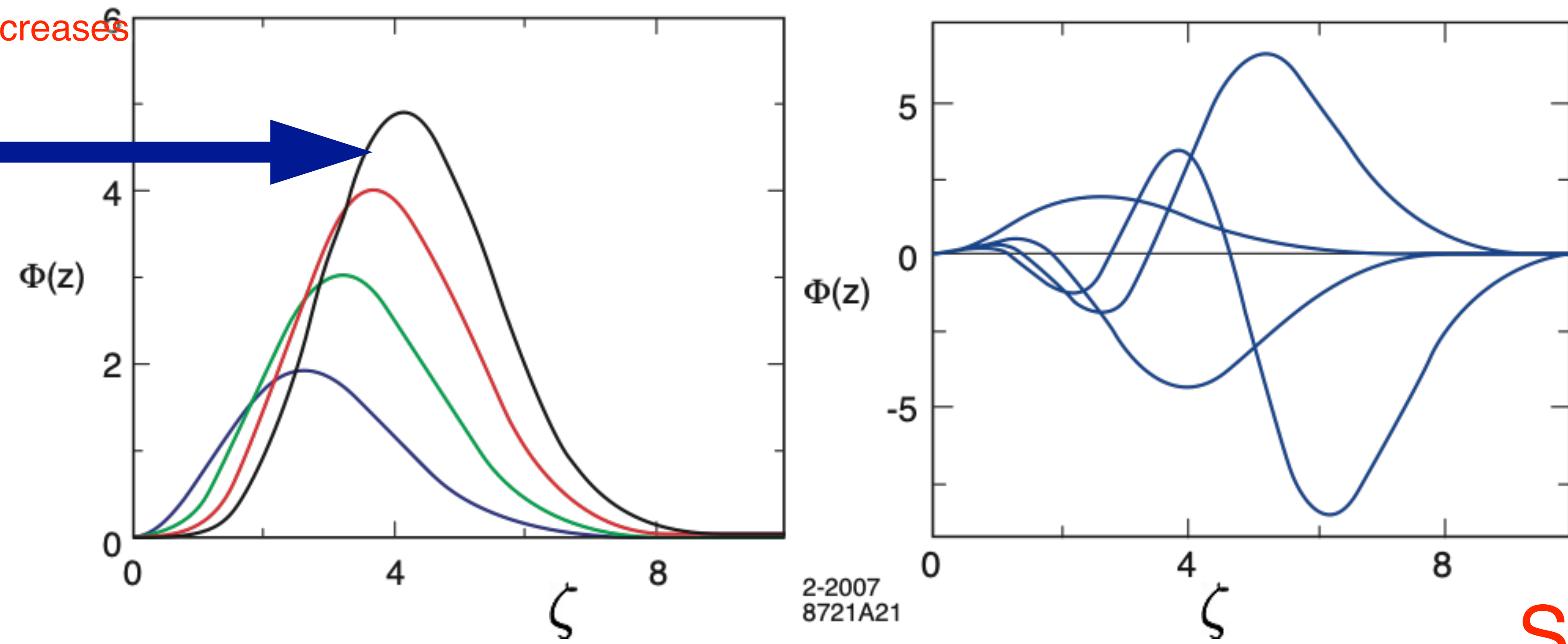

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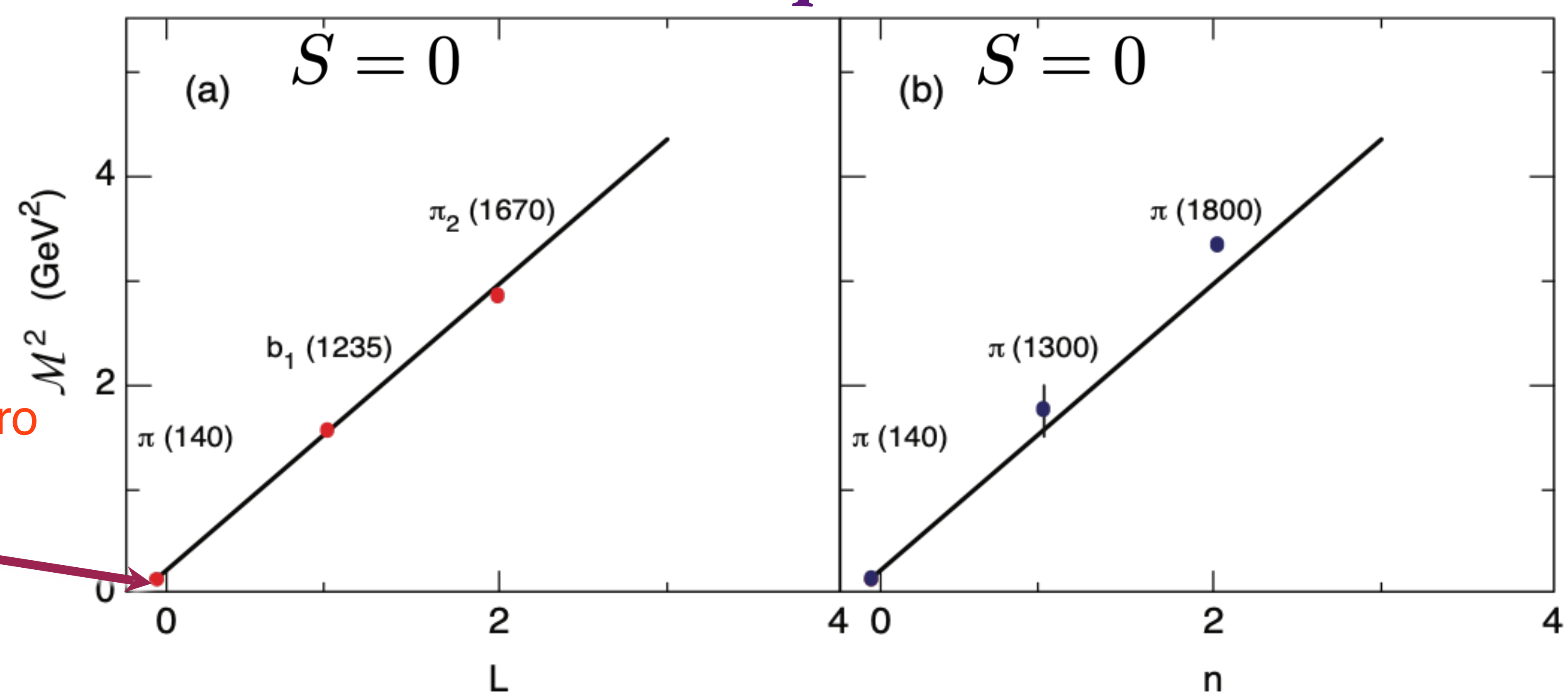

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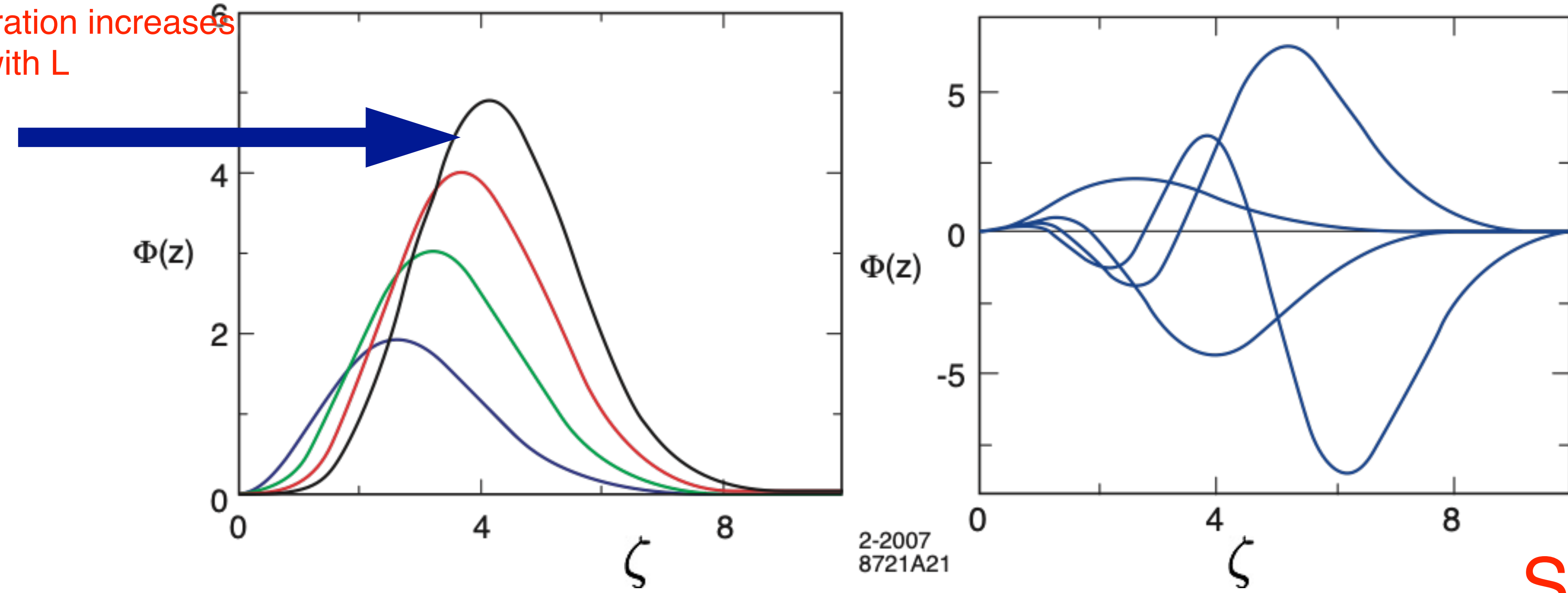

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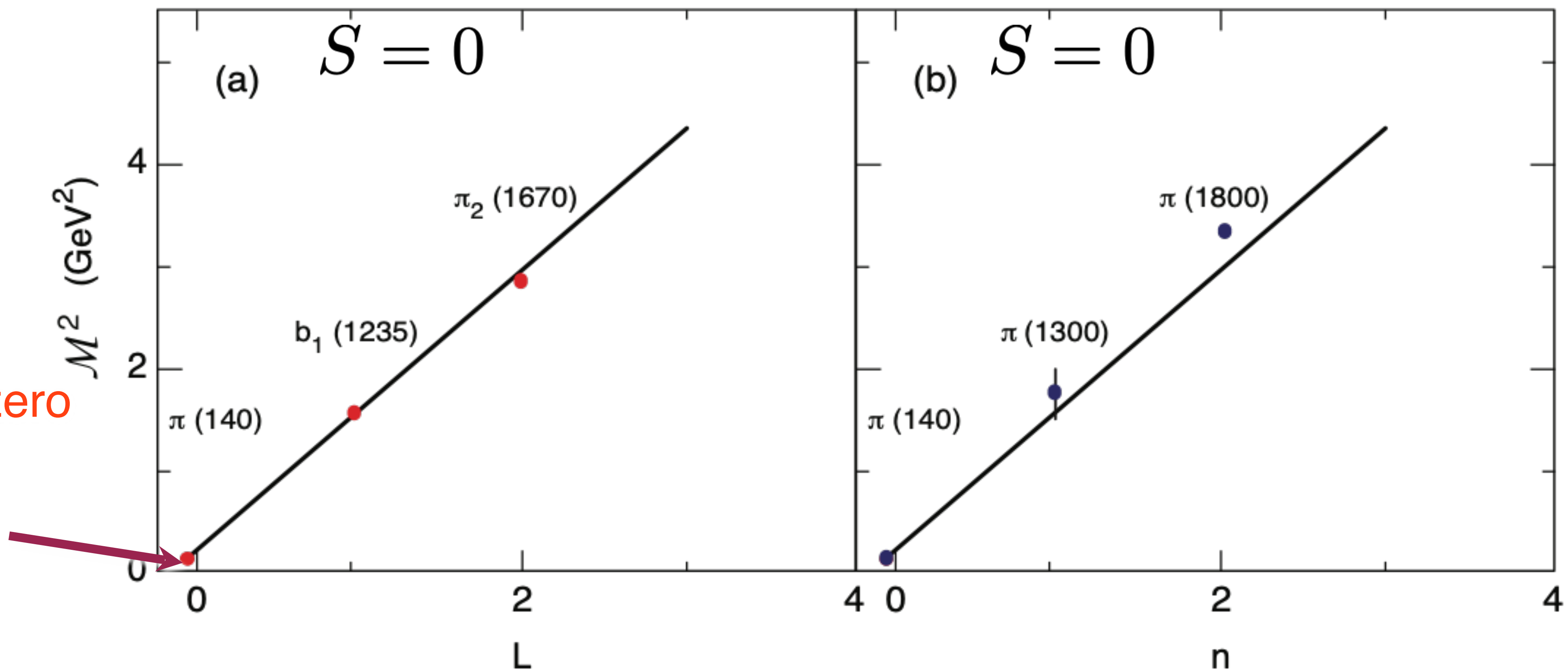

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Meson Spectrum in Soft Wall Model

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

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$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

G. de Teramond, H. G. Dosch, sjb

Meson Spectrum in Soft Wall Model

- Dilaton profile $\varphi(z) = +\kappa^2 z^2$ $\qquad z \longrightarrow \zeta$
- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
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G. de Teramond, H. G. Dosch, sjb

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G. de Teramond, H. G. Dosch, sjb

Massless pion!

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G. de Teramond, H. G. Dosch, sjb

Massless pion!

Meson Spectrum in Soft Wall Model

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Pion: Negative term for J=0 cancels positive terms from LFKE and potential

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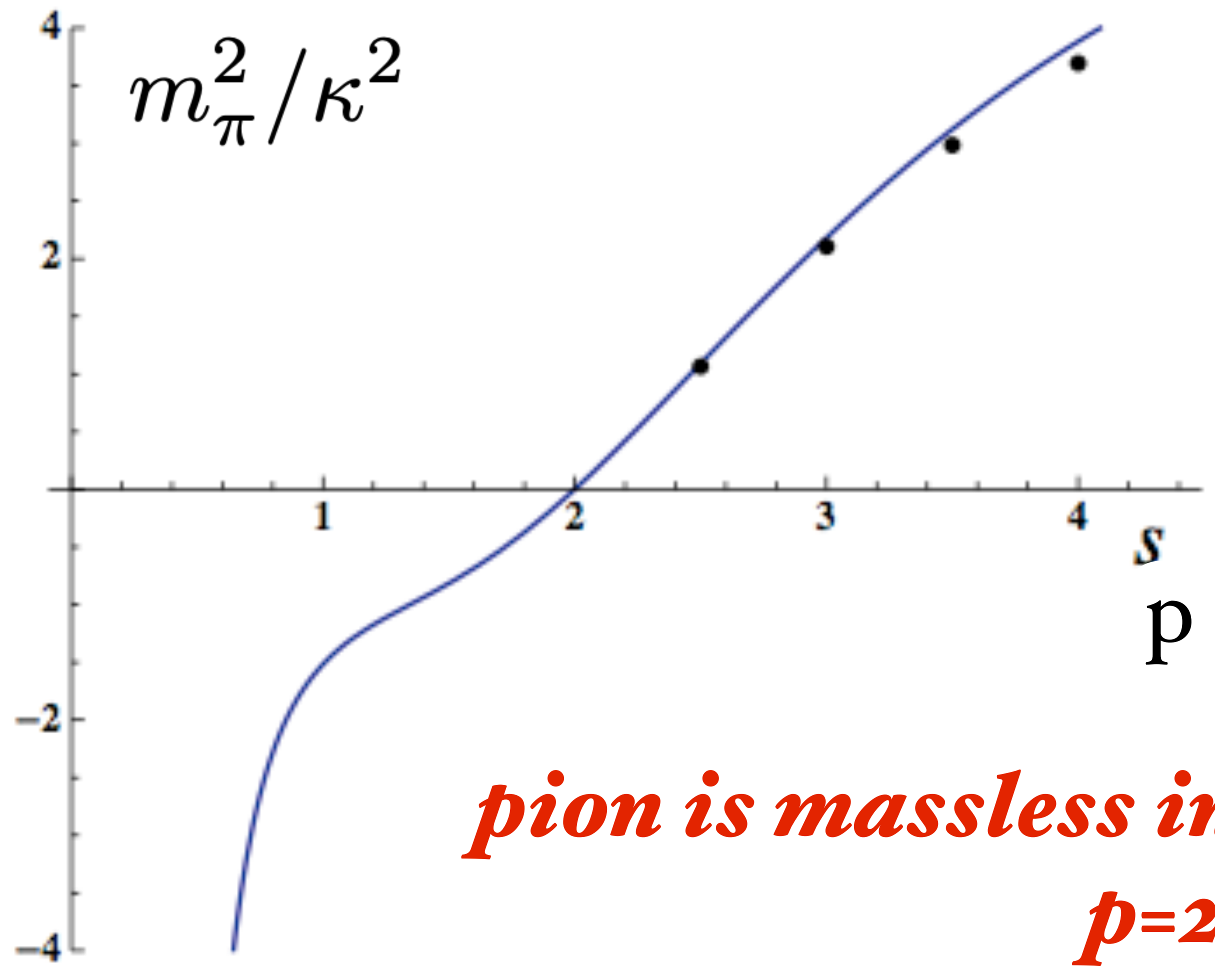
G. de Teramond, H. G. Dosch, sjb

Uniqueness of Dilaton

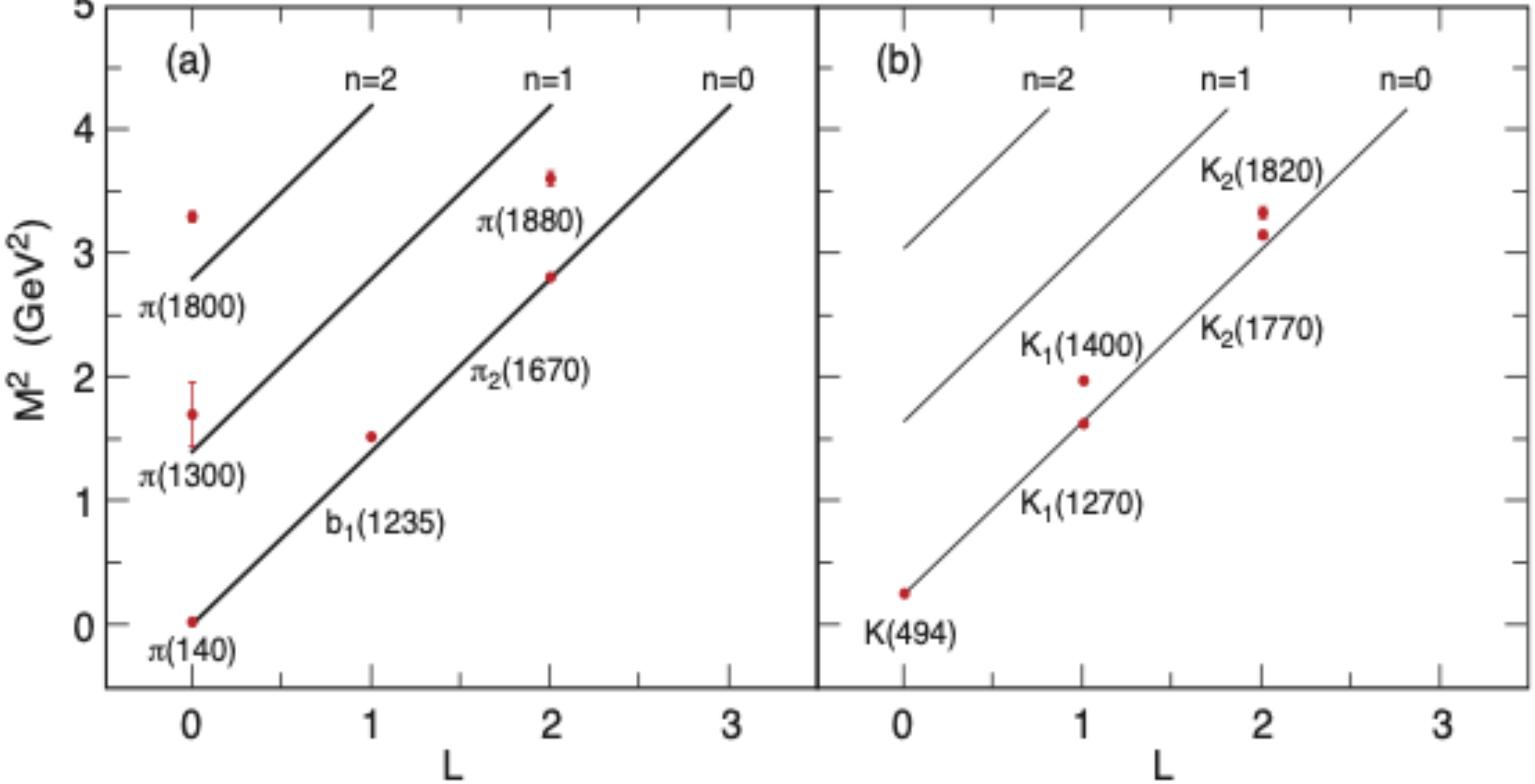
$$\varphi_p(z) = \kappa^p z^p$$

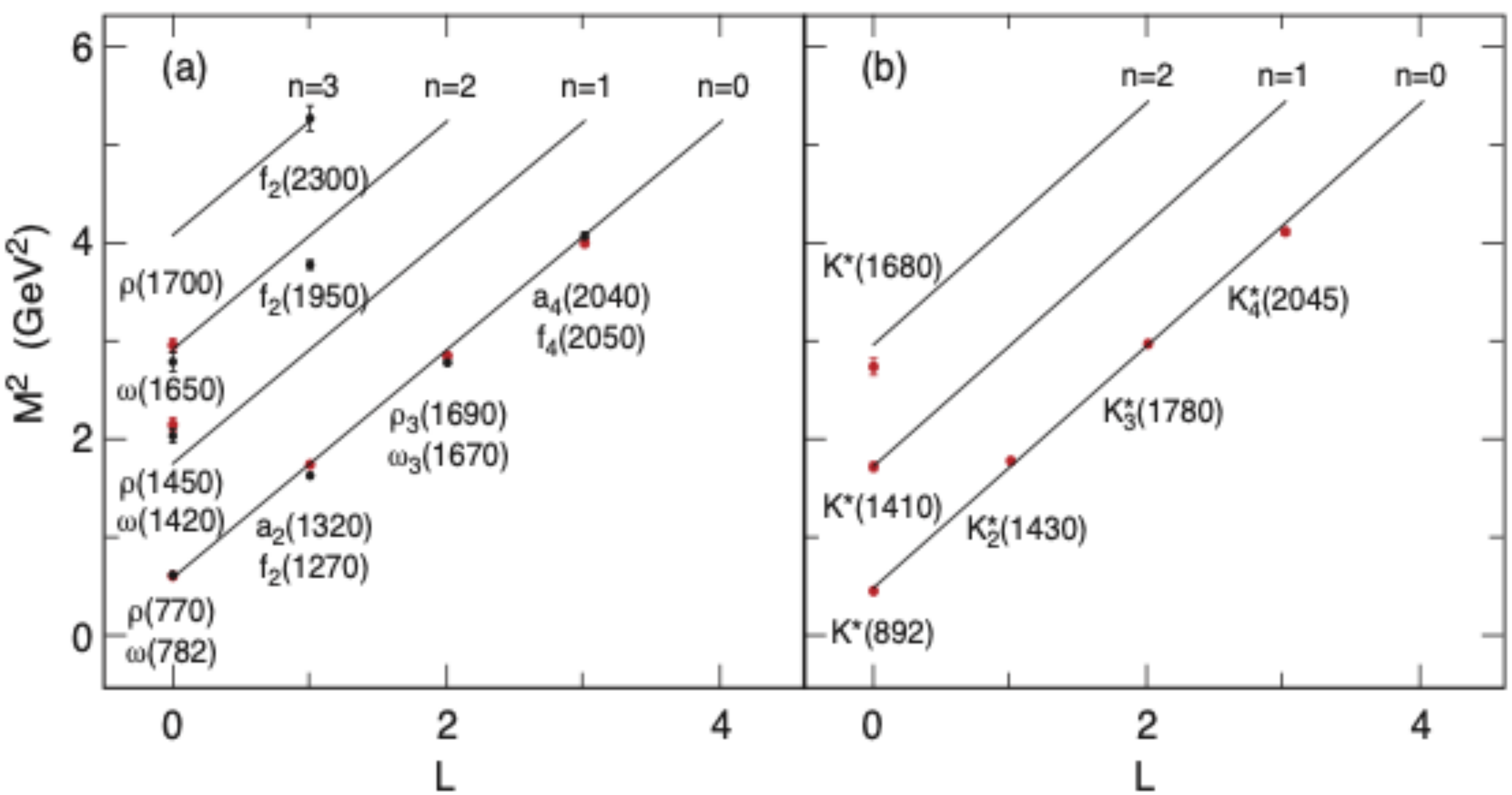


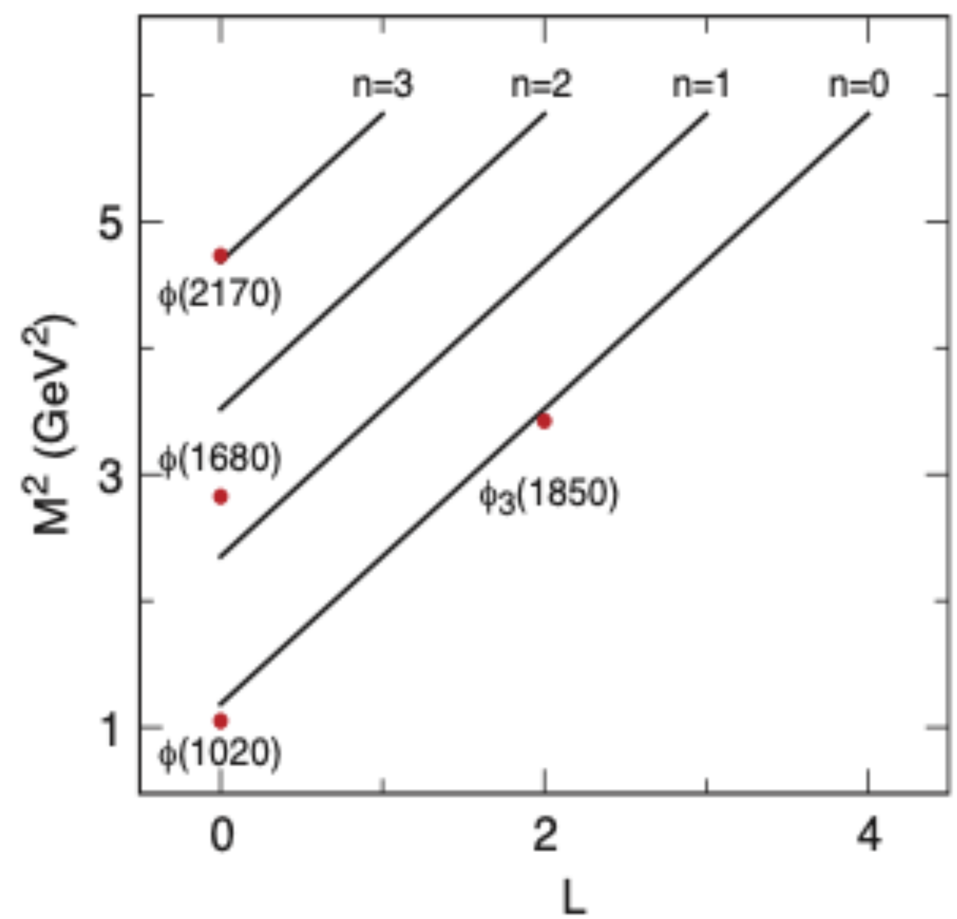
pion is massless in chiral limit iff
p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

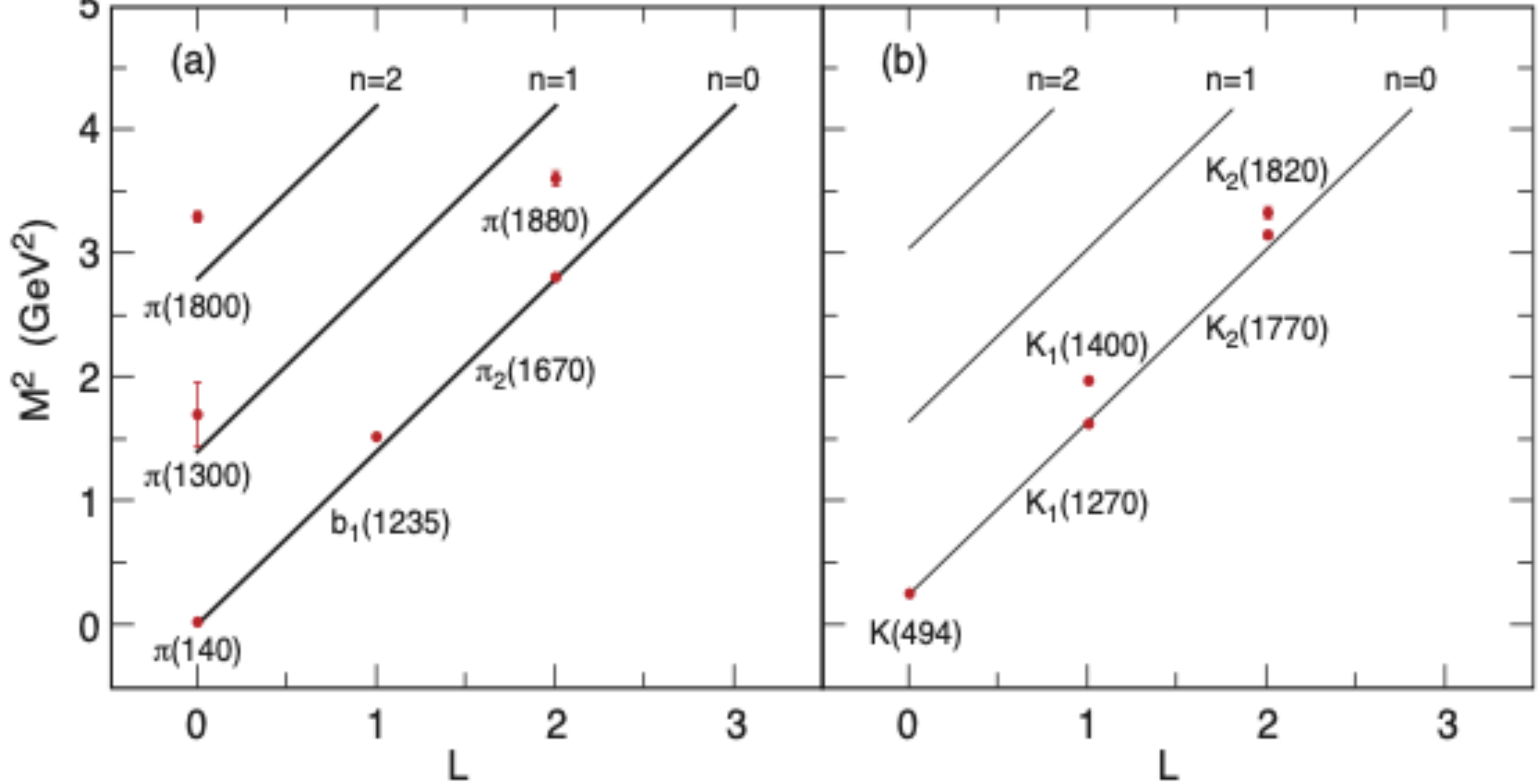


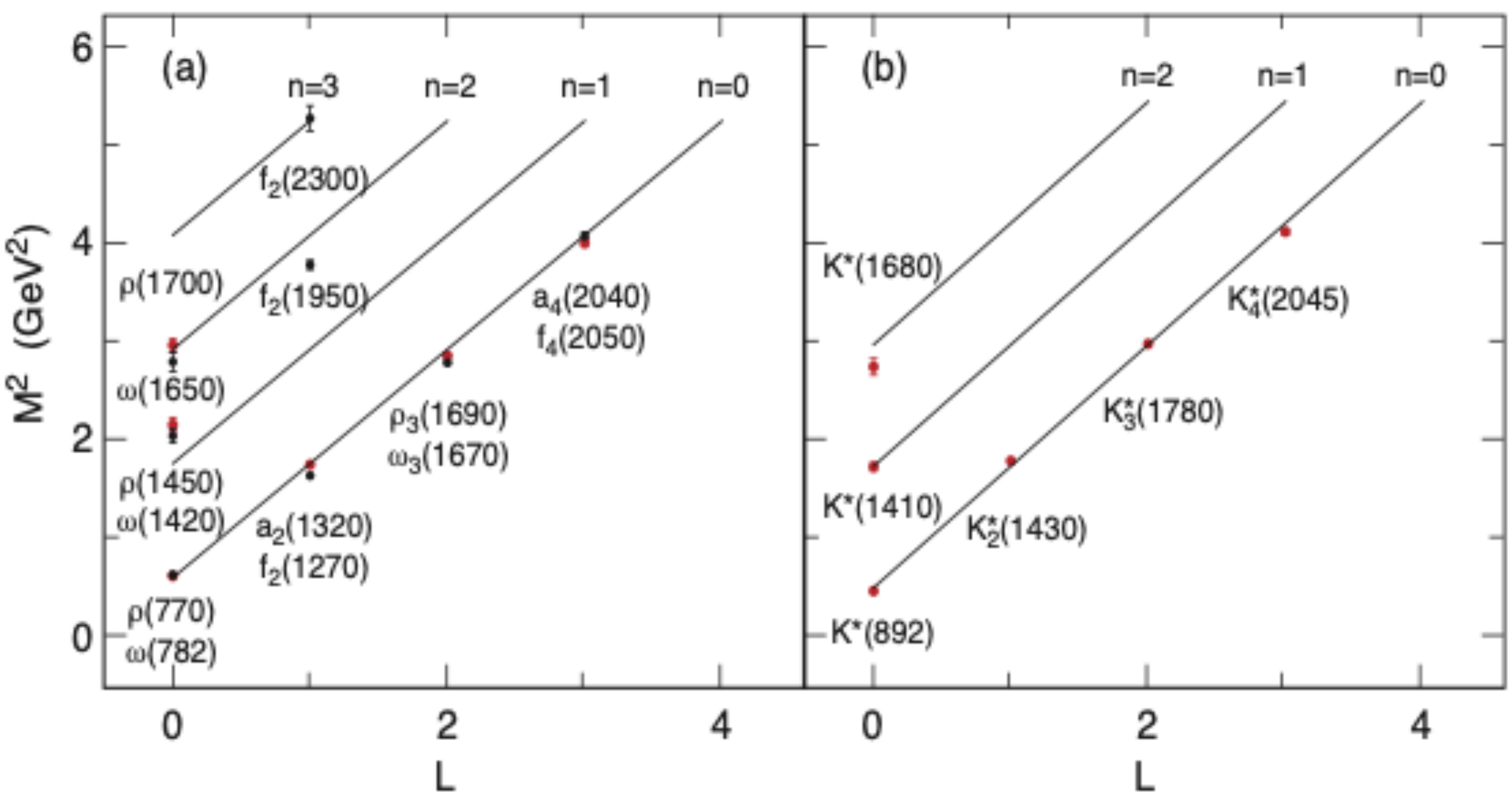


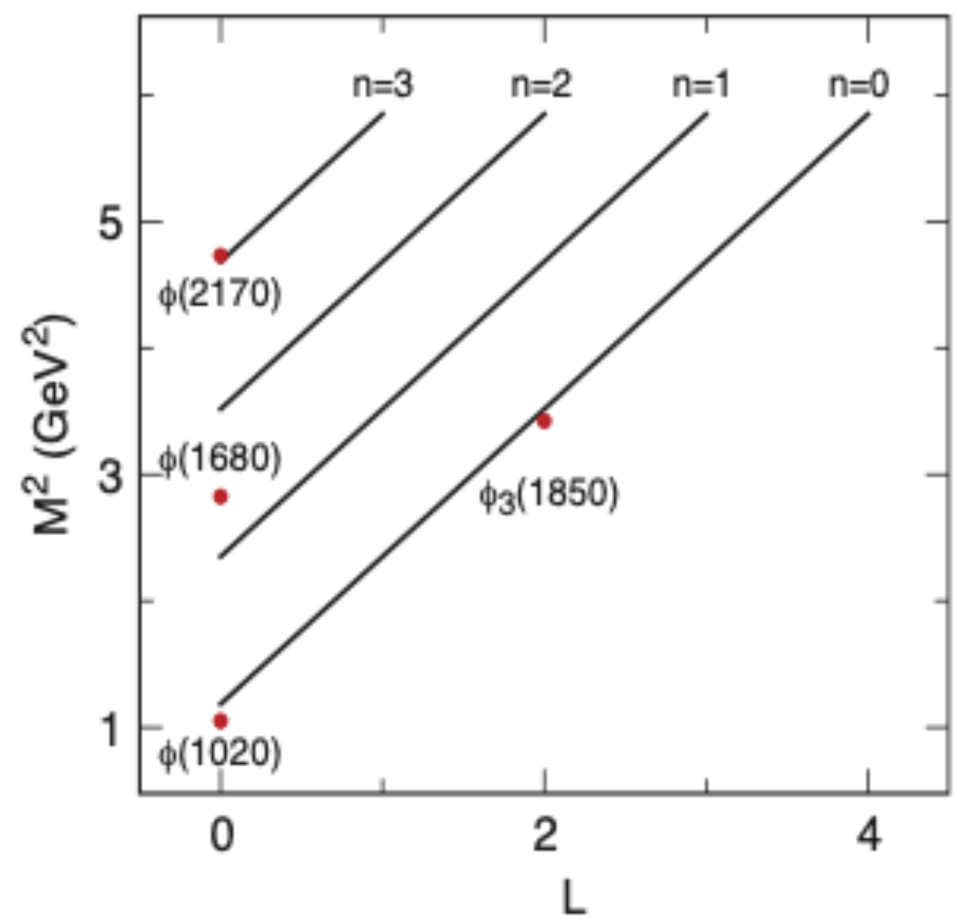


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L







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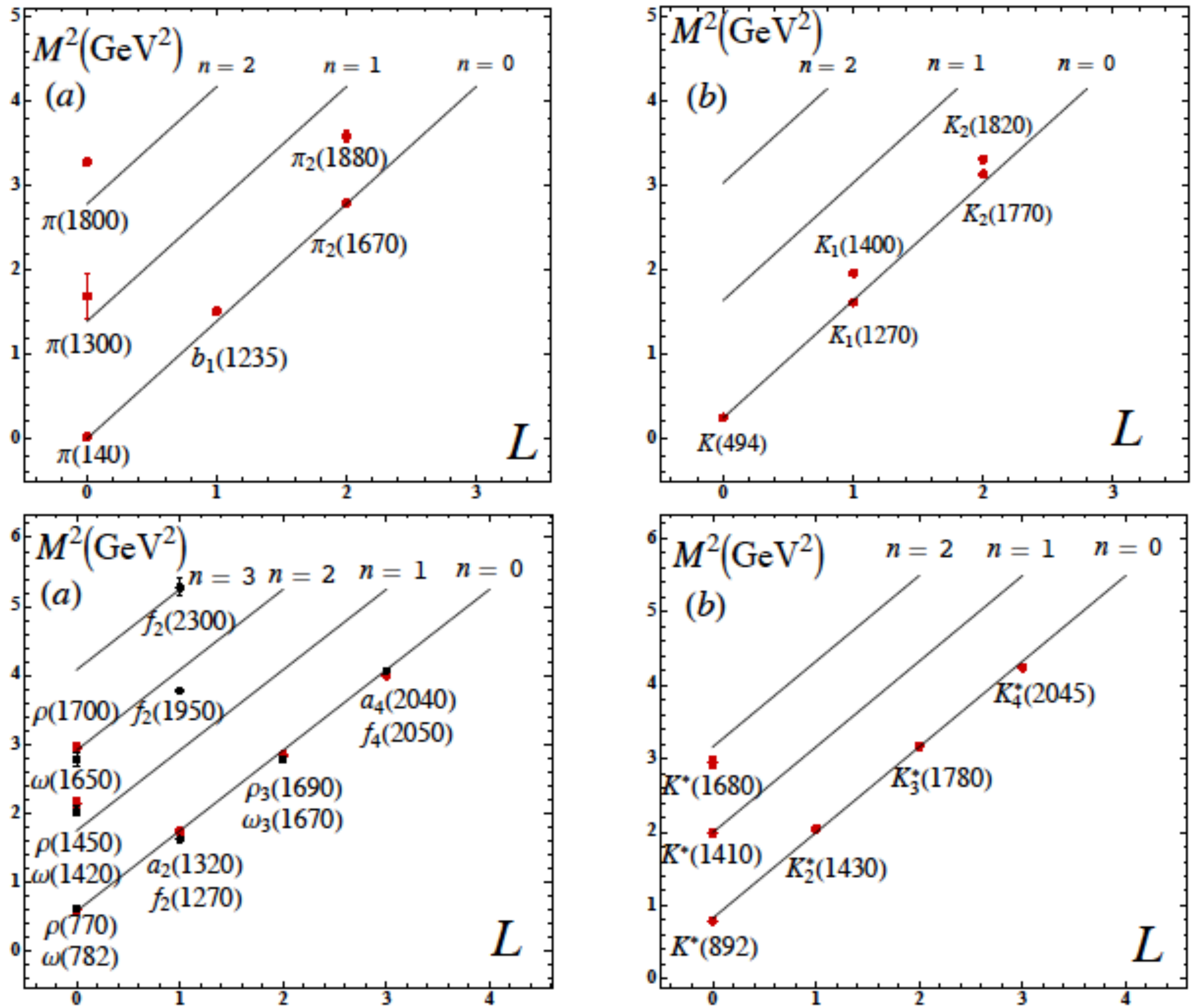
Equal Slope in n and L

De Tèramond, Dosch, sjb

$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism



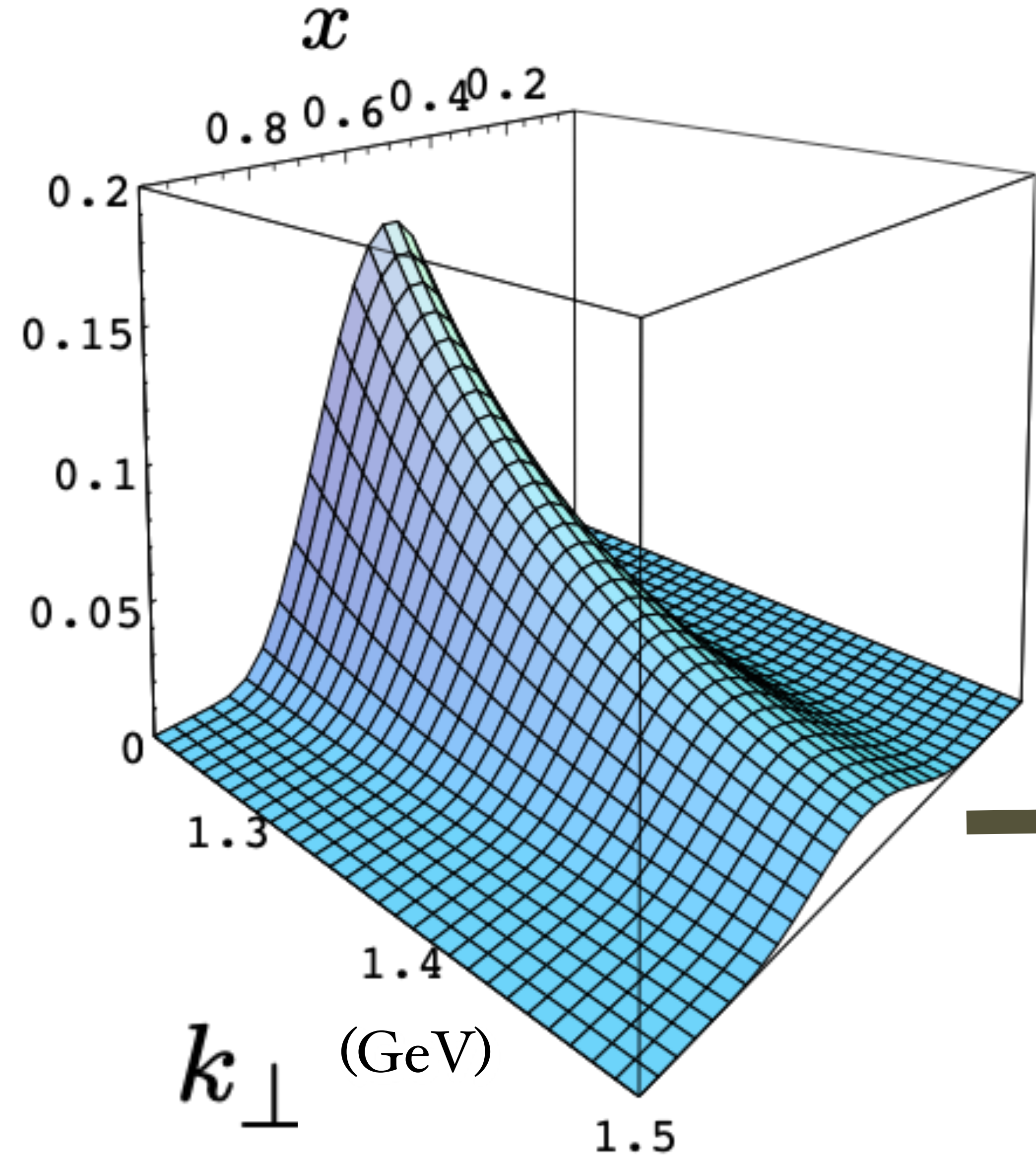
Effective mass from m(p²)

Roberts, et al.

Prediction from AdS/QCD: Meson LFWF

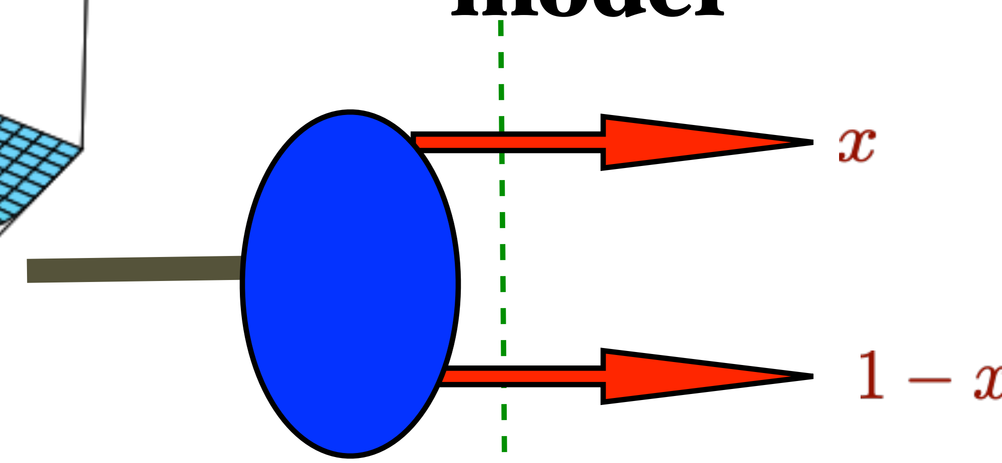
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \qquad \phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

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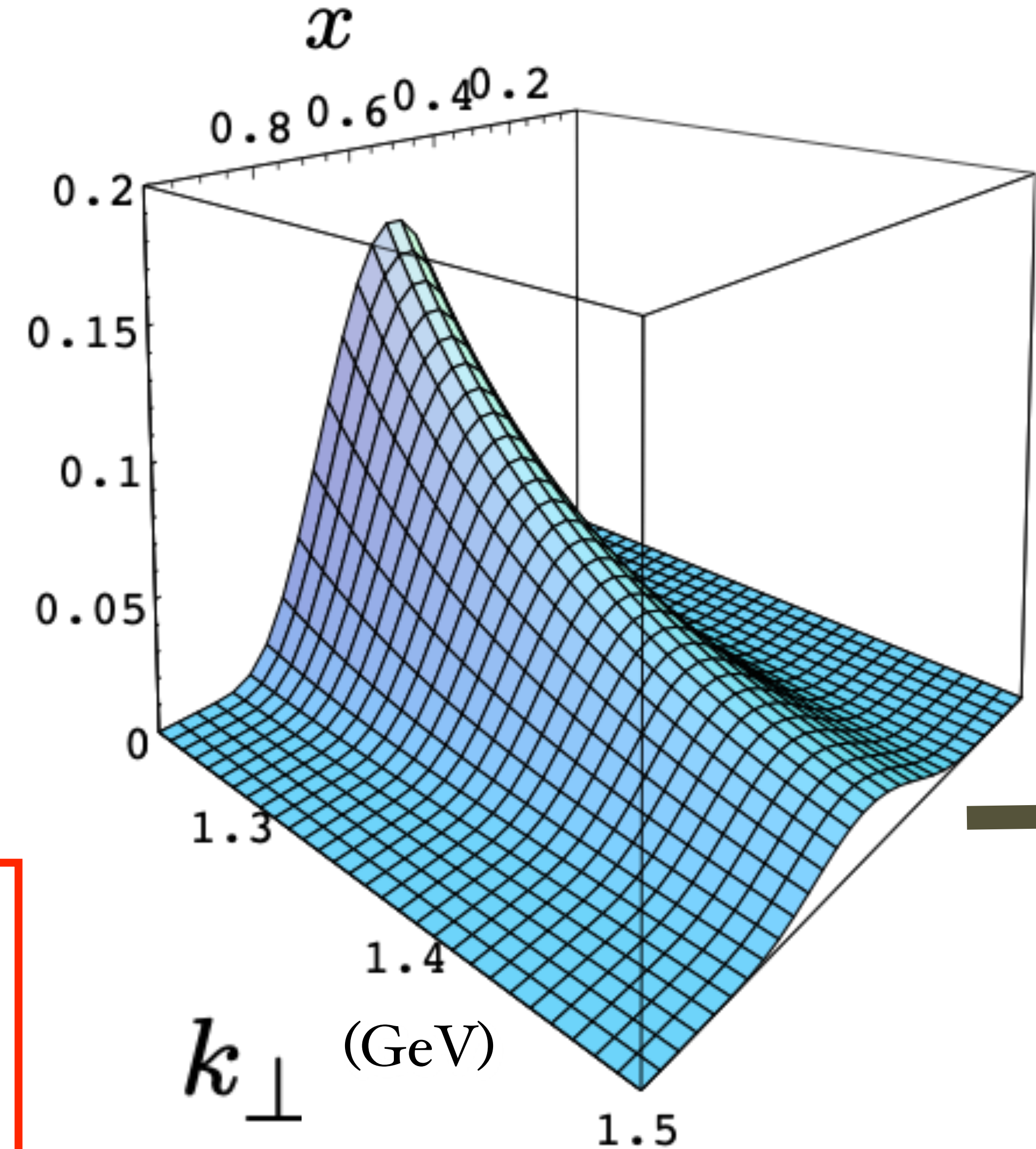

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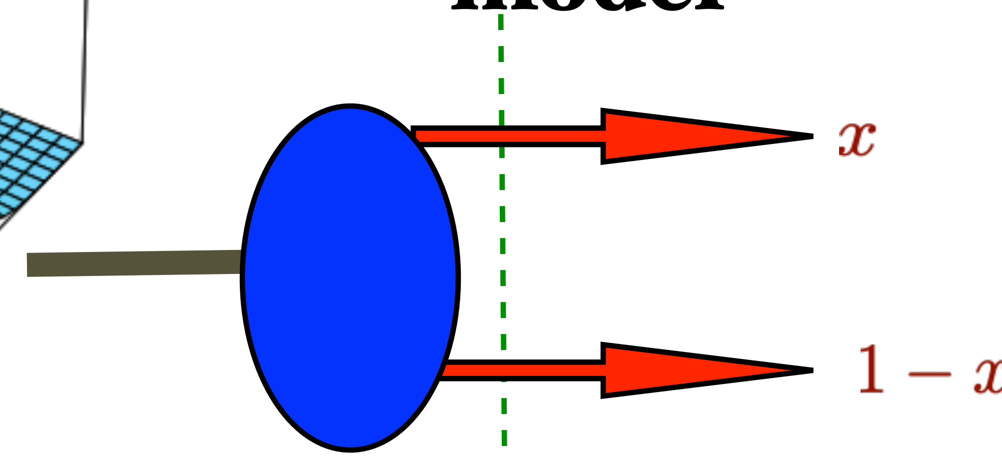

massless quarks

Note coupling

$$k_\perp^2, x$$

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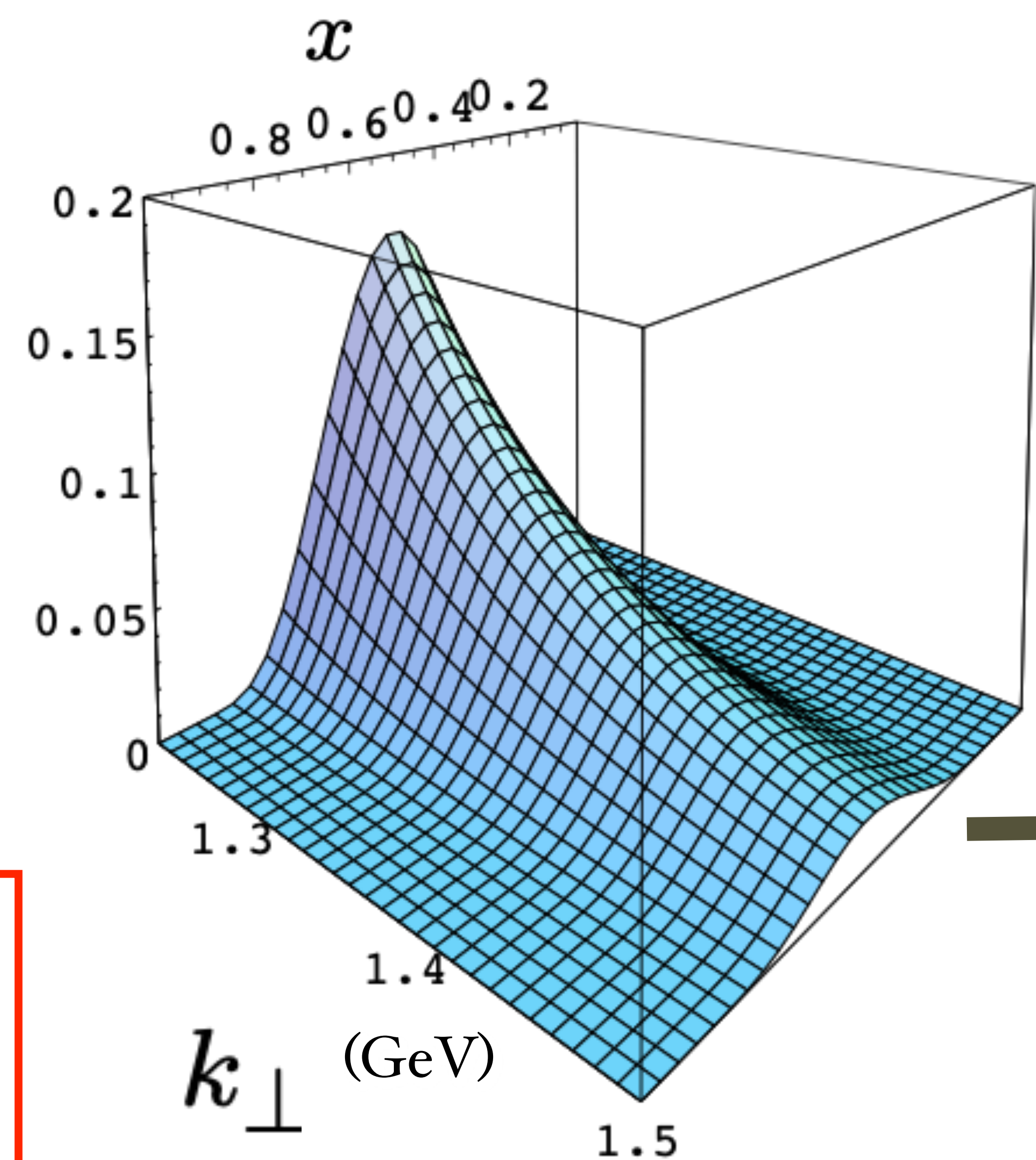

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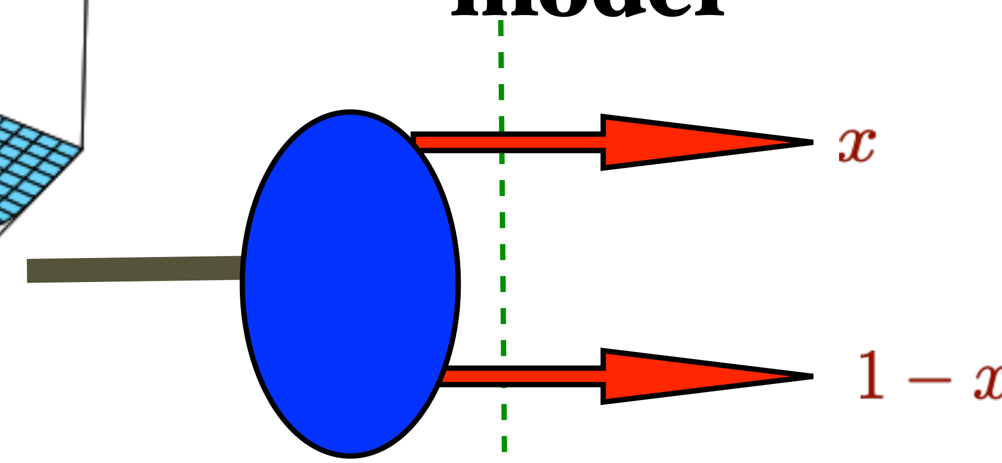

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Provides Connection of Confinement to Hadron Structure

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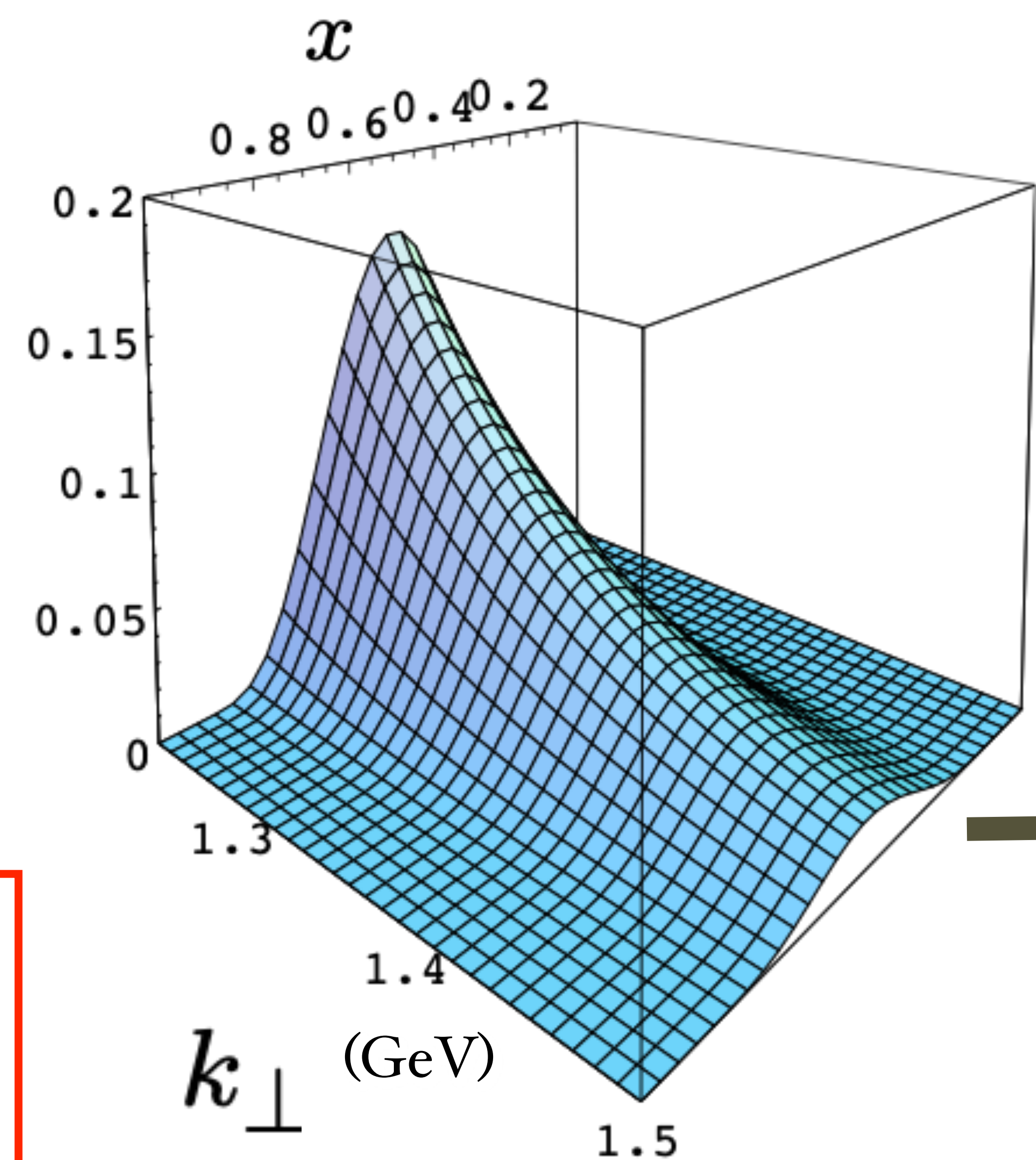

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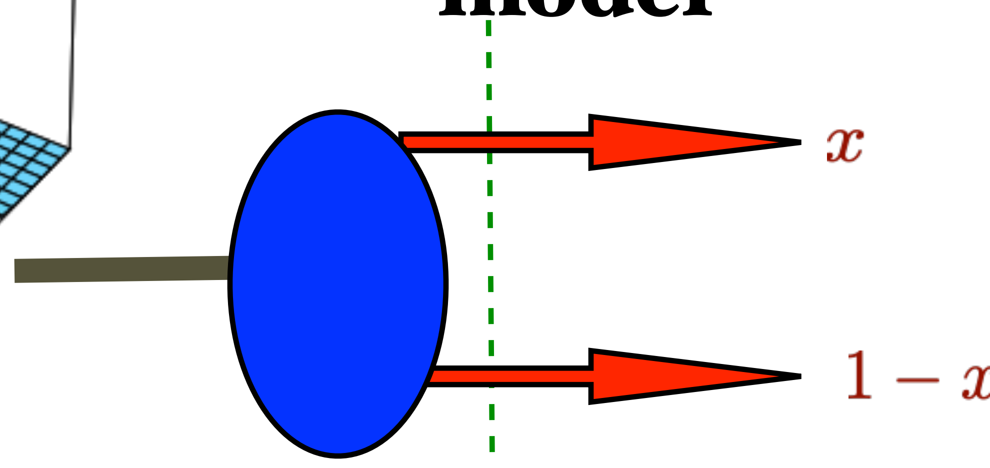

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Provides Connection of Confinement to Hadron Structure

Hadron Distribution Amplitudes

$$\phi(x_i, Q) \equiv \Pi_{i=1}^{n-1} \int^Q d^2\vec{k}_\perp \;\; \psi_n(x_i, \vec{k}_{\perp i})$$

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Lepage, SJB
Efremov, Radyushkin

ERBL Evolution of Meson Distribution Amplitude

Lepage, SJB
Efremov, Radyushkin

$$x_1 x_2 Q^2 \frac{\partial}{\partial Q^2} \tilde{\phi}(x_i, Q)$$

$$= C_F \frac{\alpha_s(Q^2)}{4\pi} \left\{ \int_0^1 [dy] V(x_i, y_i) \tilde{\phi}(y_i, Q) - x_1 x_2 \tilde{\phi}(x_i, Q) \right\},$$

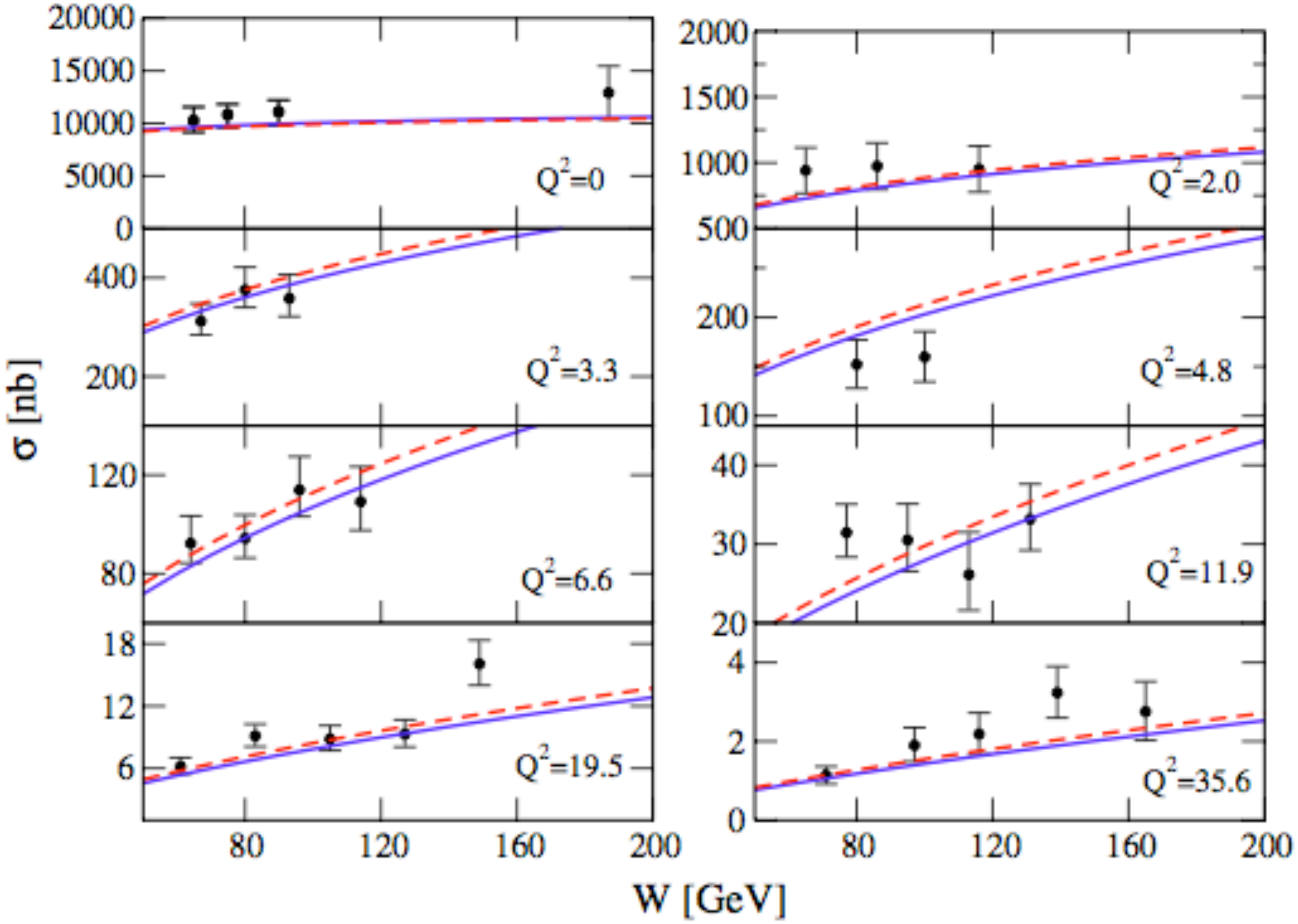
where

$$V(x_i, y_i) = 2\left[x_1 y_2 \theta(y_1 - x_1)\left(\delta_{h_1 \bar{h}_2} + \frac{\Delta}{y_1 - x_1}\right) + (1 \leftrightarrow 2) \right]$$

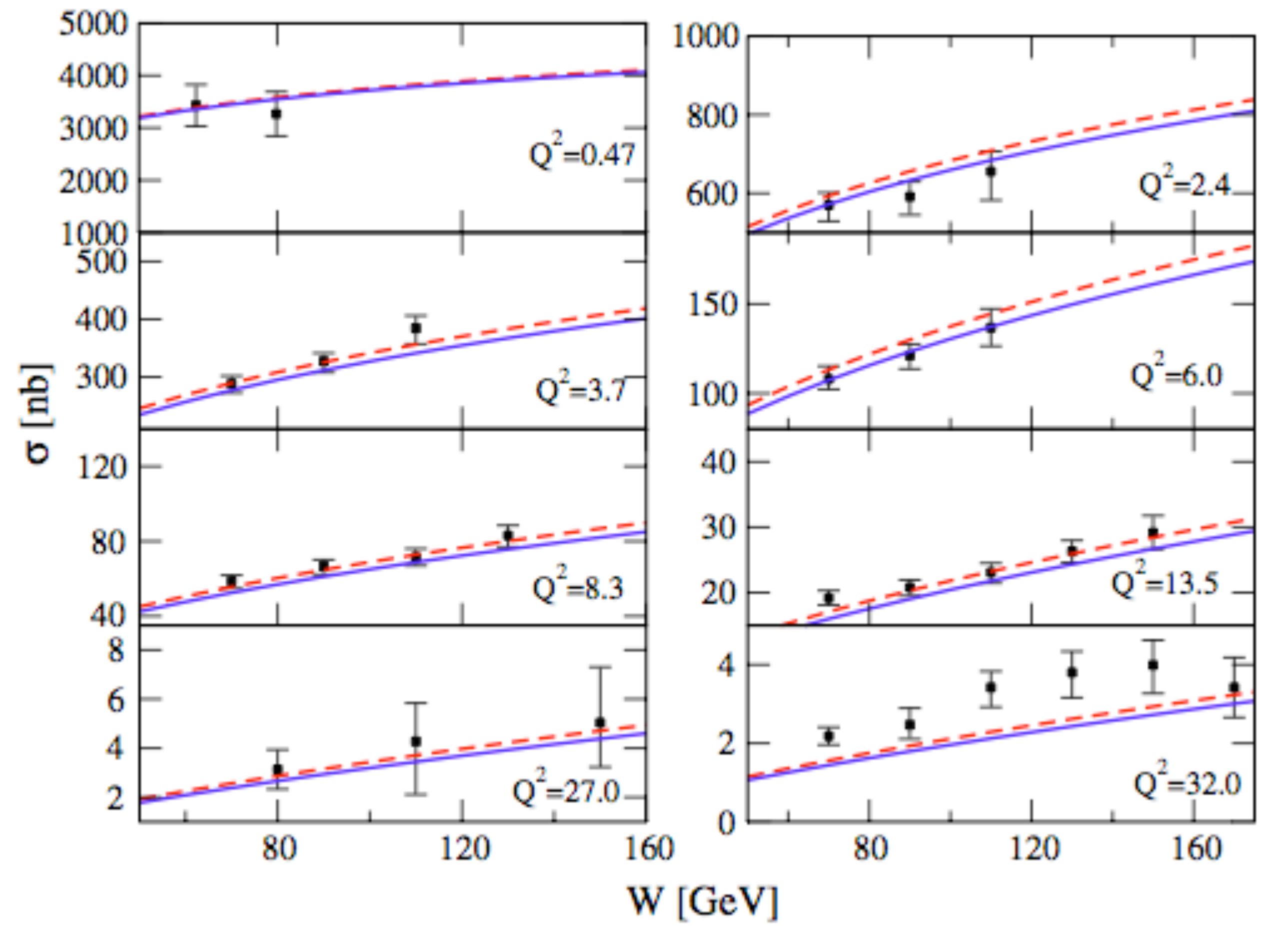
$$\lim_{Q^2 \to \infty} \phi_M(x, Q^2) \to C x(1 - x)$$

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



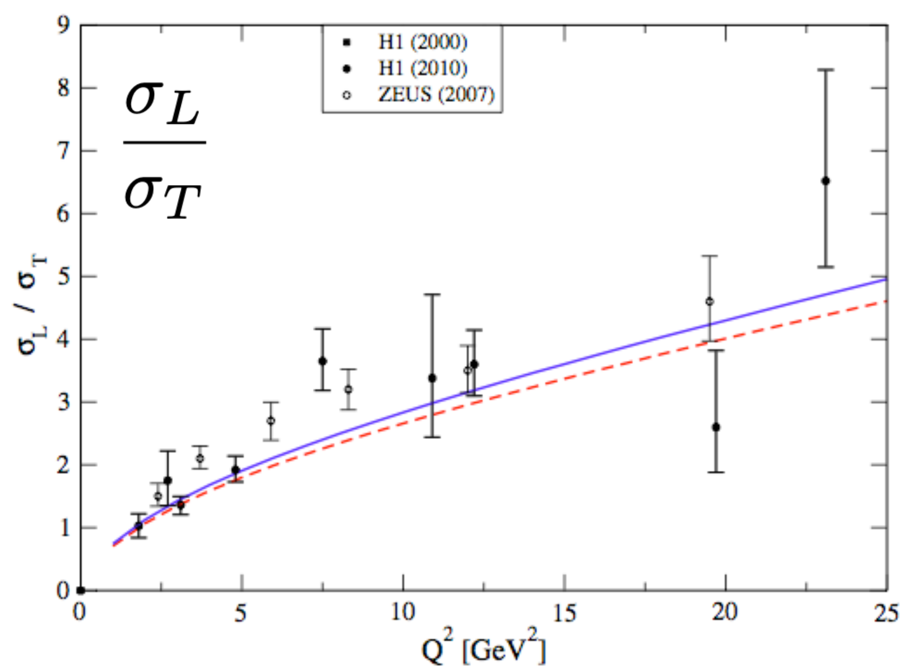
(a) H1



(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



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Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

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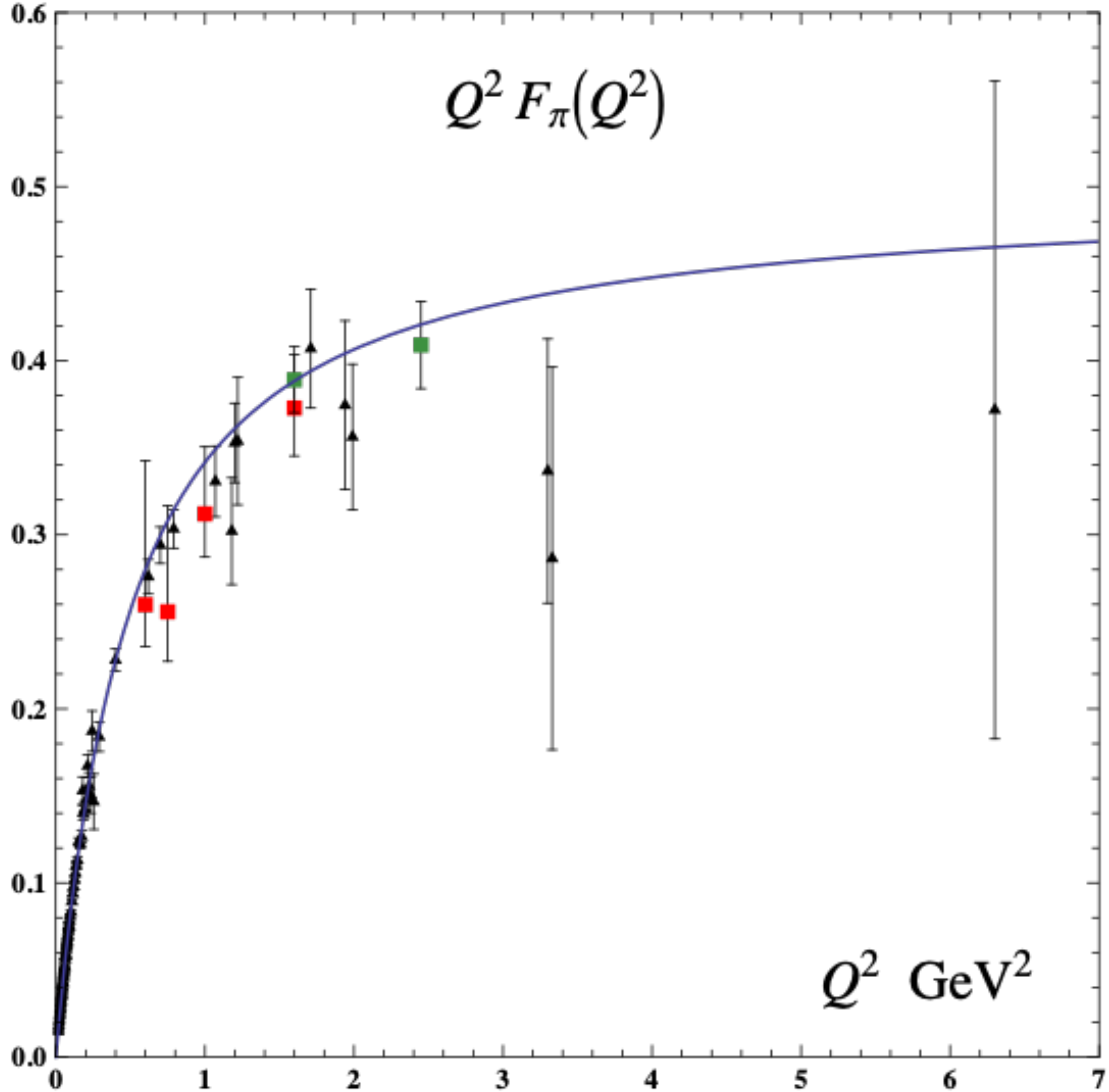
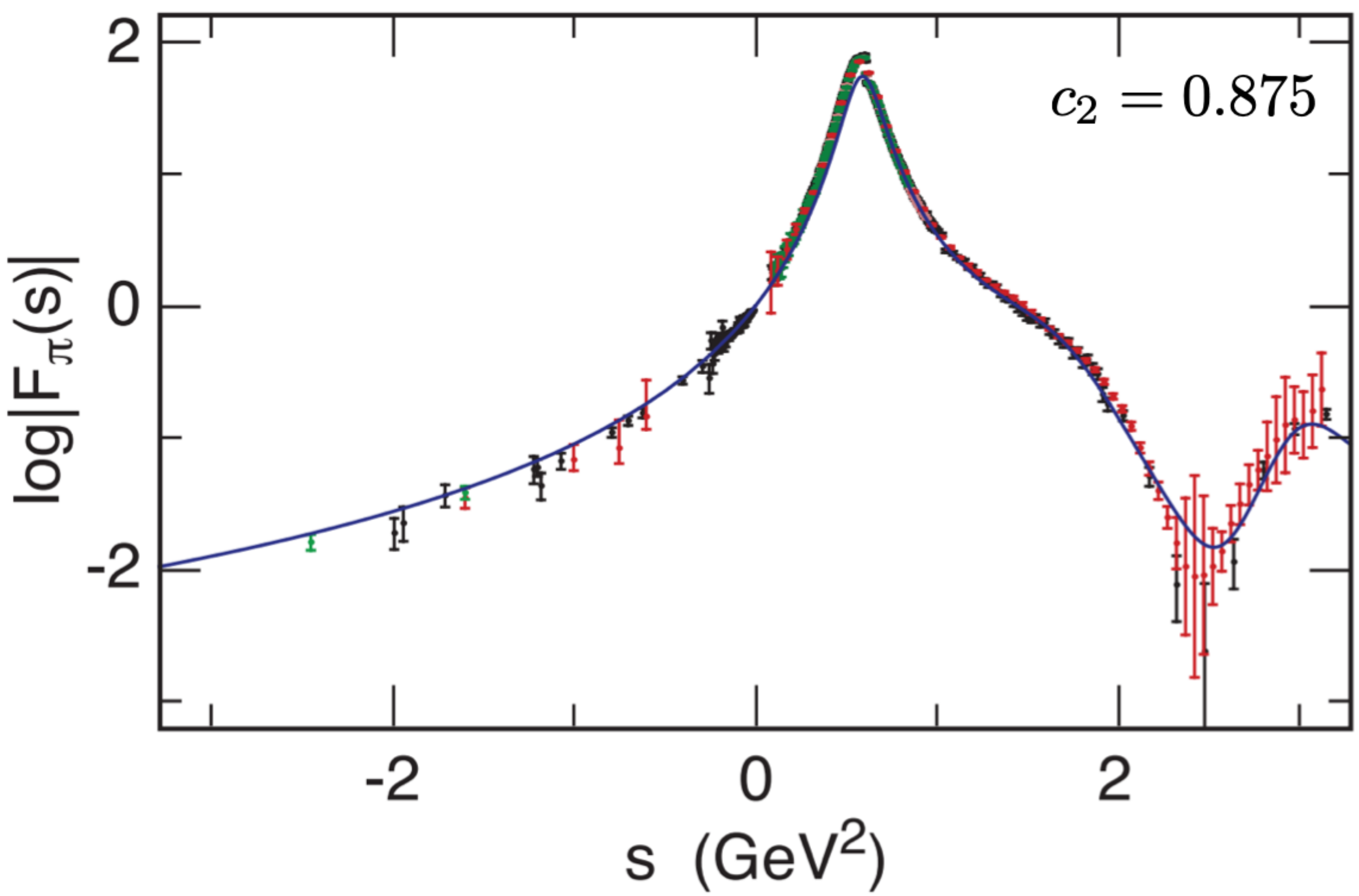


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Pion EM Form Factor

Pion form factor compared with data



$$F_\pi(t) = \sum_\tau P_\tau F_\tau(t) \qquad \sum_\tau P_\tau = 1$$

Truncated at twist-τ = 4

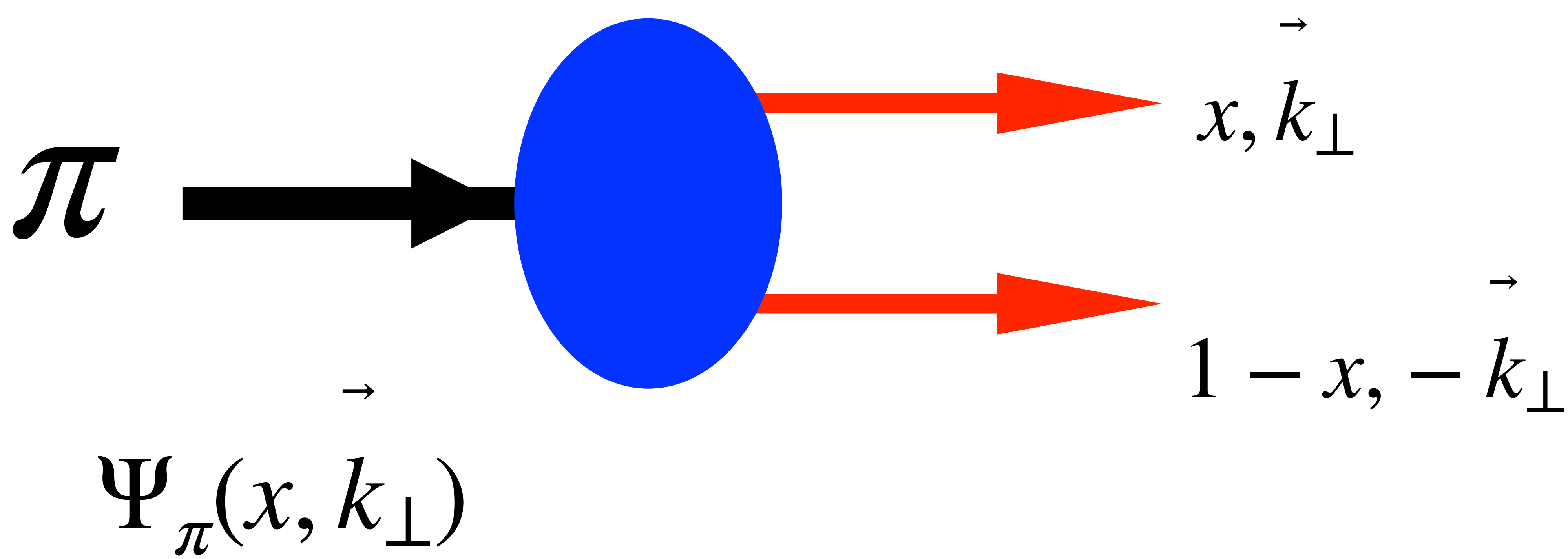
$$F_\pi(t) = c_2 F_{\tau=2}(t) + (1 - c_2) F_{\tau=4}(t)$$

G.F. de Téramond and S.J. Brodsky, Proc. Sci. LC2010 (2010) 029.
S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015). [Sec. 6.1.5]

The Pion's Valence Light-Front Wavefunction

- Relativistic Quantum-Mechanical Wavefunction of the pion eigenstate

- Independent of the observer's or pion's motion
- No Lorentz contraction; causal
- ***Confined*** quark-antiquark bound state



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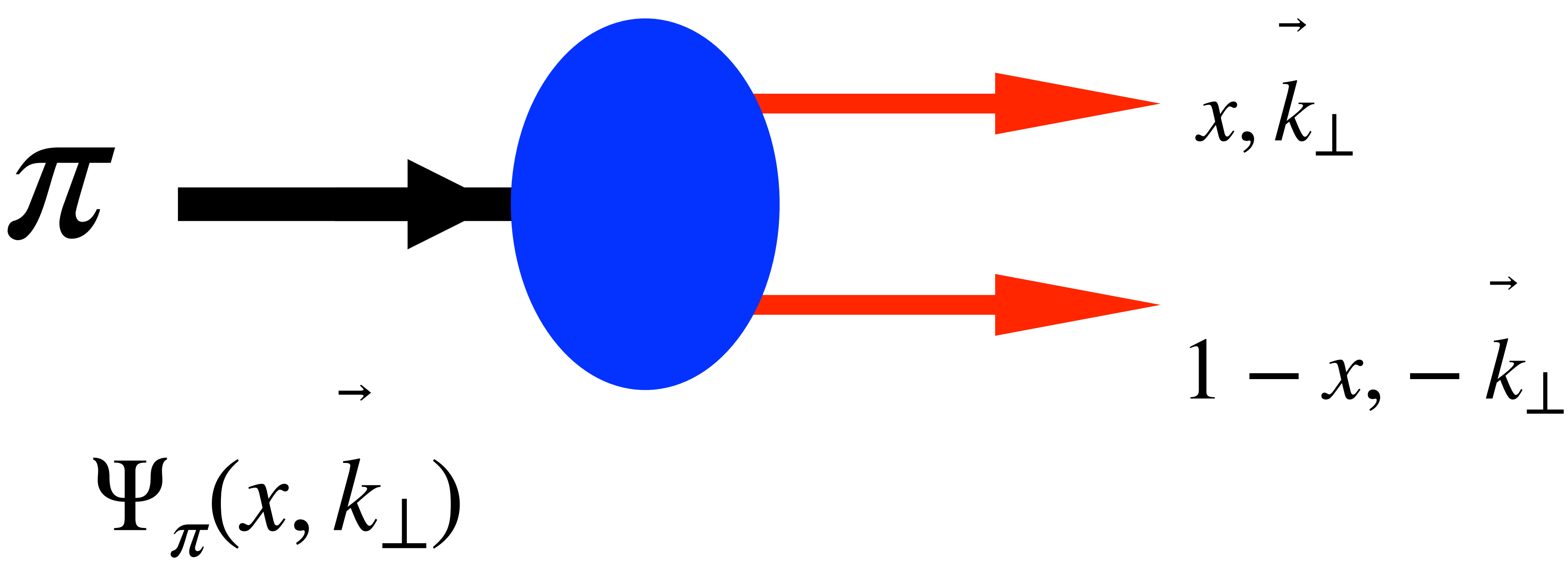

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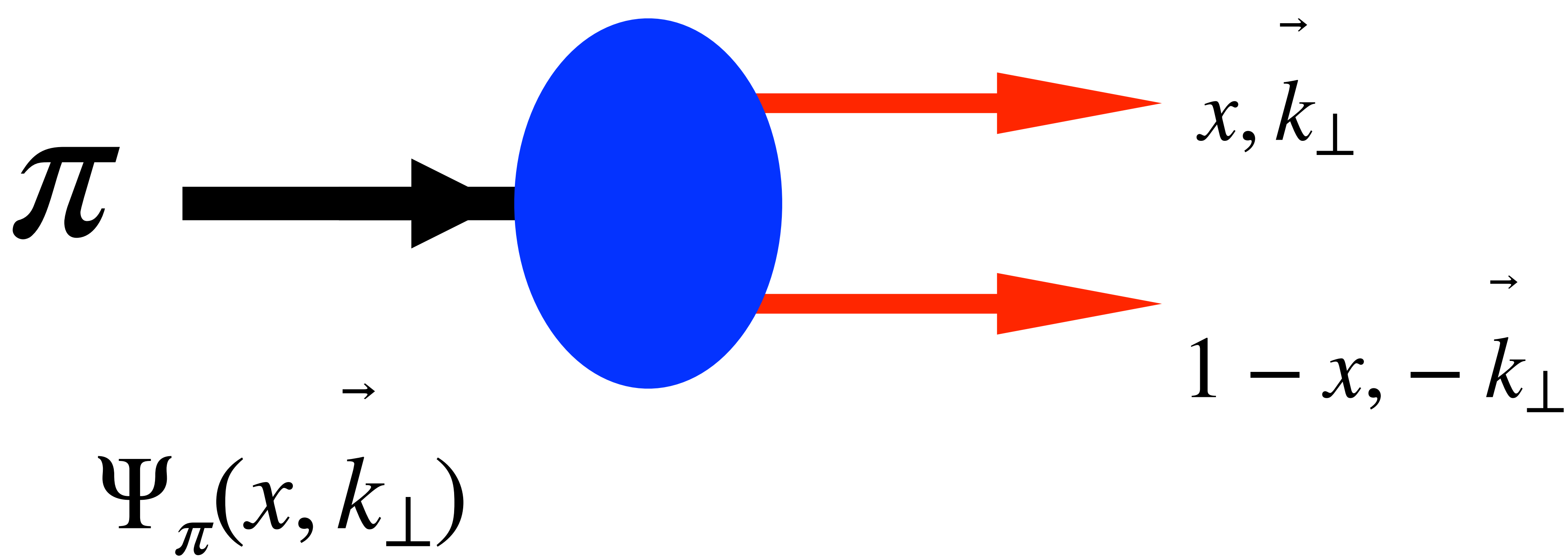

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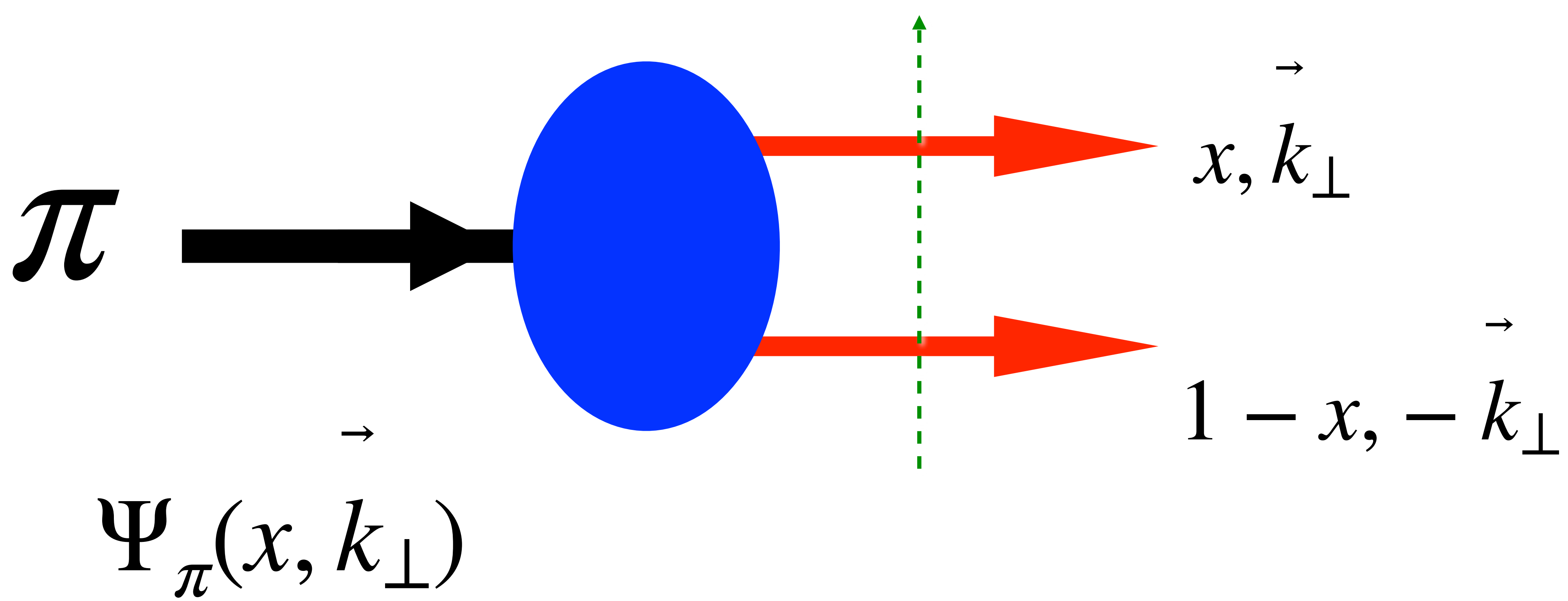

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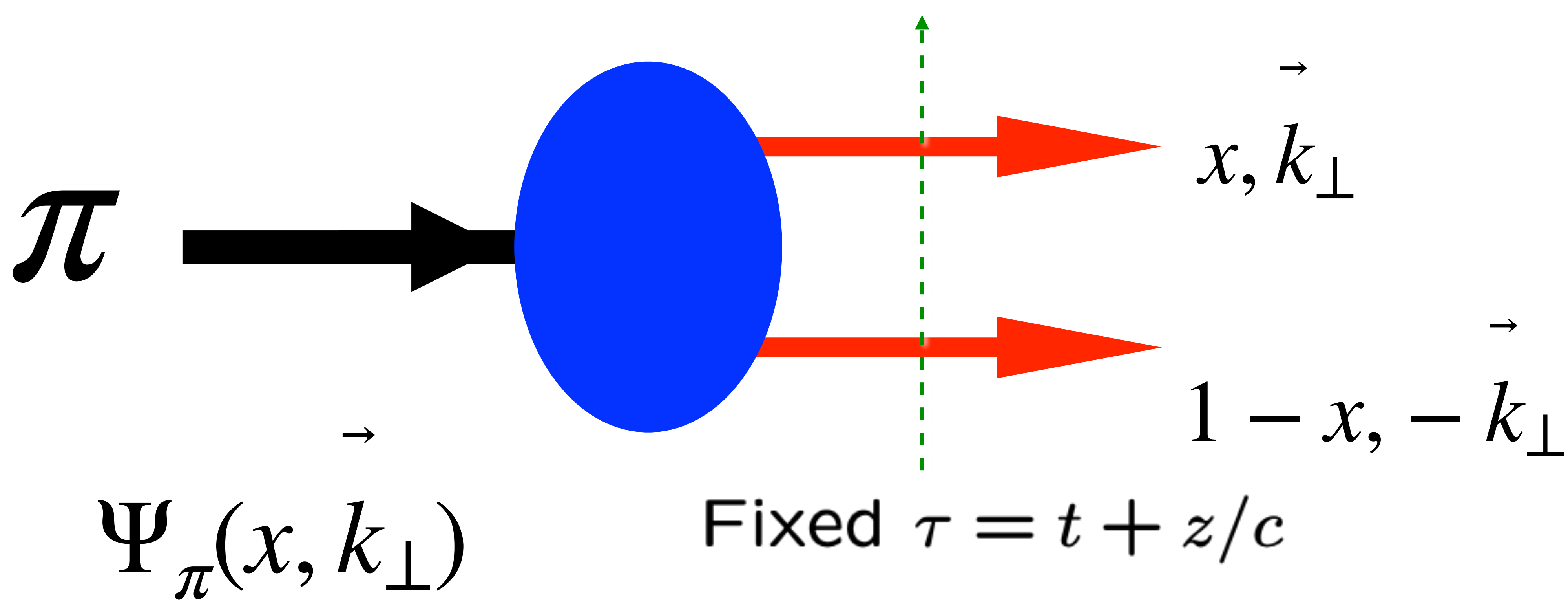
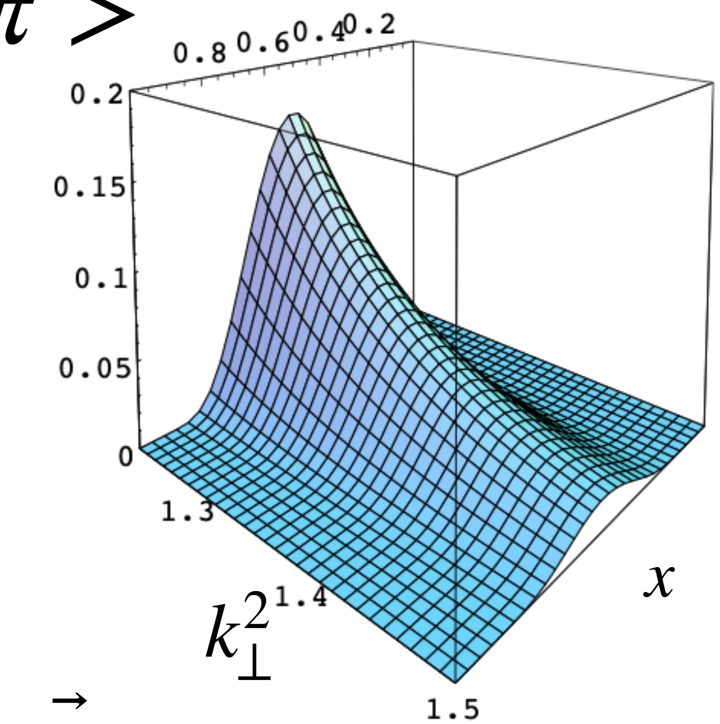


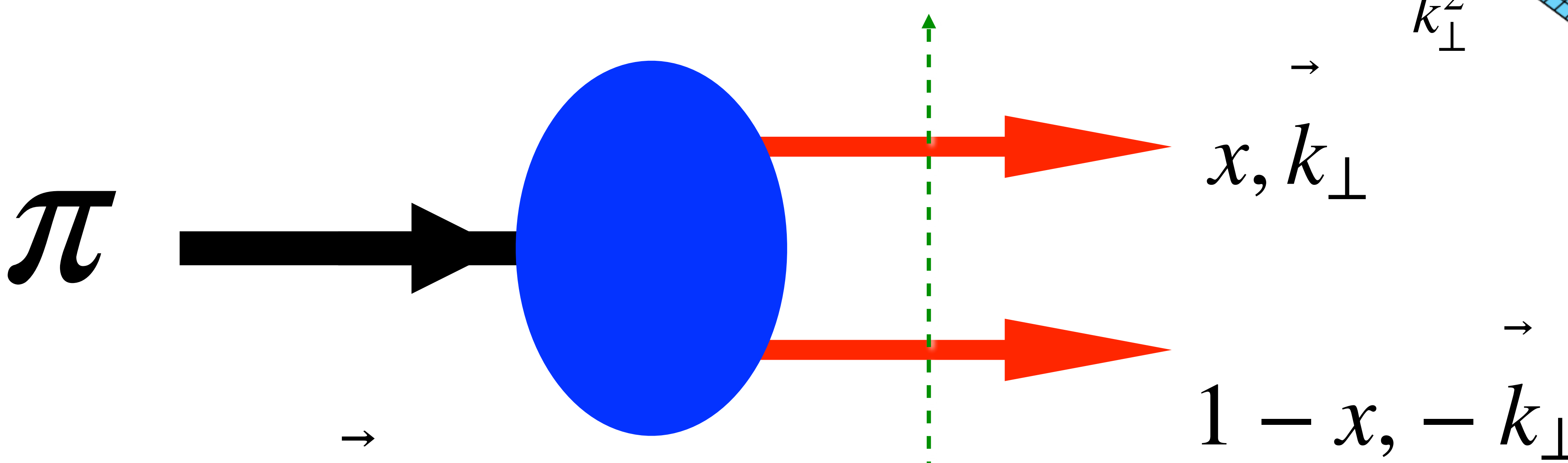
The Pion's Valence Light-Front Wavefunction

- Relativistic Quantum-Mechanical Wavefunction of the pion eigenstate $H^{QCD}_{LF} | \pi > = m^2_\pi | \pi >$

$$\Psi_\pi(x, \vec{k}_\perp) = < q(x, \vec{k}_\perp) \, \bar{q} \, (1 - x, -\vec{k}_\perp) \, | \pi >$$



- Independent of the observer's or pion's motion
- No Lorentz contraction; causal
- ***Confined*** quark-antiquark bound state

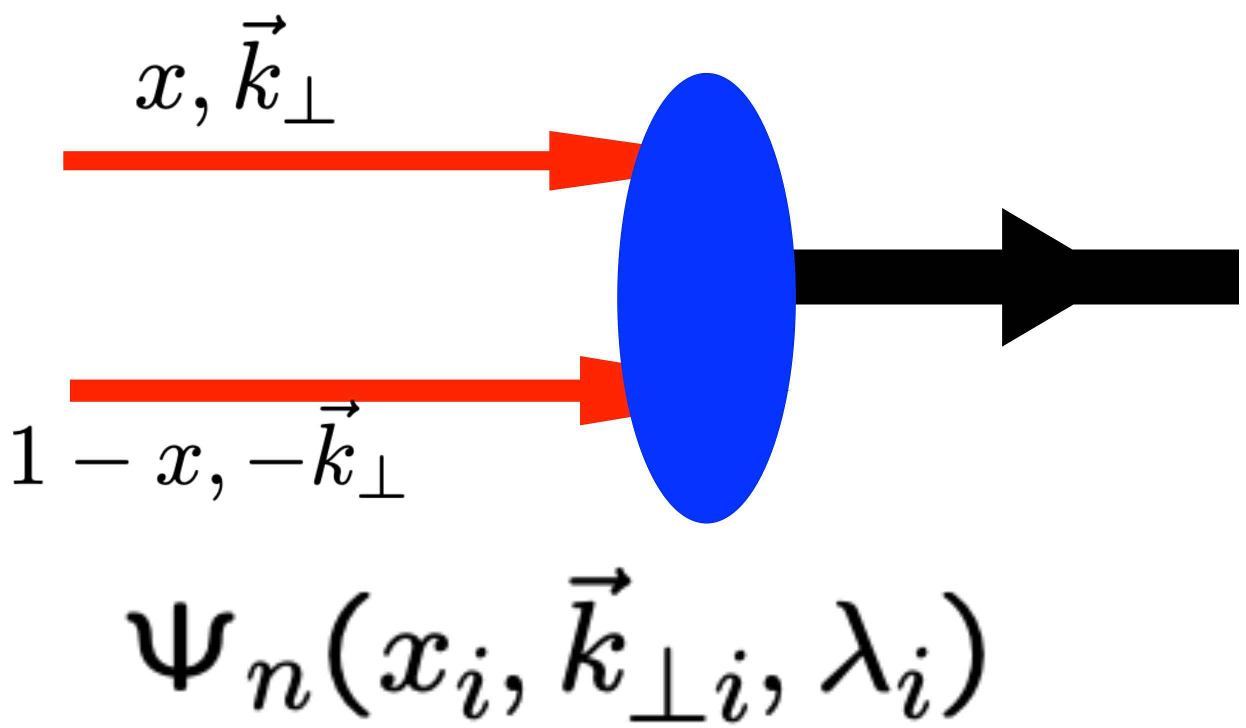


$\Psi_\pi(x, \vec{k}_\perp)$

Fixed $\tau = t + z/c$

- Light Front Wavefunctions: $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$



"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

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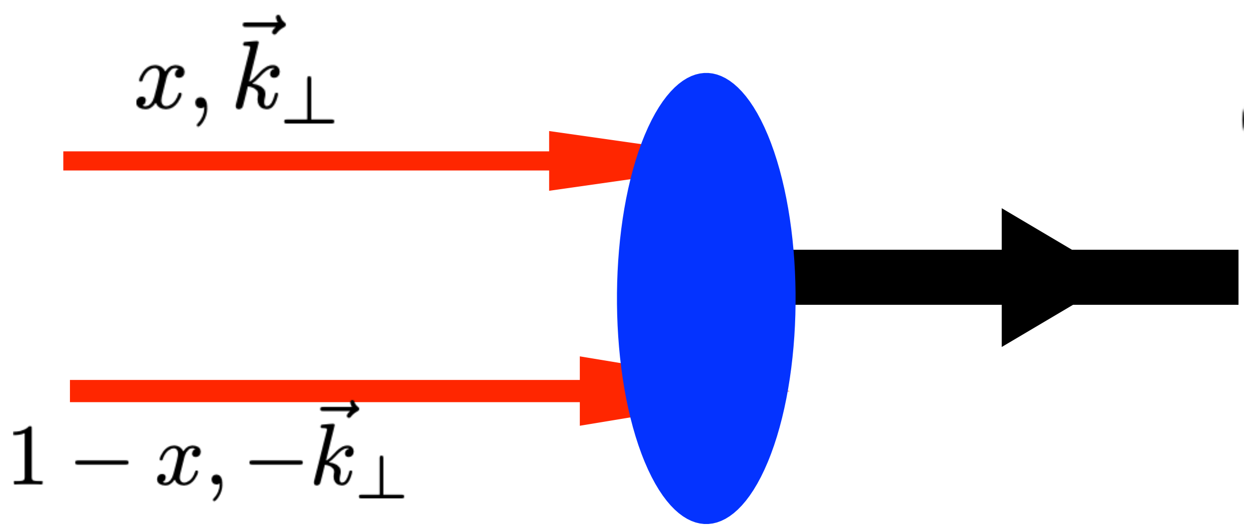
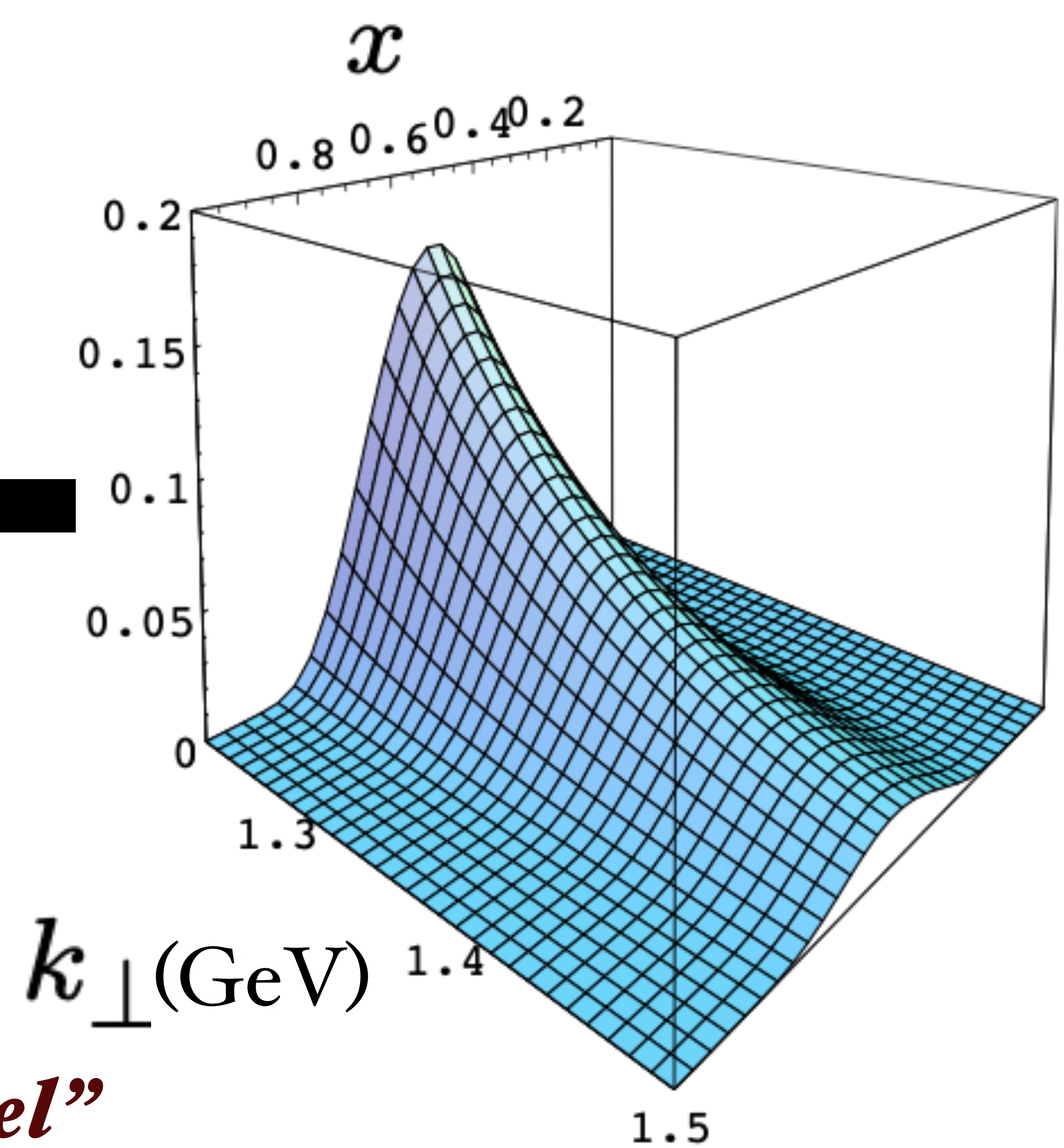


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



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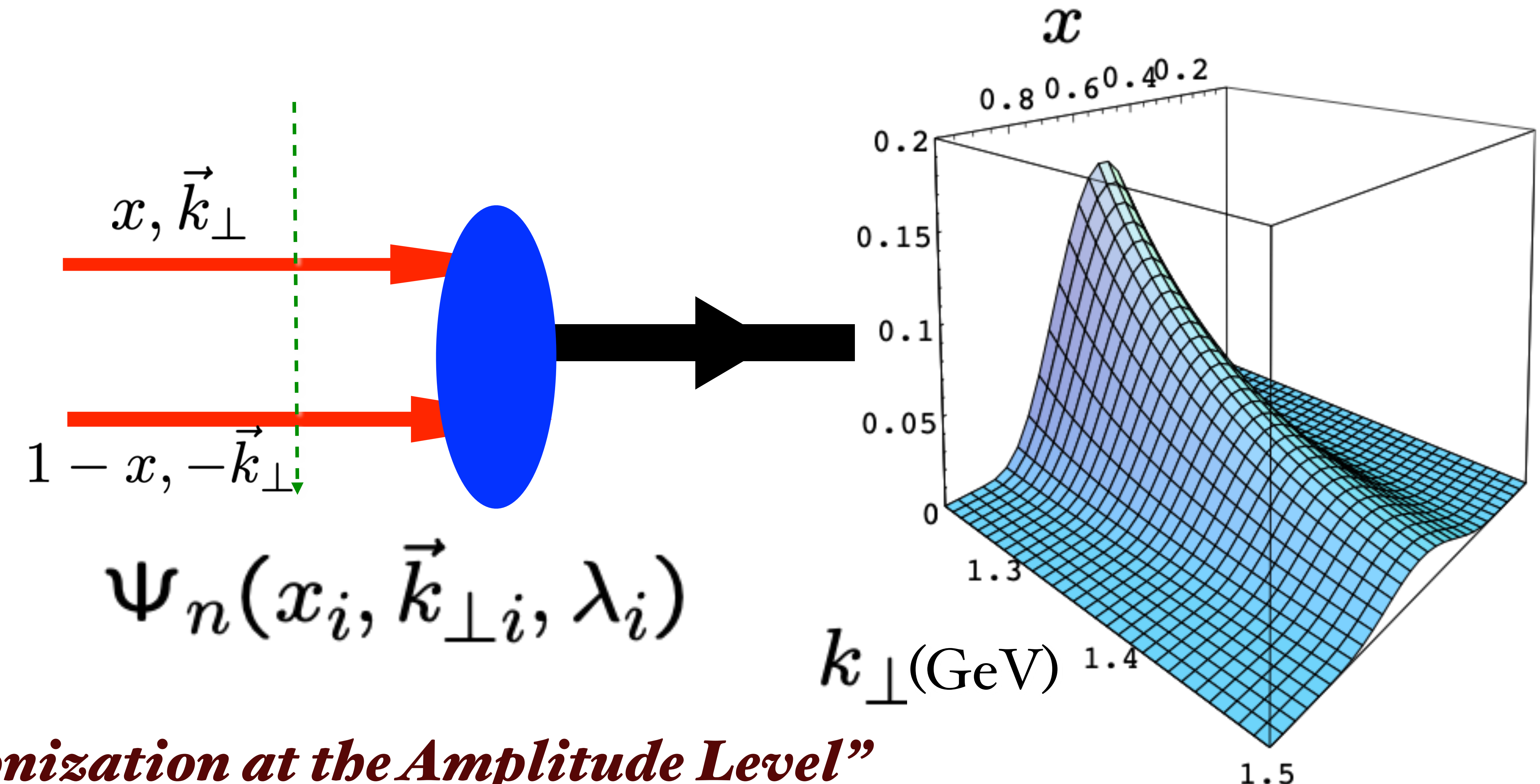

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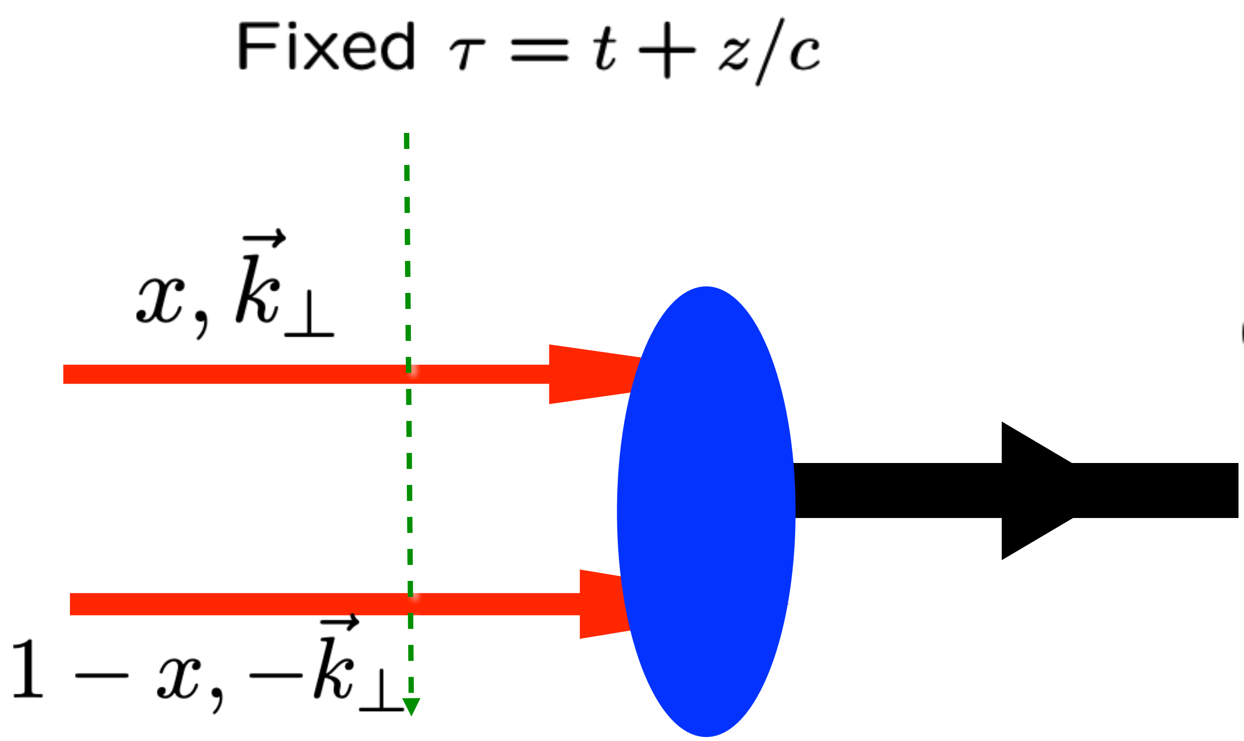

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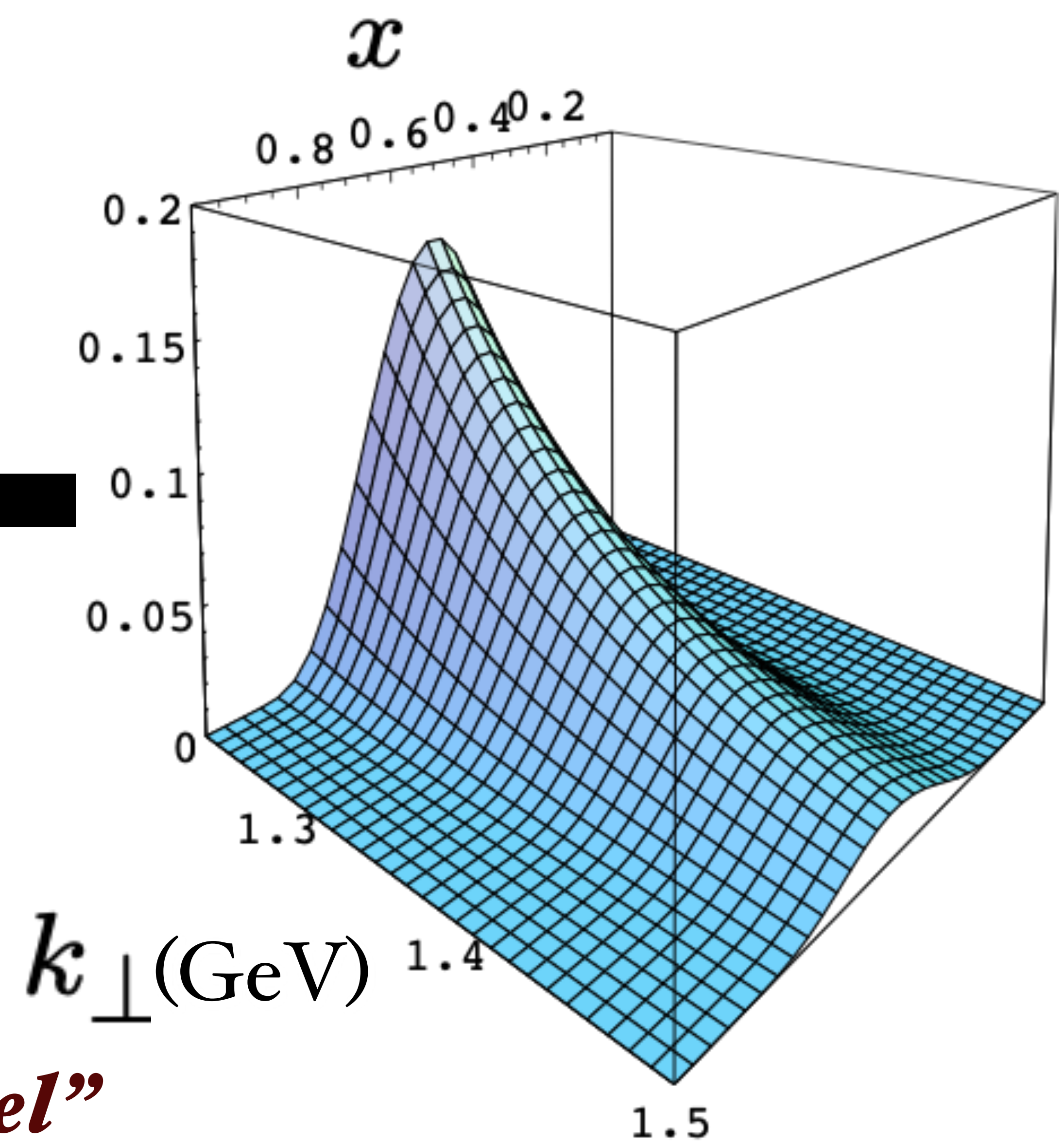

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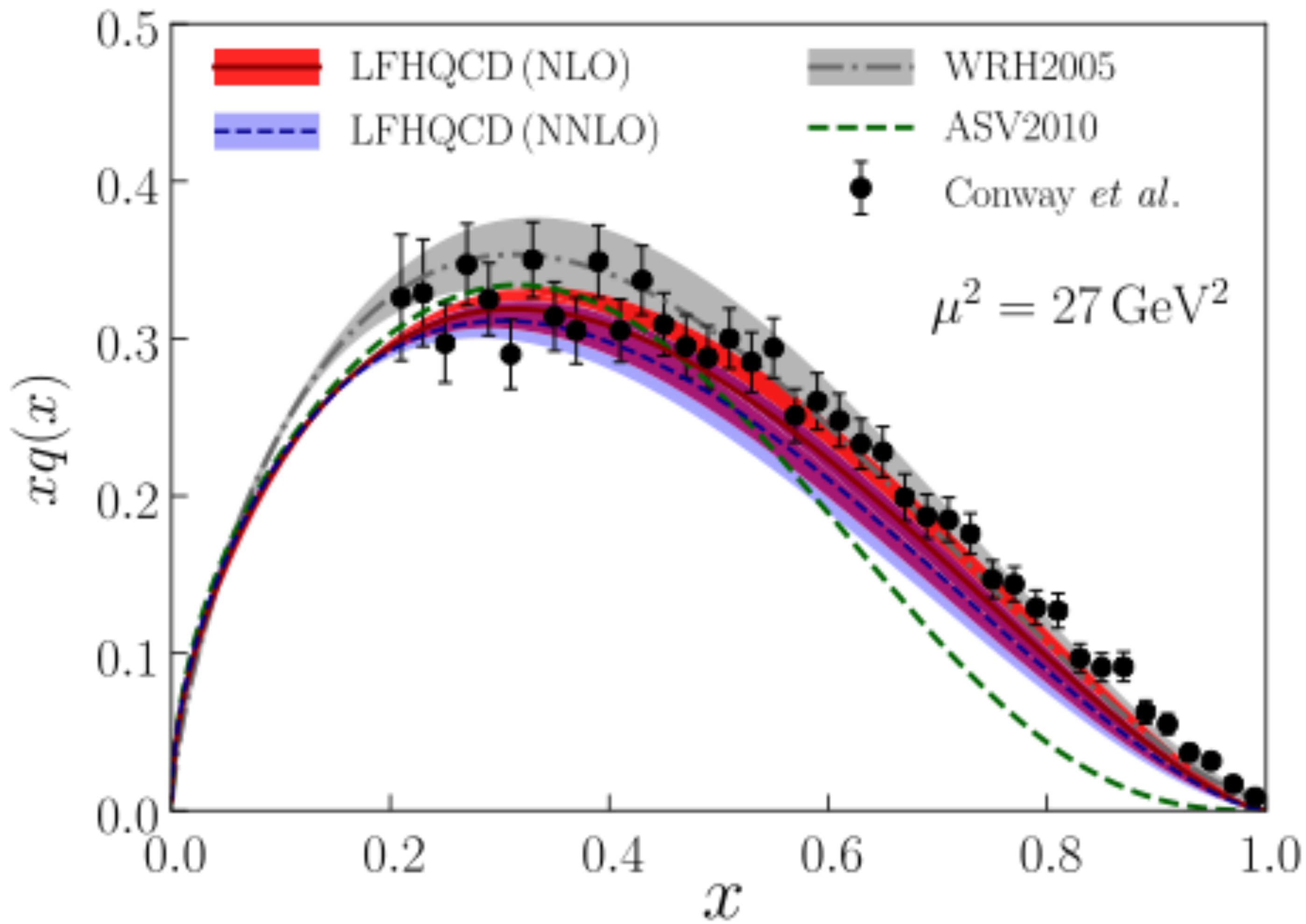


Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1 \pm 0.2$ GeV at NLO and the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Te´ramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

PHYSICAL REVIEW LETTERS 120, 182001 (2018)

$|\pi^+ >= |u\bar{d} >$

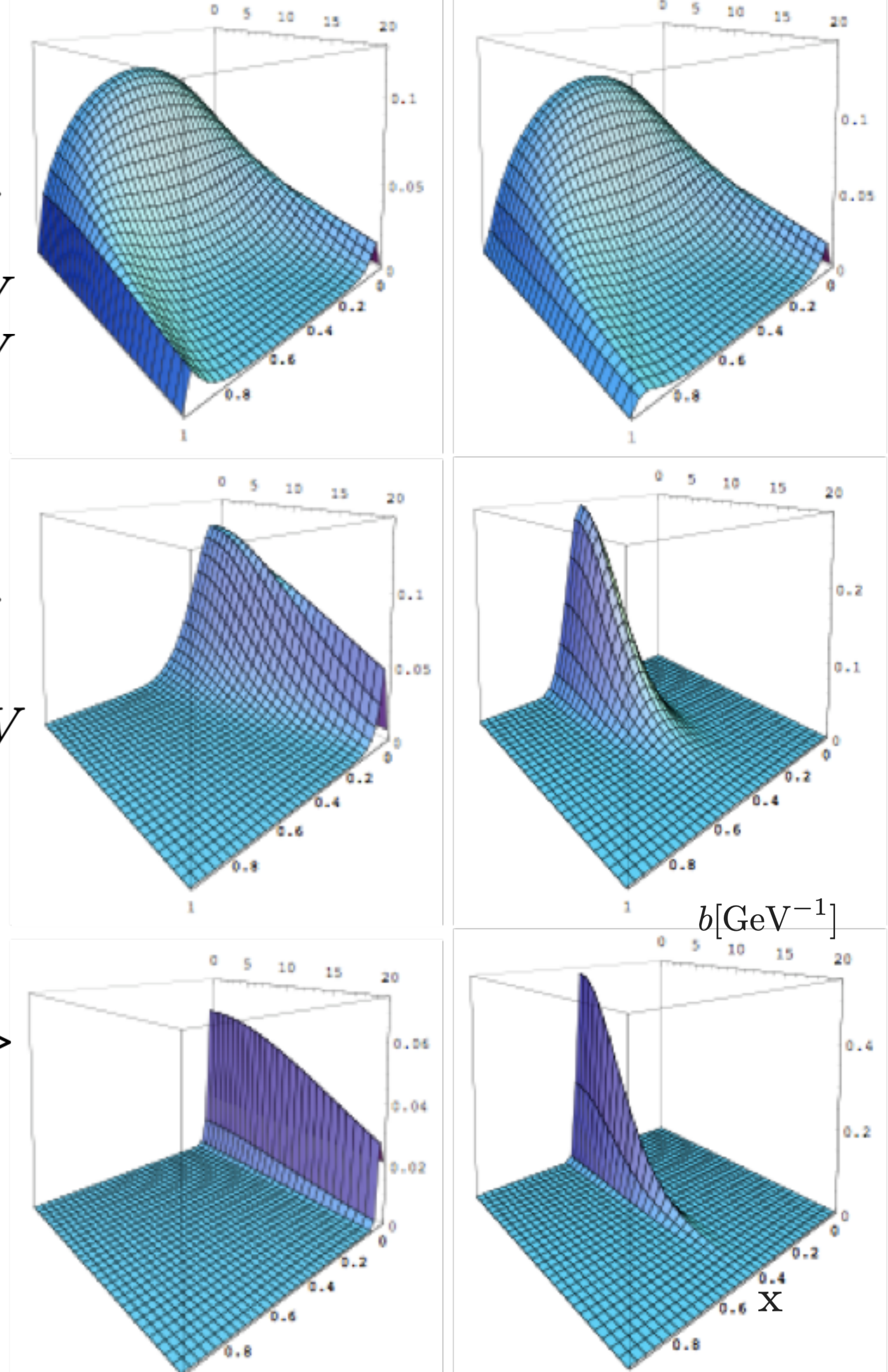
$m_u = 2\ MeV$
$m_d = 5\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$



$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|\eta_c >= |c\bar{c} >$

$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

Remarkable Features of Light-Front Schrödinger Equation

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

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Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

$\Updownarrow$

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

QCD Lagrangian

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

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Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

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Unique confinement potential!

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau) >= i\frac{\partial}{\partial\tau}|\psi(\tau) >$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

- **Dosch, de Teramond, sjb**

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

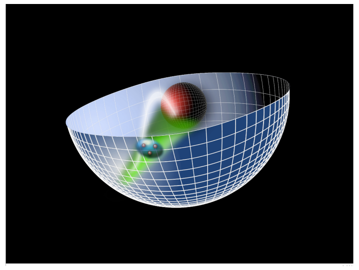
- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

LFHQCD: Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time τ*
- *Causality: Information within causal horizon: Light-Front*
- *Light-Front Holography: AdS$_5$ = LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$



- *Introduce Mass Scale κ while retaining the Conformal Invariance of the Action (dAFF)*
- *Unique Dilaton in AdS$_5$:* $e^{+\kappa^2 z^2}$
- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\boxed{\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]}$$

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[\mathrm{H},\mathrm{D}] = \mathrm{i}\ \mathrm{H}, \quad [\mathrm{H},\ \mathrm{K}] = 2\ \mathrm{i}\ \mathrm{D}, \quad [\mathrm{K},\ \mathrm{D}] = -\ \mathrm{i}\ \mathrm{K}$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

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[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

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de Téramond, Dosch, Lorcé, sjb

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

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S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

de Téramond, Dosch, Lorcé, sjb

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Meson-Baryon Degeneracy for $L_M=L_B+1$

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Same κ!

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LF Holography

Baryon LFWFs

Superconformal Quantum Mechanics

- Nucleon LF modes

$$
\begin{aligned}
\psi_+(\zeta)_{n,L} &= \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)\\
\psi_-(\zeta)_{n,L} &= \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)
\end{aligned}
$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\int_0^\infty d\zeta \int_0^1 dx\psi_+^2(\zeta^2,x) = \int_0^\infty d\zeta \int_0^1 dx\psi_-^2(\zeta^2,x) = \tfrac{1}{2}$$

$$J^z = +1/2 : \frac{1}{\sqrt{2}}[\,|\,S_q^z = +1/2, L^z = 0 > + \,|\,S_q^z = -1/2, L^z = +1 >\,]$$

Nucleon spin carried by quark orbital angular momentum

LF Holography

Baryon LFWFs

Superconformal Quantum Mechanics

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Quark Chiral Symmetry of Eigenstate!

Nucleon: Equal Probability for L=0, 1

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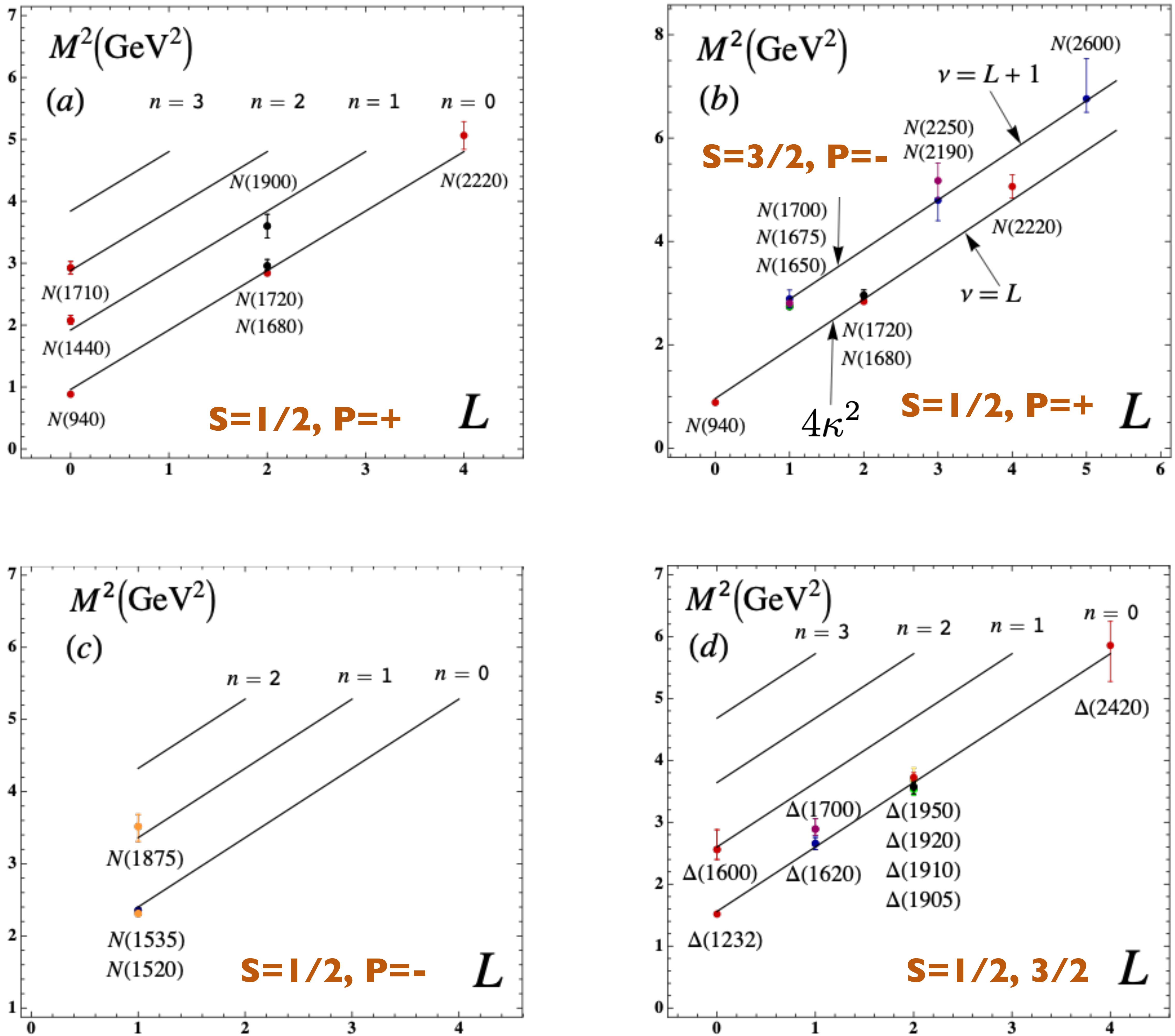
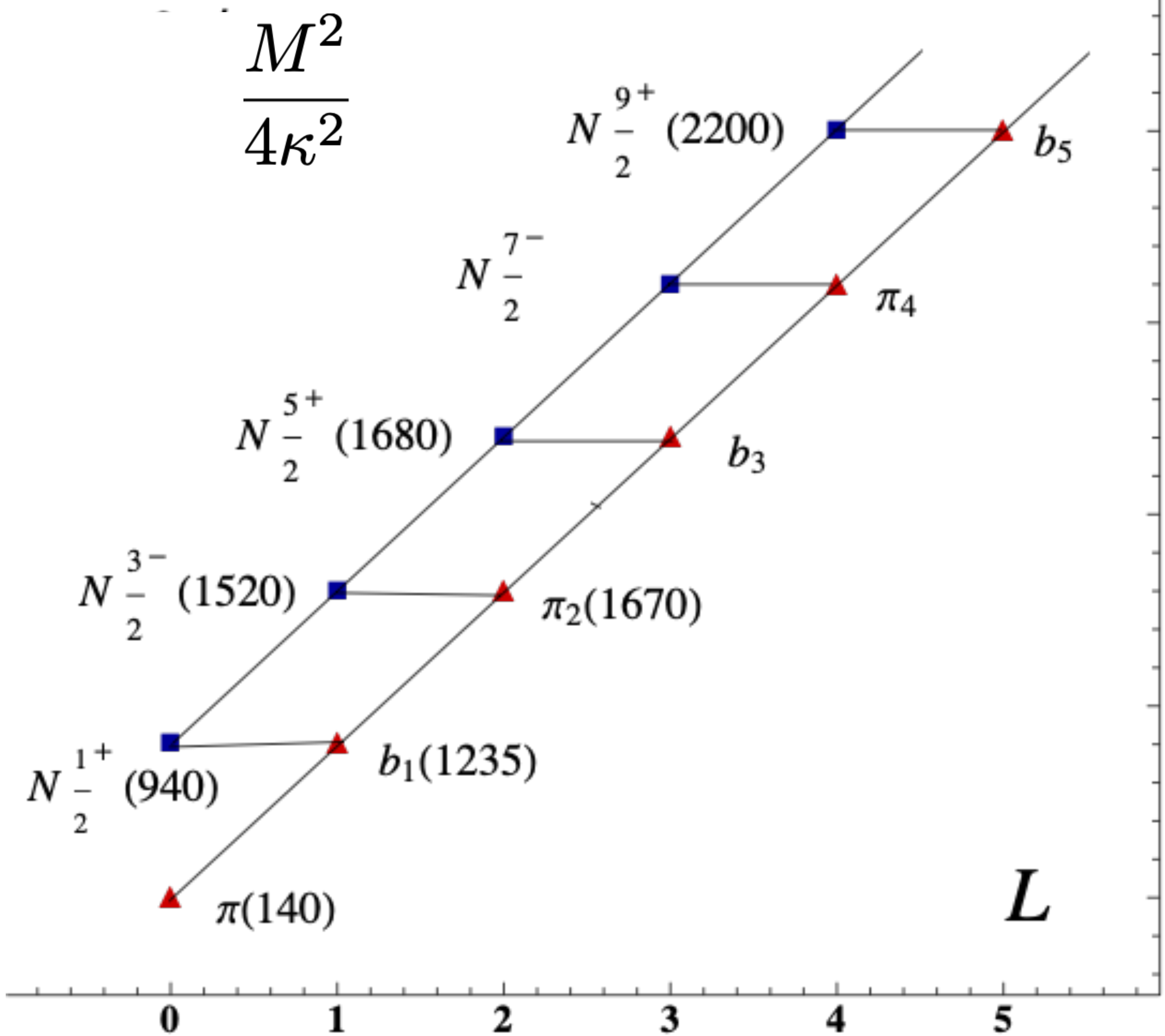


$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
$\nu = L + 1$
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

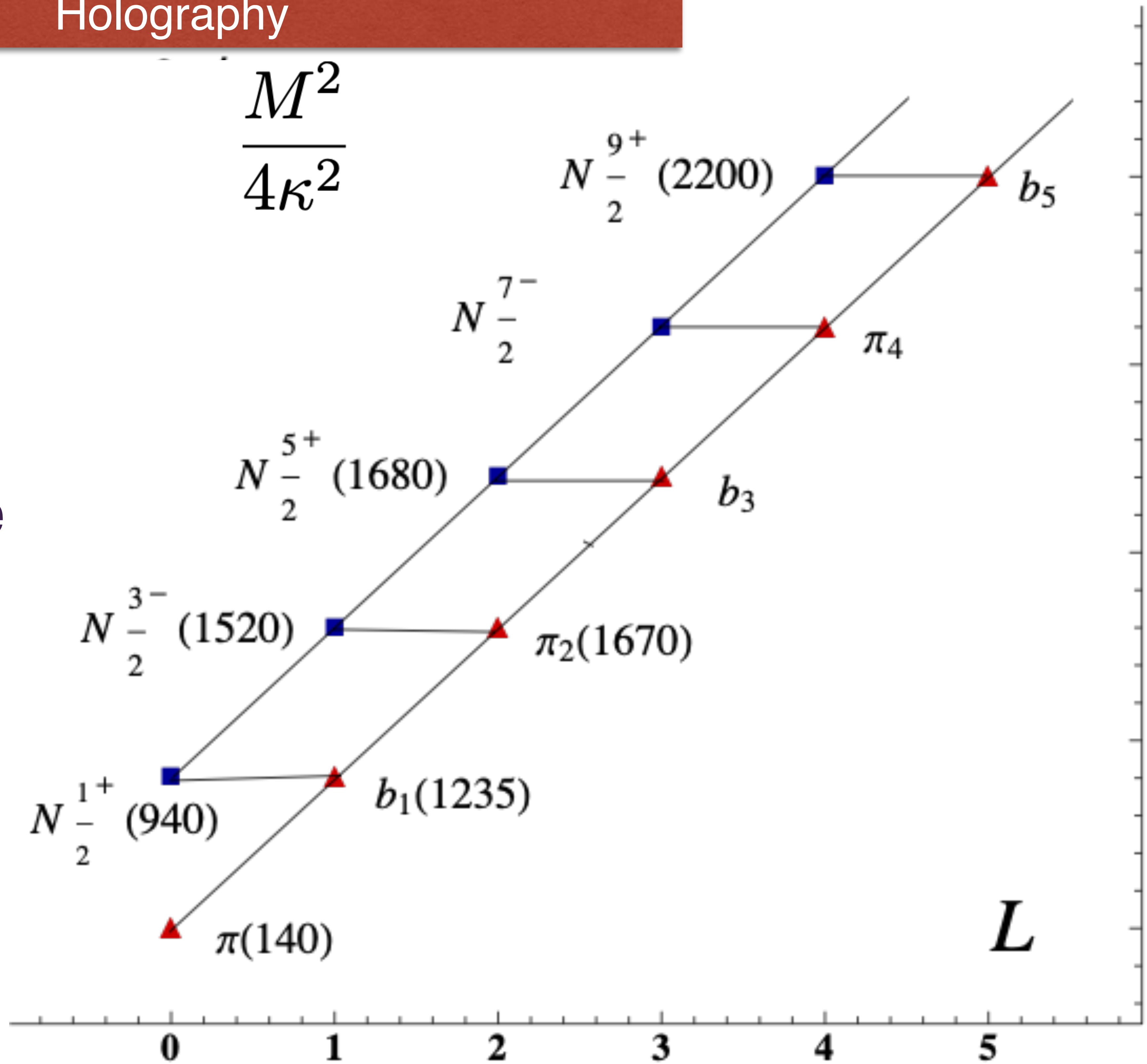
de Téramond, Dosch, Lorcé, sjb

Same slope



de Téramond, Dosch, Lorcé, sjb

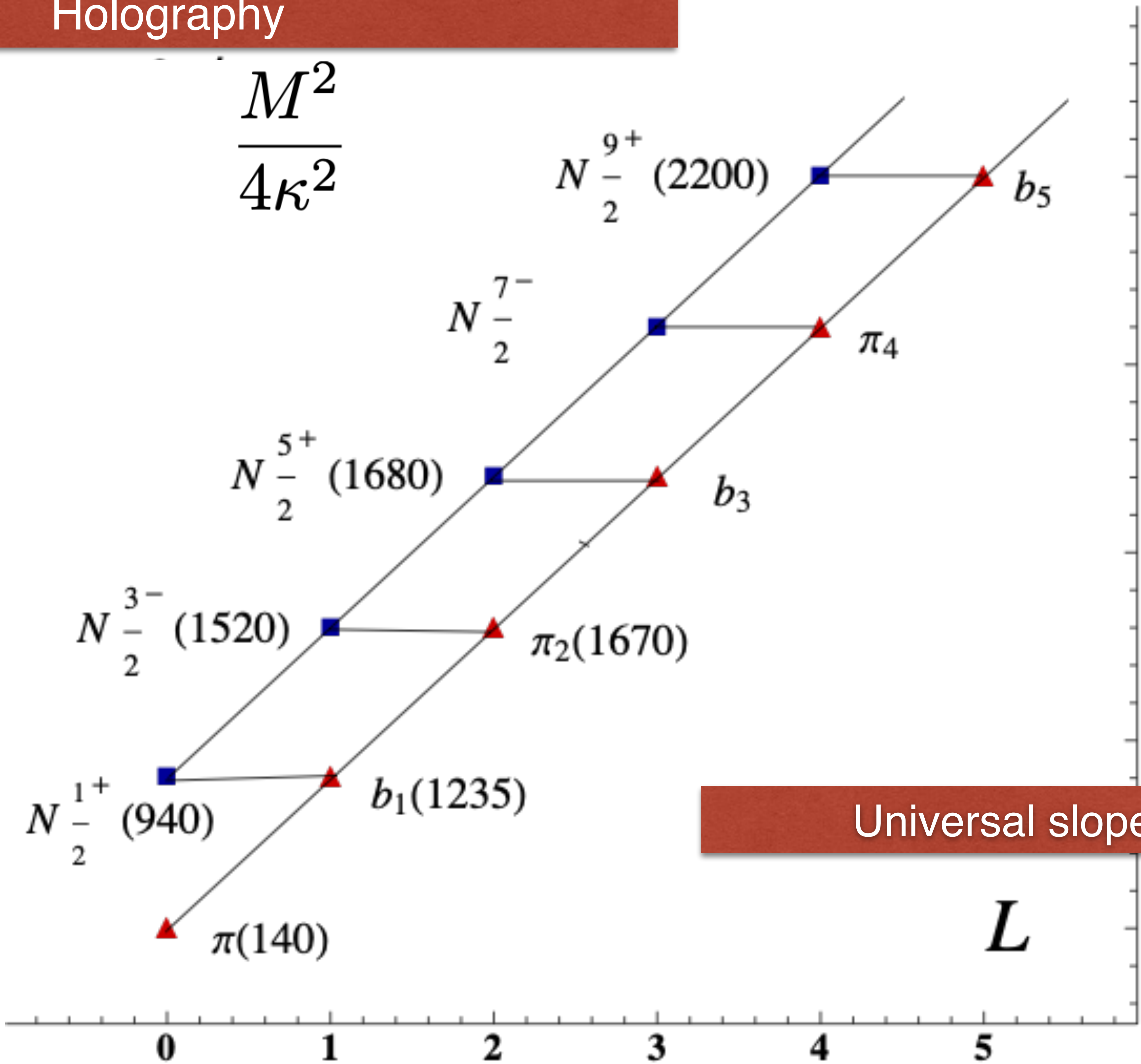
Same slope



Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

Same slope

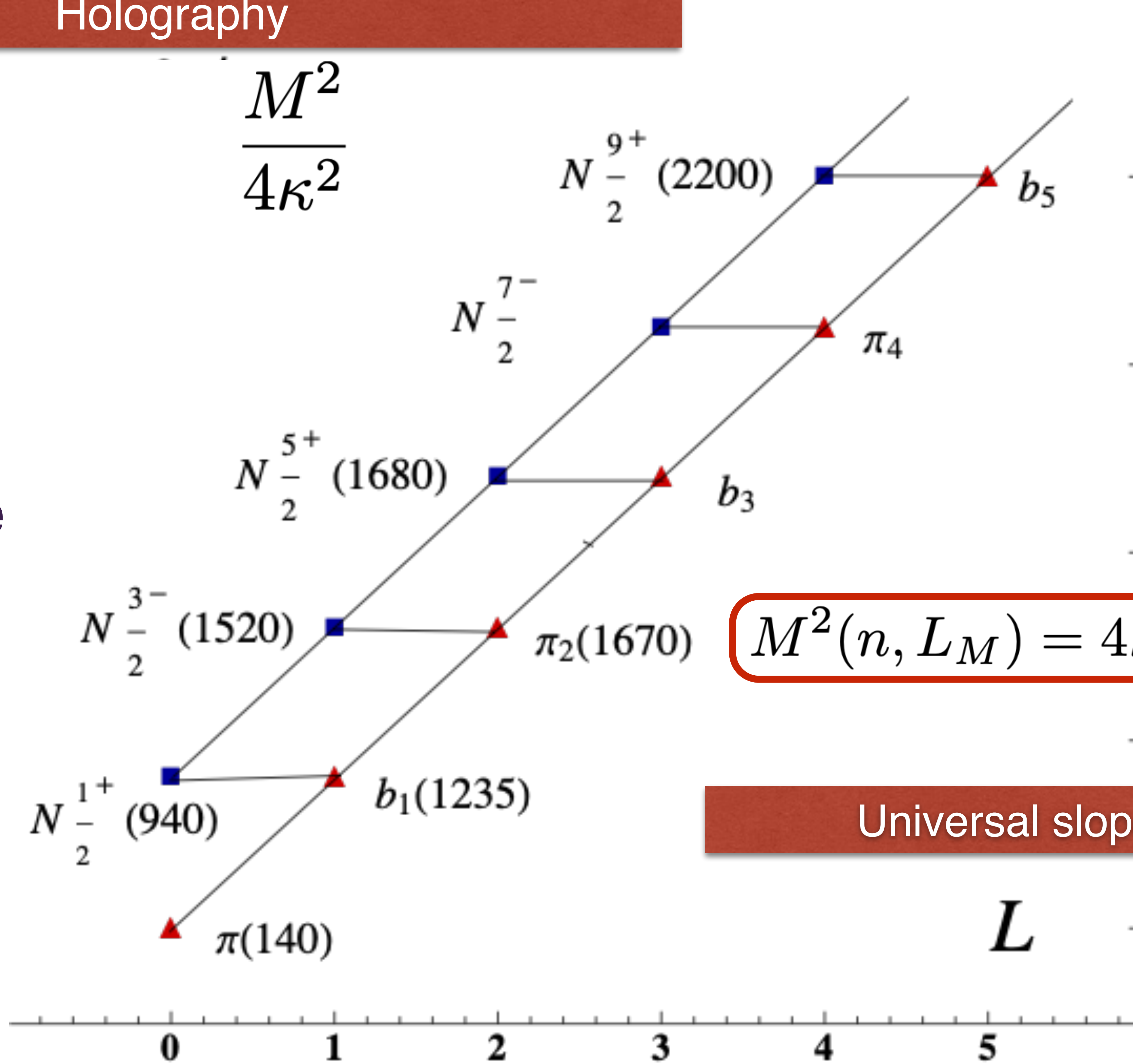


Universal slopes in n, L

Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

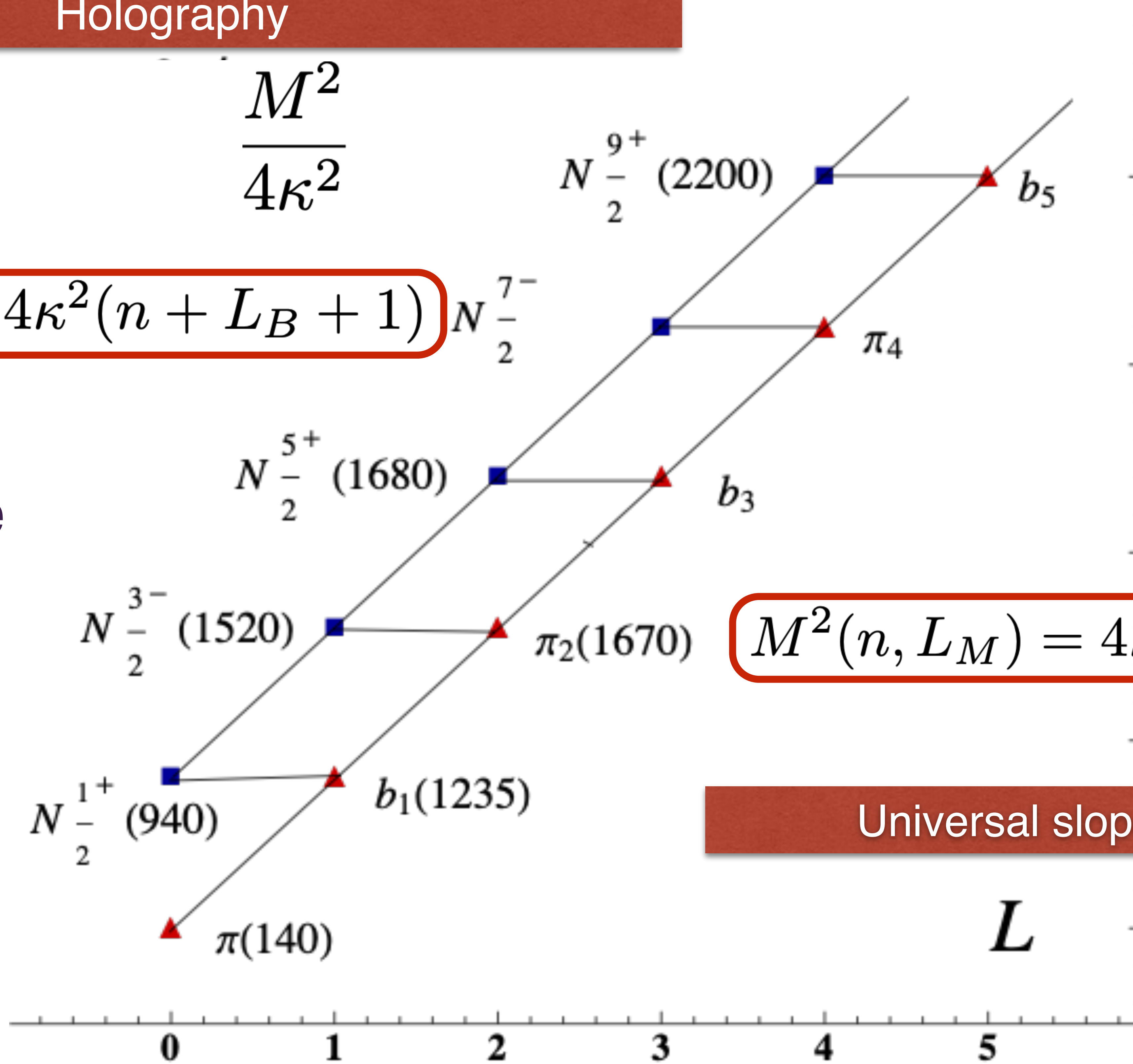
Universal slopes in n, L

Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

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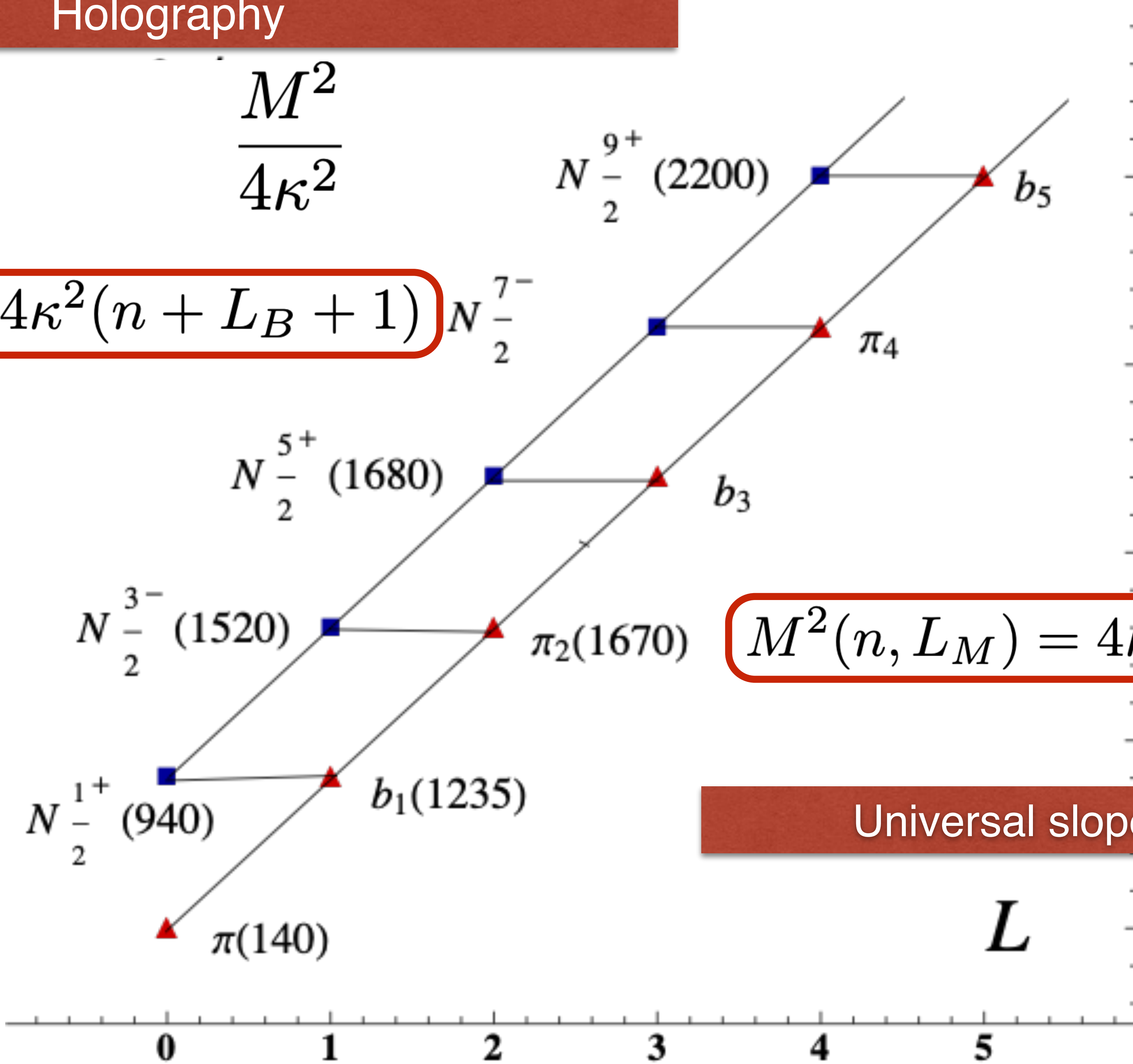
Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

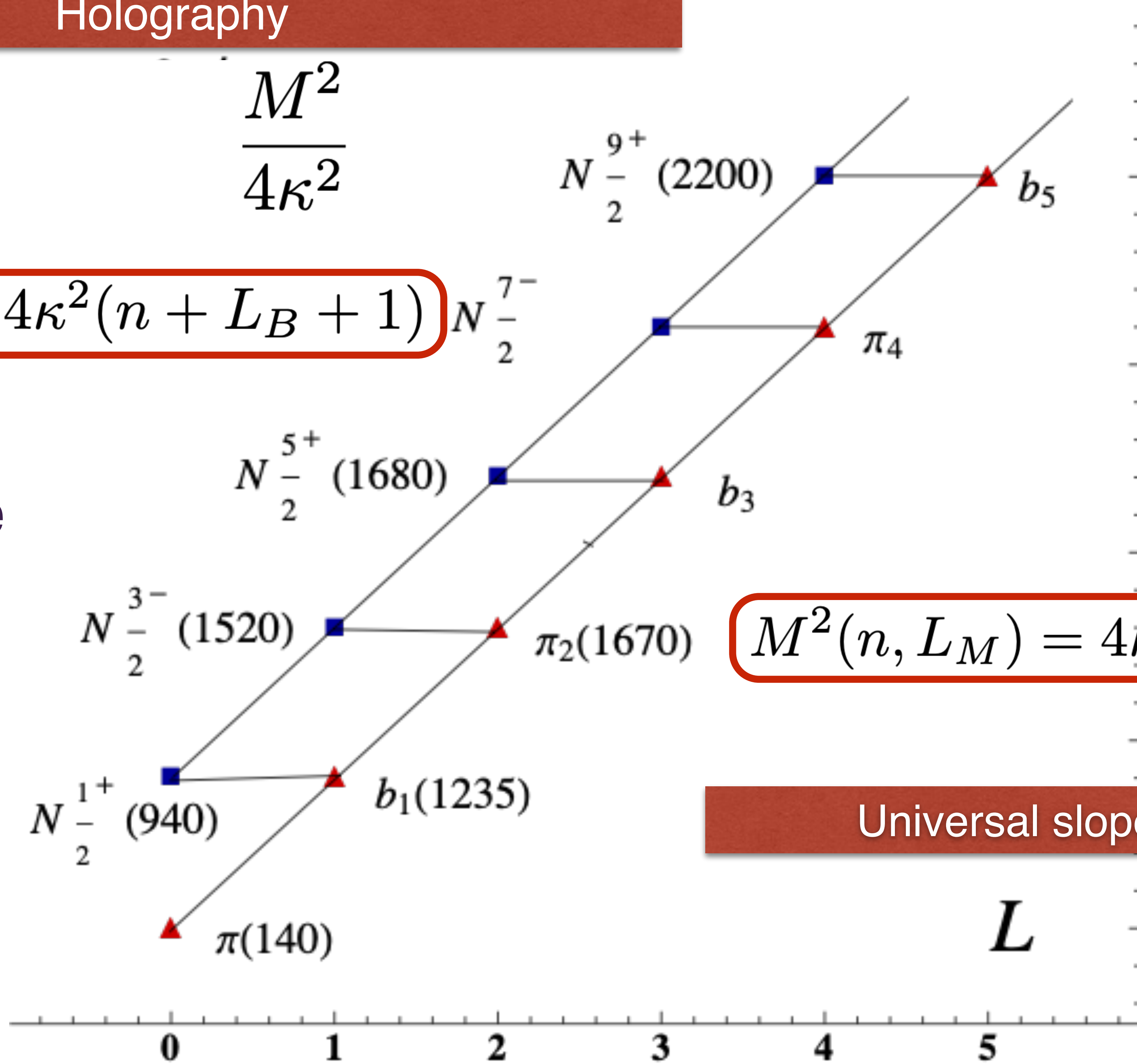
Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

$$\boxed{M^2(n, L_B) = 4\kappa^2(n + L_B + 1)}$$

Same slope

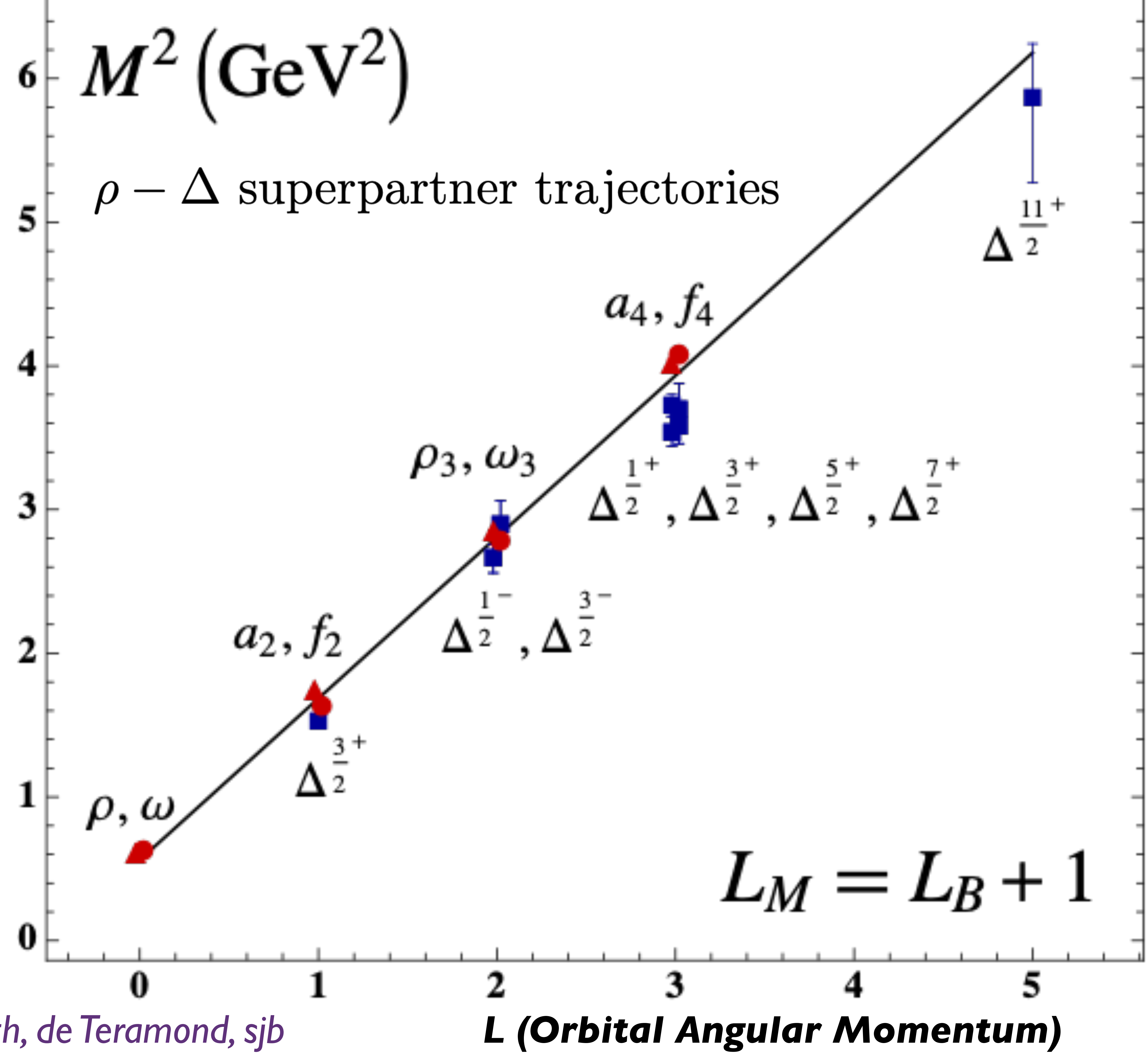


$$\boxed{M^2(n, L_M) = 4\kappa^2(n + L_M)}$$

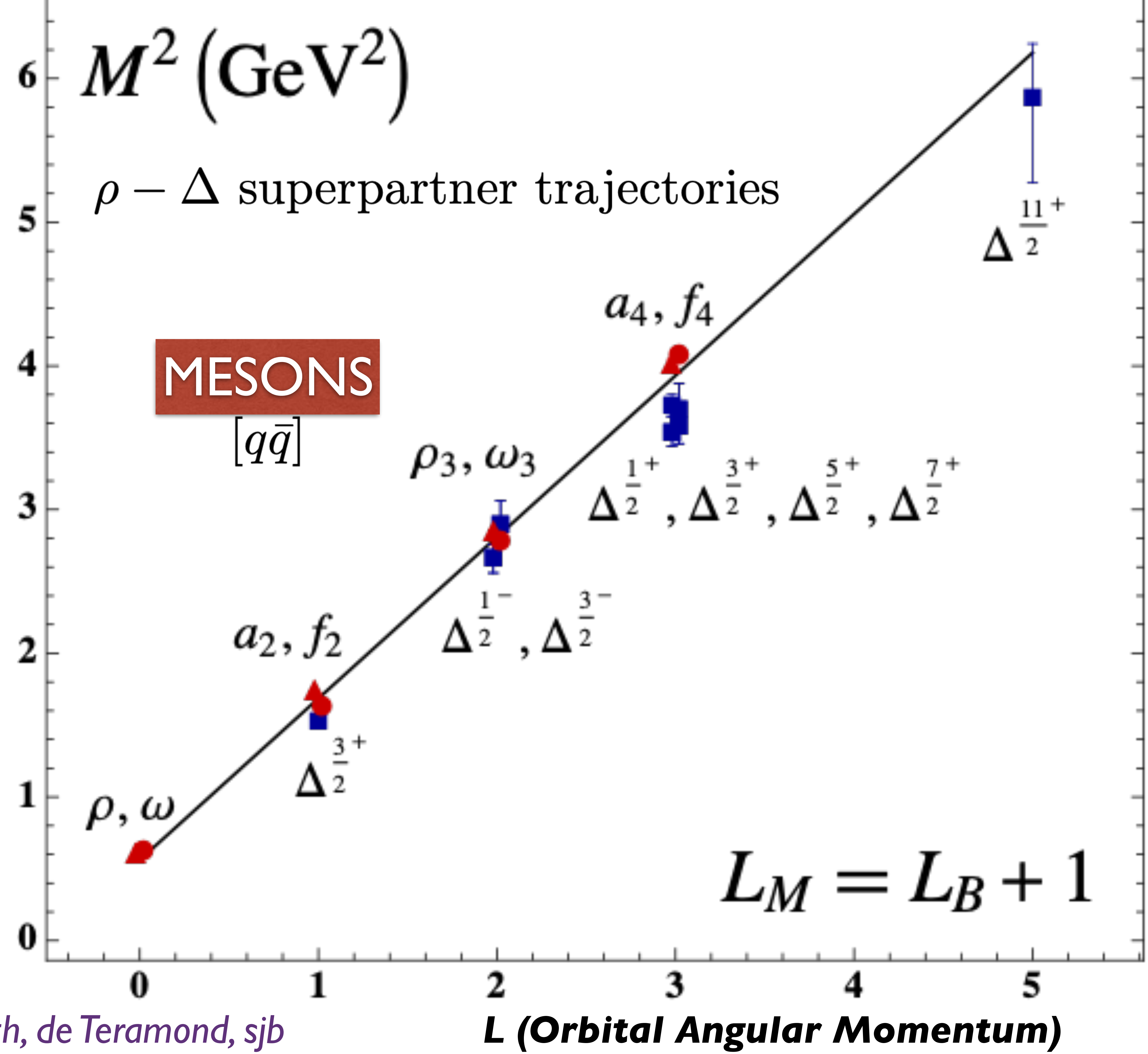
Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

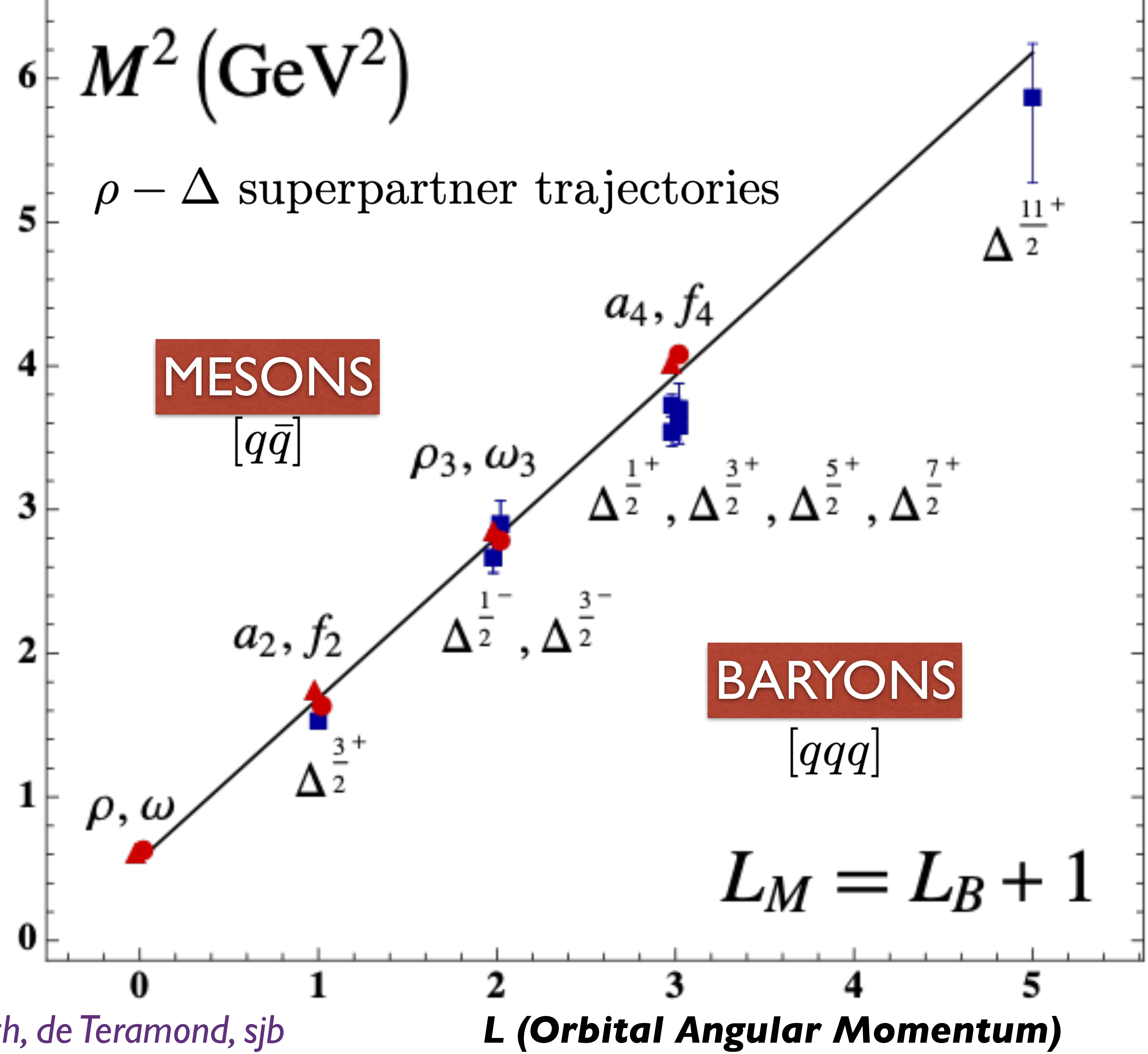
Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



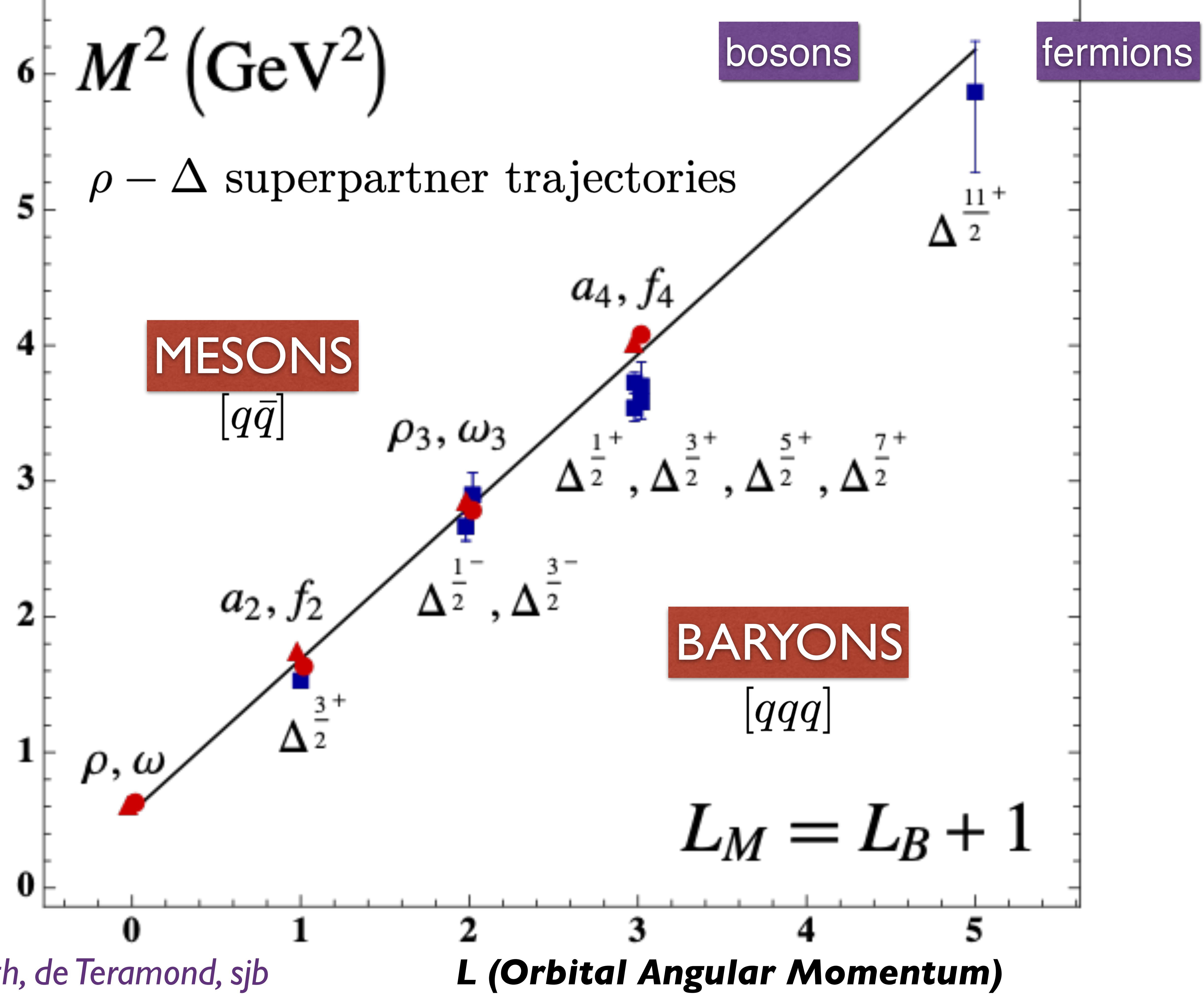
Dosch, de Teramond, sjb



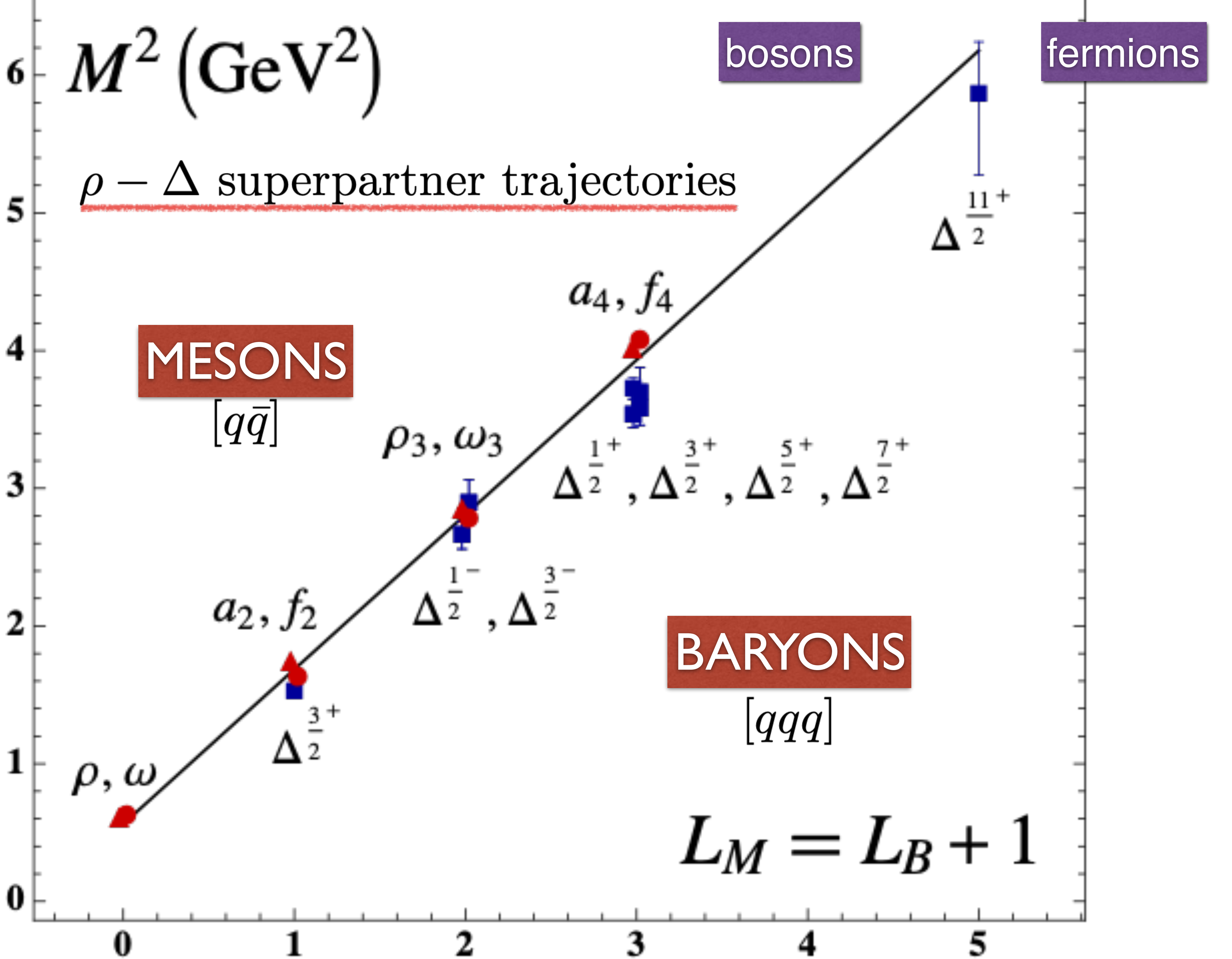
Dosch, de Teramond, sjb



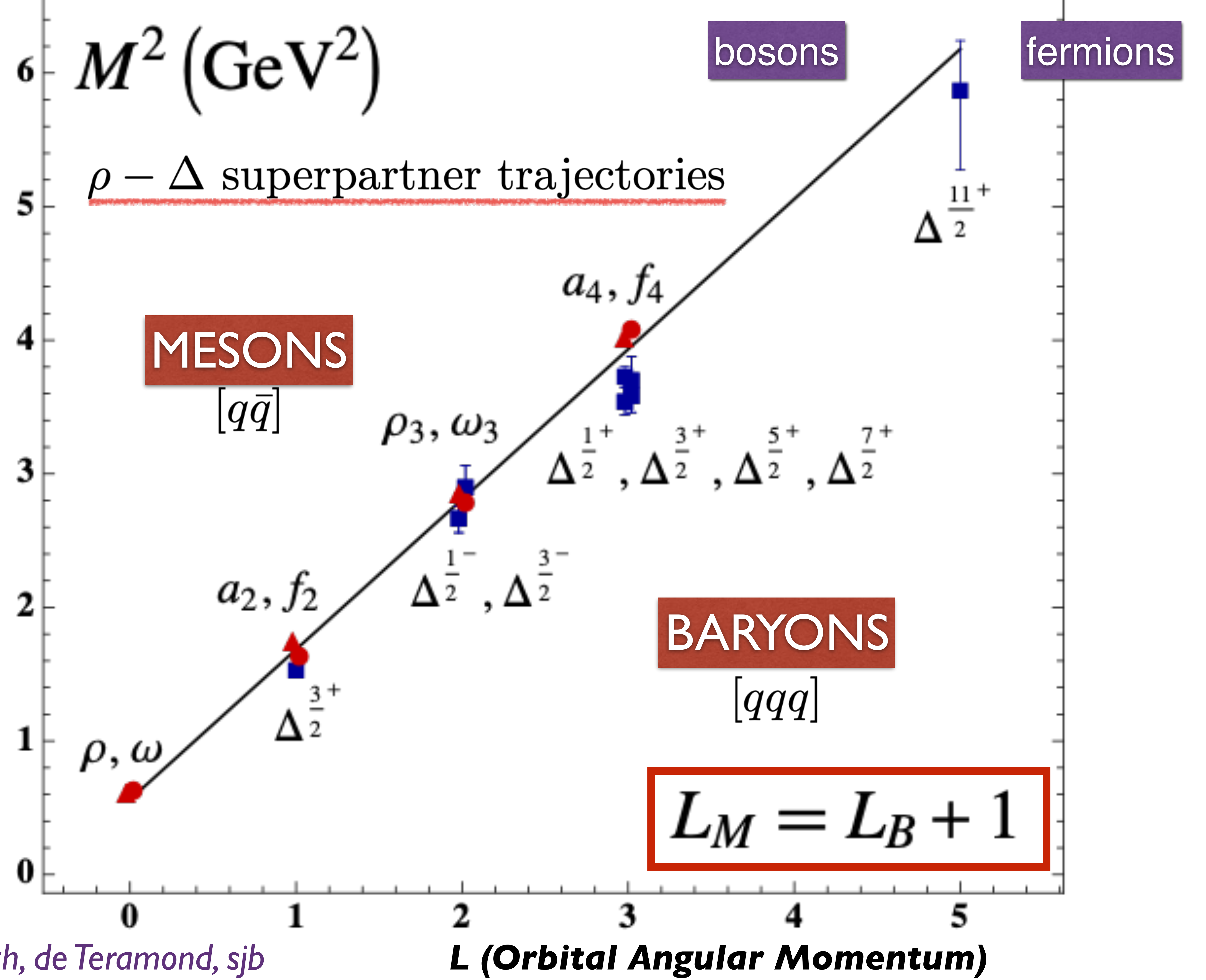
Dosch, de Teramond, sjb



Dosch, de Teramond, sjb



Dosch, de Teramond, sjb



Dosch, de Teramond, sjb

de Tèramond, Dosch, Lorce', sjb

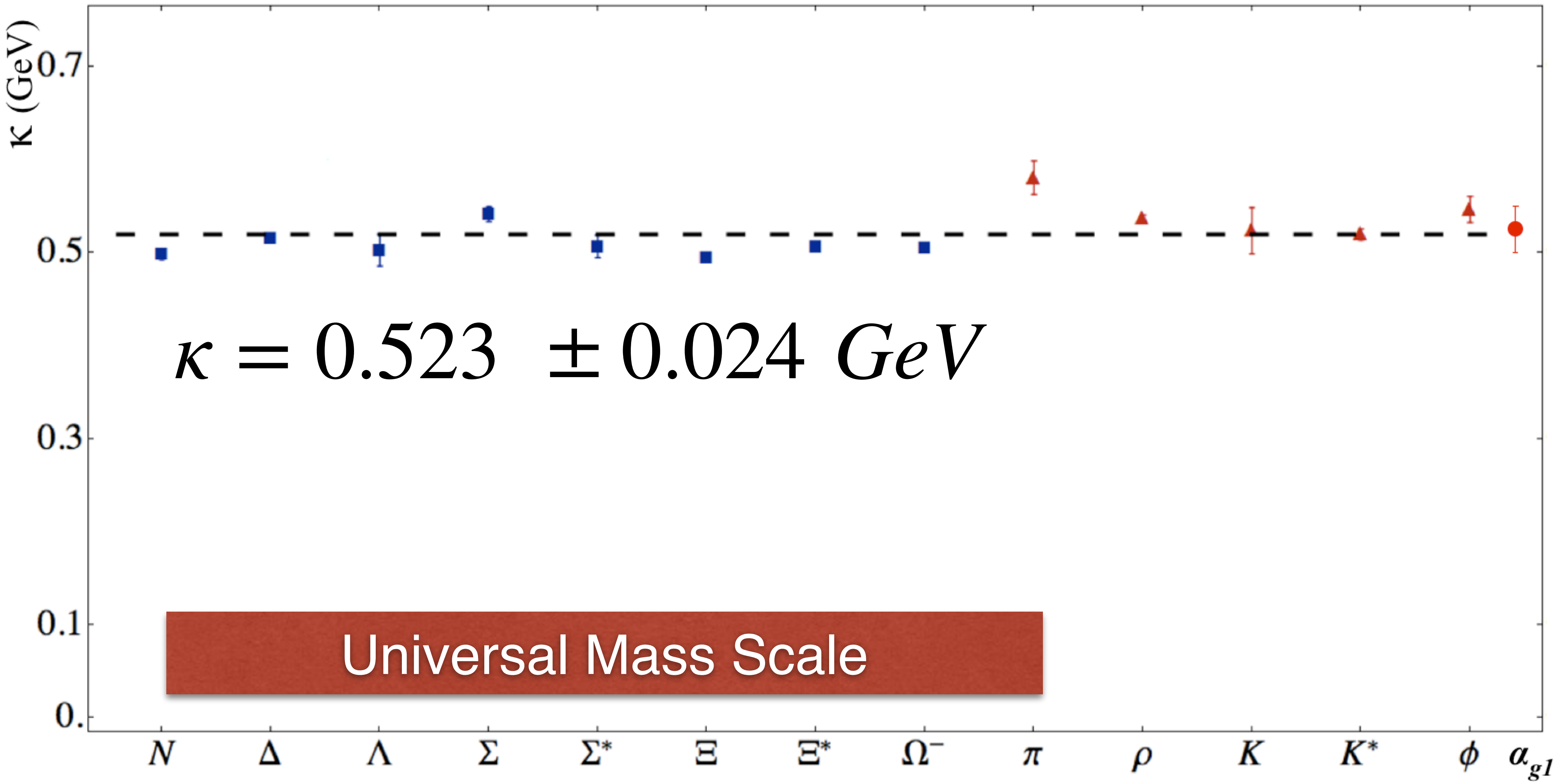
Mesons

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Baryons

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

κ



Fit to the slope of Regge trajectories, including radial excitations

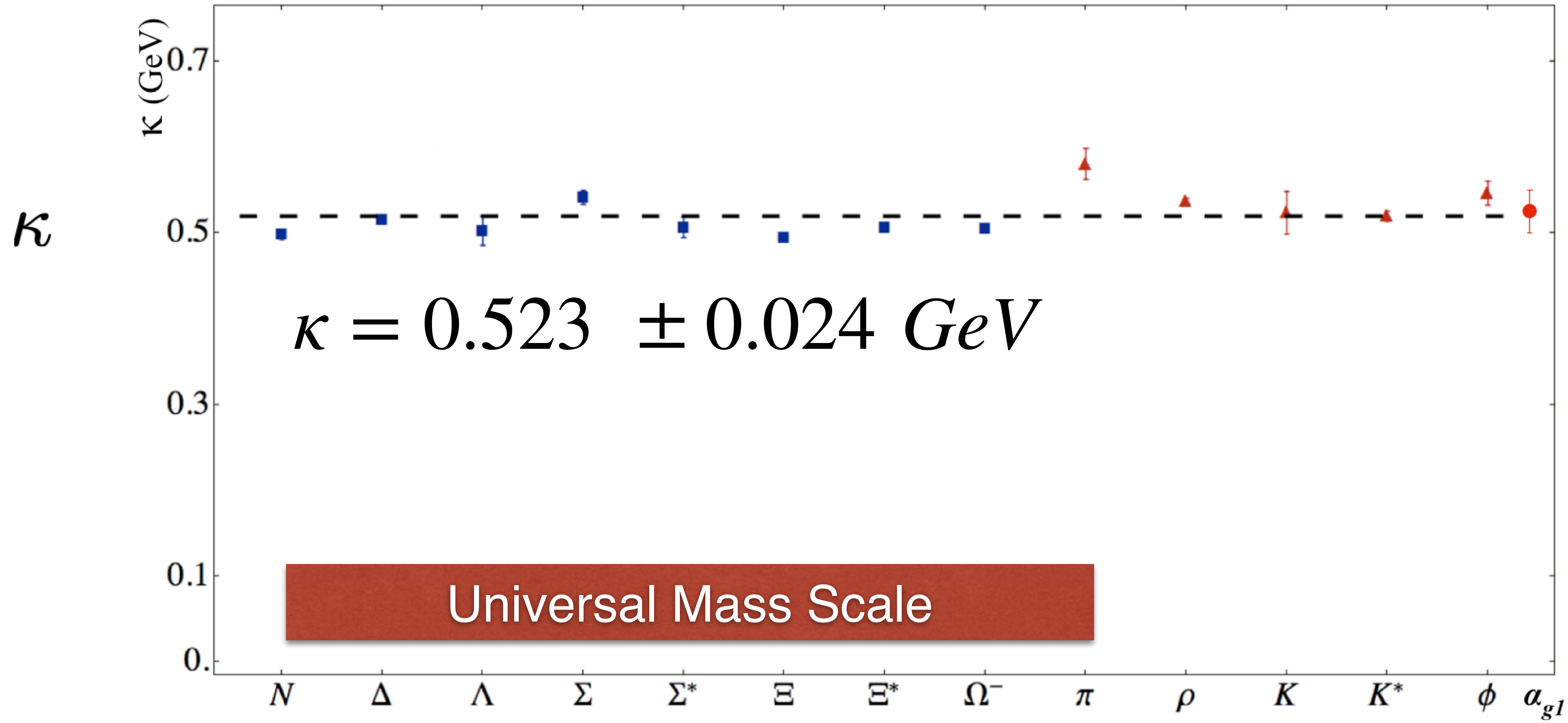
de Tèramond, Dosch, Lorce', sjb

Mesons

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Baryons

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$



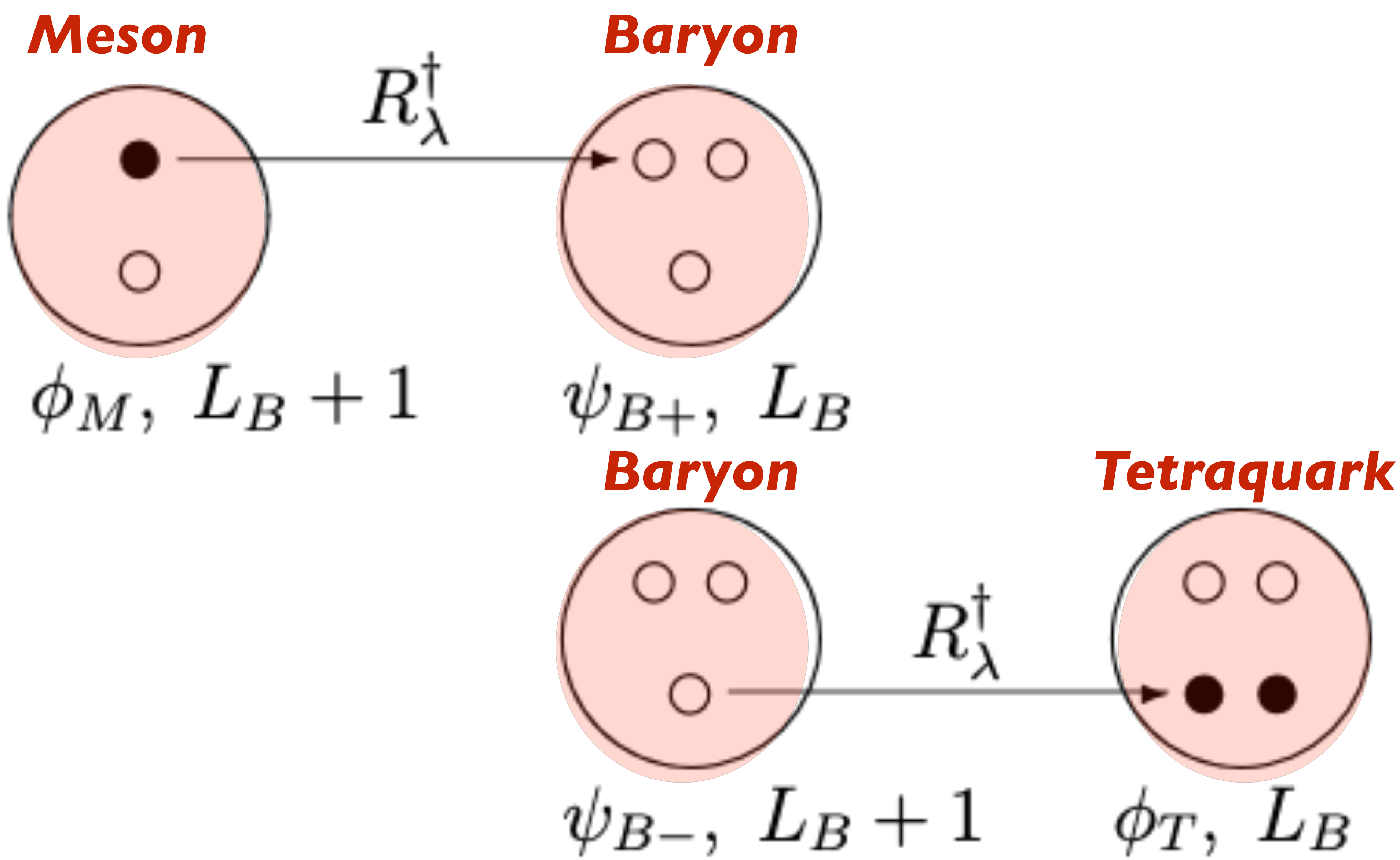
Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons in n and L: Supersymmetric feature of hadron physics

Superconformal Algebra

2X2 Hadronic Multiplets

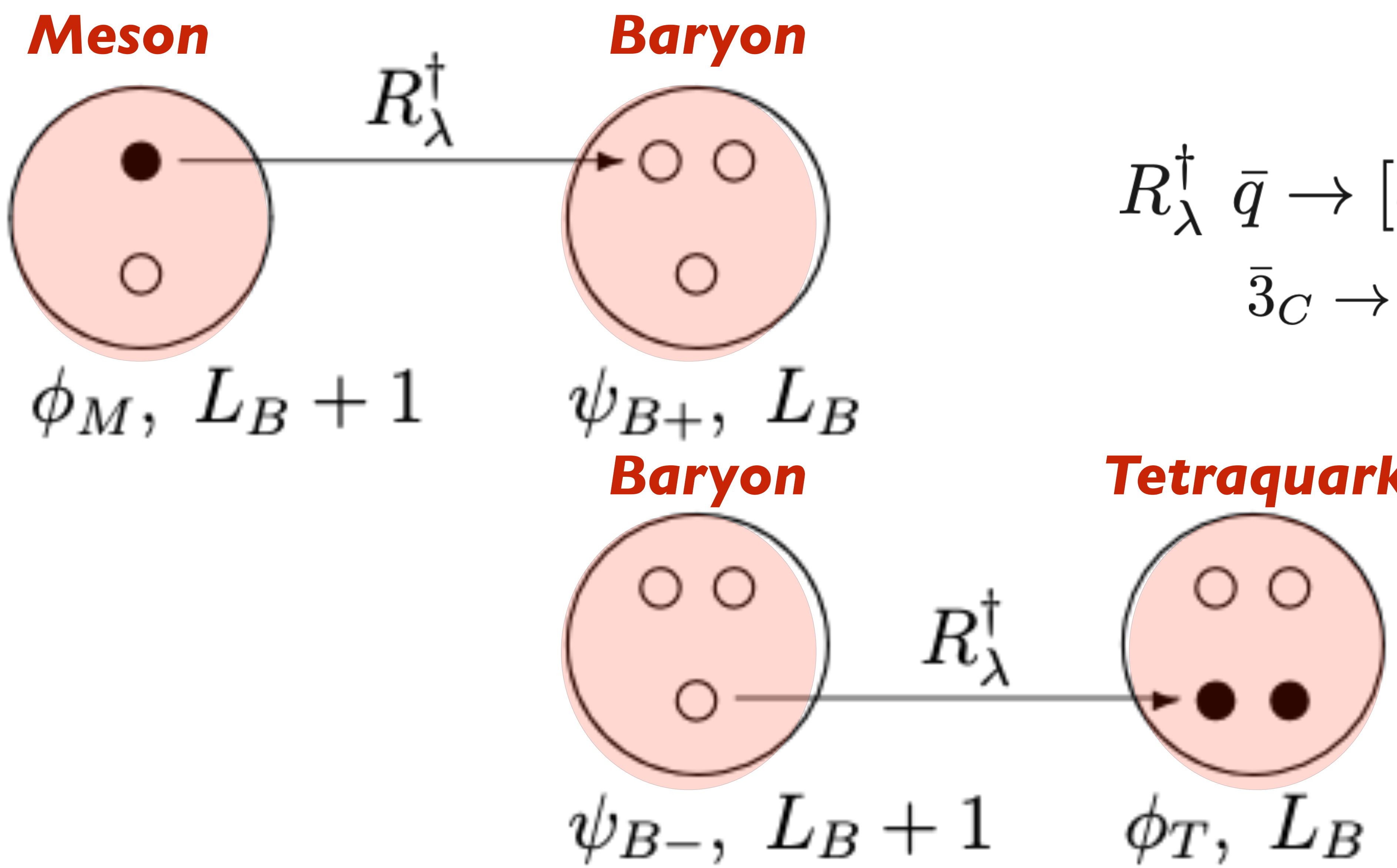
Bosons, Fermions with Equal Mass!



Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



$$R^\dagger_\lambda \; \bar{q} \to [qq]$$

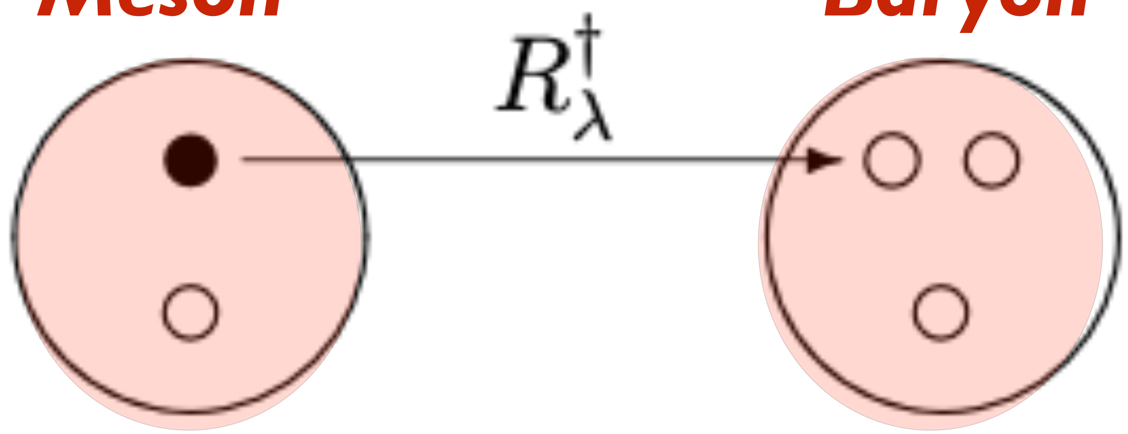
$$\bar{3}_C \to \bar{3}_C$$

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!

Meson **Baryon**

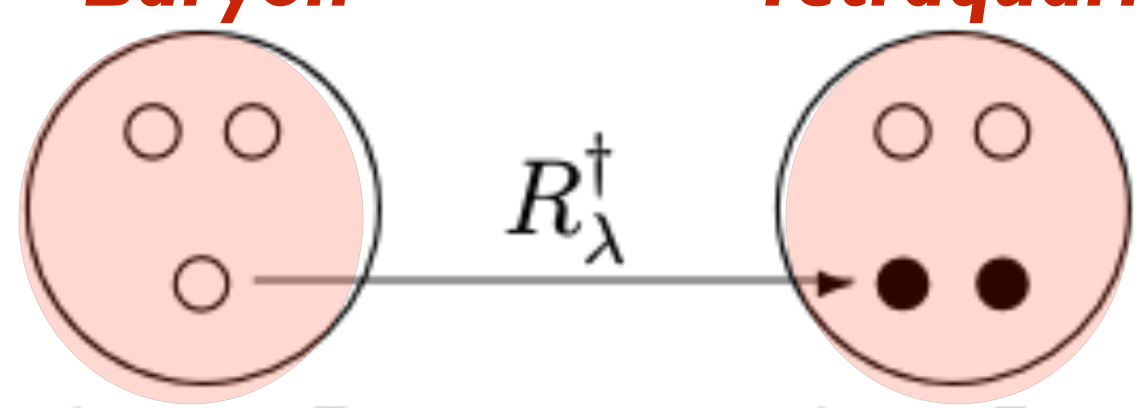


$\phi_M, \ L_B + 1$ $\psi_{B+}, \ L_B$

$R^\dagger_\lambda \ \bar{q} \to [qq]$

$\bar{3}_C \to \bar{3}_C$

Baryon **Tetraquark**



$\psi_{B-}, \ L_B + 1$ $\phi_T, \ L_B$

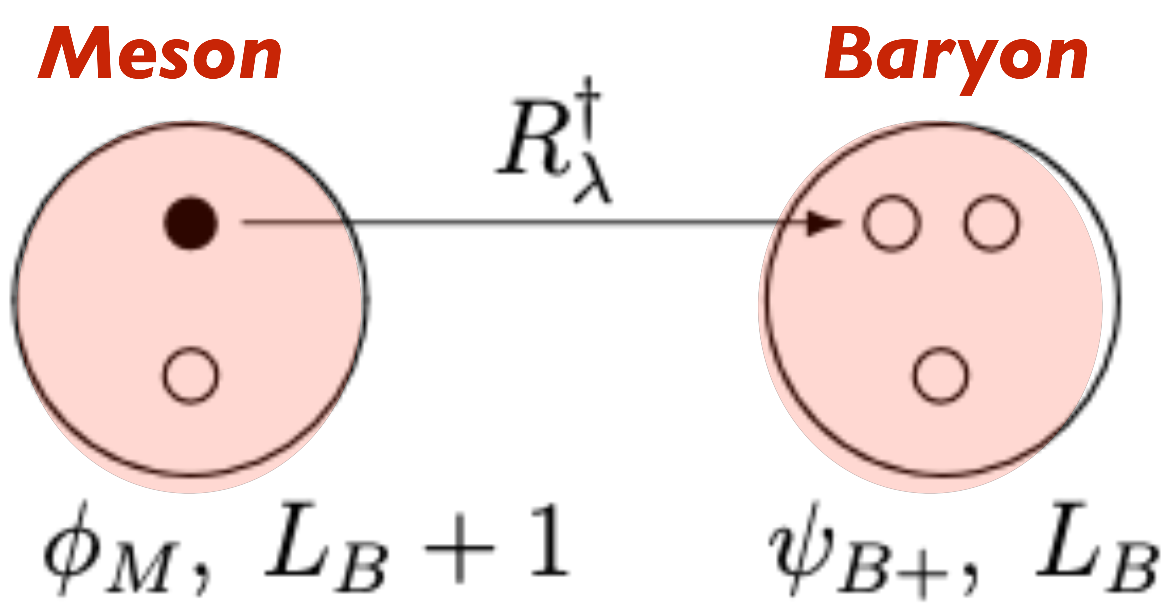
$R^\dagger_\lambda \ q \to [\bar{q}\bar{q}]$

$3_C \to 3_C$

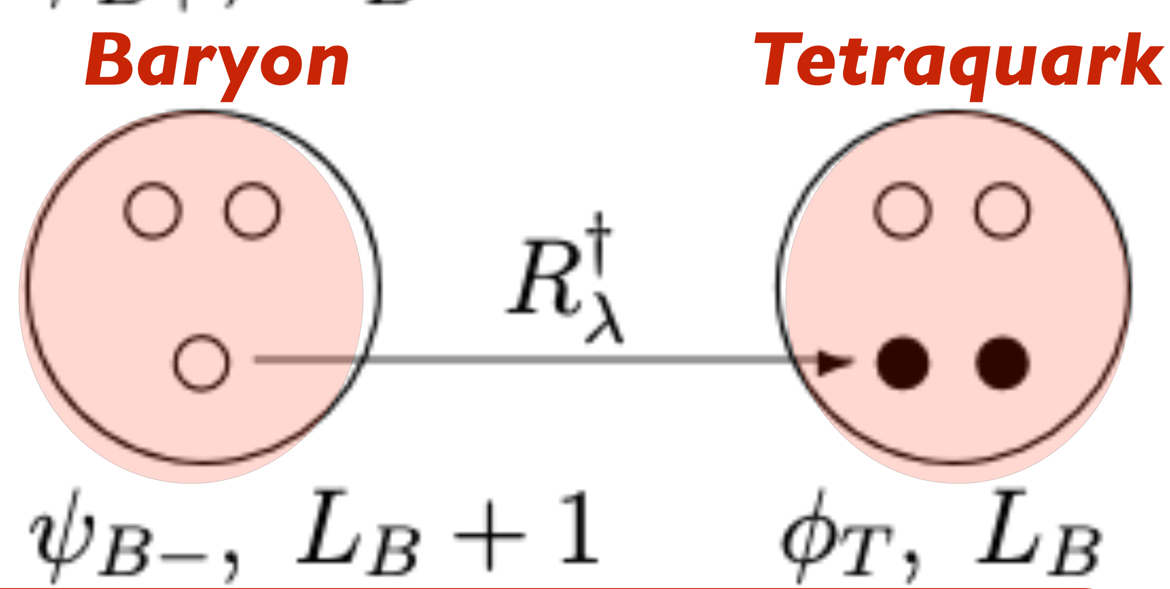
Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



$$R^\dagger_\lambda \; \bar{q} \to [qq]$$
$$\bar{3}_C \to \bar{3}_C$$



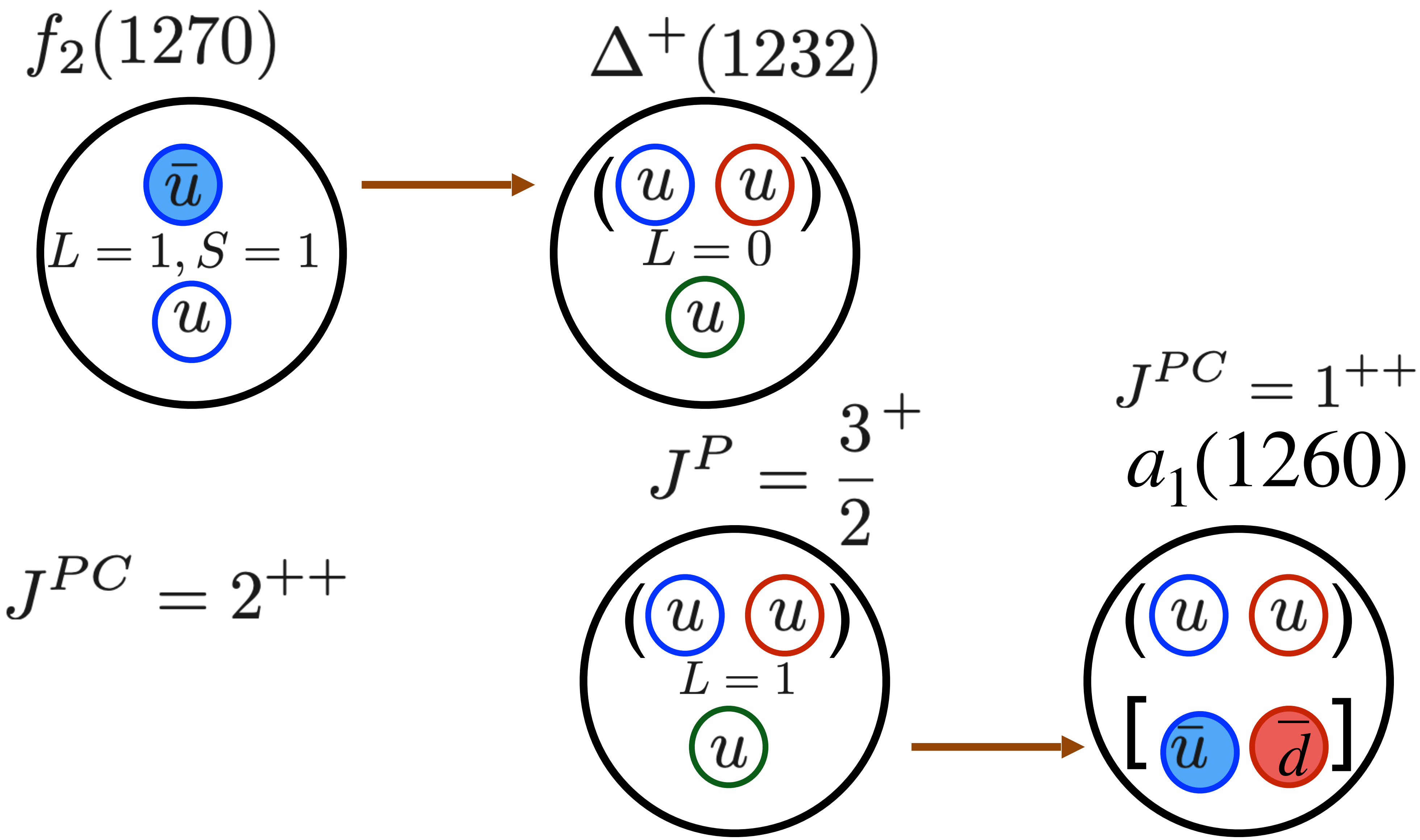
$$R^\dagger_\lambda \; q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Superconformal Algebra 4-Plet

$$R_\lambda^\dagger \quad \bar{q} \to (qq) \quad S = 1$$
$$\bar{3}_C \to \bar{3}_C$$

Vector ()+ Scalar [] Diquarks

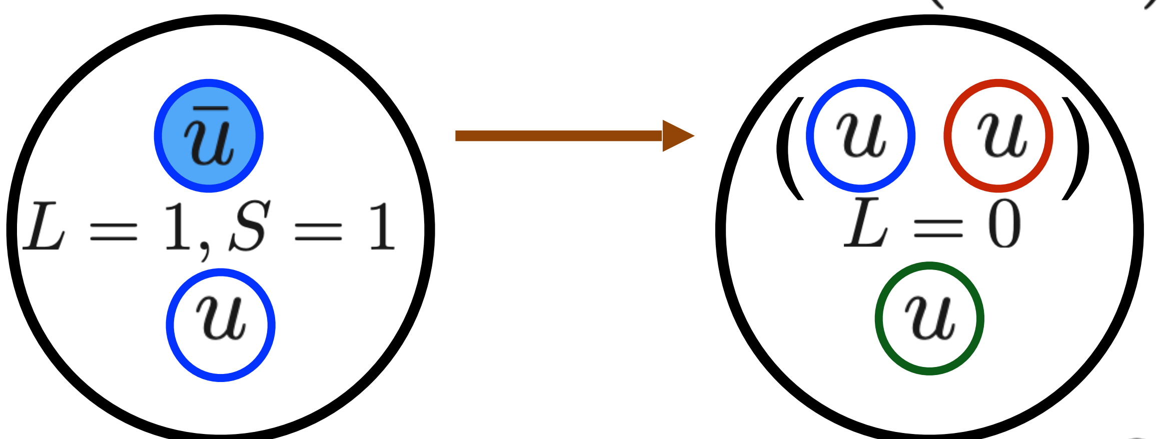


Superconformal Algebra 4-Plet

$$R^\dagger_\lambda \quad \bar{q} \to (qq) \quad S = 1$$
$$\bar{3}_C \to \bar{3}_C$$

Vector ()+ Scalar [] Diquarks

$f_2(1270)$ $\Delta^+(1232)$

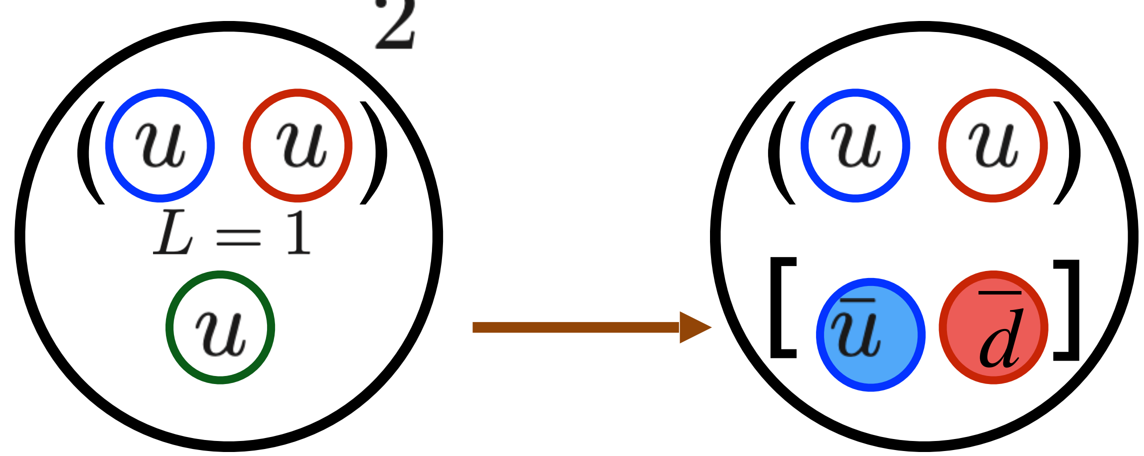


$$J^{PC} = 1^{++}$$

$$J^P = \frac{3}{2}^+$$

$$a_1(1260)$$

$$J^{PC} = 2^{++}$$

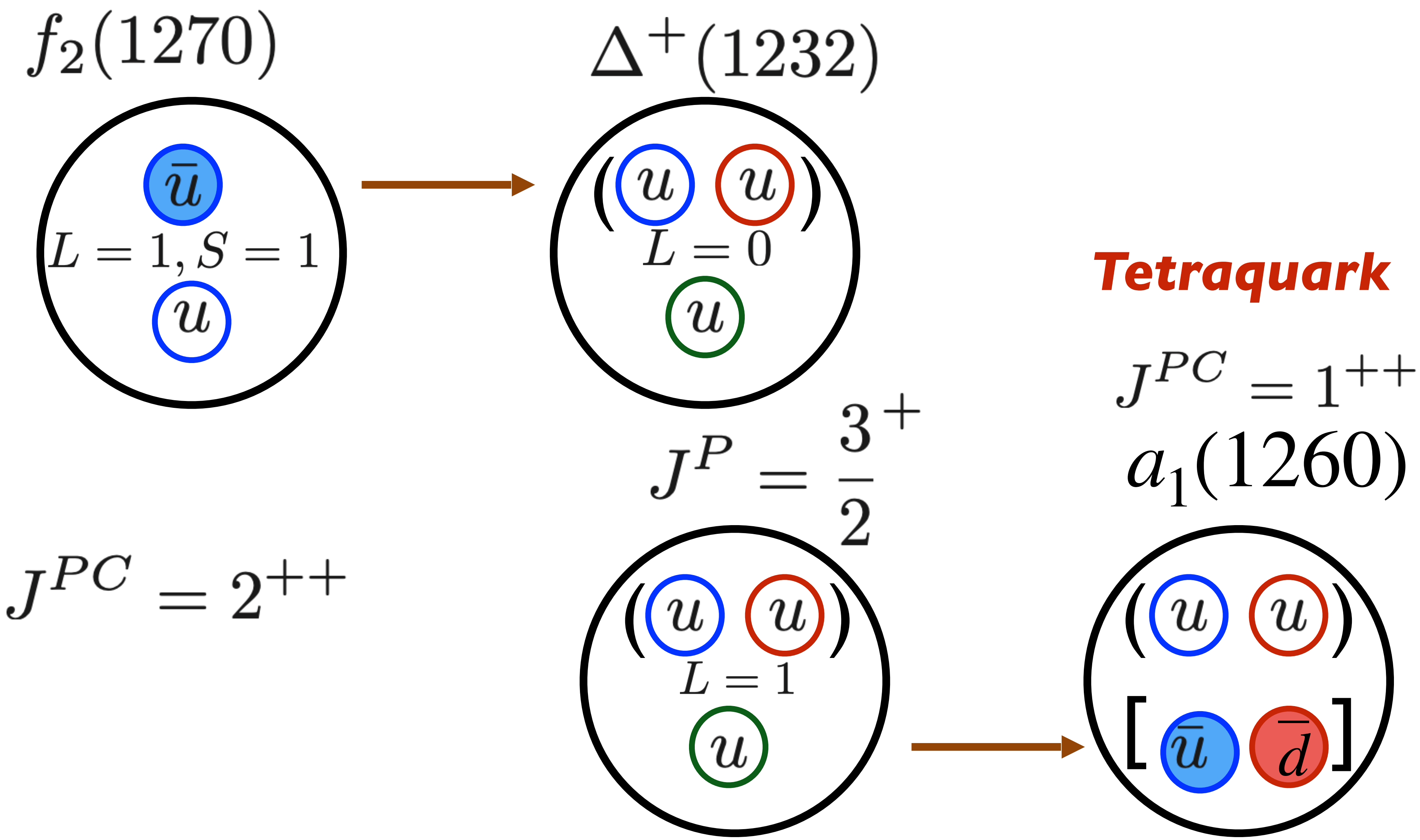


$$R^\dagger_\lambda \quad q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

Superconformal Algebra 4-Plet

$$R^{\dagger}_{\lambda} \quad \bar{q} \to (qq) \quad S = 1$$
$$\bar{3}_C \to \bar{3}_C$$

Vector ()+ Scalar [] Diquarks



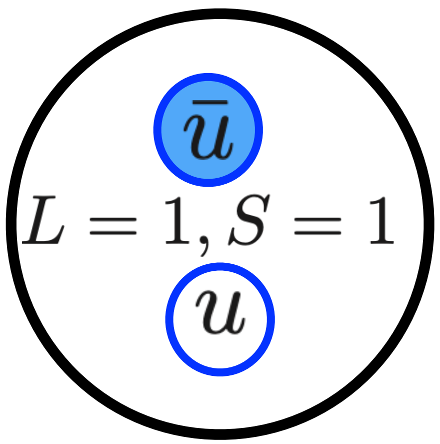
$$R^{\dagger}_{\lambda} \quad q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

Superconformal Algebra 4-Plet

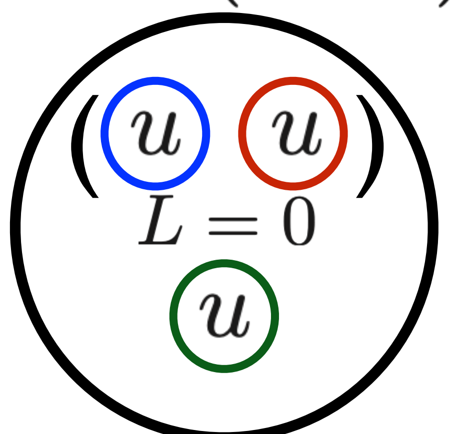
$$R_\lambda^\dagger \quad \bar{q} \to (qq) \quad S = 1$$
$$\bar{3}_C \to \bar{3}_C$$

Vector ()+ Scalar [] Diquarks

$f_2(1270)$



$\Delta^+(1232)$



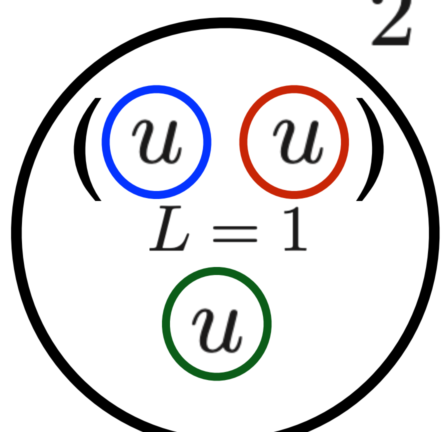
Tetraquark

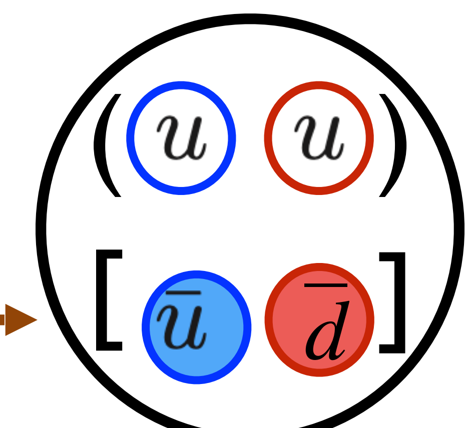
$$J^{PC} = 1^{++}$$

$$a_1(1260)$$

$$J^P = \frac{3}{2}^+$$

$$J^{PC} = 2^{++}$$





$$R_\lambda^\dagger \quad q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

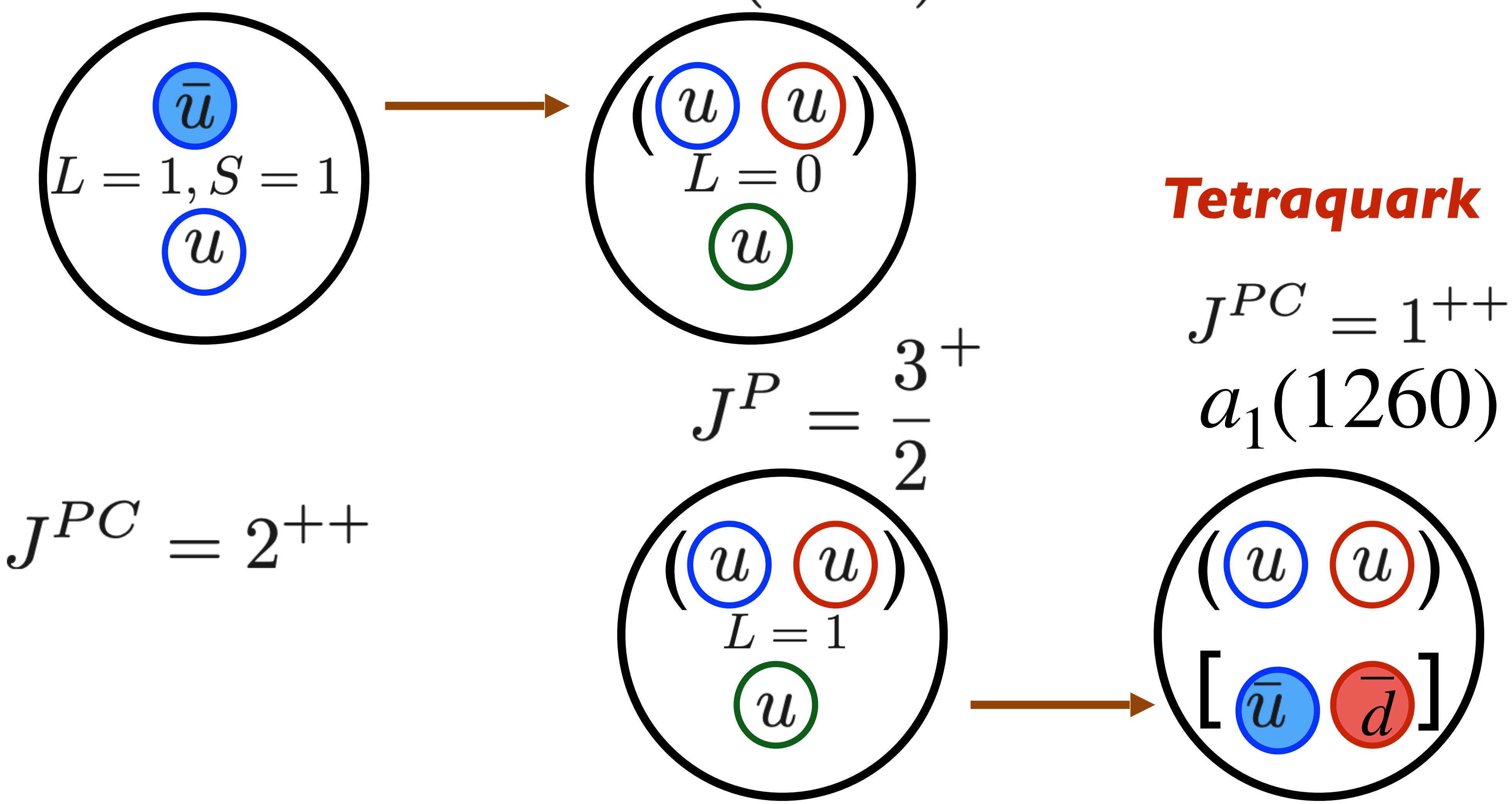
Meson

Superconformal Algebra 4-Plet

$$R_\lambda^\dagger \quad \bar{q} \to (qq) \quad S = 1$$
$$\bar{3}_C \to \bar{3}_C$$

Vector ()+ Scalar [] Diquarks

$f_2(1270)$ $\Delta^+(1232)$



$$R_\lambda^\dagger \quad q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

Meson **Baryon**

Light-Front Holography: First Approximation to QCD

- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L**
- **Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770)$, $\omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320)$, $f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690)$, $\omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)$?
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040)$, $f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)$?
$\bar{q}s$	$0^{-(+)}$	$\bar{K}(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$\bar{K}_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$\bar{K}_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$\bar{K}_1^*(\sim 1700)$?
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)$?
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)$?
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)$?
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)$?
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)$?

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- ***Universal quark light-front kinetic energy***

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

- ***Universal quark light-front potential energy***

$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- ***Universal Constant Contribution from AdS and Superconformal Quantum Mechanics***

$$\Delta\mathcal{M}^2_{spin} = 2\kappa^2(L + 2S + B - 1)$$

hyperfine spin-spin

Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

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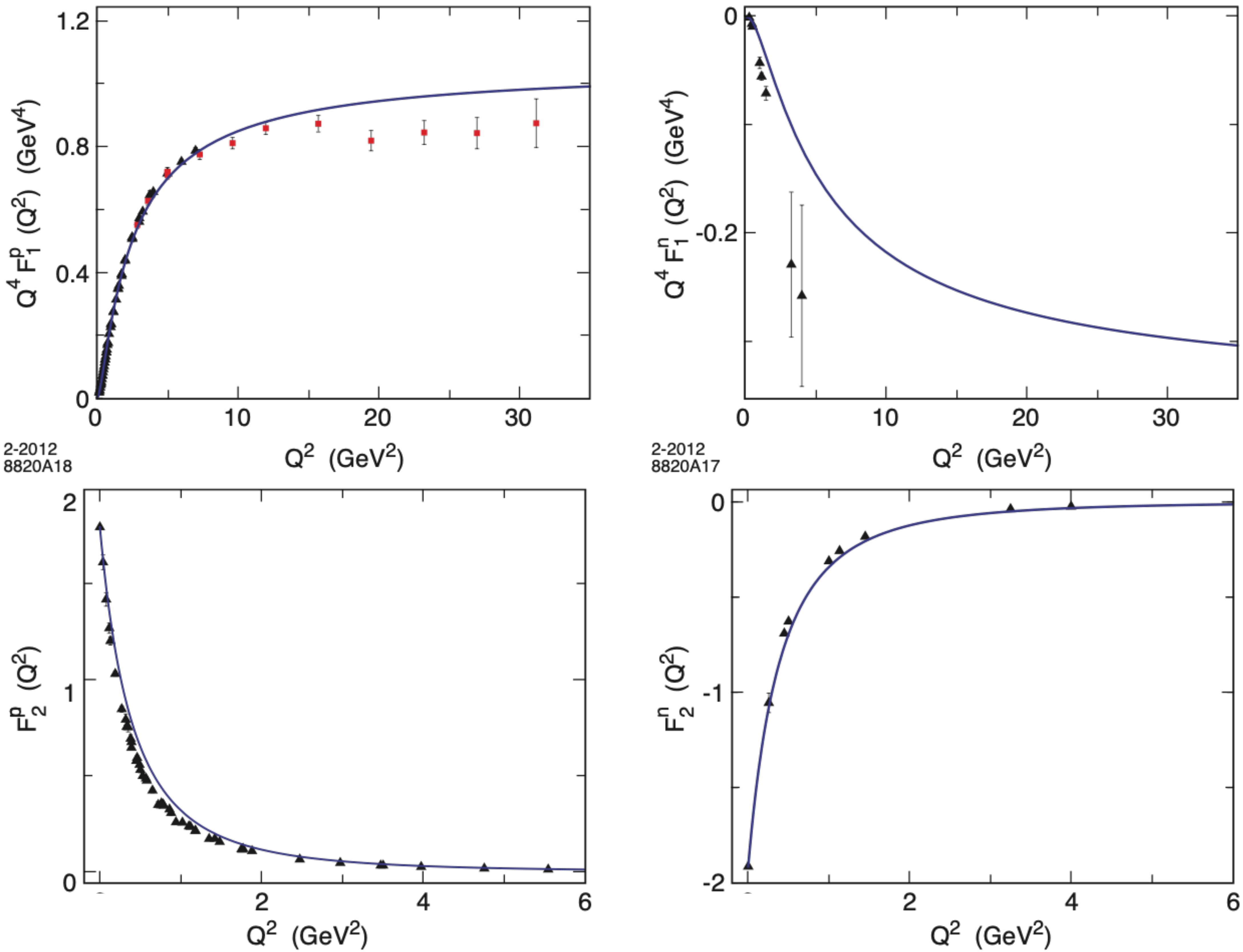
$$\Delta\mathcal{M}^2_{spin} = 2\kappa^2(L + 2S + B - 1)$$

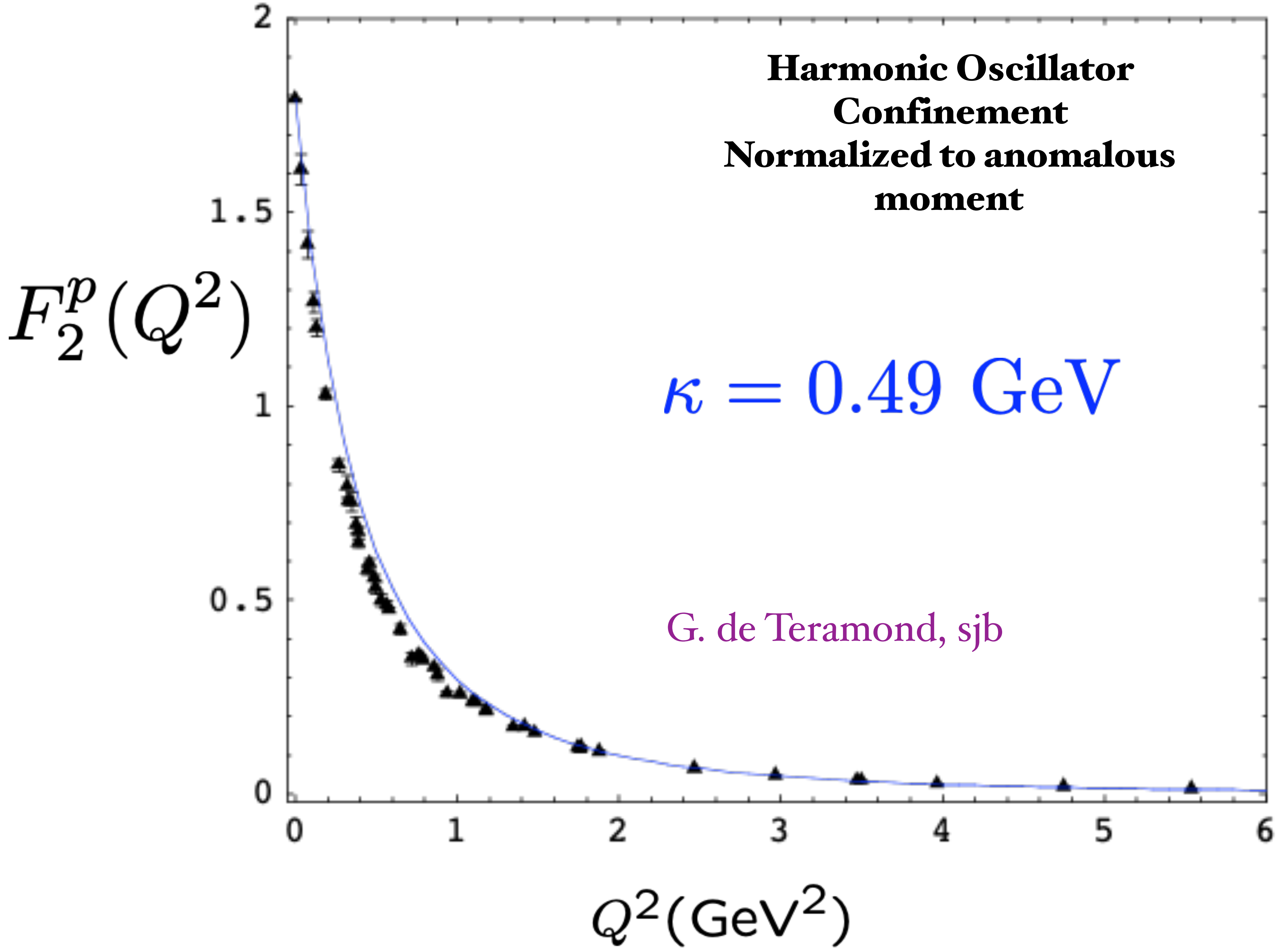
hyperfine spin-spin

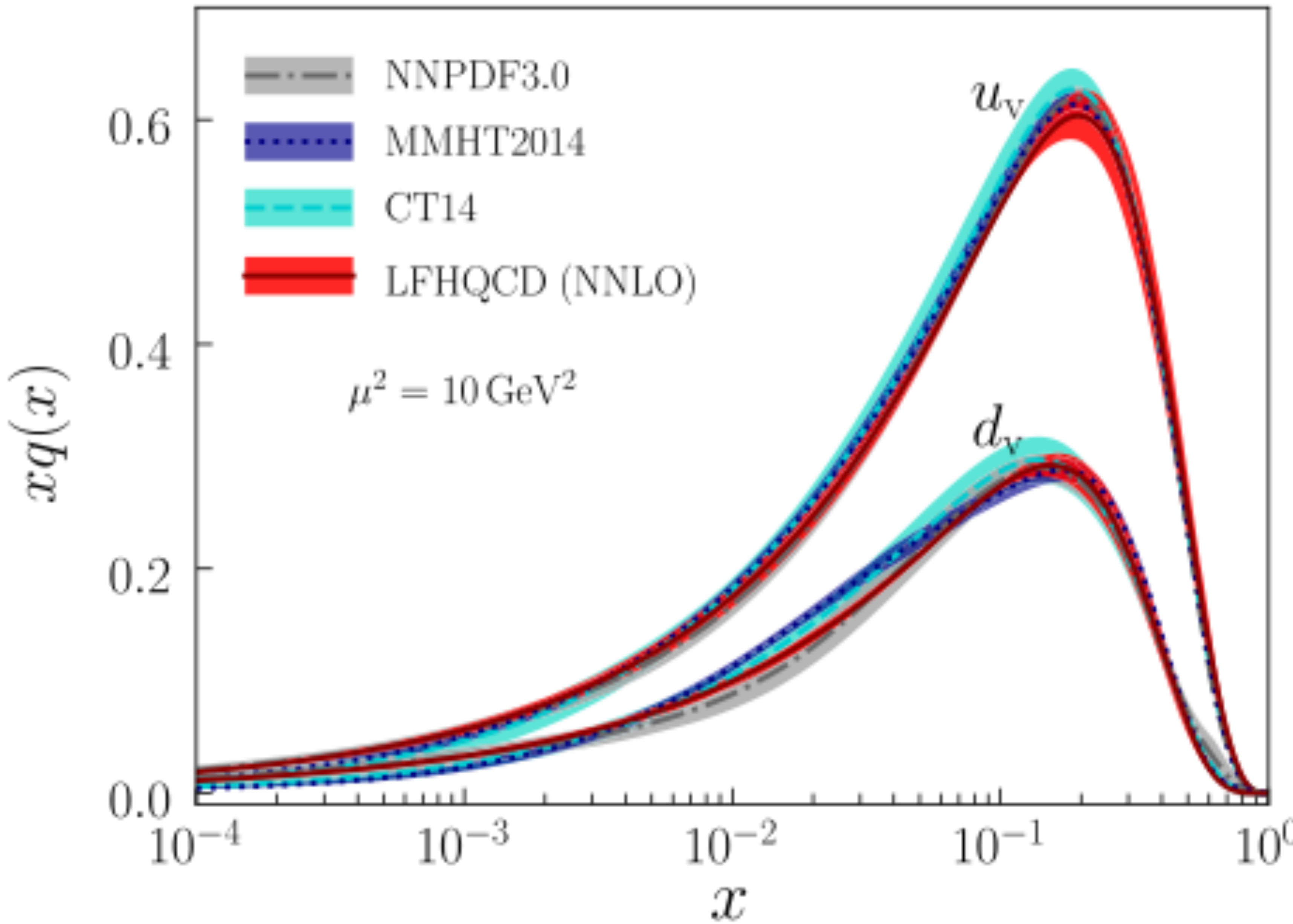
Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



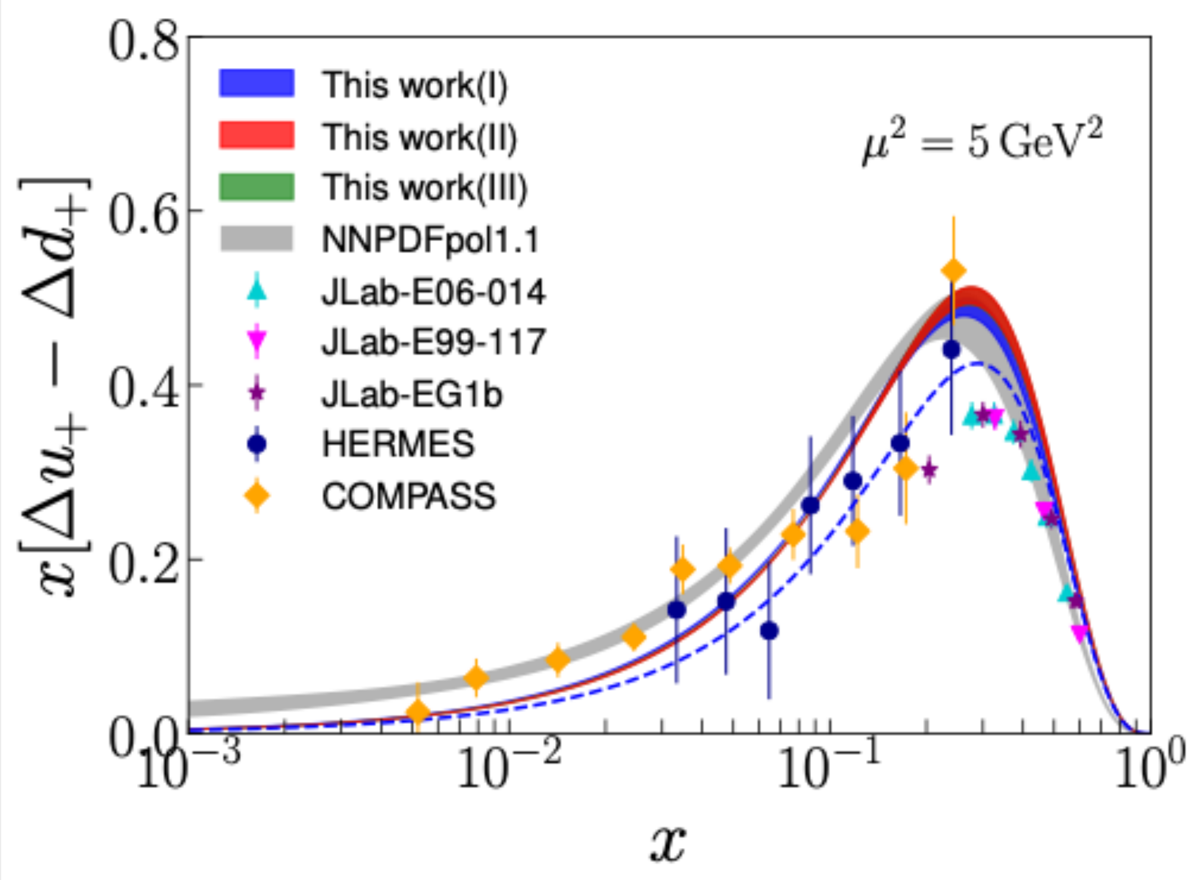


Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Te´ramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

Tianbo Liu, Raza Sabbir Sufian, Guy F. de T´eramond, Hans Gunter Dösch, Alexandre Deur, sjb



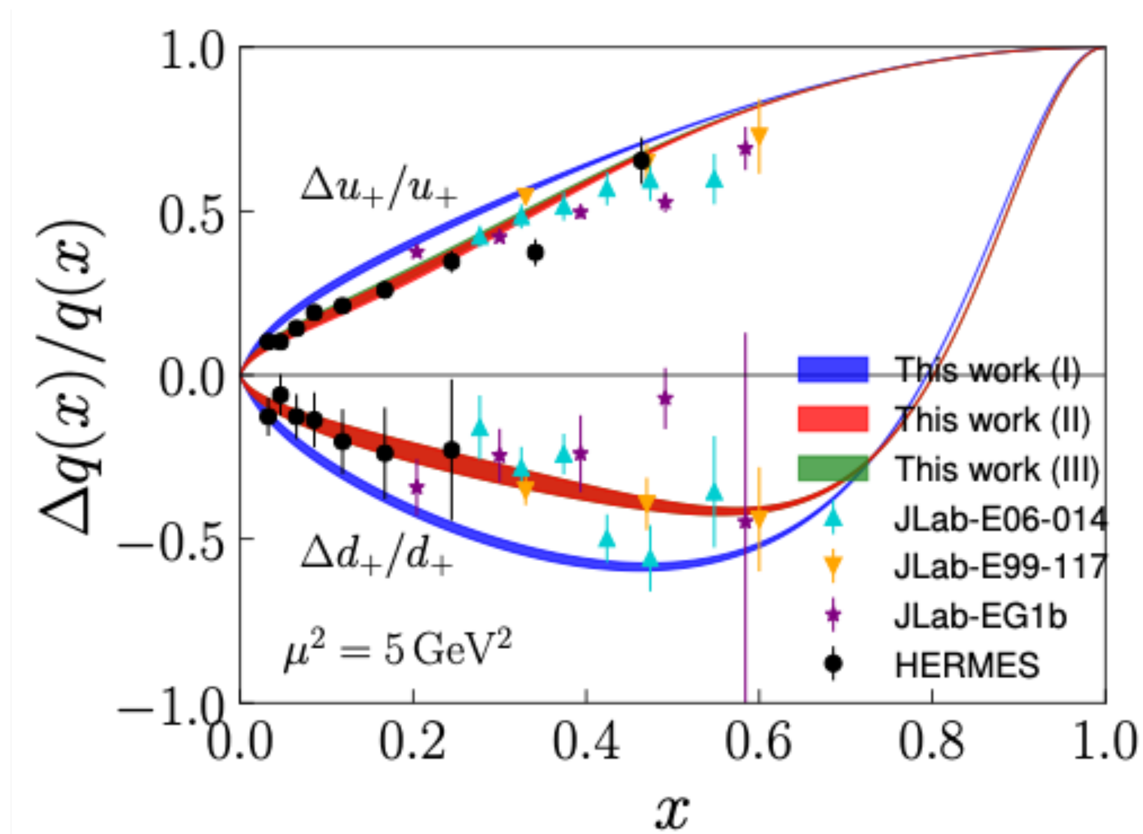
Polarized distributions for the

isovector combination $x[\Delta u_+(x) - \Delta d_+(x)]$

$d_+(x) = d(x) + \bar{d}(x) \qquad u_+(x) = u(x) + \bar{u}(x)$

Proton Structure Functions

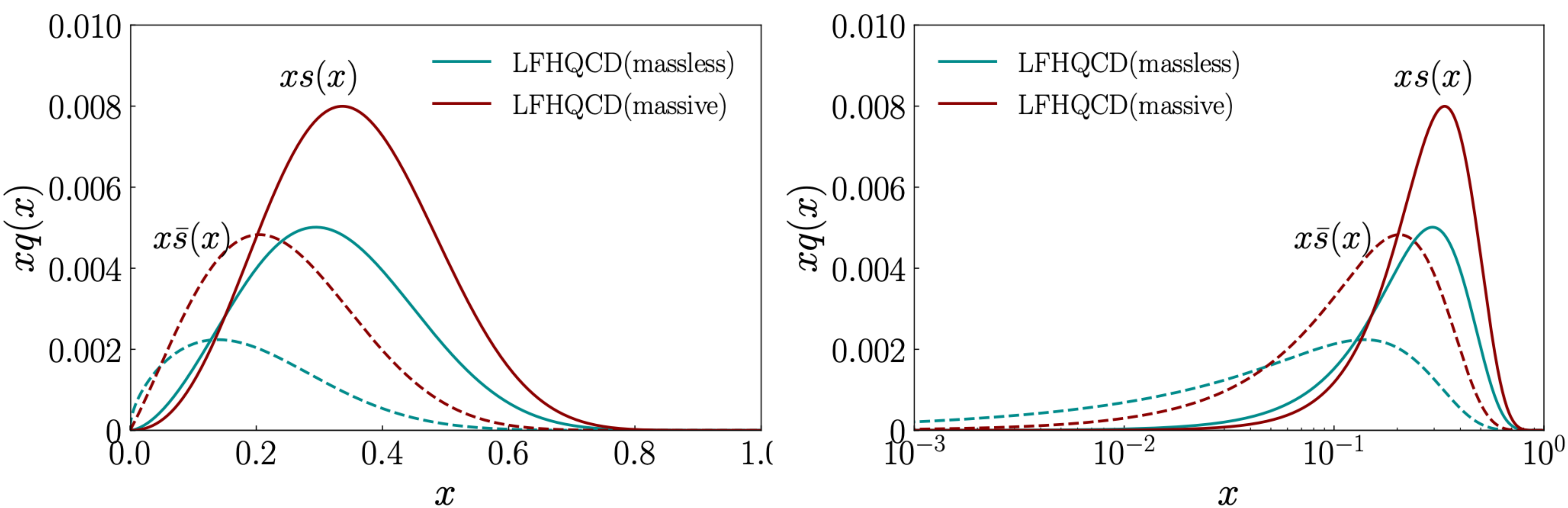
$$\Delta q(x) = q_\uparrow(x) - q_\downarrow(x)$$



Strange and Antistrange Distributions

Input: nonzero lattice axial form factor

Duality with $|K\Lambda>$ meson-nucleon fluctuations

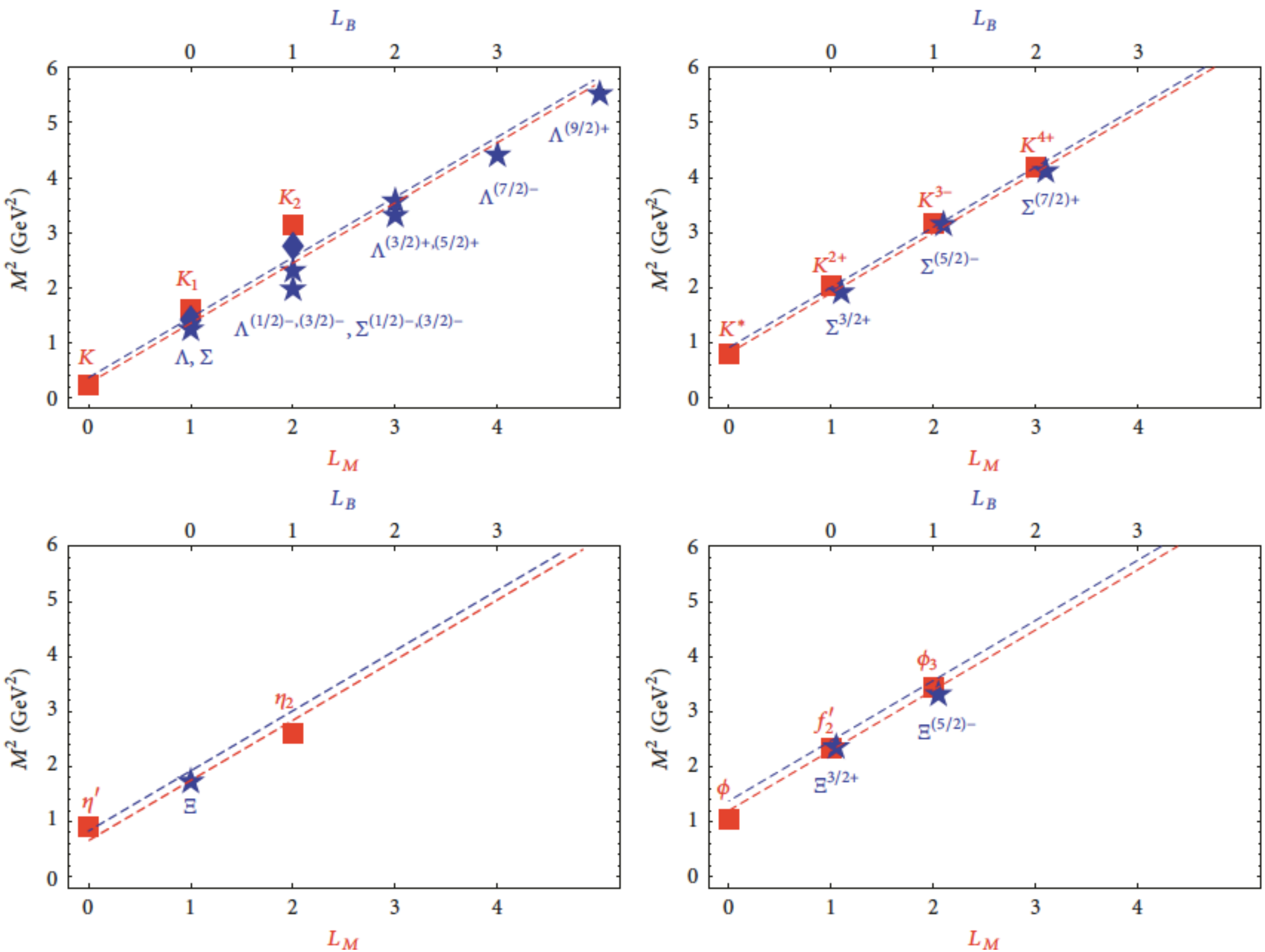


Phys. Rev. D 98, 114004 (2018).

R. S. Sufian, T.Liu, de Teramond, Dosch, Deur, Islam, Ma, sjb

de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



de Téramond, Dosch, Lorcé, sjb

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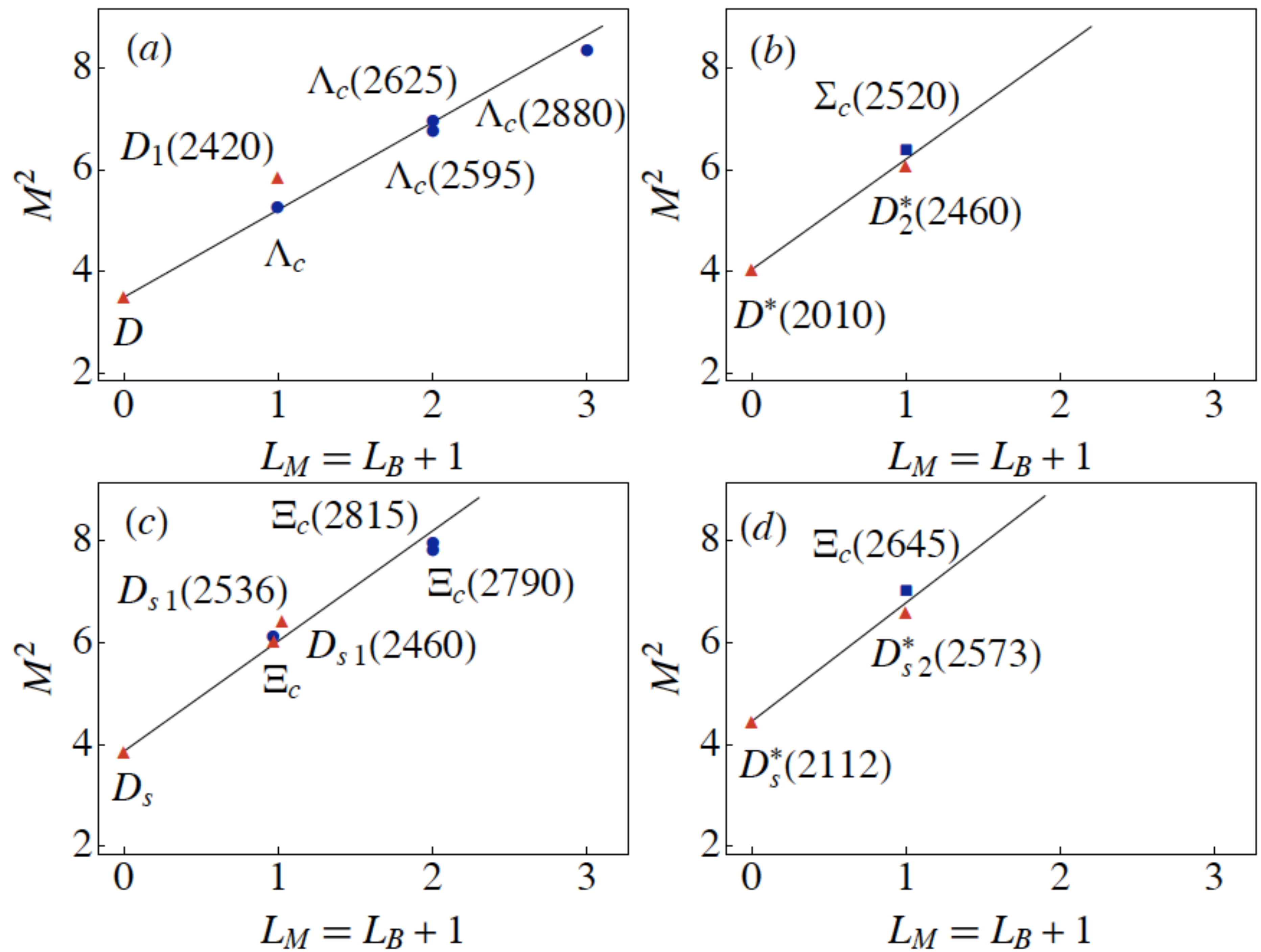

Heavy charm quark mass does not break supersymmetry

Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$\bar{D}_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

M. Nielsen, sjb

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beautiful agreement!

M. Nielsen, sjb

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$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
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$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
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$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
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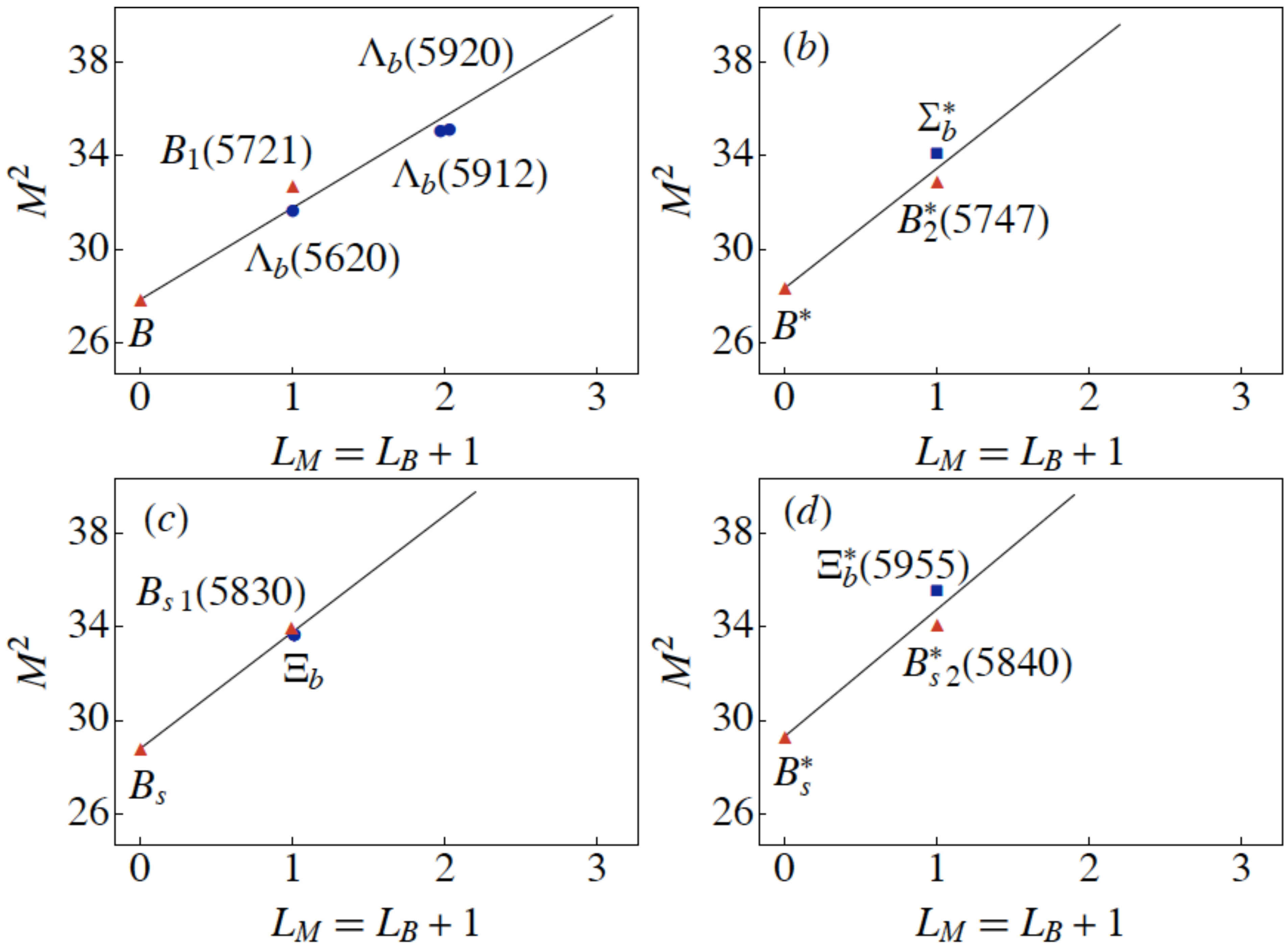
predictions

beautiful agreement!

M. Nielsen, sjb

de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector

  [H. G. Dosch, GdT, S. J. Brodsky, PRD **92**, 074010 (2015), PRD **95**, 034016 (2017)]

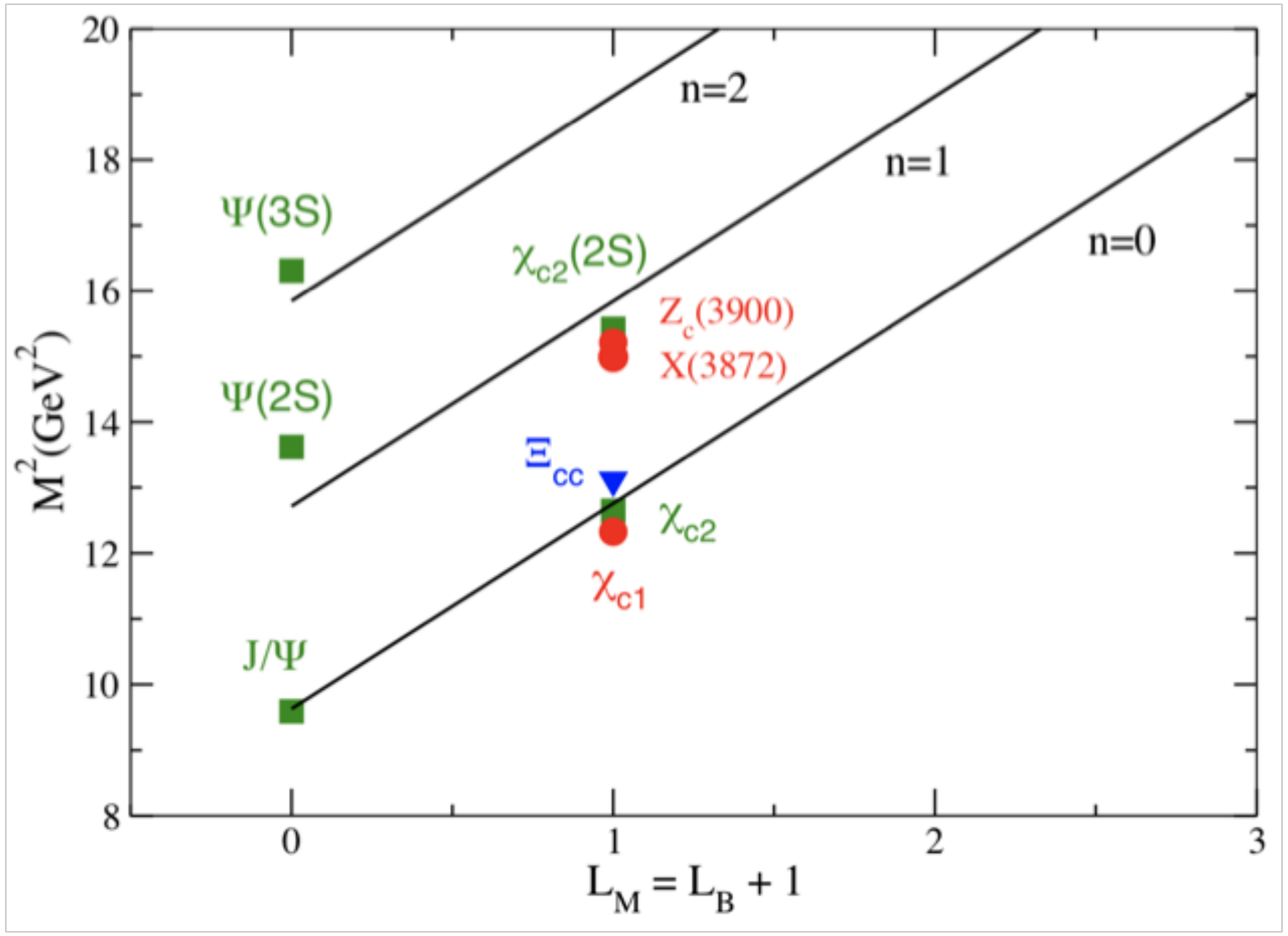
- Extension to the double-heavy hadronic sector

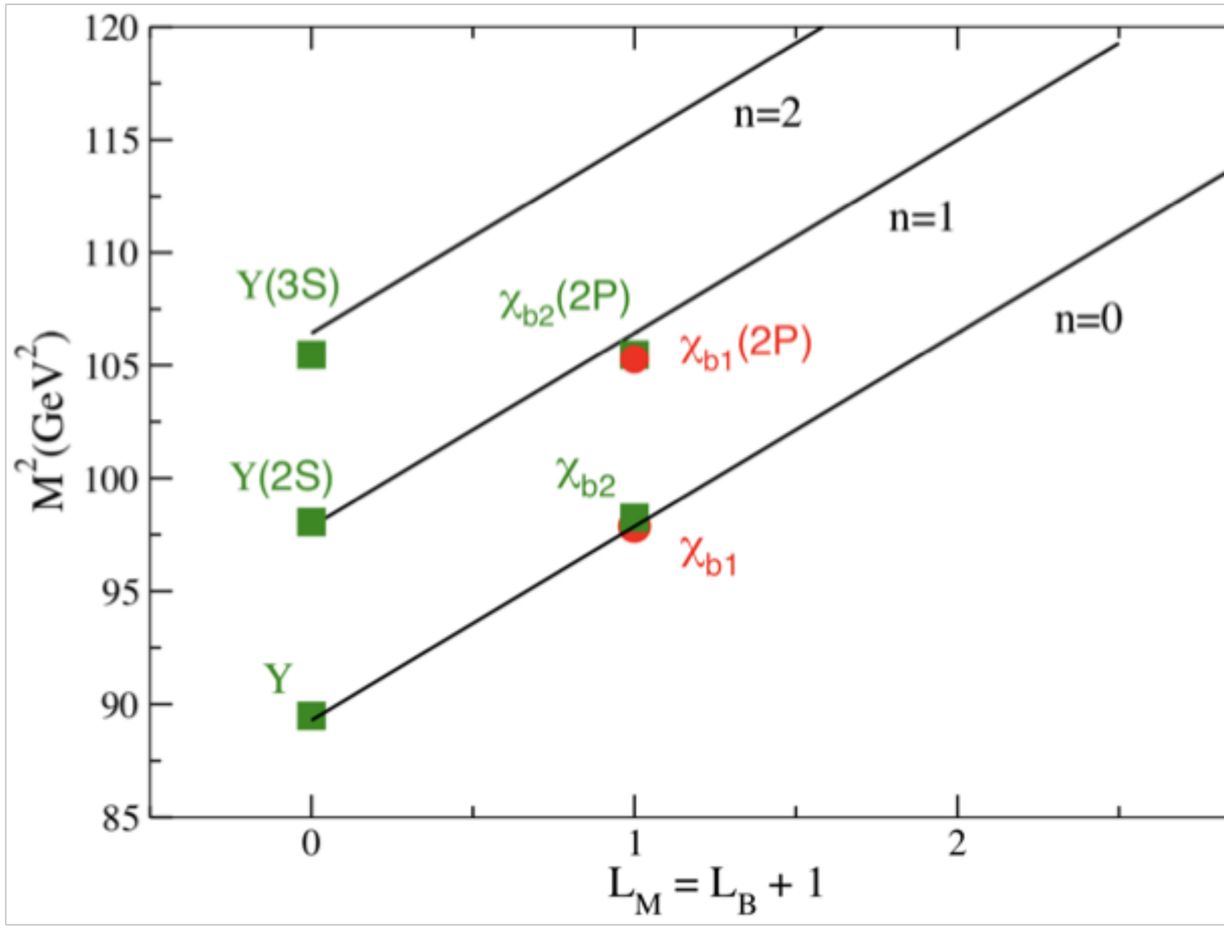
  [M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

  [M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra, L. Zou, PRD **98**, 034002 (2018)]

- Extension to the isoscalar hadronic sector

  [L. Zou, H. G. Dosch, GdT,S. J. Brodsky, arXiv:1901.11205 [hep-ph]]

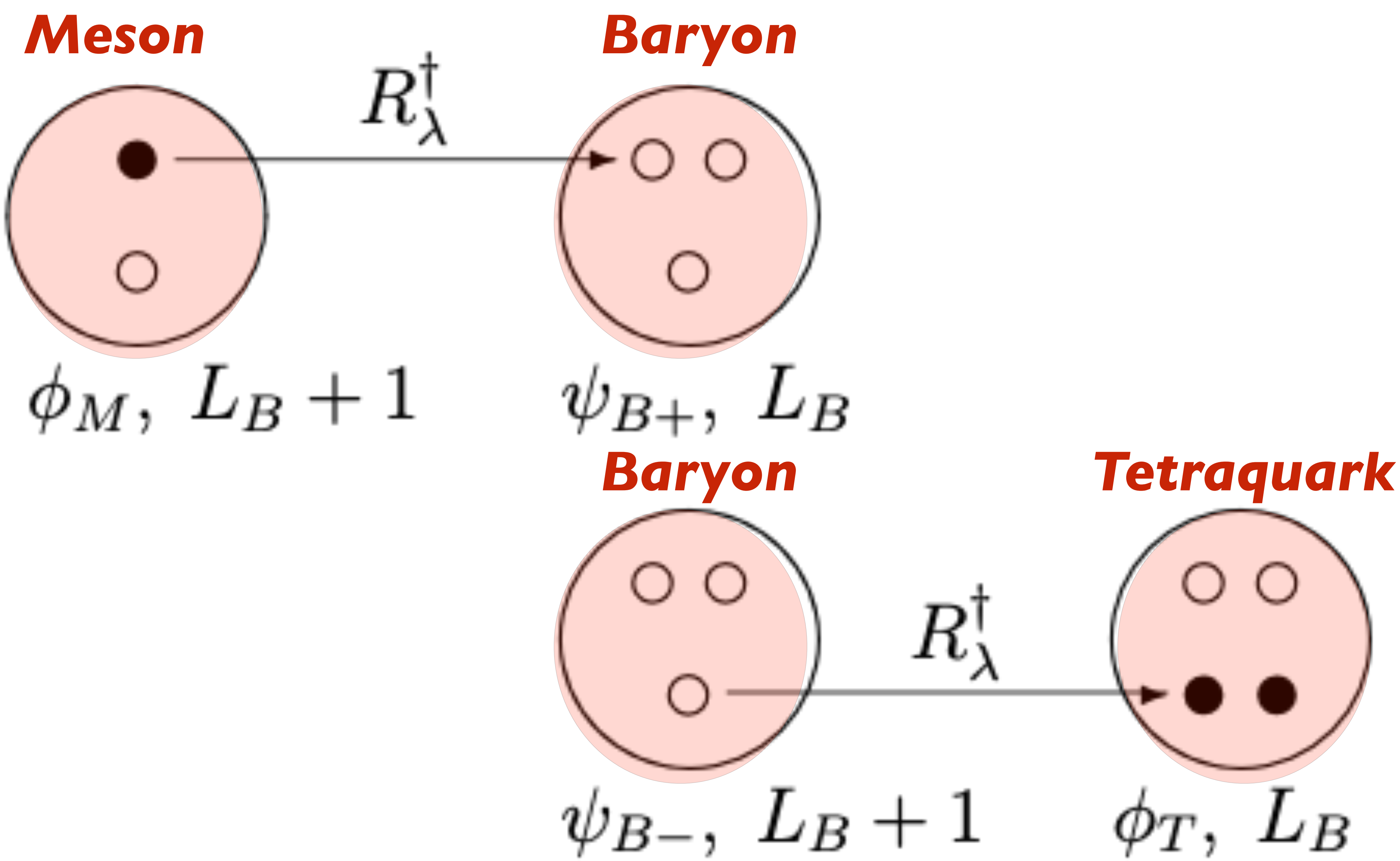




Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



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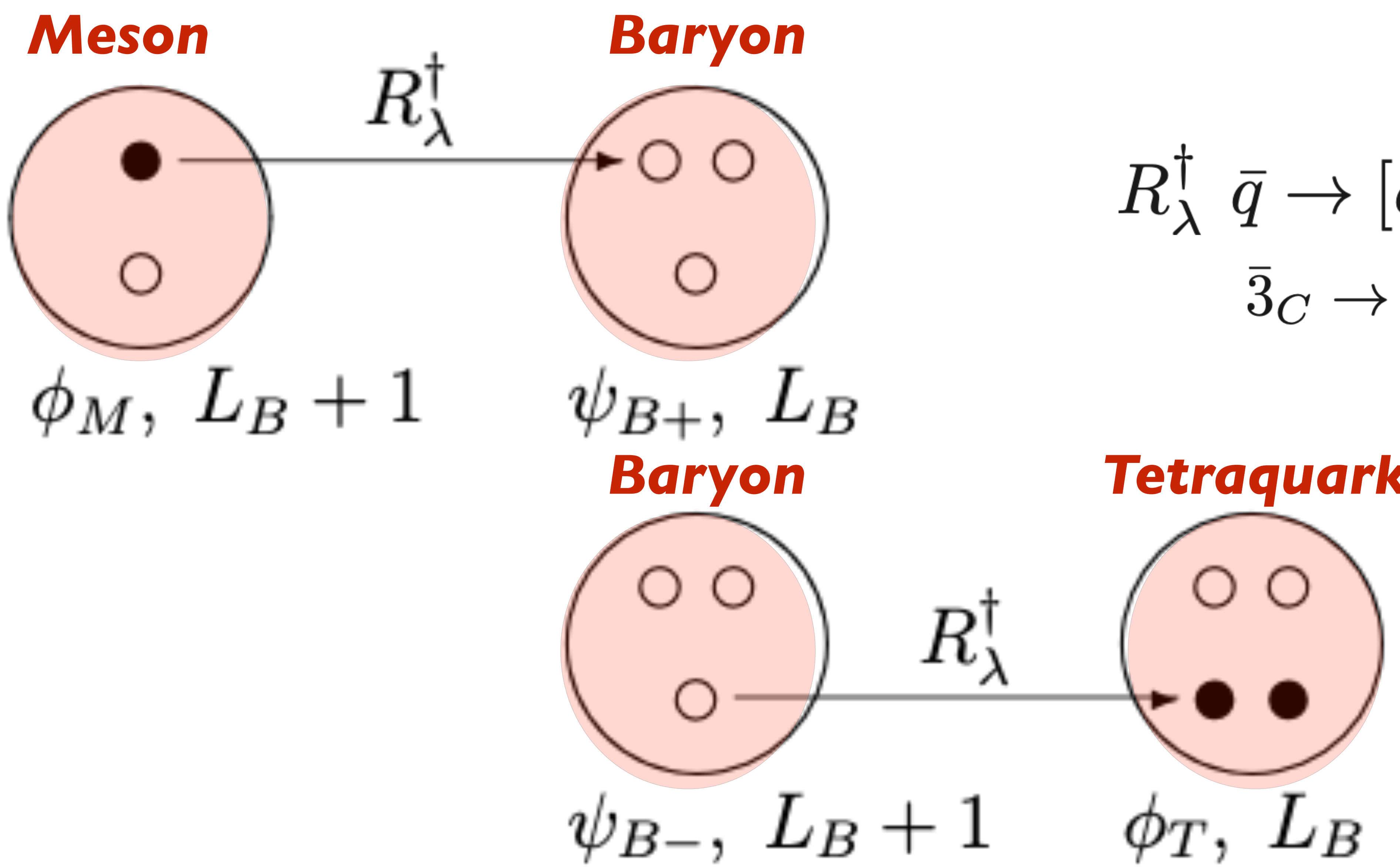


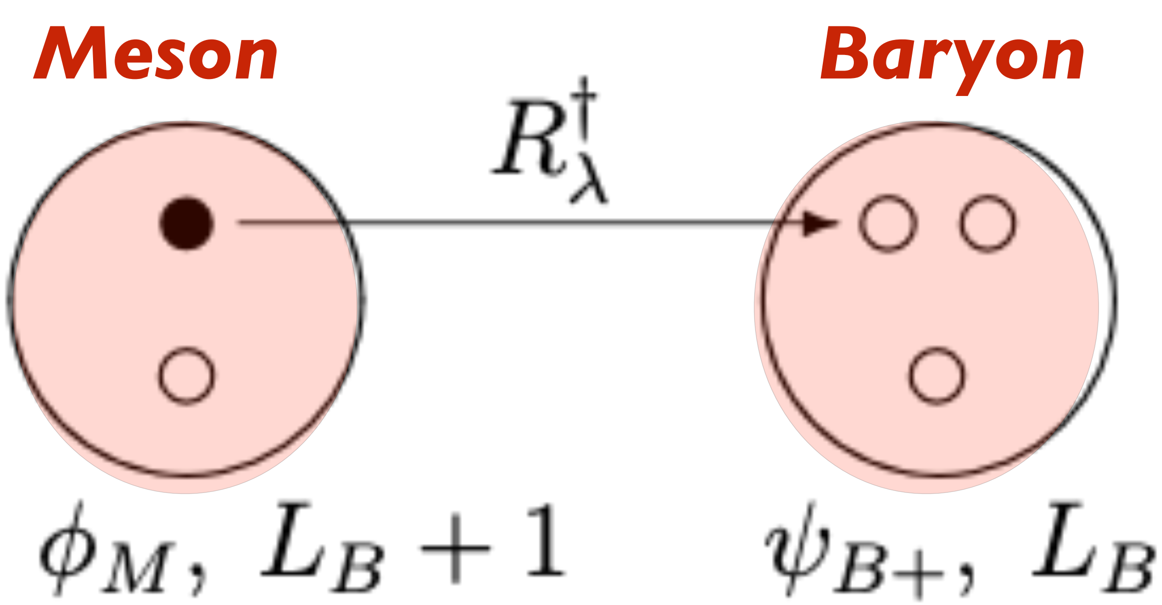
$$R^\dagger_\lambda \; \bar{q} \rightarrow [qq]$$

$$\bar{3}_C \rightarrow \bar{3}_C$$

Superconformal Algebra

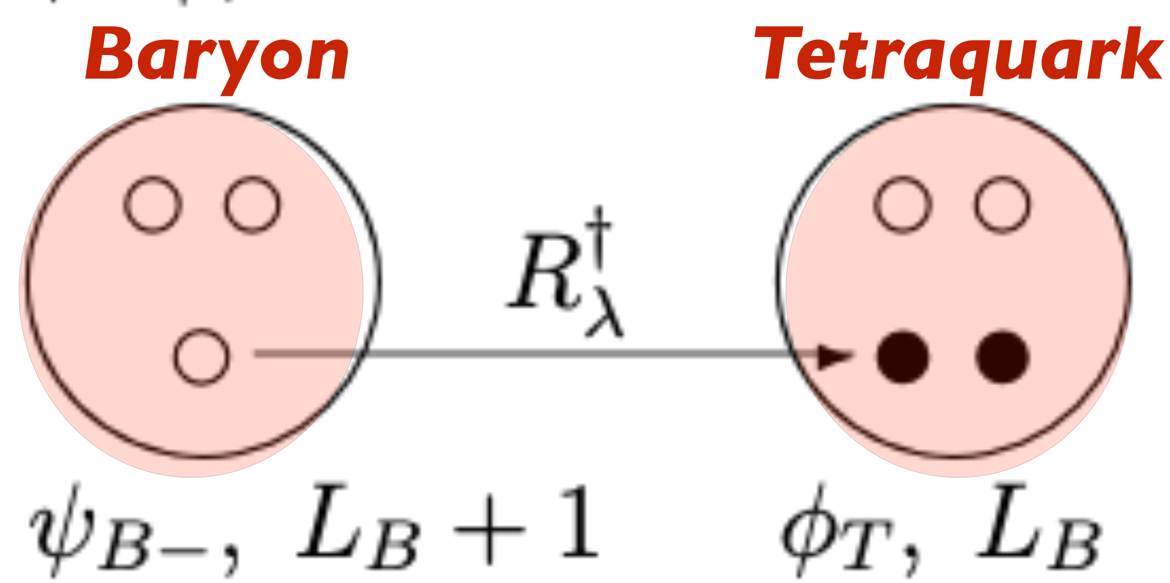
2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



$R^\dagger_\lambda \; \bar{q} \rightarrow [qq]$

$\bar{3}_C \rightarrow \bar{3}_C$



$R^\dagger_\lambda \; q \rightarrow [\bar{q}\bar{q}]$

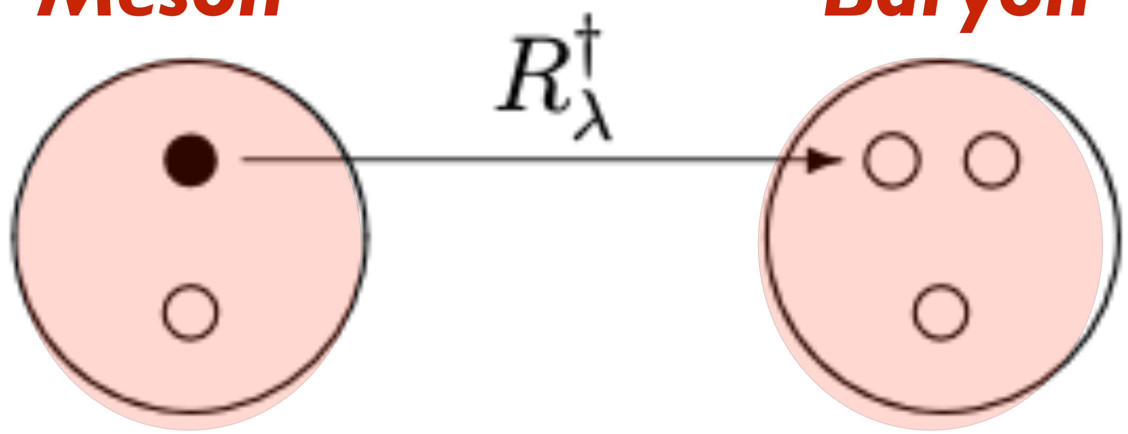
$3_C \rightarrow 3_C$

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!

Meson **Baryon**

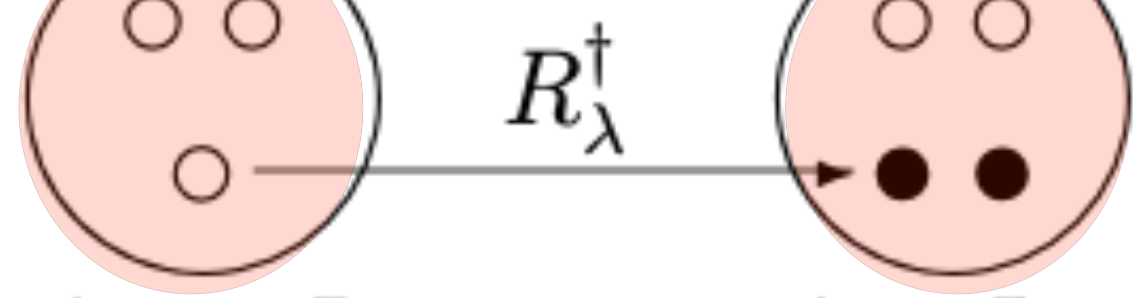


$\phi_M, \; L_B + 1$ $\qquad$ $\psi_{B+}, \; L_B$

$R_\lambda^\dagger \; \bar{q} \to [qq]$

$\bar{3}_C \to \bar{3}_C$

Baryon **Tetraquark**



$R_\lambda^\dagger \; q \to [\bar{q}\bar{q}]$

$3_C \to 3_C$

$\psi_{B-}, \; L_B + 1$ $\qquad$ $\phi_T, \; L_B$

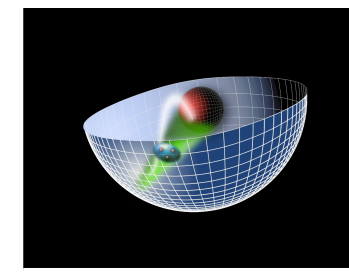
Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time* **τ**

- *Causality: Information within causal horizon: Light-Front*

- *Light-Front Holography: AdS₅ = LF (3+1)*



$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

- *Introduce Mass Scale* **κ** *while retaining the Conformal Invariance of the Action (dAFF)*

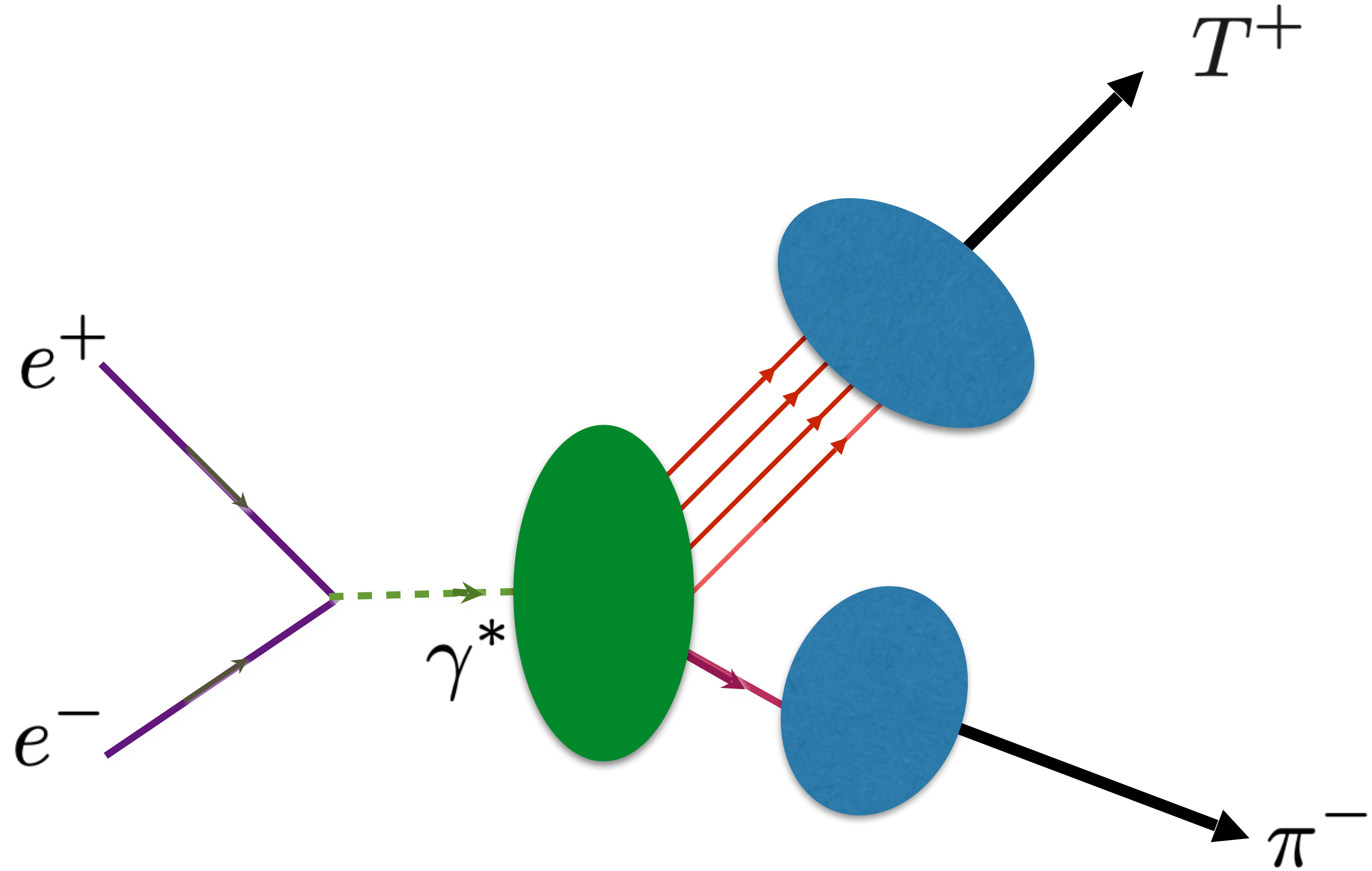
- *Unique Dilaton in AdS₅:* $e^{+\kappa^2 z^2}$

- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$

- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\boxed{\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]}$$

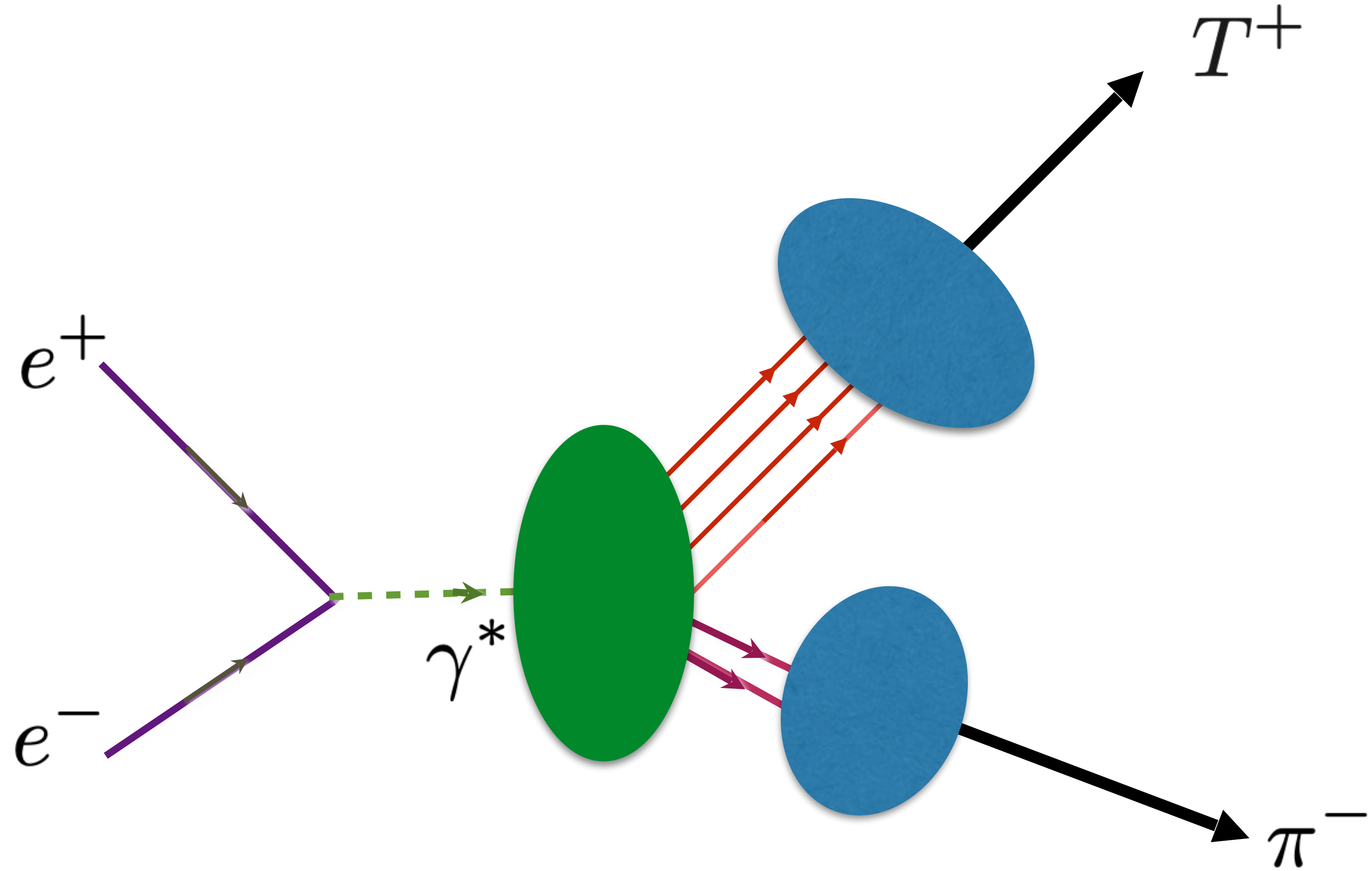
$$\sigma(e^+e^- \to MT) \propto \frac{1}{s^{N-1}} \quad N = 6$$



Use counting rules to identify composite structure

Lebed, sjb

$$\sigma(e^+e^- \to MT) \propto \frac{1}{s^{N-1}} \quad N = 6$$



Use counting rules to identify composite structure

Lebed, sjb

Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

de Téramond, Dosch, Lorcé, sjb

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad\qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

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where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge

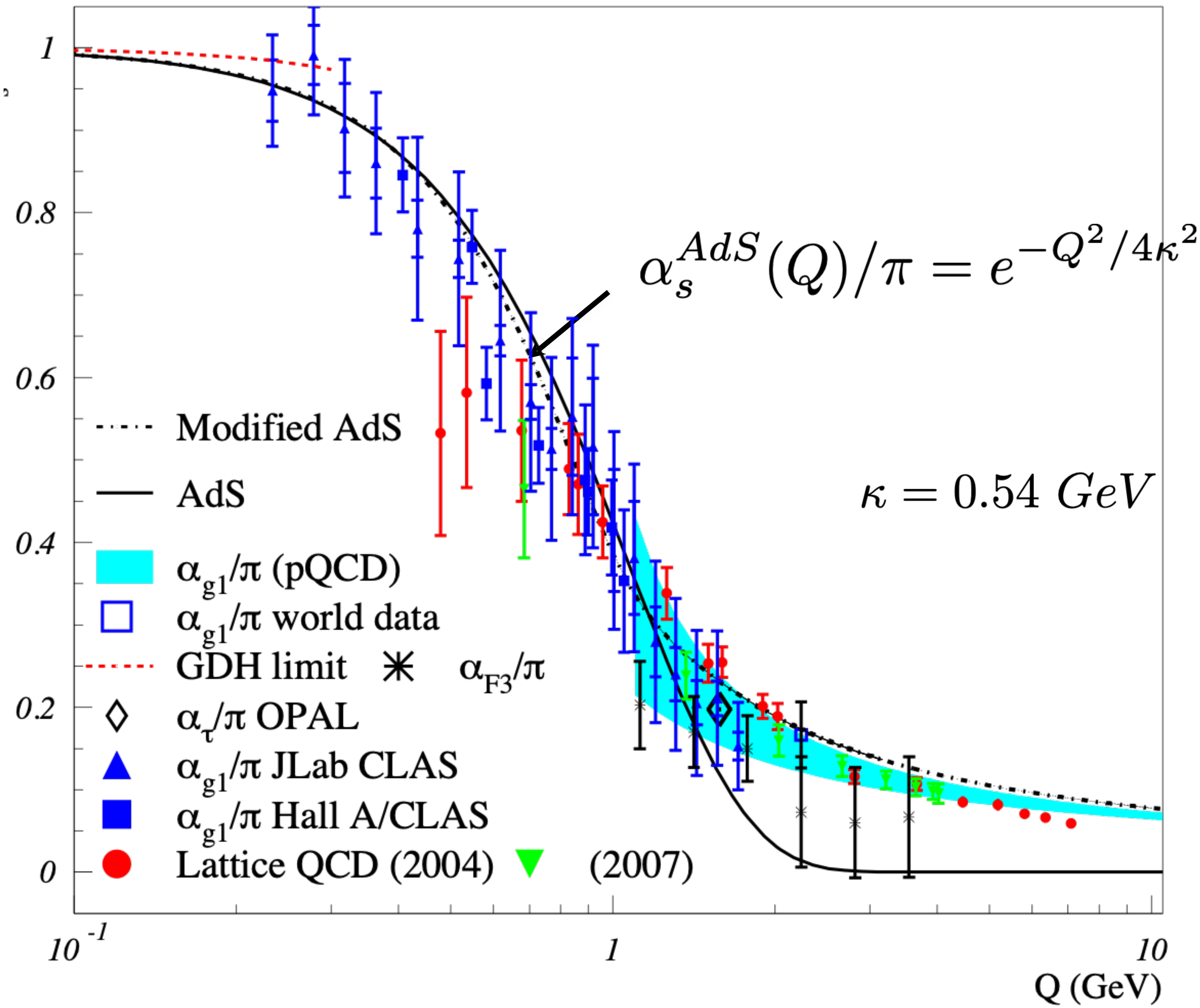
$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal*** β_0, β_1

Analytic, defined at all scales, IR Fixed Point

$$\frac{\alpha_s(Q)}{\pi}$$



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

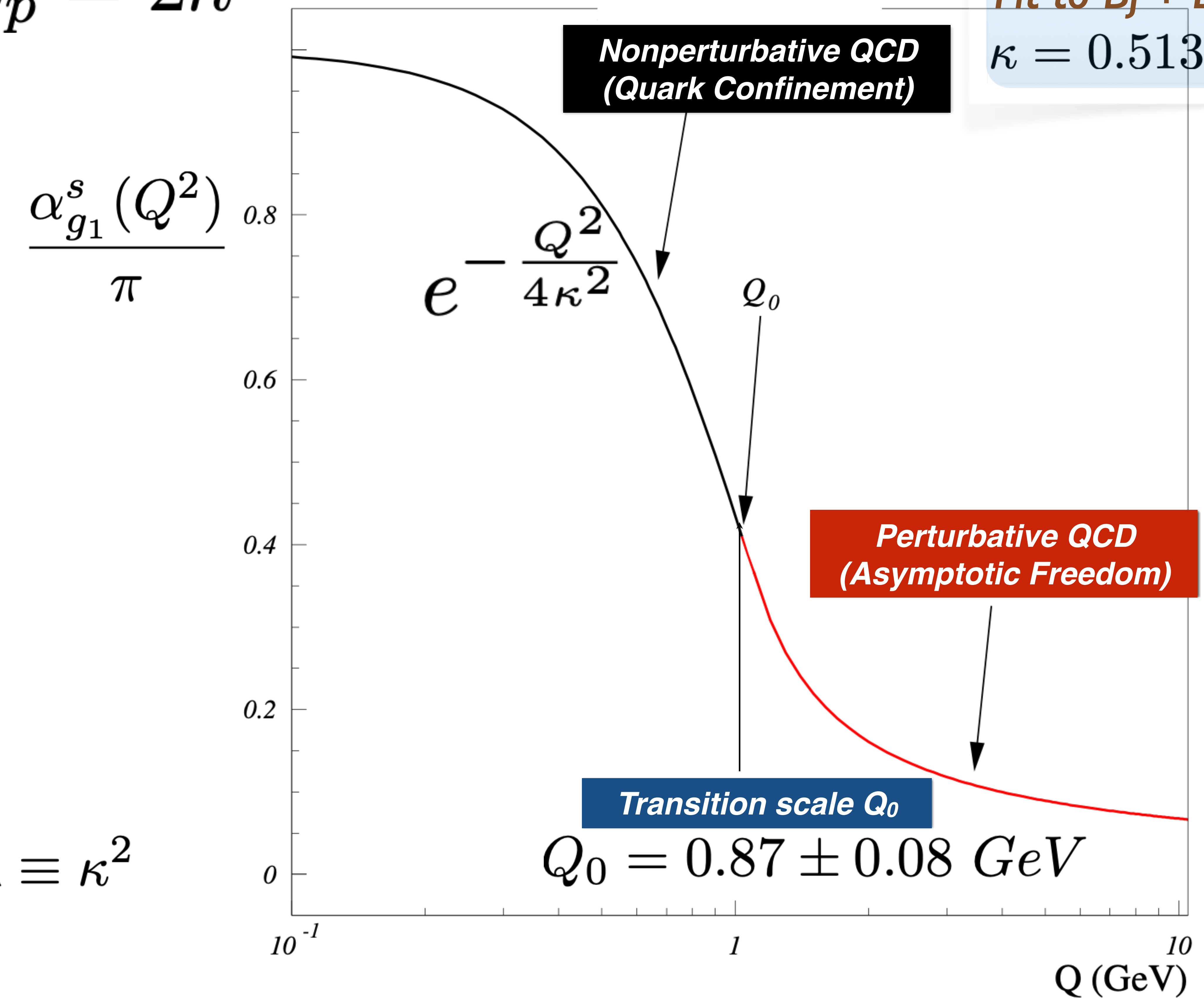
$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$



$$\lambda \equiv \kappa^2$$

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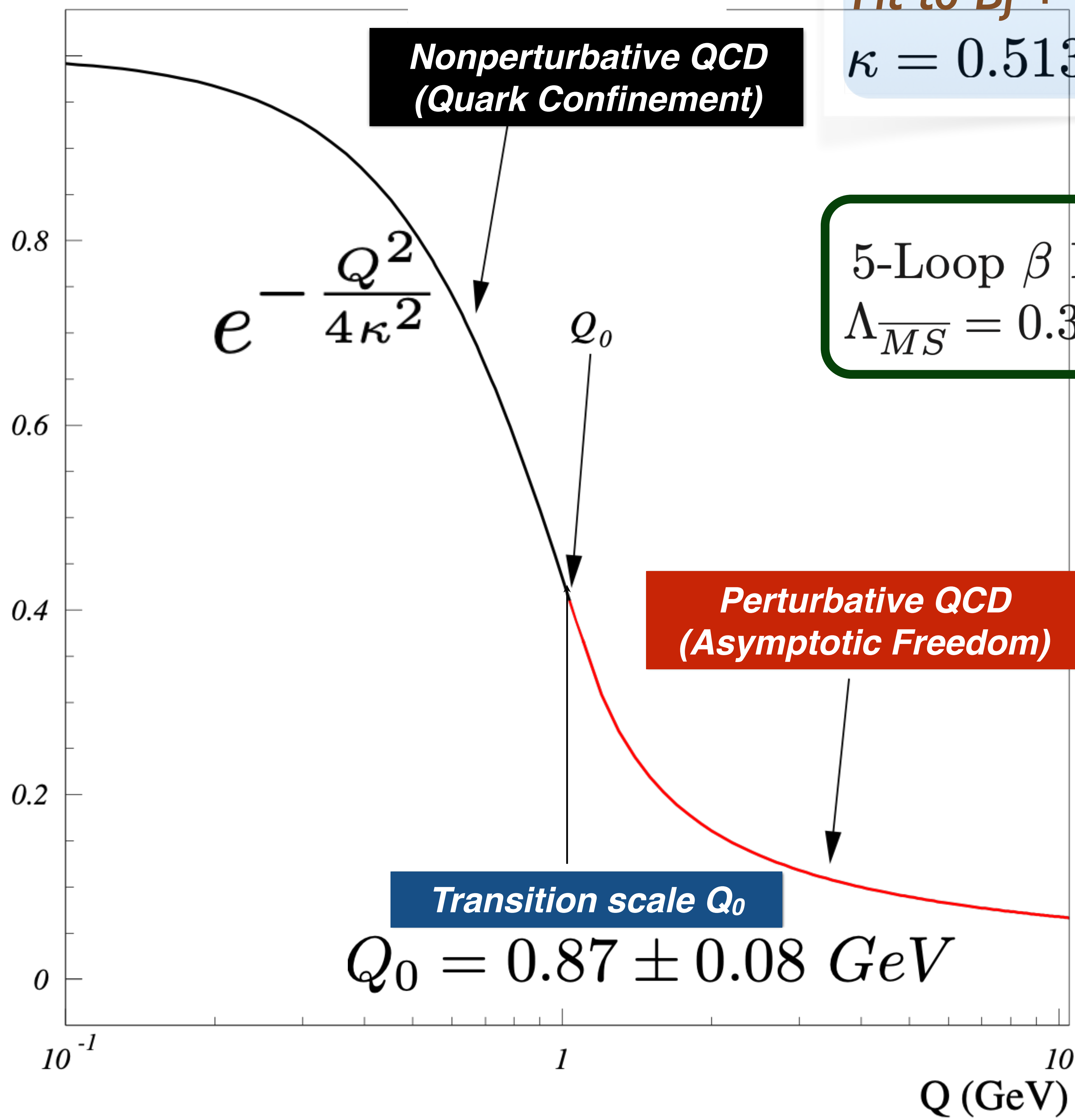
5-Loop β Prediction:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



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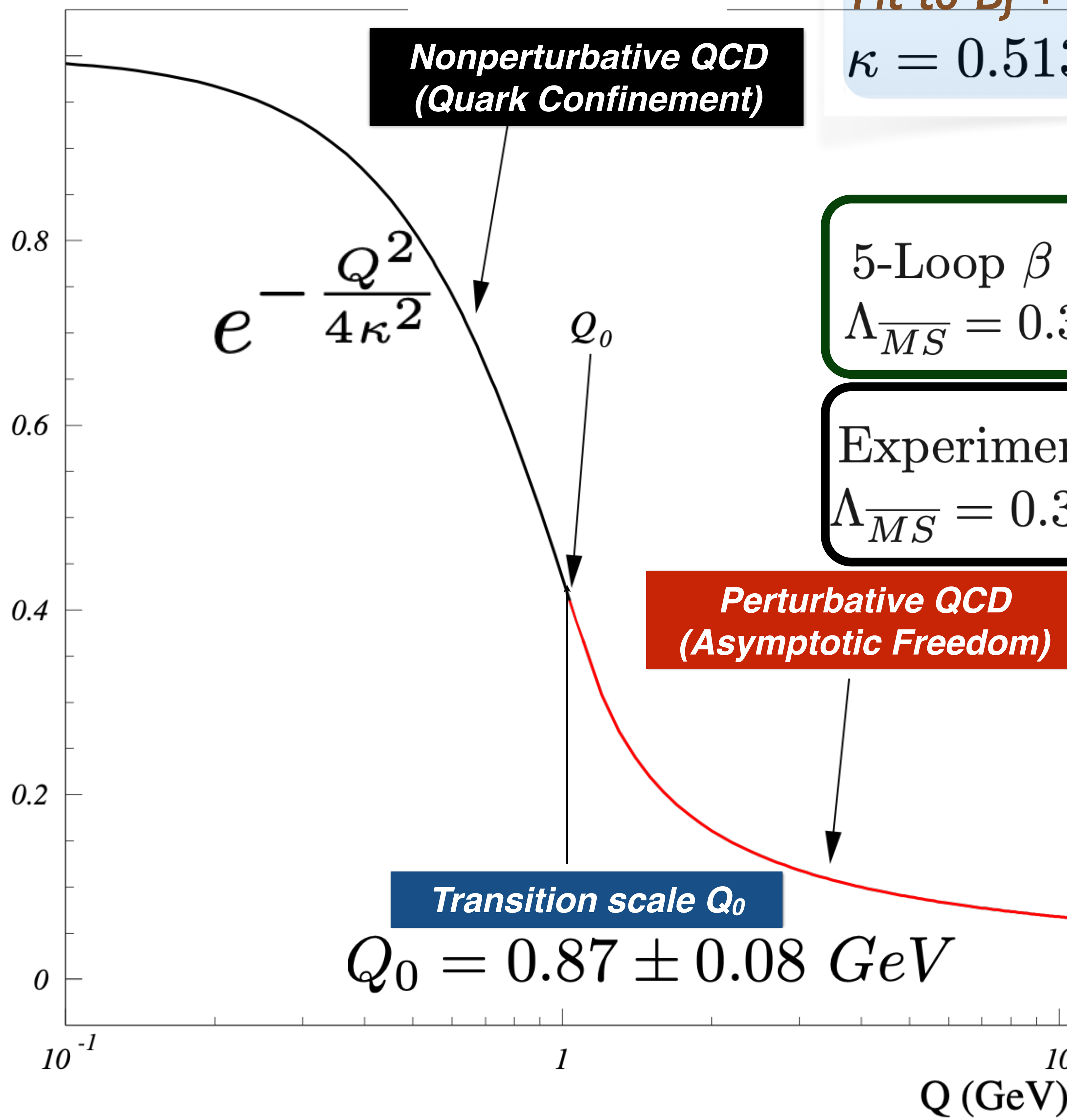
Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



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Deur, de Tèramond, sjb

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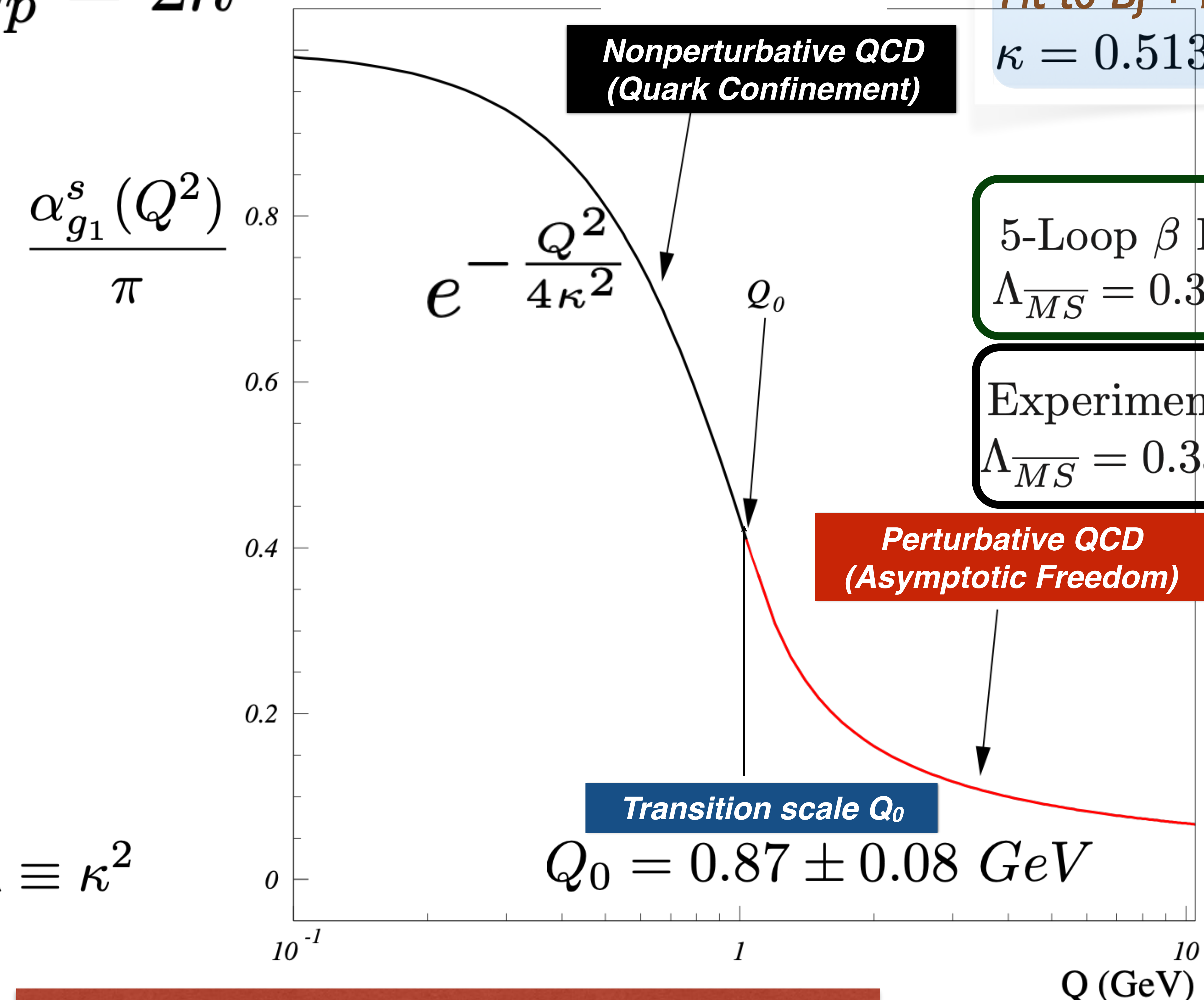
Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$



$$\lambda \equiv \kappa^2$$

Reverse Dimensional Transmutation!

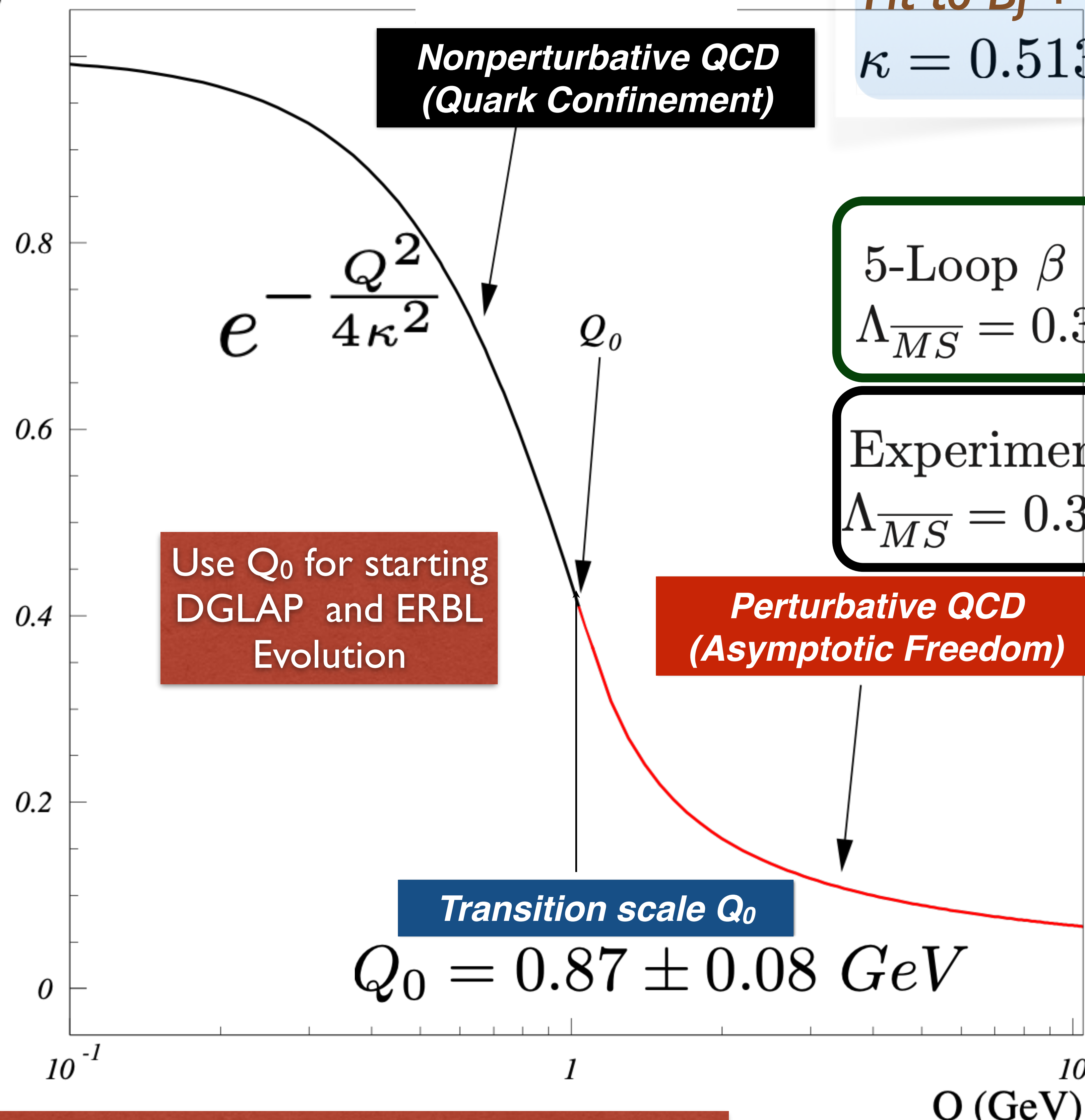
All-Scale QCD Coupling

Deur, de Tèramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



Fit to Bj + DHG Sum Rules:
$\kappa = 0.513 \pm 0.007\ GeV$

5-Loop β Prediction:
$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$

Experiment:
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Reverse Dimensional Transmutation!

$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

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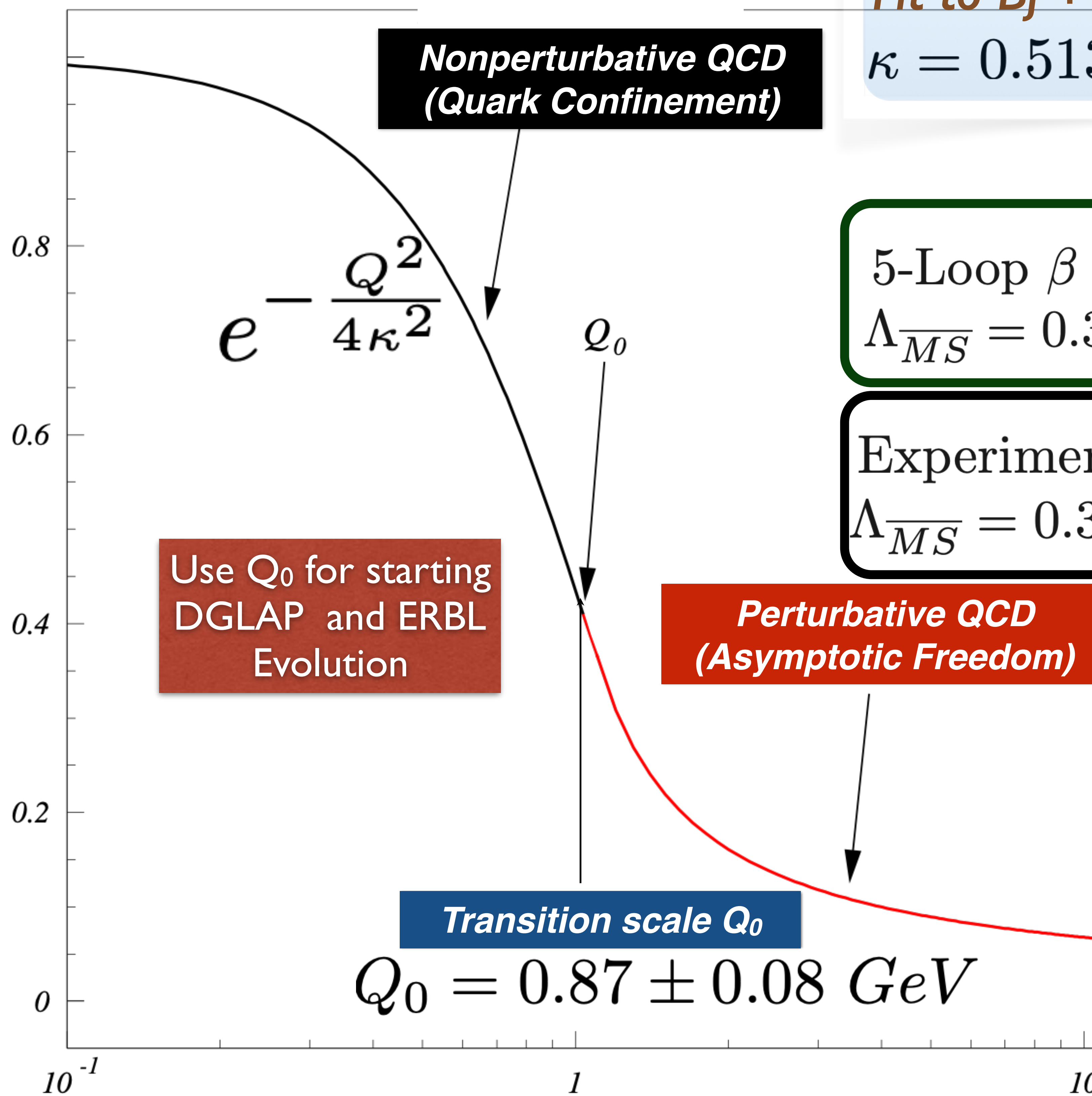


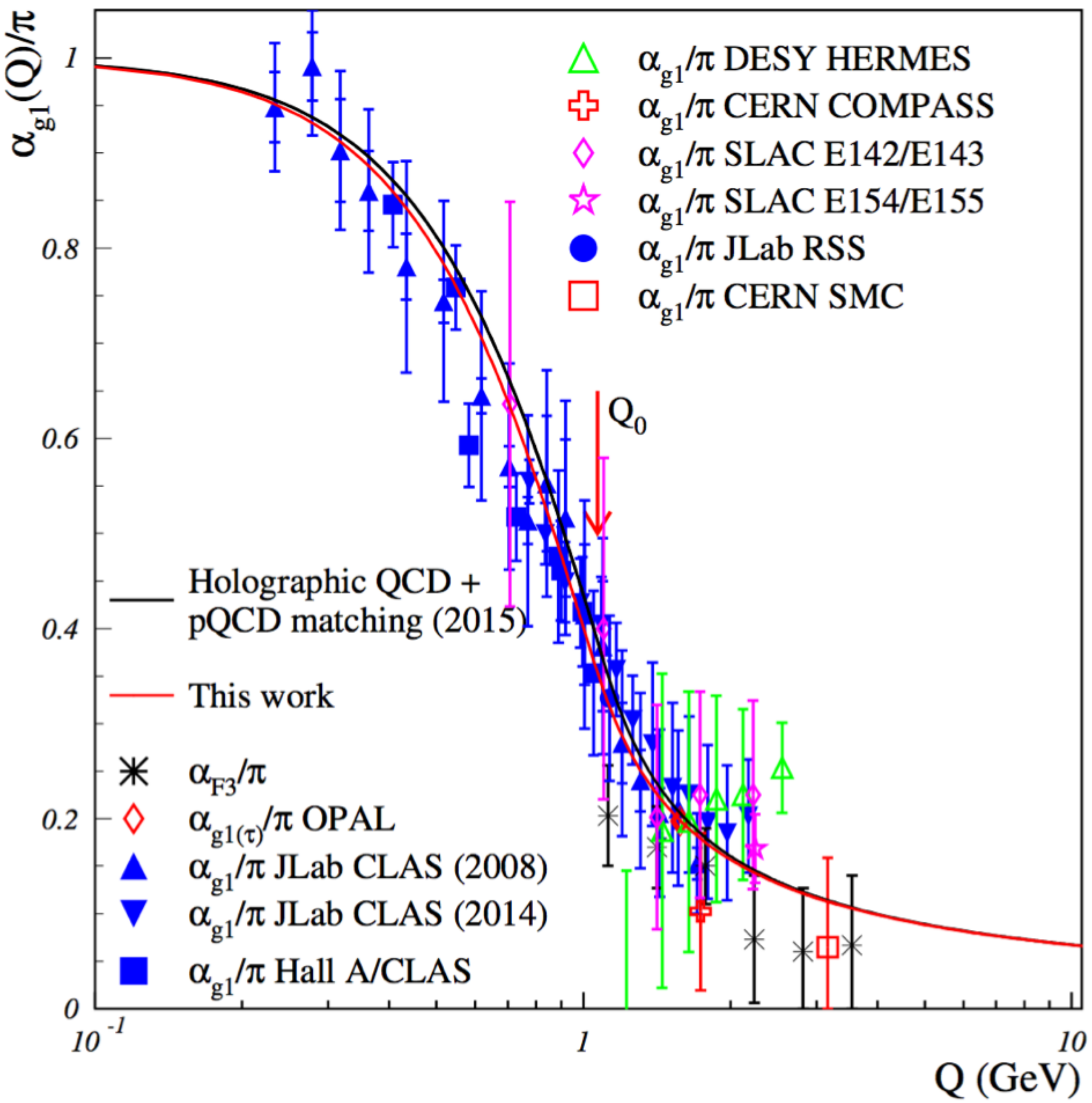
$$Q_0 = 0.87 \pm 0.08\ GeV$$

$$\lambda \equiv \kappa^2$$

$\overline{MS}$ *scheme*

Reverse Dimensional Transmutation!

Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A}\int_0^1 dx\, g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD (valid at low-Q^2)

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp\left(-Q^2/4\kappa^2\right)$$

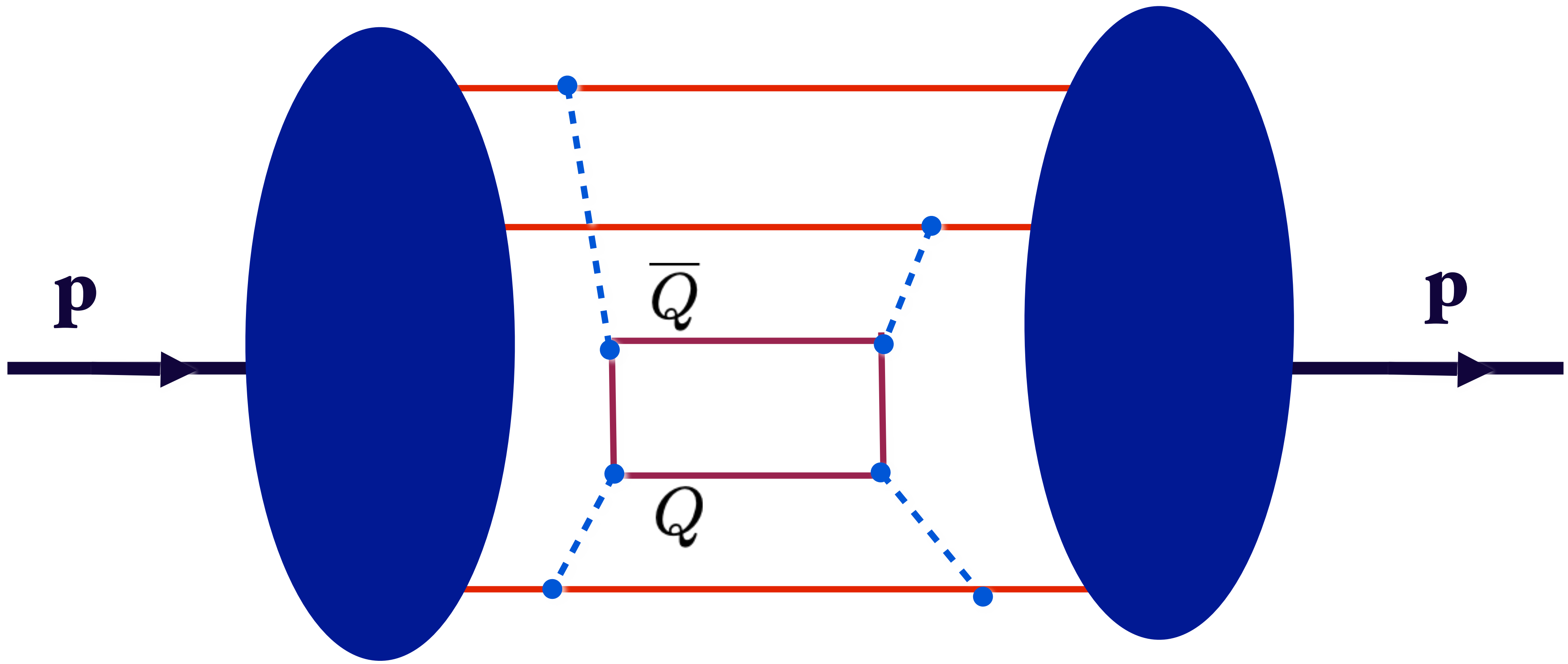
Imposing continuity for α and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond, Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

Analytic, defined at all scales, IR Fixed Point

Hoyer, Peterson, Sakai, sjb

Proton Self Energy
Intrinsic Heavy Quarks



$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

Rigorous OPE Analysis

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.

Hoyer, Peterson, Sakai, sjb

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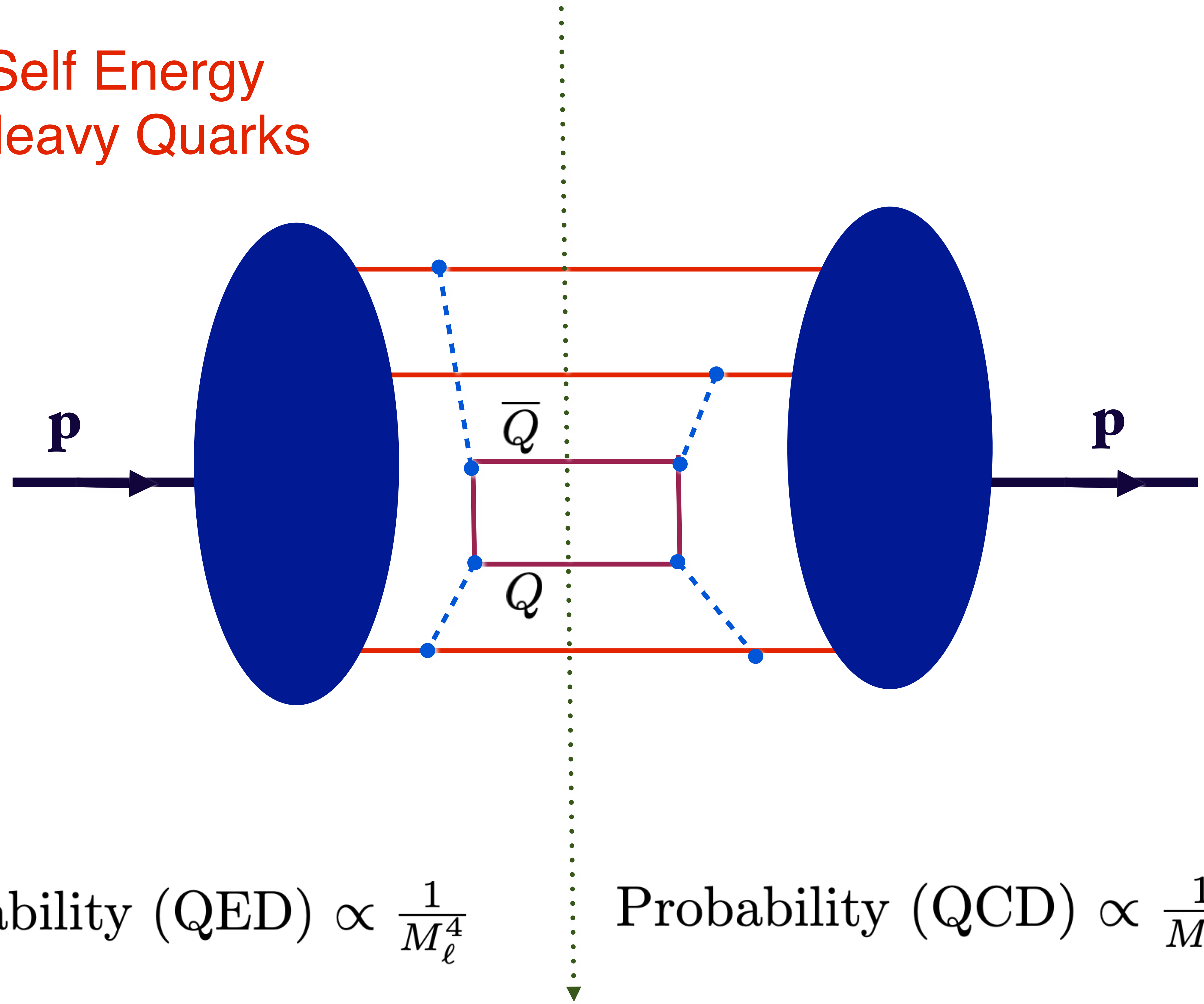

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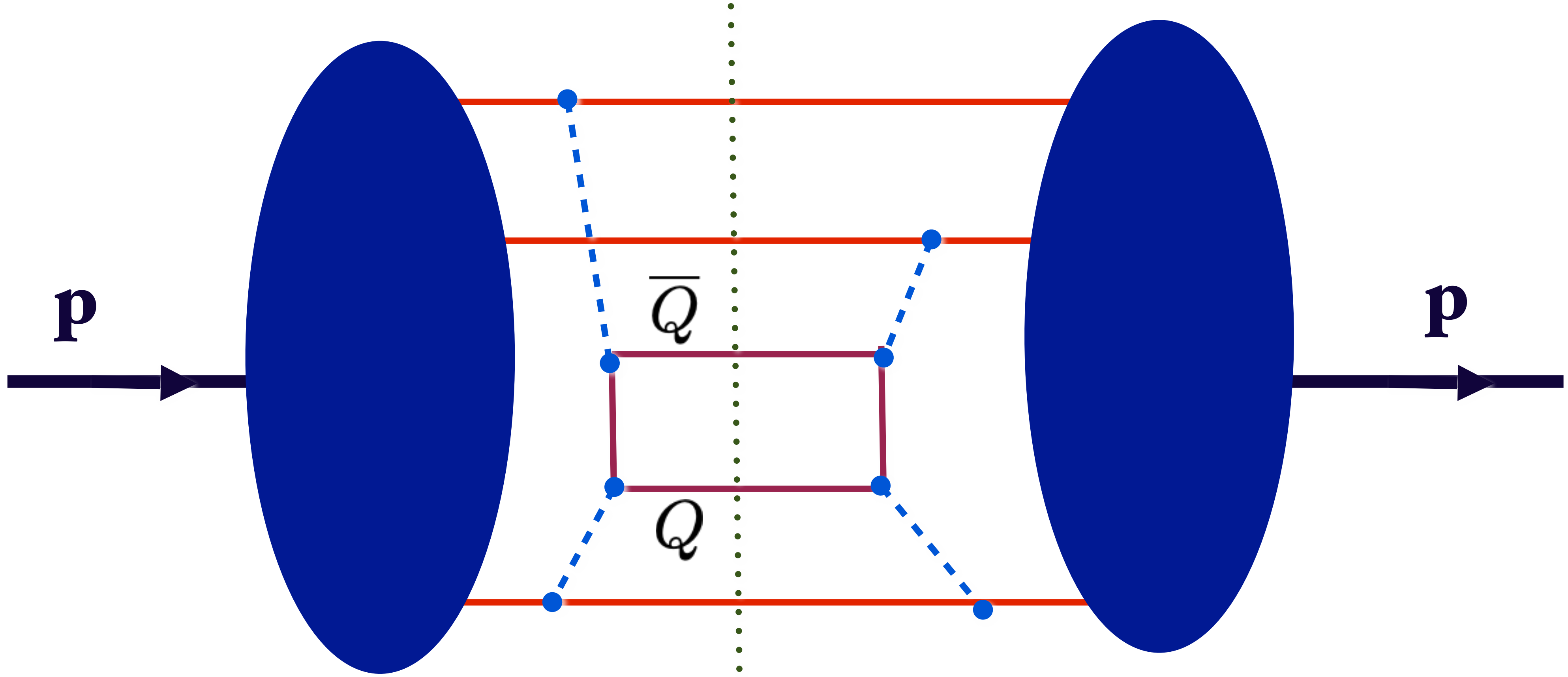
Fixed LF time

Proton Self Energy
Intrinsic Heavy Quarks



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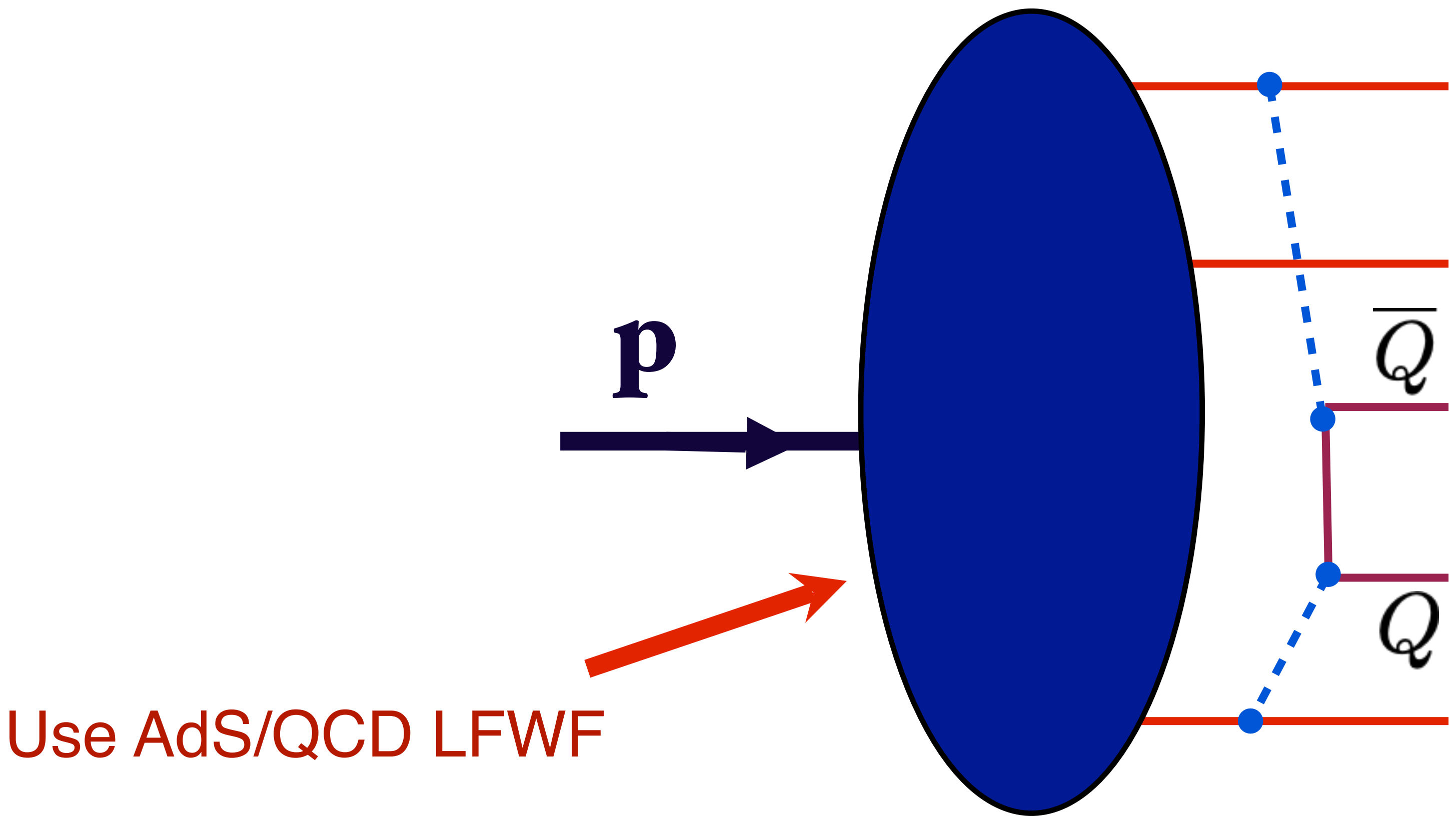
Proton 5-quark Fock State :
Intrinsic Heavy Quarks



$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

Minimal off-shellness!

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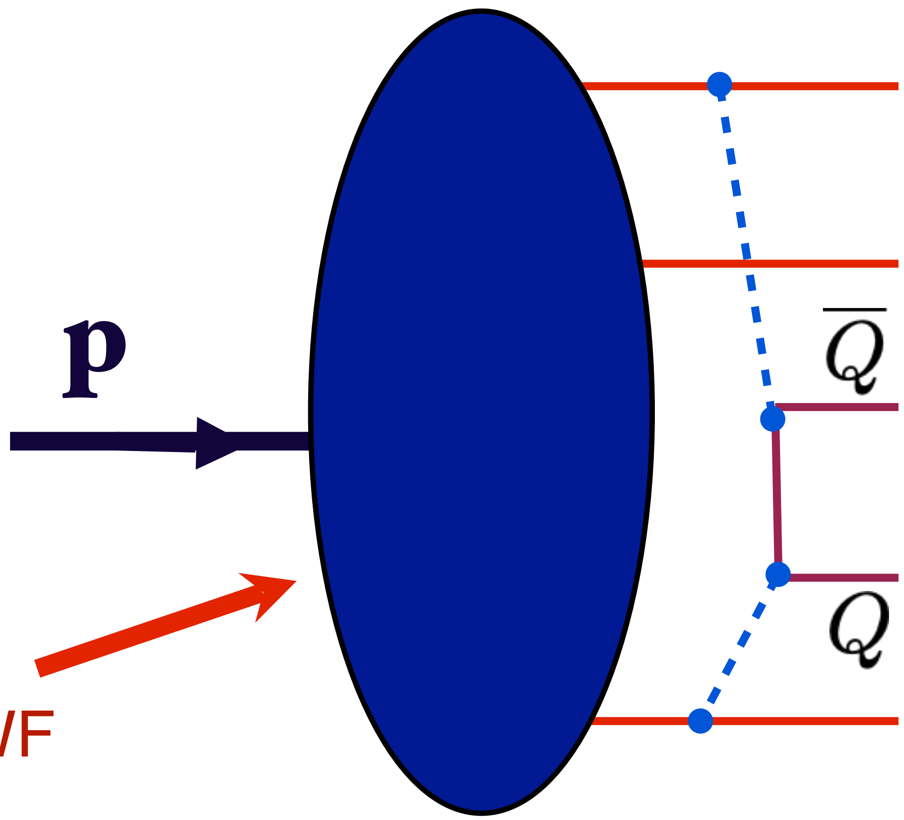

Use AdS/QCD LFWF

QCD predicts
Intrinsic Heavy Quarks
at high x!

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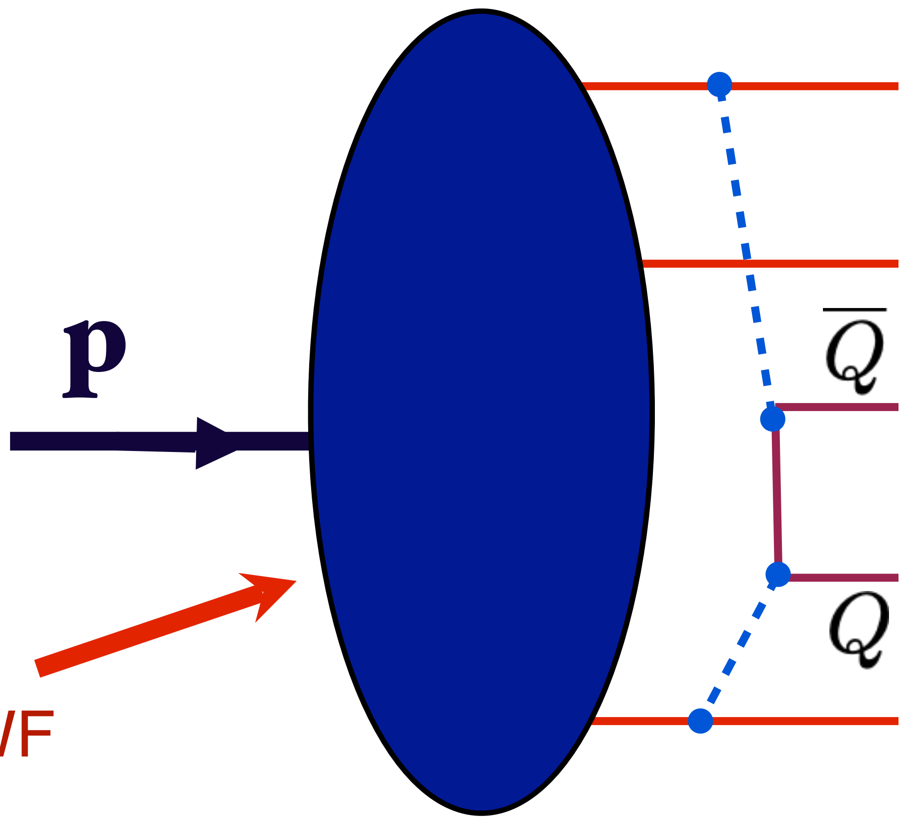
Proton 5-quark Fock State :
Intrinsic Heavy Quarks

$g \to Q\bar{Q}$ at low x: High $\mathcal{M}^2$



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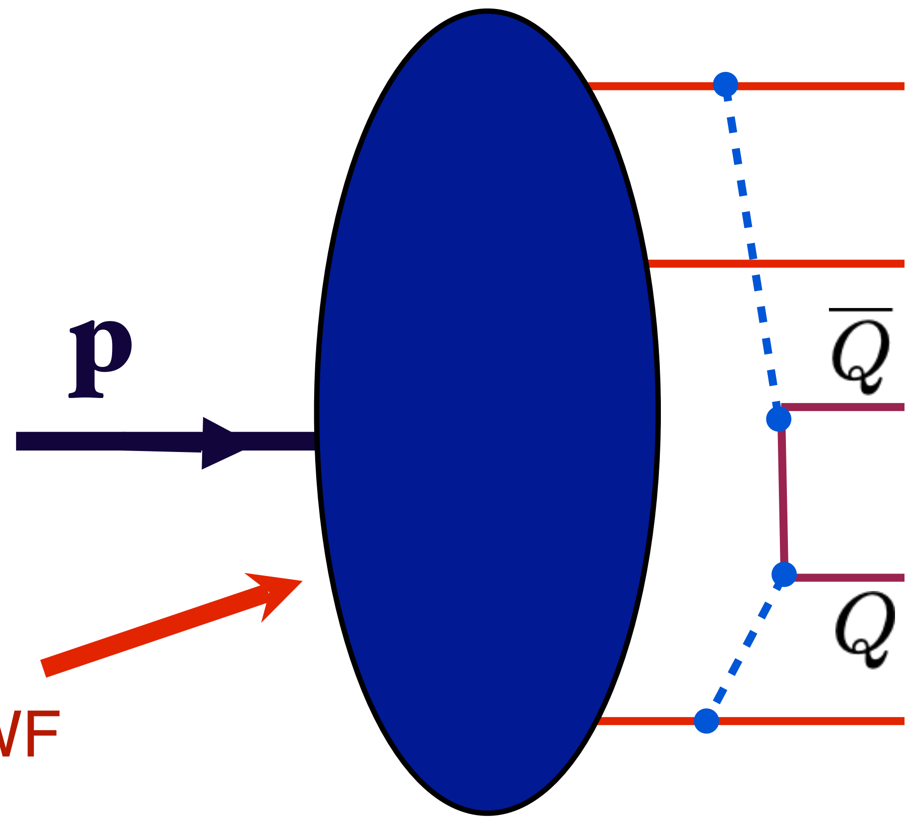

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Collins, Ellis, Gunion, Mueller, sjb
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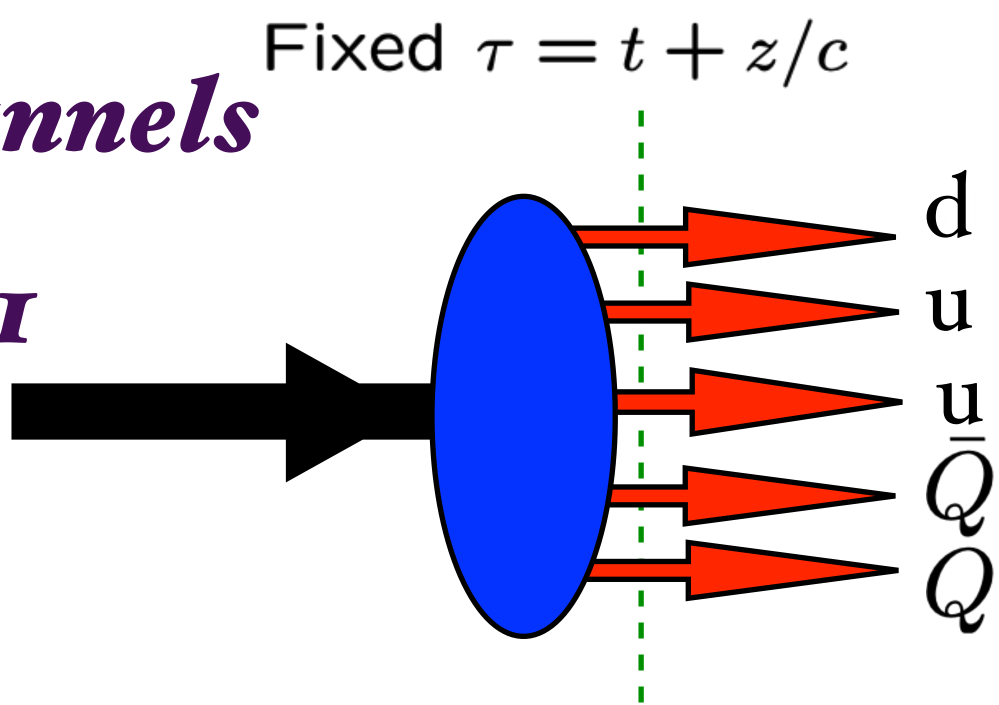
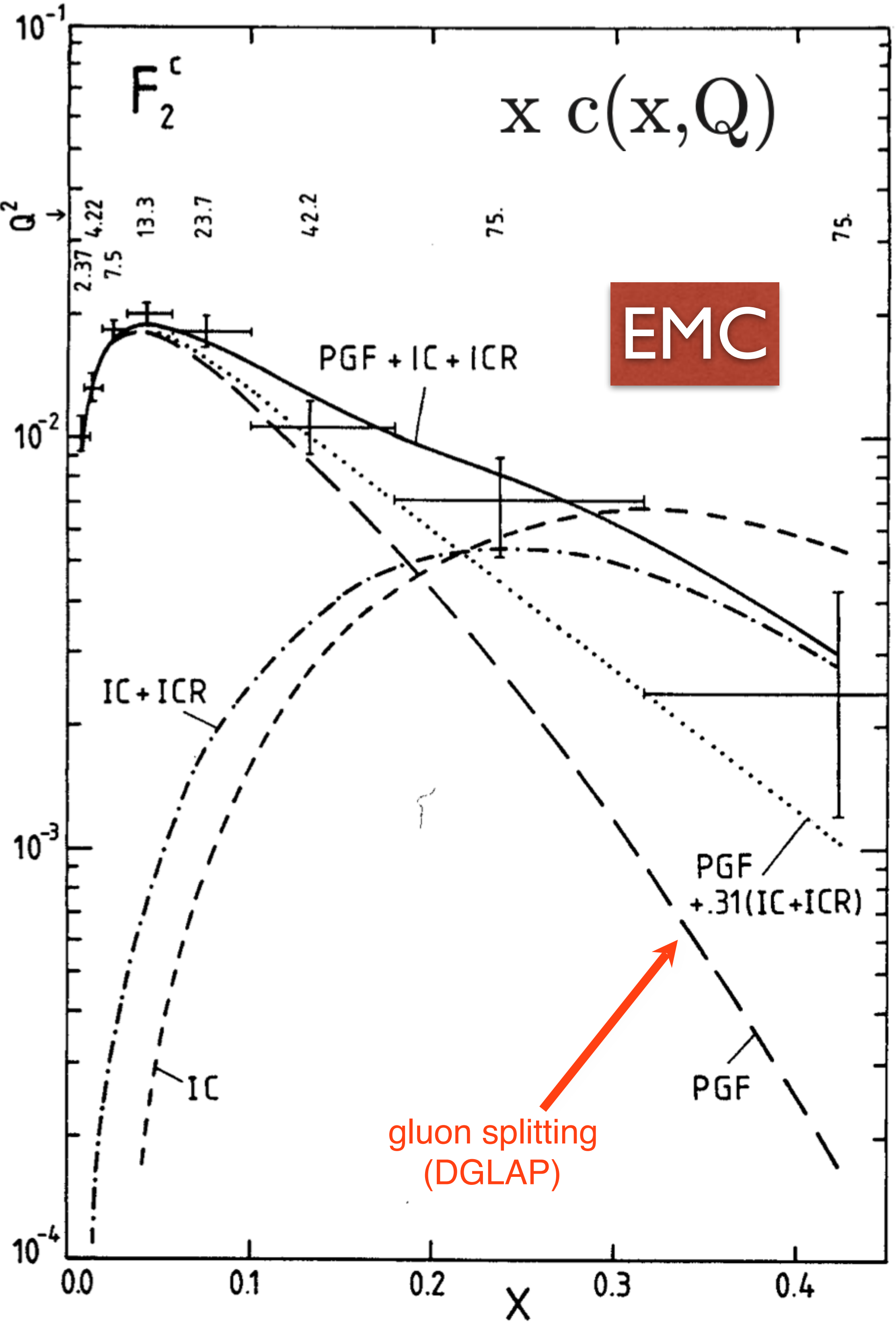
Properties of Non-Perturbative Five-Quark Fock-State

- ***Dominant configuration: mininum off-shell, same rapidity***
- ***Heavy quarks have most of the LF momentum*** $< x_Q > \propto \sqrt{m_Q^2 + k_\perp^2}$
- ***Correlated with proton quantum numbers***
- ***Duality with meson-baryon channels***
- ***strangeness asymmetry at x > 0.1***
- ***Maximally energy efficient***



Measurement of Charm Structure Function!

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



Evidence for Intrinsic Charm

$$< x_{c\bar{c}} >_p \simeq 1\%$$

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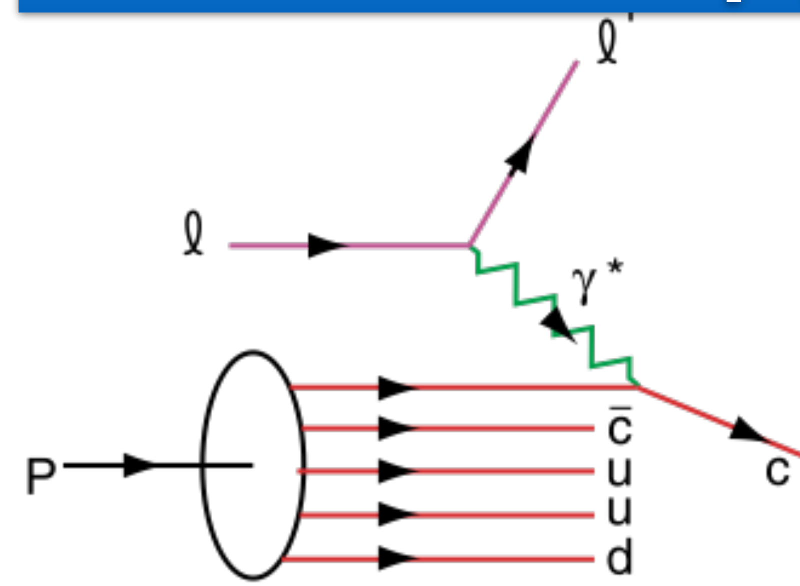

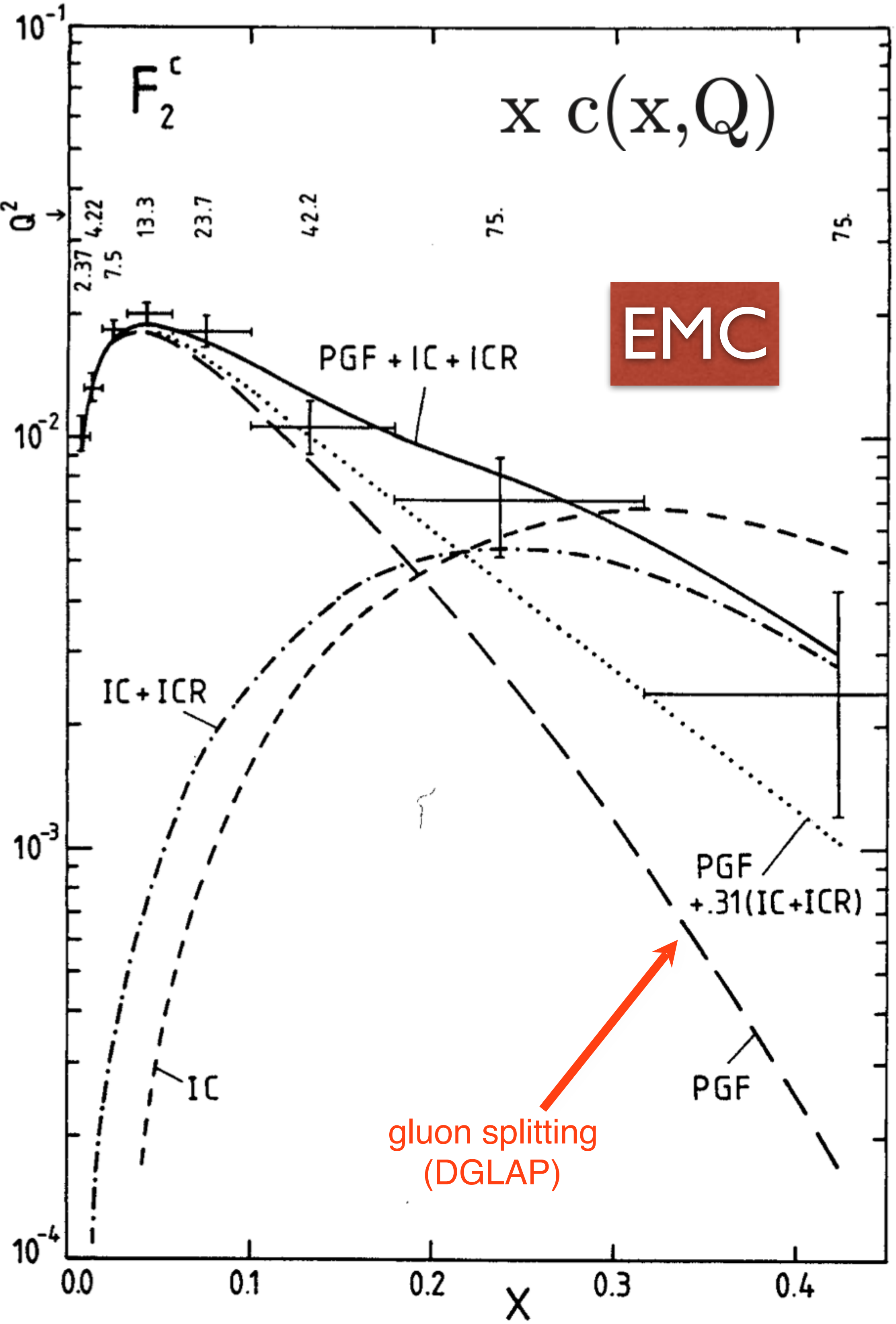

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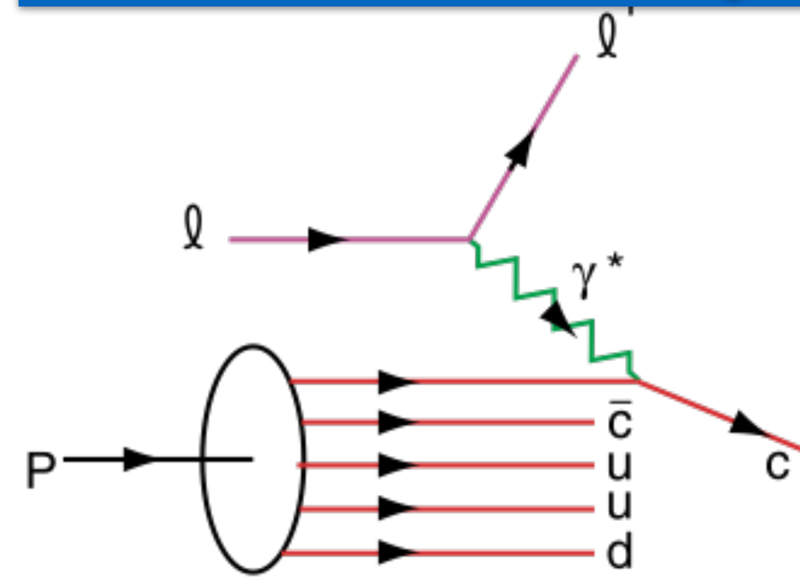

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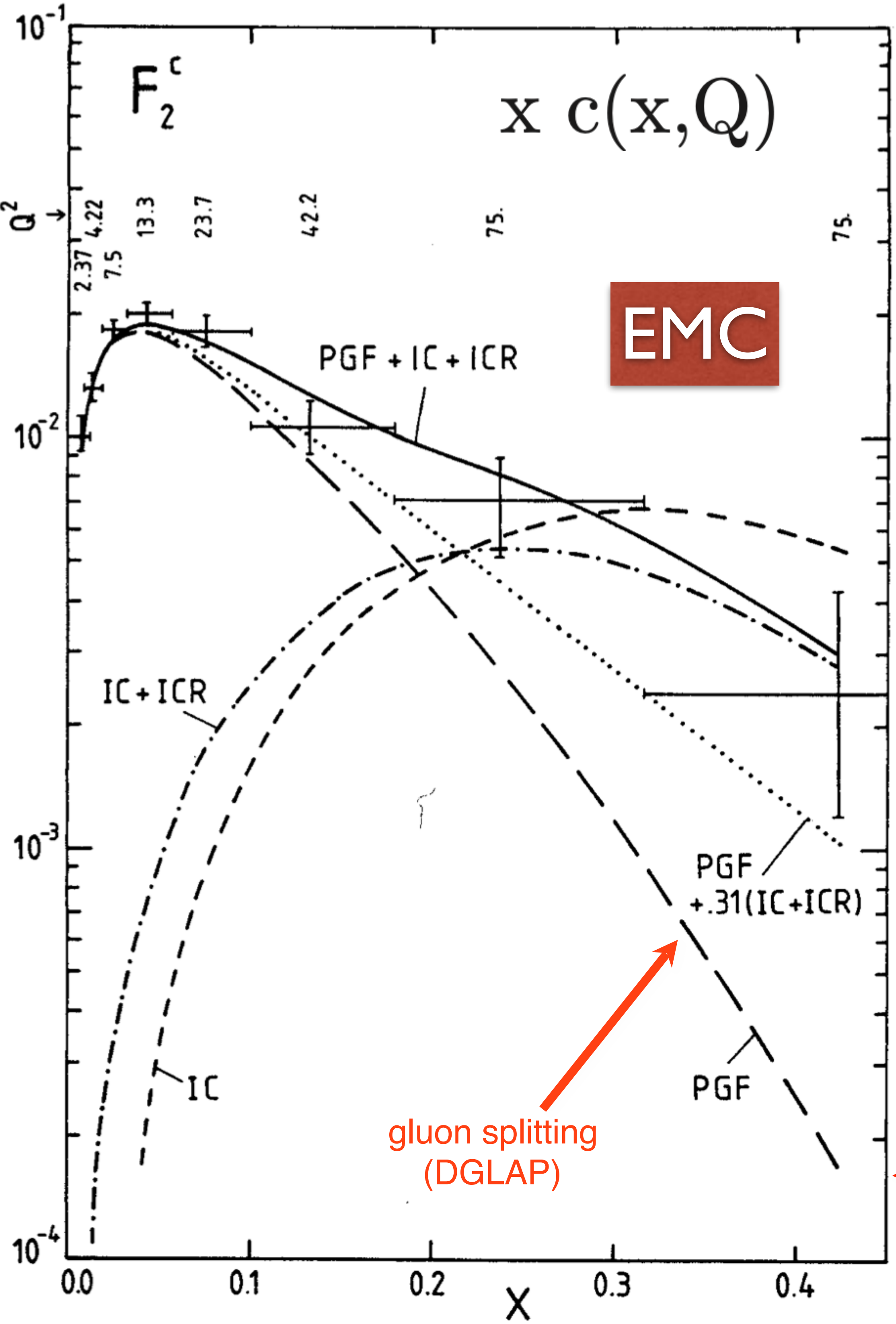

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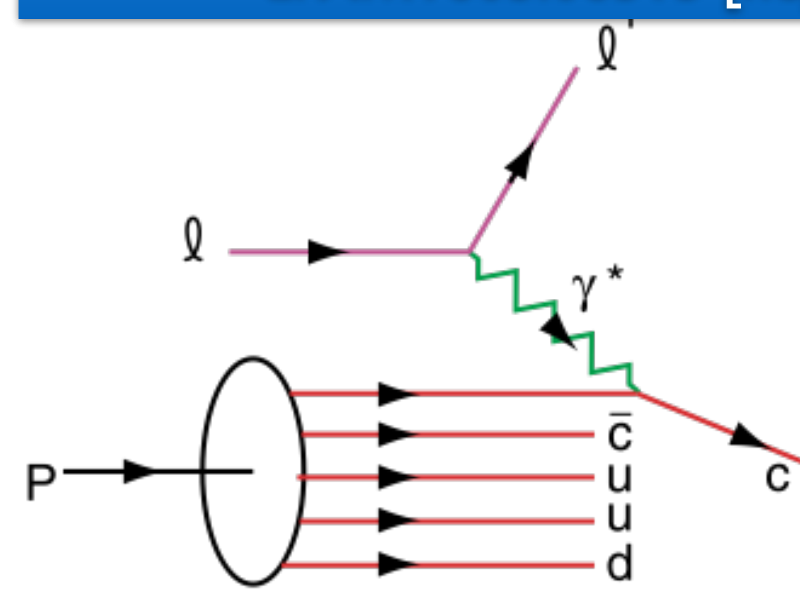

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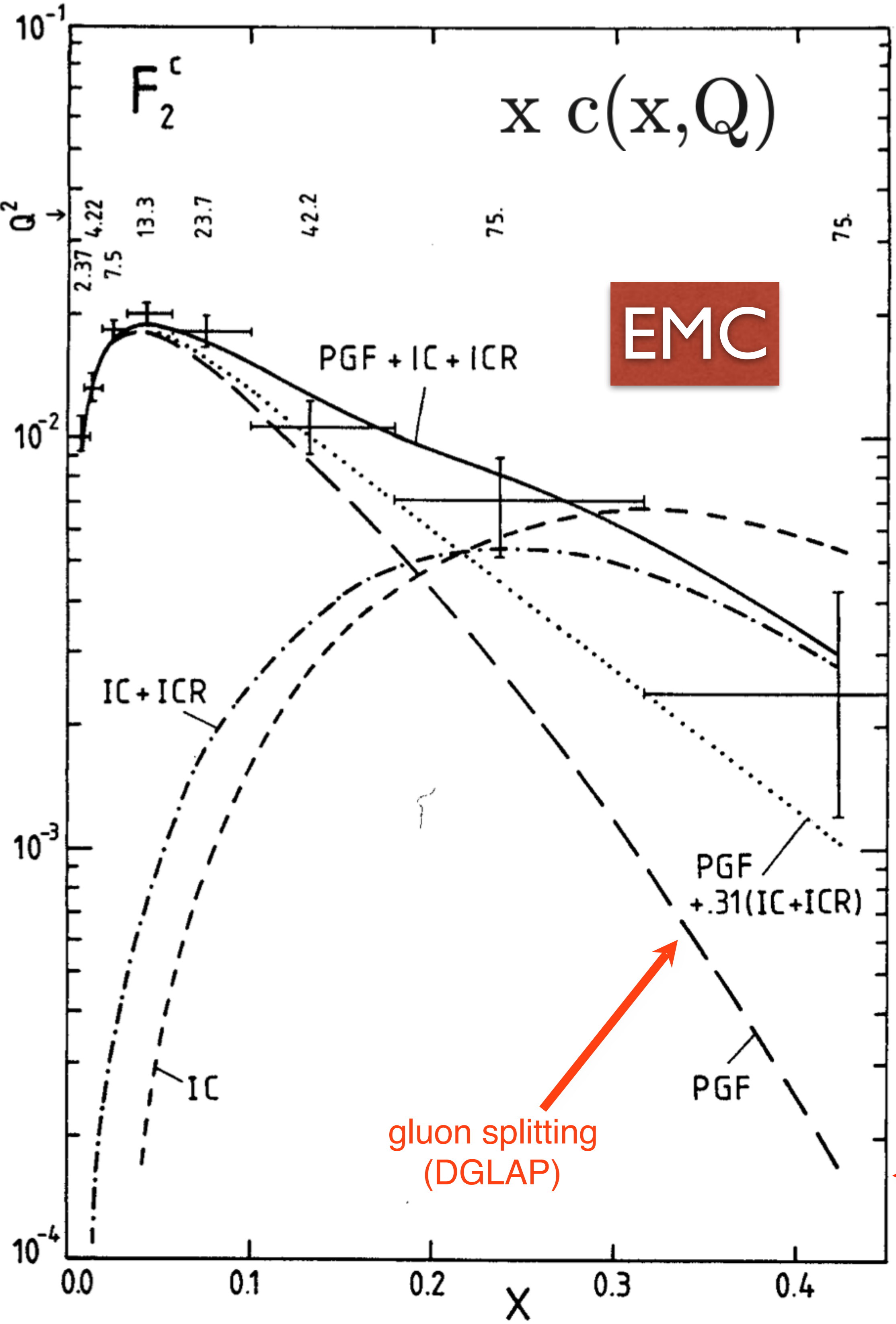

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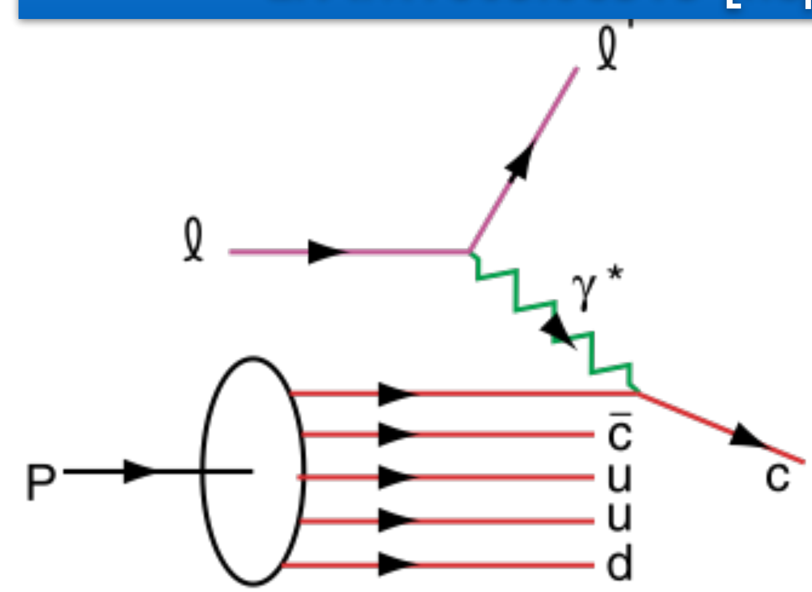
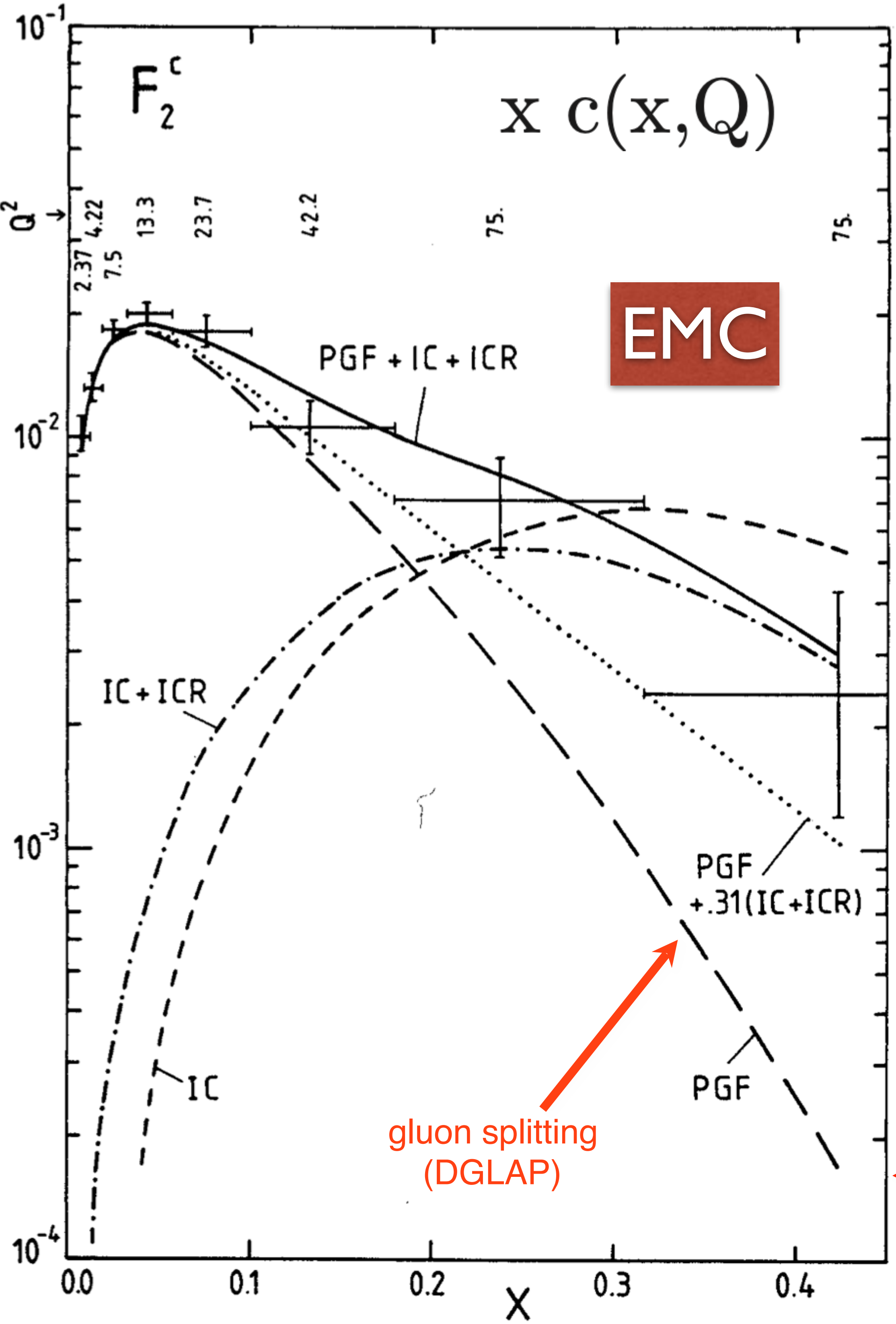


DGLAP / Photon-Gluon Fusion: factor of 30 too small



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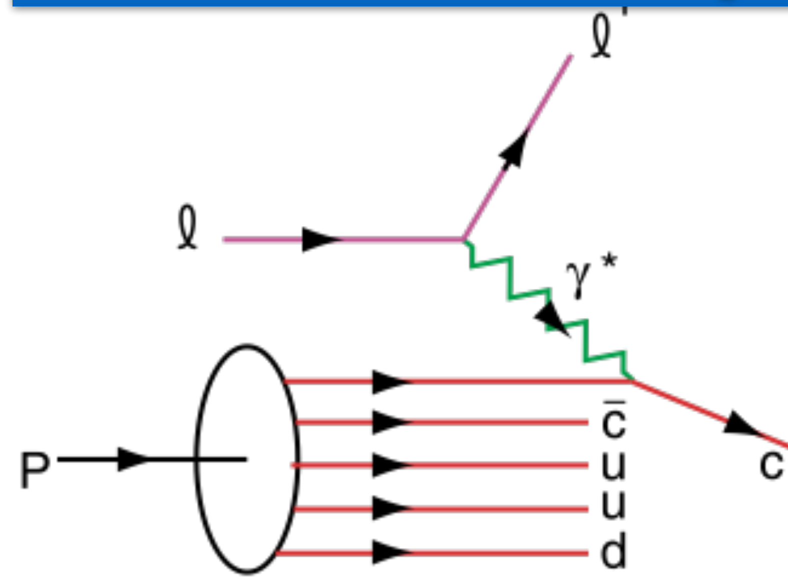
factor of 30 !



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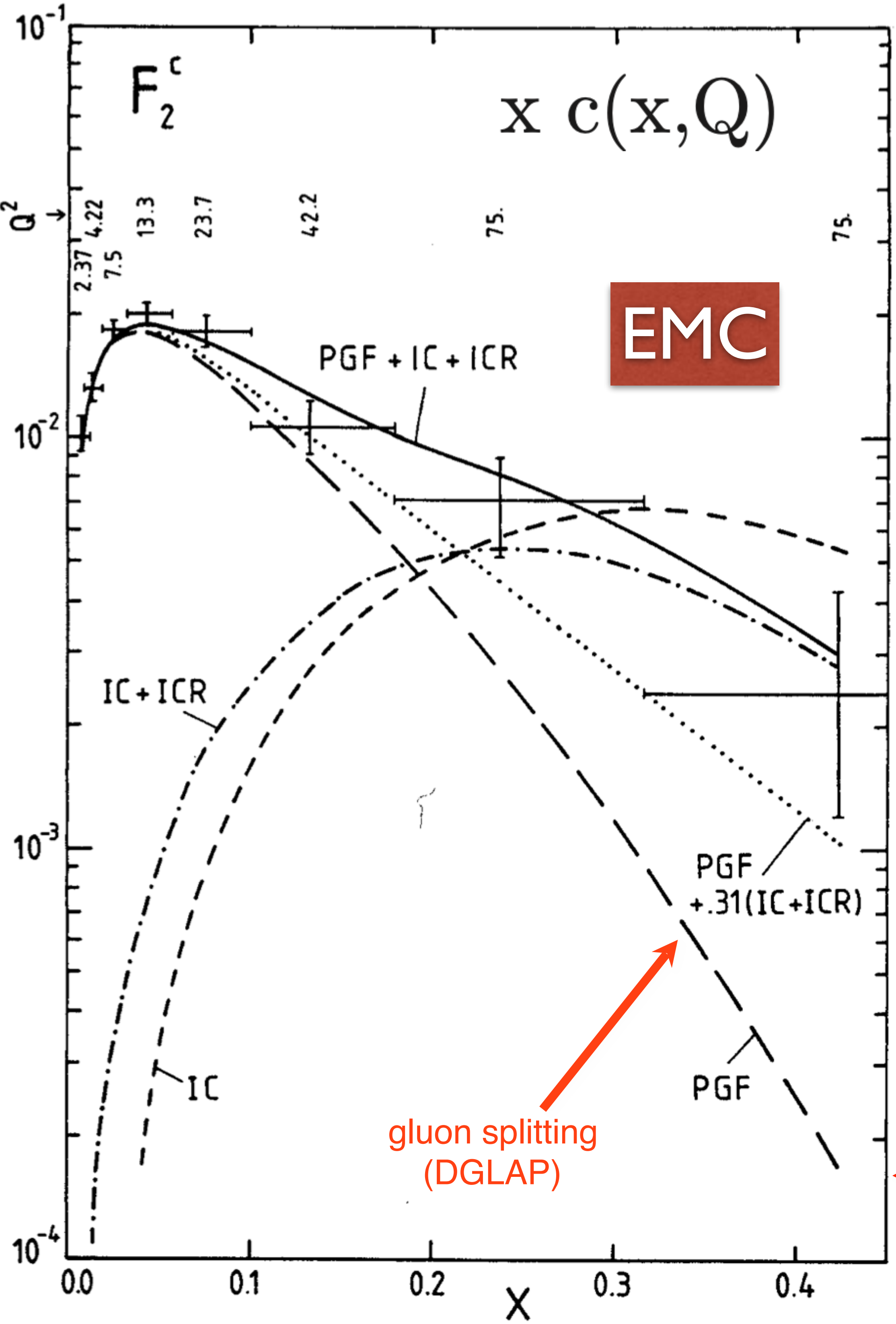

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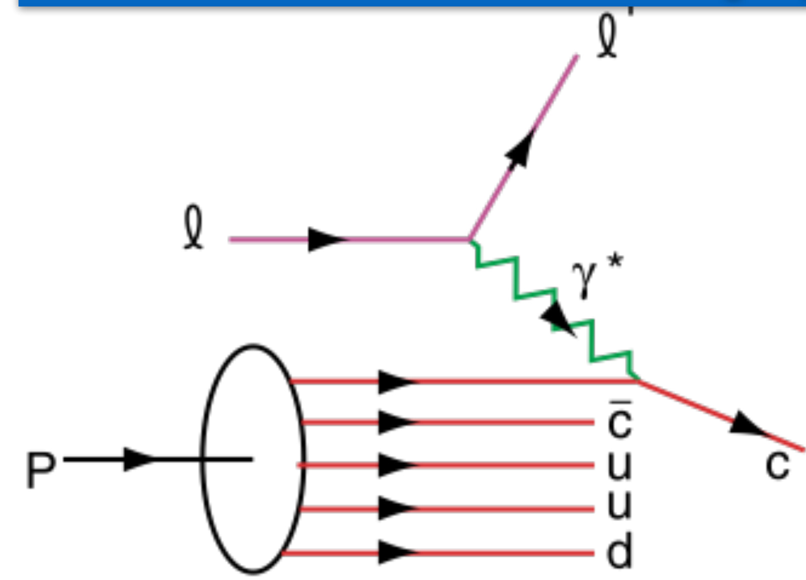


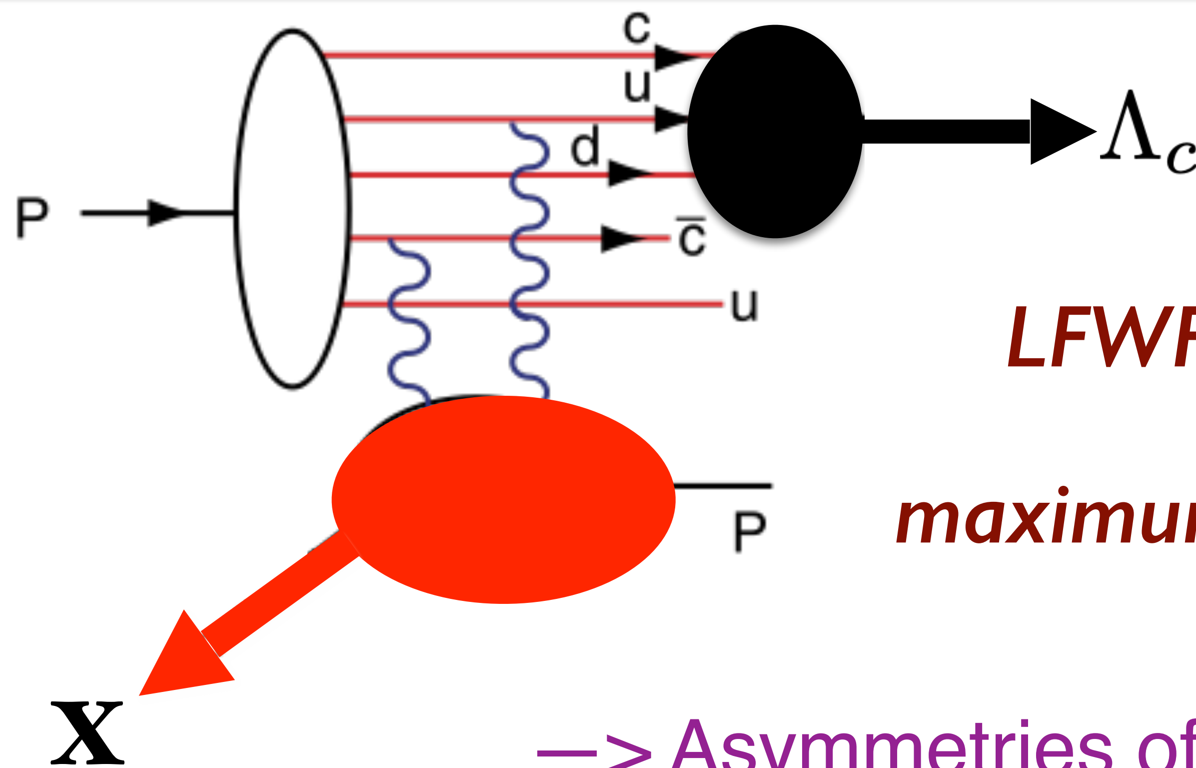
DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

Coalesece of comovers produces high x_F heavy hadrons

High x_F hadrons combine most of the comovers, fewest spectators



$$\psi_H(x_i, \vec{k}_{\perp i}, \lambda_i)$$

LFWF maximum at equal rapidity

maximum at minimal invariant mass

—> Asymmetries of leading hadrons

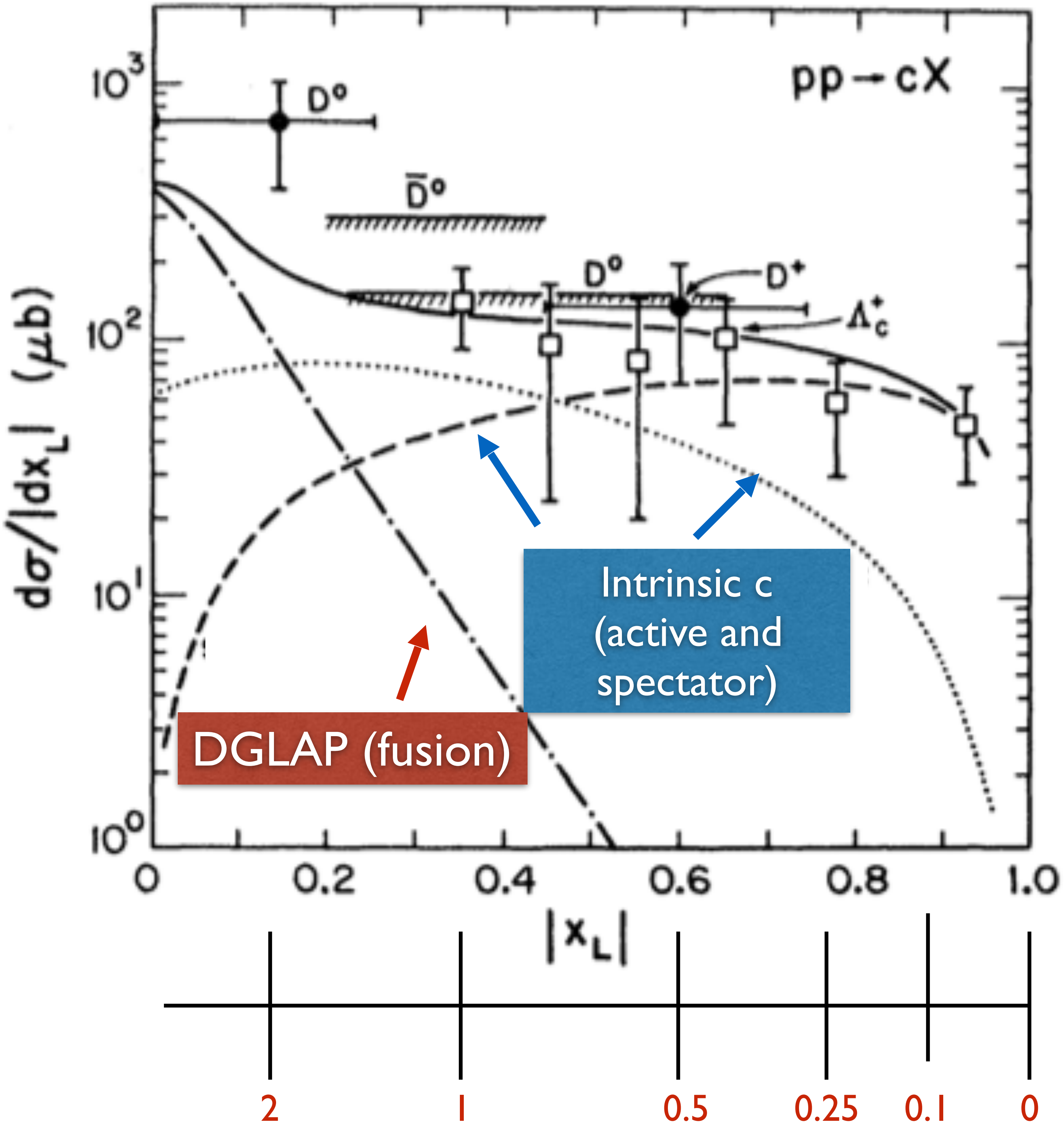
Spectator counting rules

$$\frac{dN}{dx_F} \propto (1 - x_F)^{2n_{spect}-1}$$

Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ, Λ_c and other Charm Hadrons at High x_F

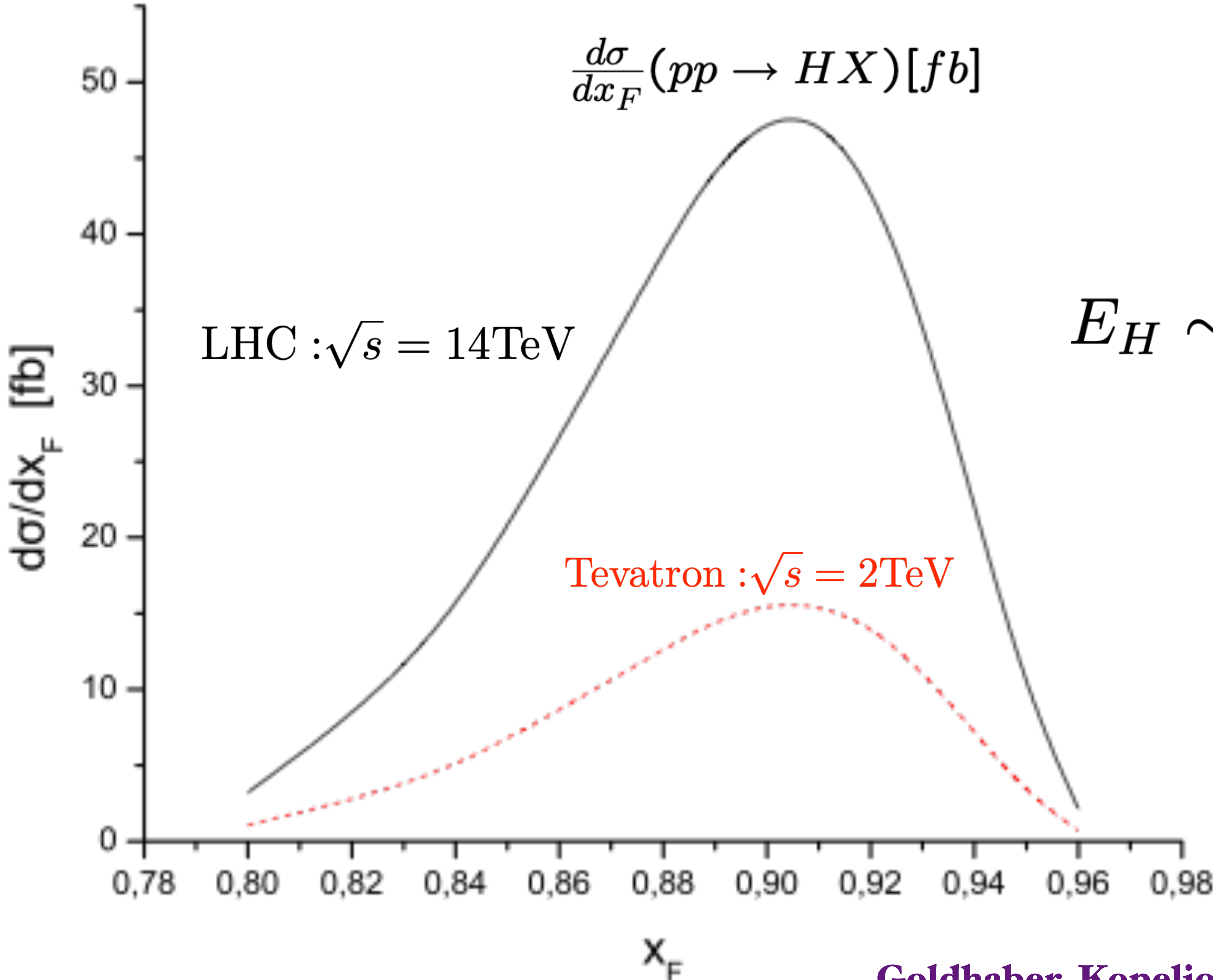
Vogt, sjb

Barger, Halzen, Keung
PRD 25 (1981)



$\Delta y = \log x$

Intrinsic Heavy Quark Contribution to Inclusive Higgs Production



$$E_H \sim 0.9 E_p$$

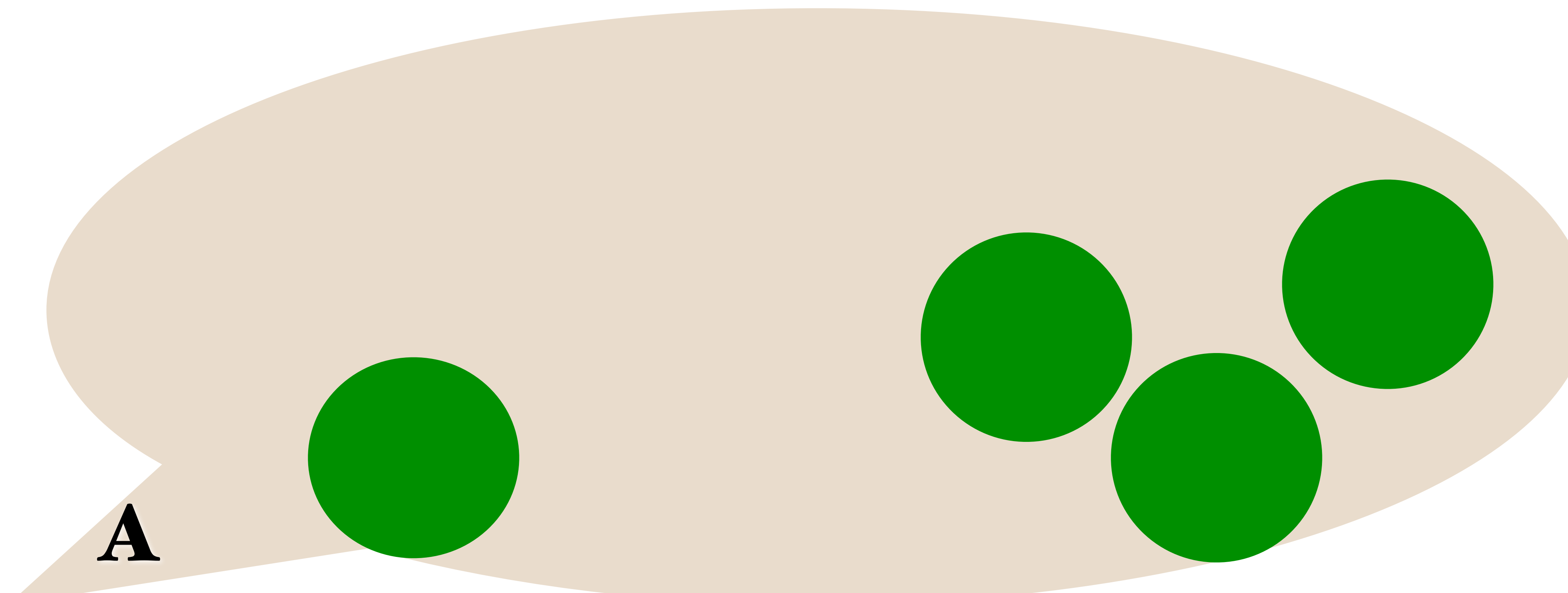
Goldhaber, Kopeliovich, Schmidt, sjb

$$\text{Measure } H \to ZZ^* \to \mu^+\mu^-\mu^+\mu^-.$$

High x_F

Color-Opaque IC Fock state interacts on nuclear front surface

Kopeliovich, Schmidt, Soffer, sjb



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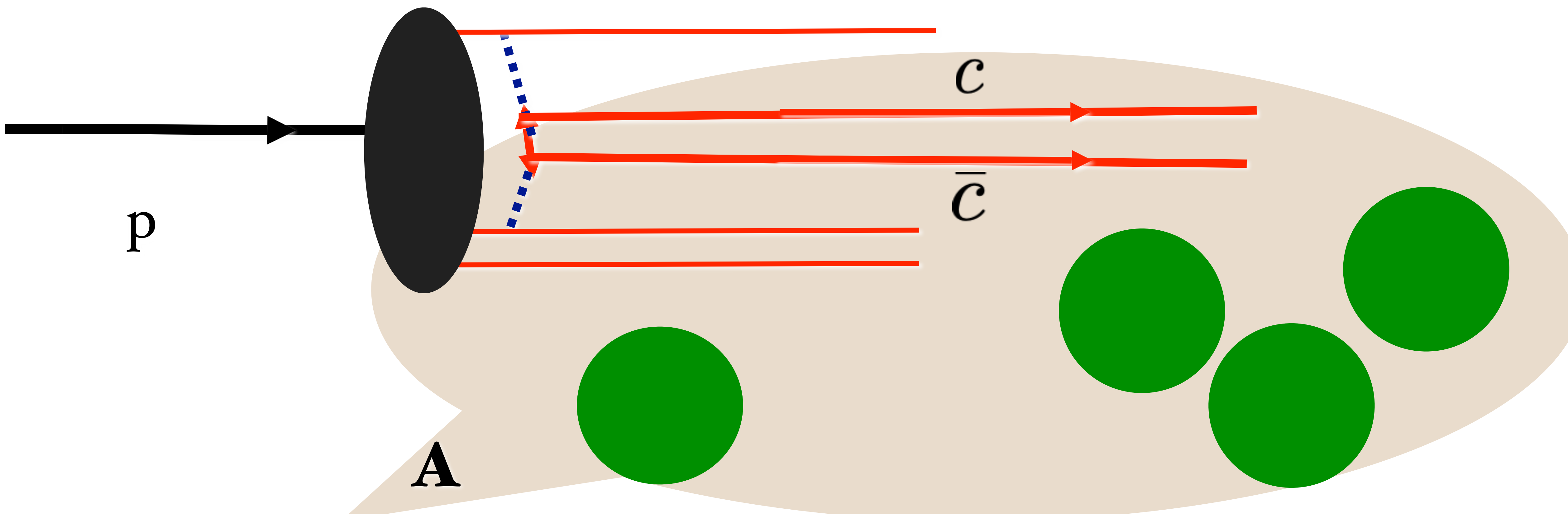

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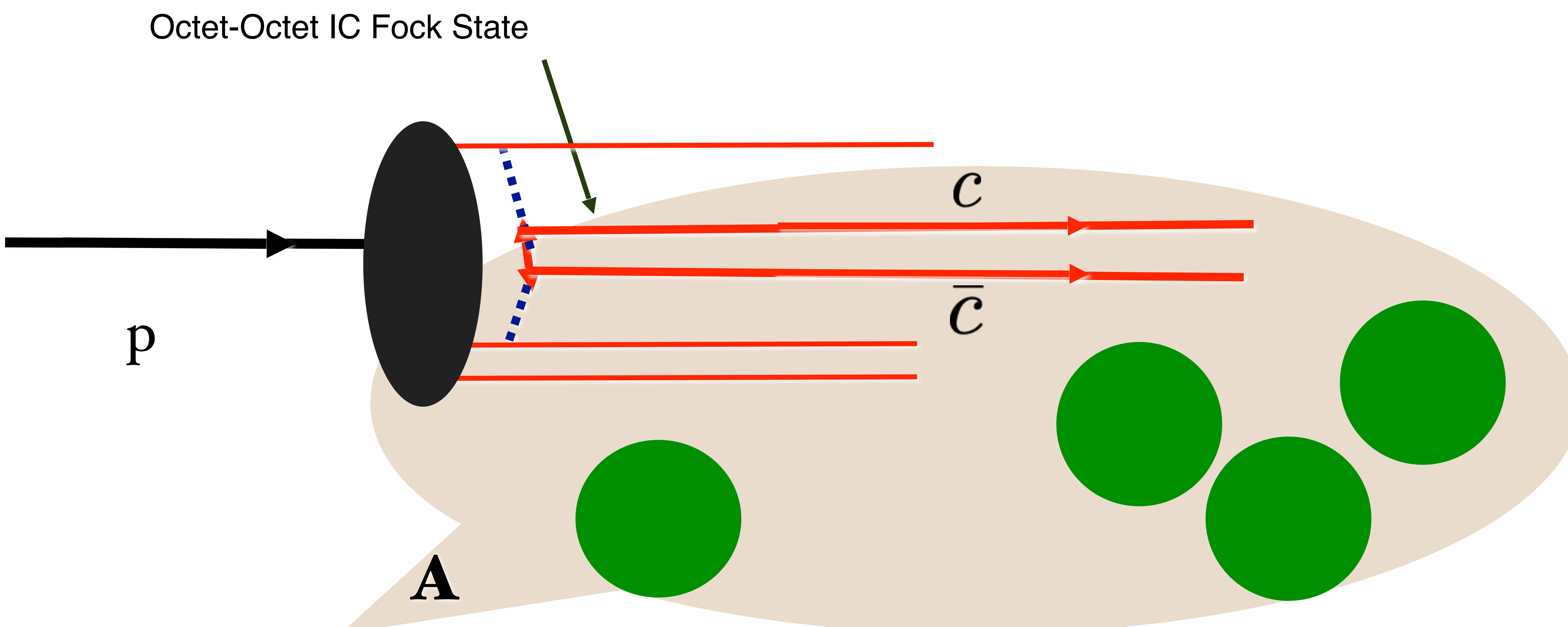

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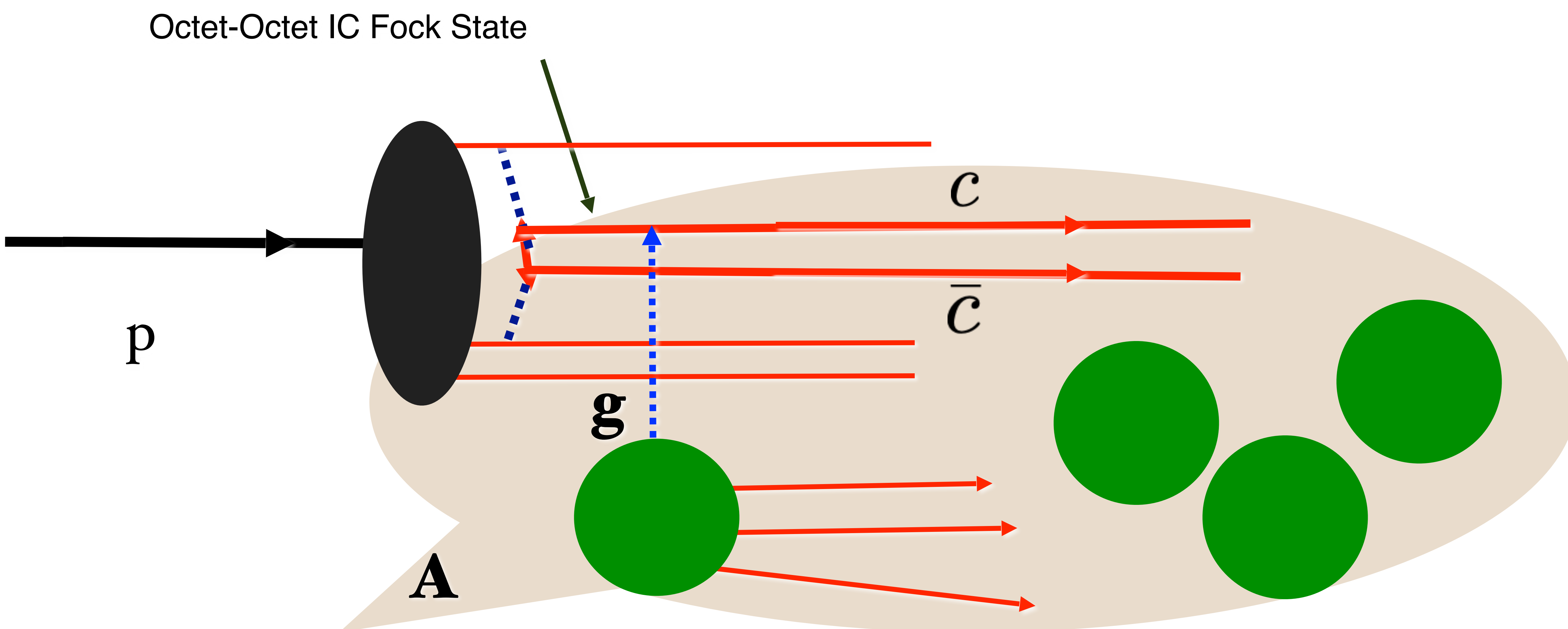
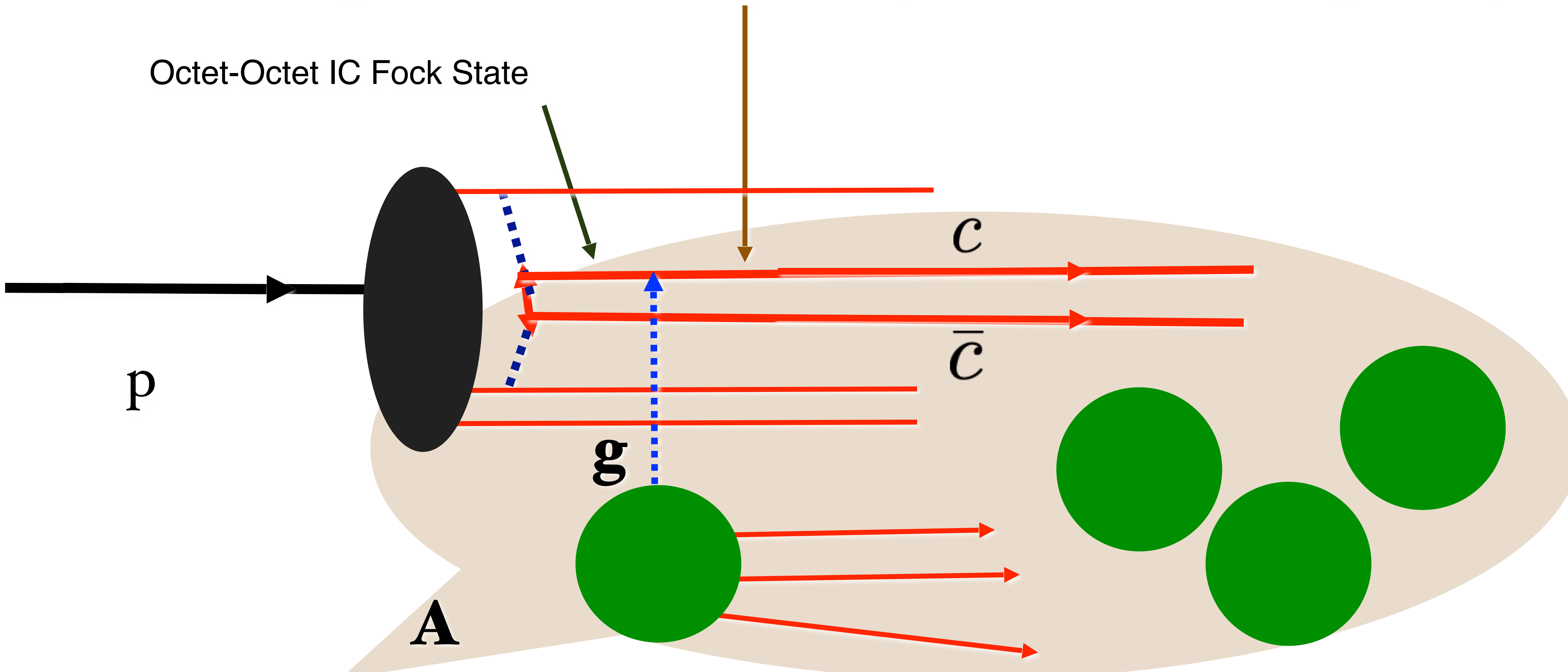


Kopeliovich, Schmidt, Soffer, sjb

Color-Opaque IC Fock state interacts on nuclear front surface

Scattering on front-face nucleon produces color-singlet $c\bar{c}$ pair



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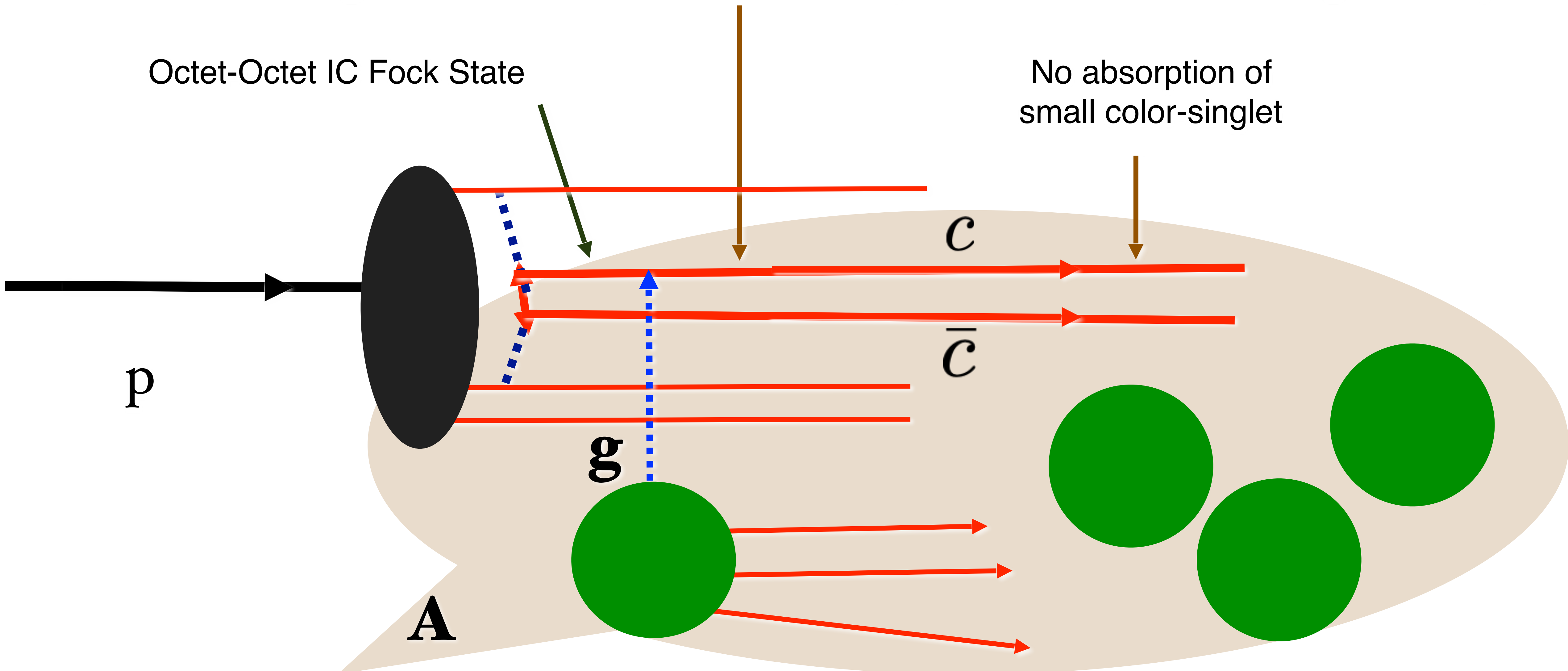

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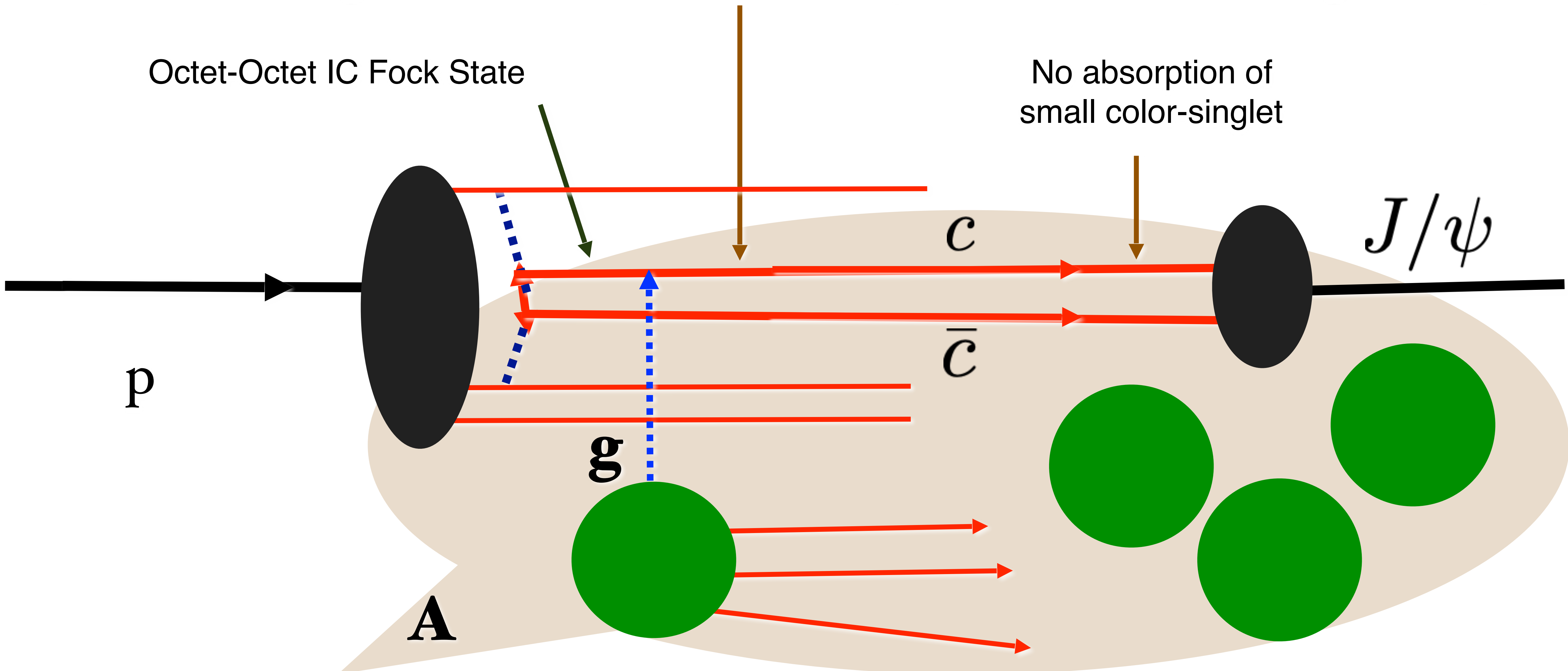

High x_F

Kopeliovich, Schmidt, Soffer, sjb

Color-Opaque IC Fock state interacts on nuclear front surface

$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \to J/\psi X)$$

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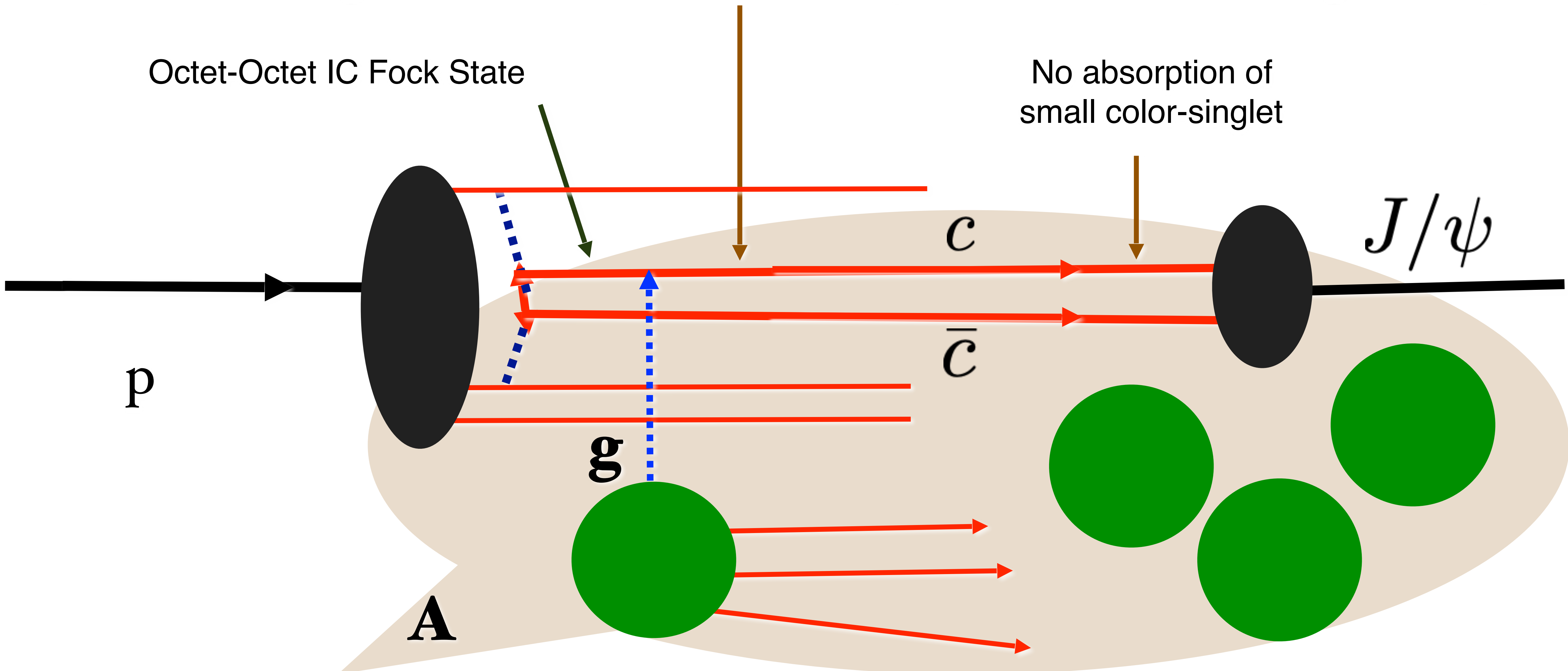
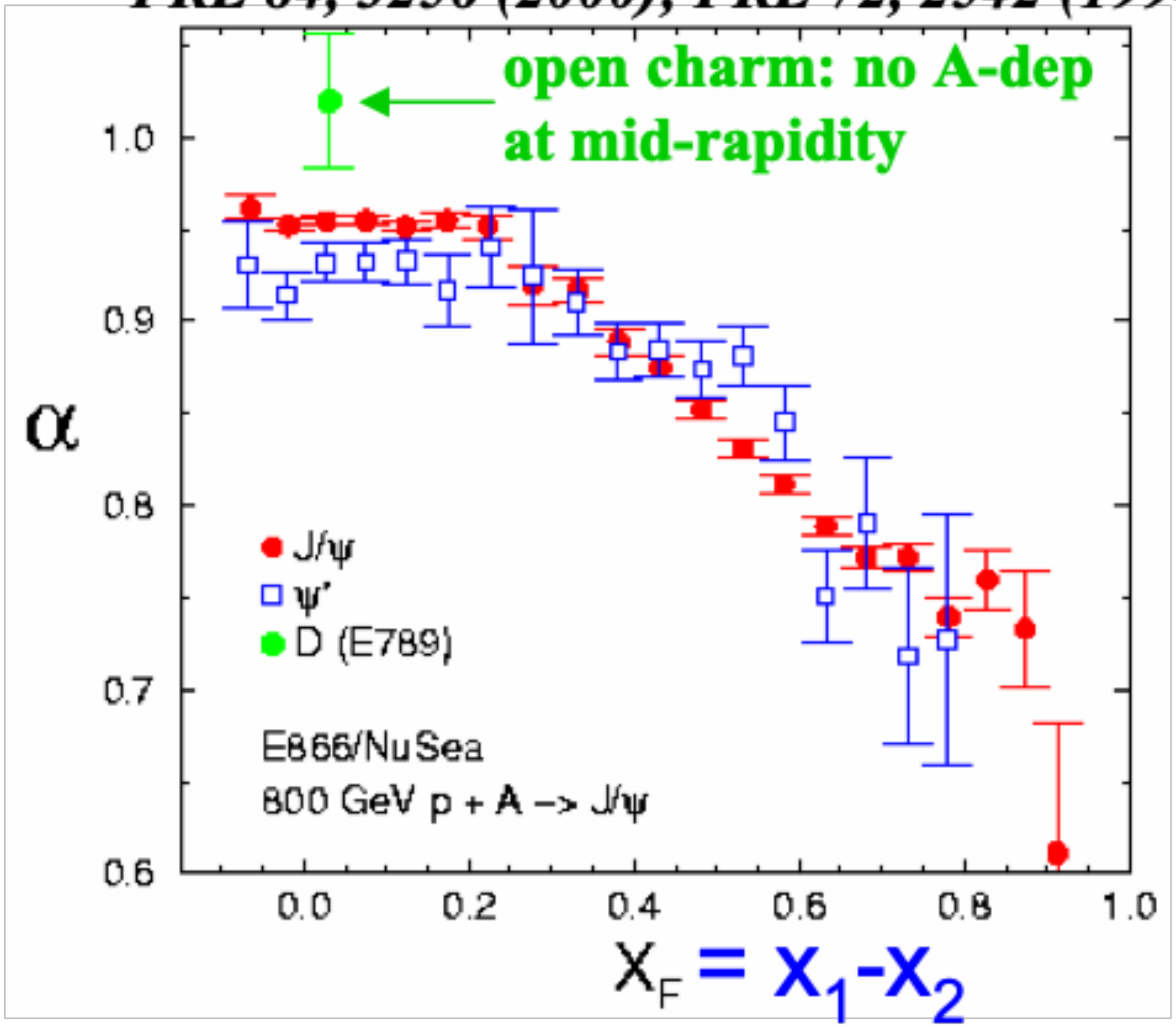


800 GeV p-A (FNAL) $\sigma_A = \sigma_p * A^\alpha$

PRL 84, 3256 (2000); PRL 72, 2542 (1994)

M. Leitch



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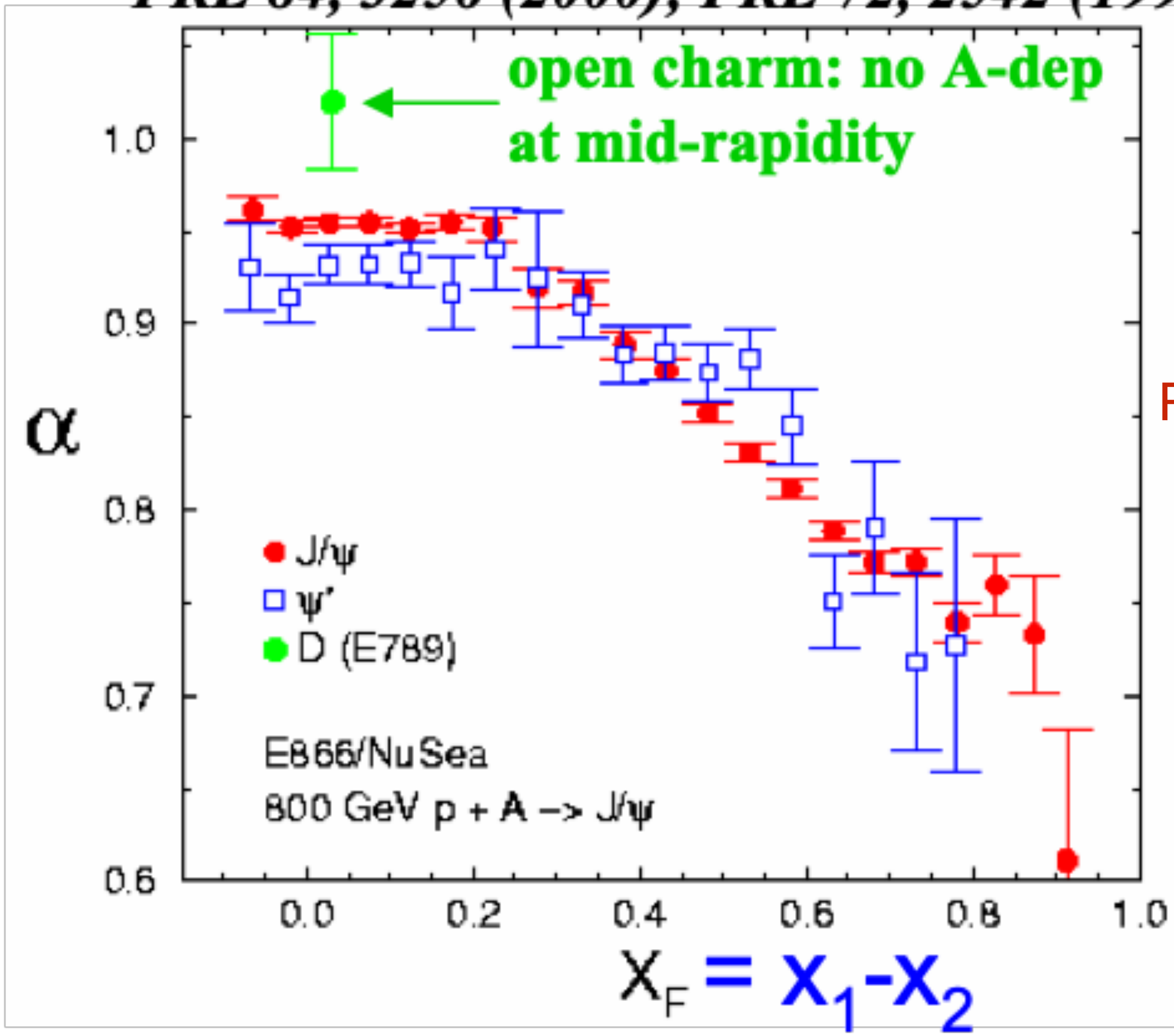

M. Leitch

$$\frac{d\sigma}{dx_F}(pA \to J/\psi X)$$

Remarkably Strong Nuclear Dependence for Fast Charmonium

M. Leitch

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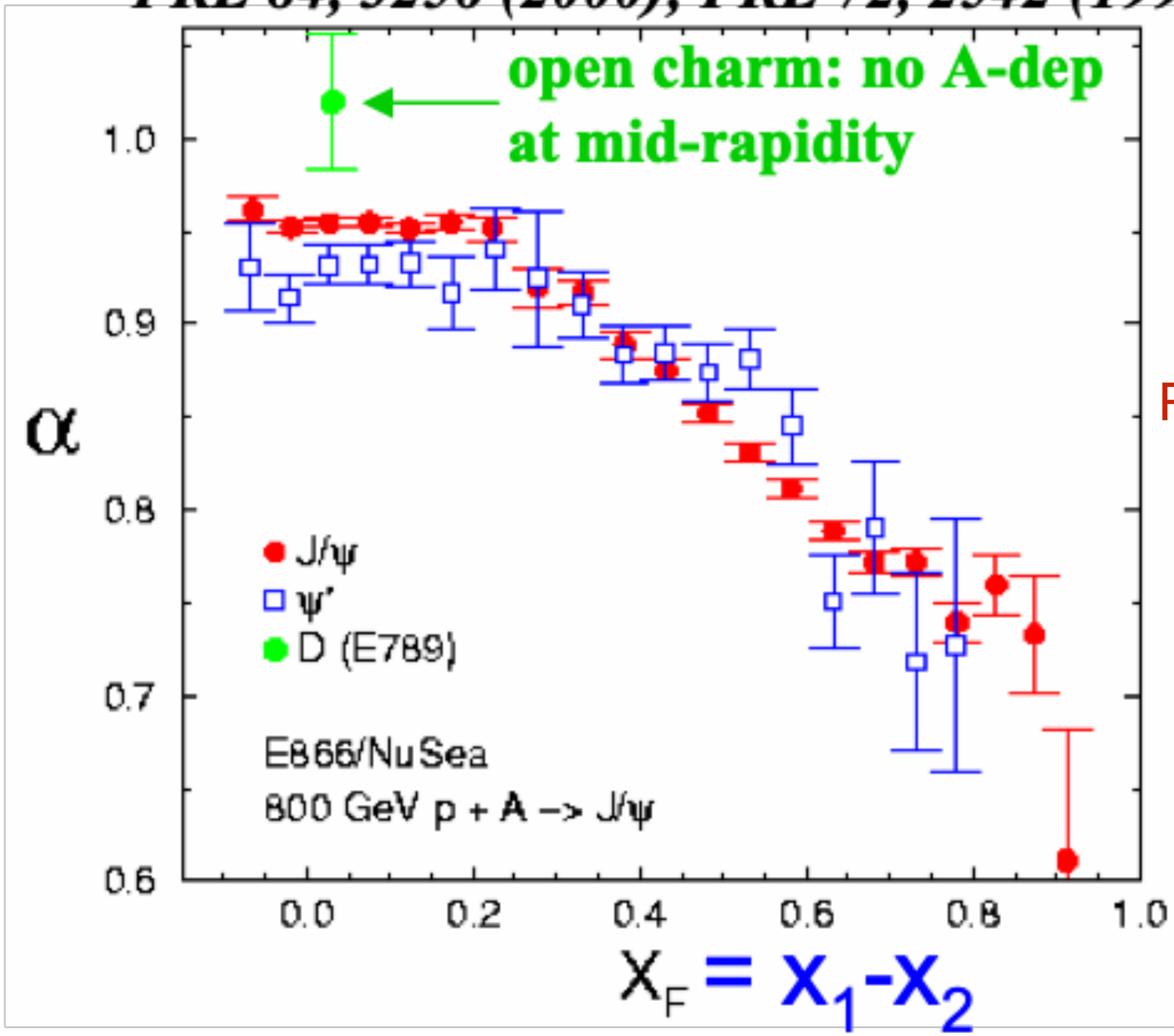

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Remarkably Strong Nuclear Dependence fo
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Violation of PQCD Factorization!

Violation of factorization in charm hadroproduction.
P. Hoyer, **M. Vanttinen** (**Helsinki U.**) , **U. Sukhatme** (**Illinois U., Chicago**) . **HU-TFT-90-14, May 1990. 7pp.**
Published in Phys.Lett.B246:217-220,1990

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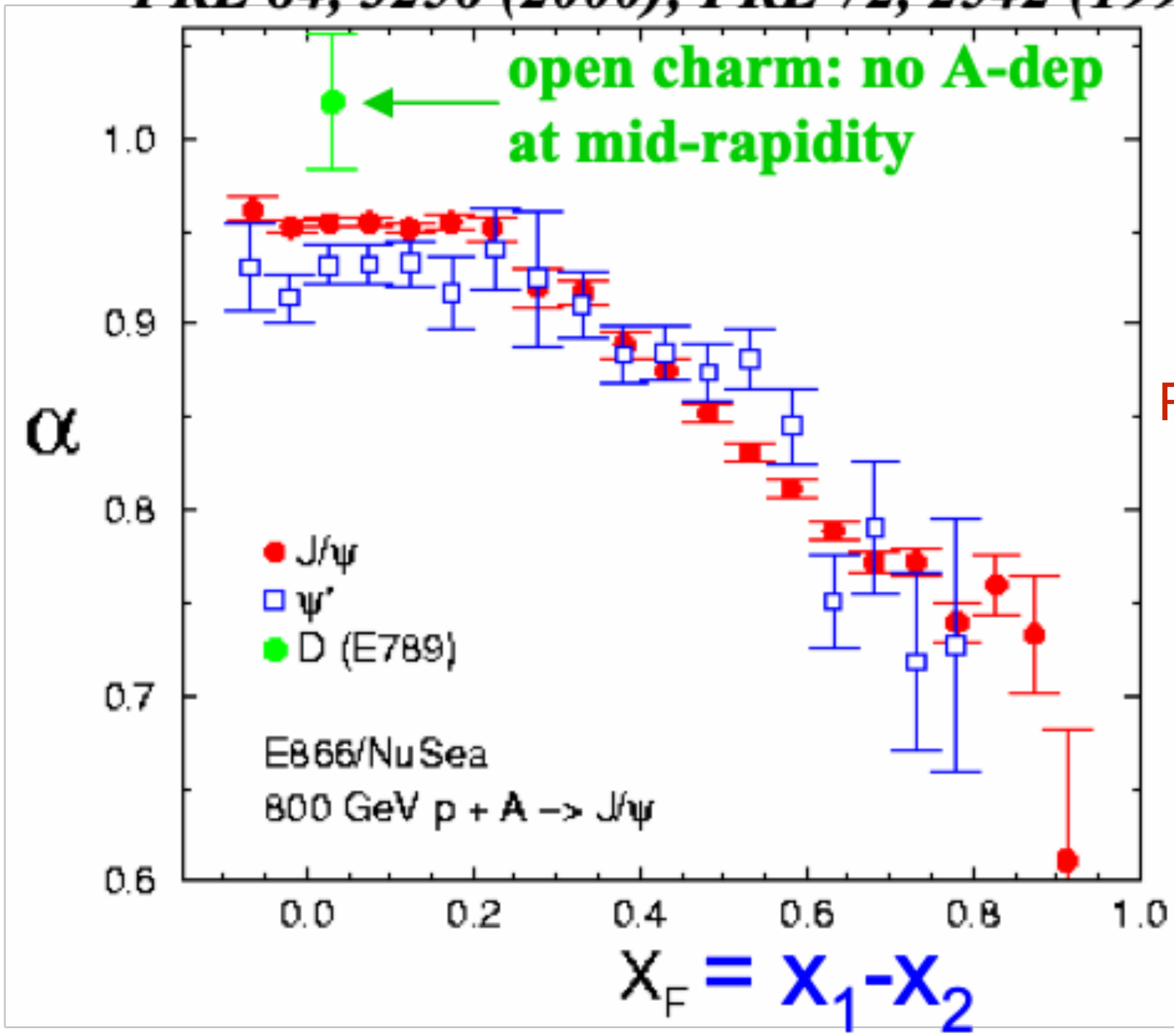

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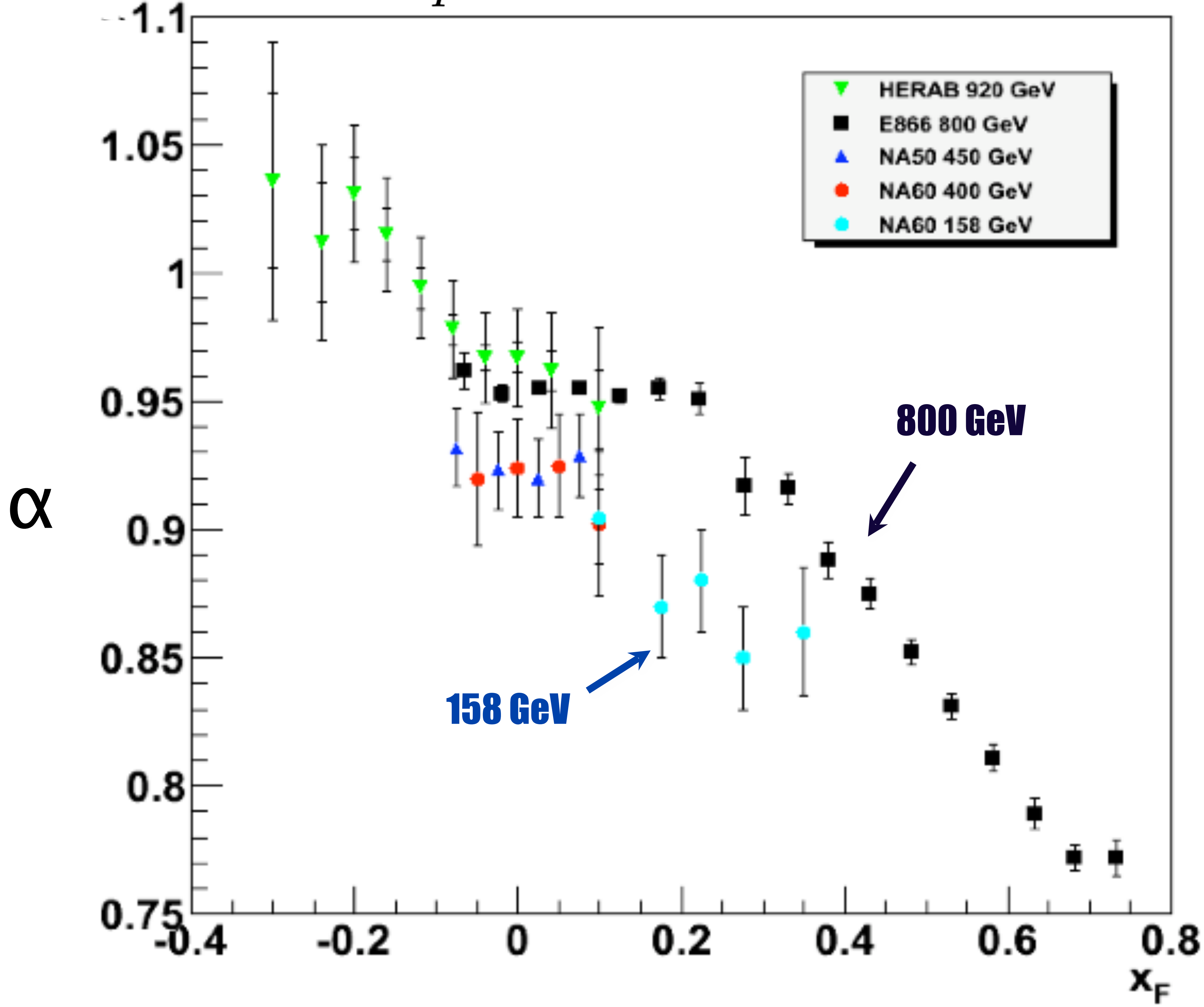
IC Explains large excess of quarkonia at large x_F, A-dependence

NA60 pA data @ 158GeV

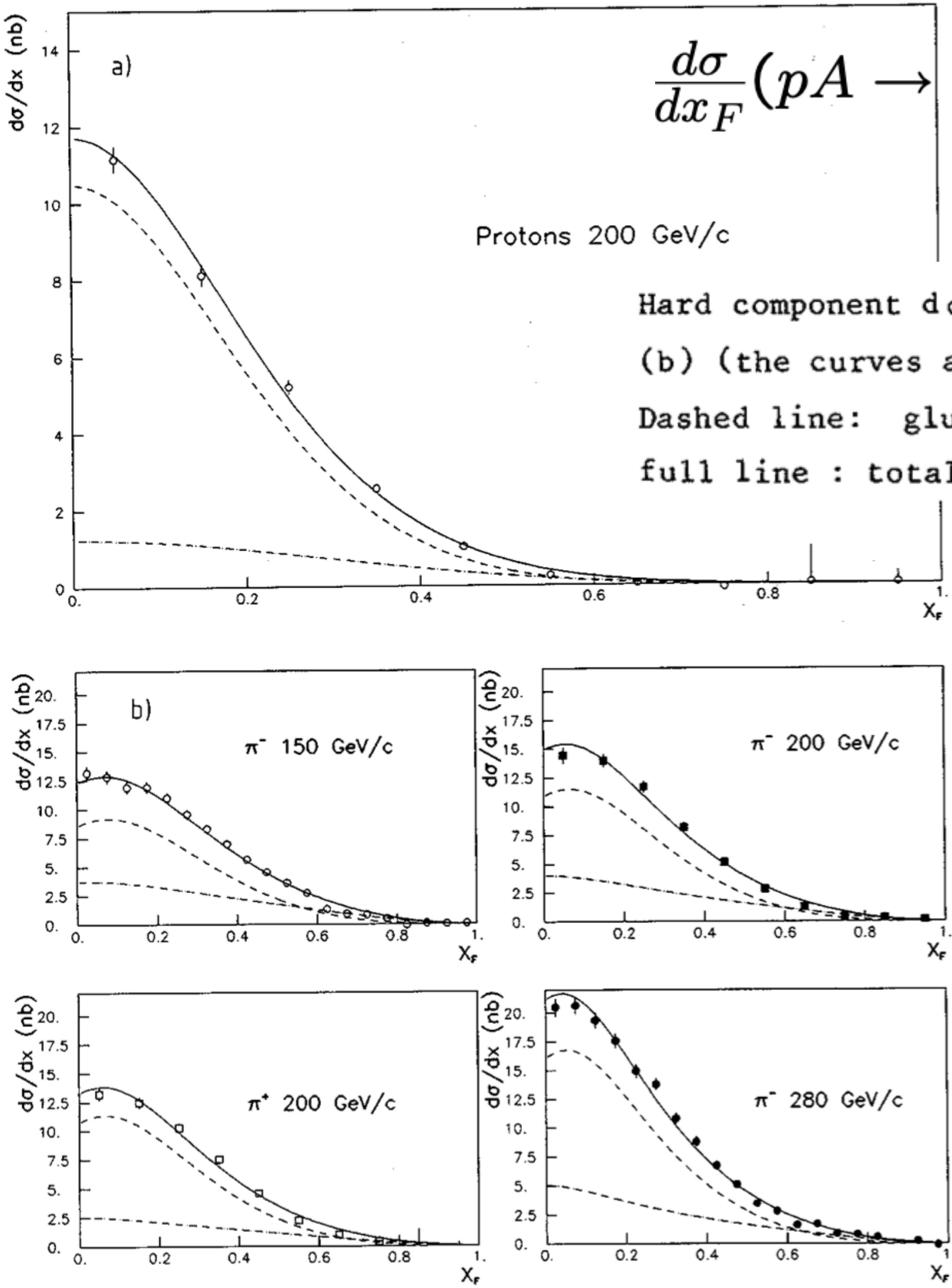
$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) \propto A^{\alpha}$$



Clear dependence on x_F and beam energy

Dramatic change in nuclear dependence

$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3}\frac{d\sigma_{2/3}}{dx_F}$$



Hard component $d\sigma_h/dx_F$ for incident protons (a) and pions (b) (the curves are the result of the fit described in the text. Dashed line: gluon-gluon fusion; dash-dotted line : $q\bar{q}$ fusion; full line : total).

A^1 component consistent with sum of gg and $\bar{q}q$ fusion

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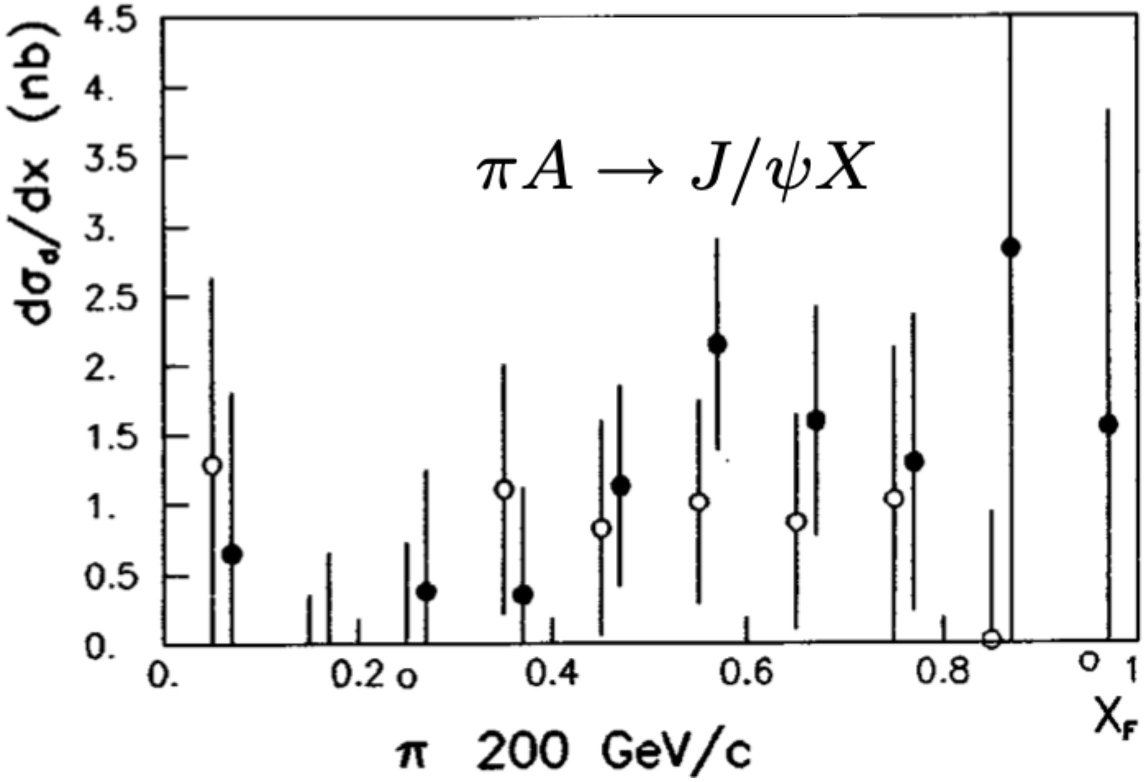
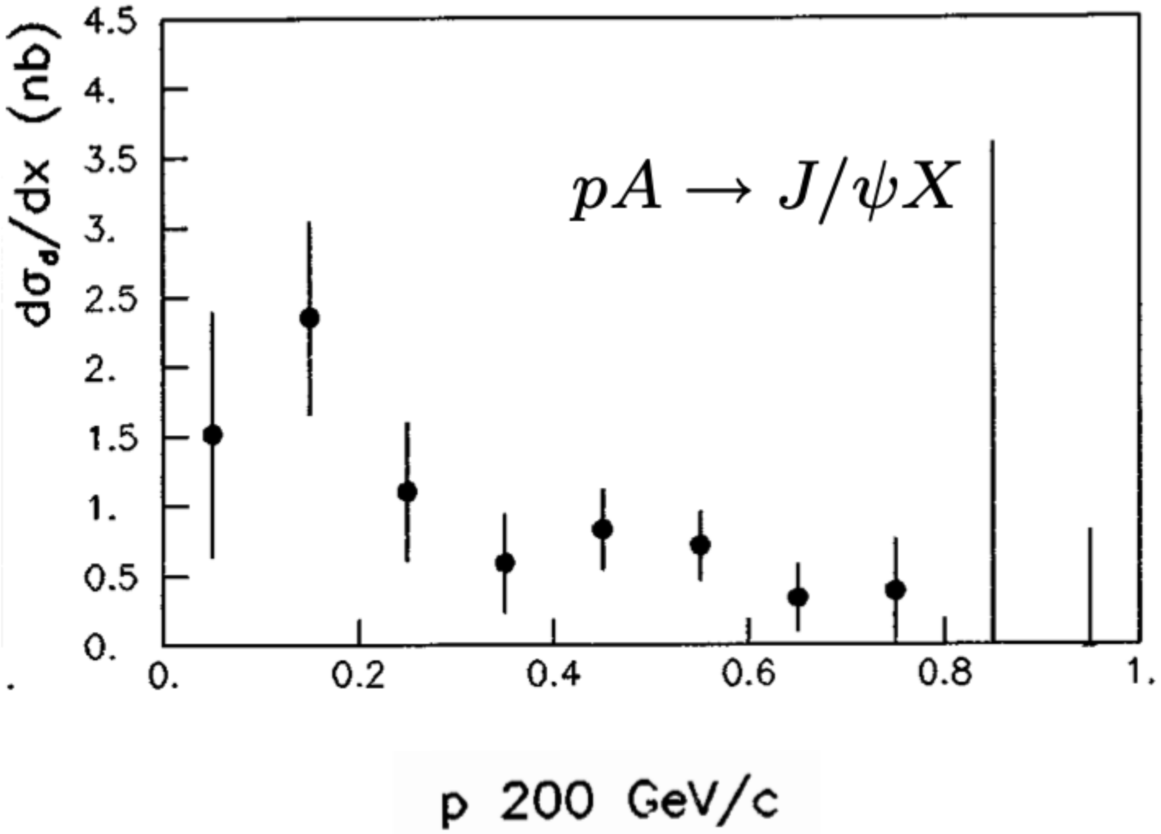


$A^{2/3}$ component



J. Badier et al, NA3

Excess beyond conventional PQCD subprocesses

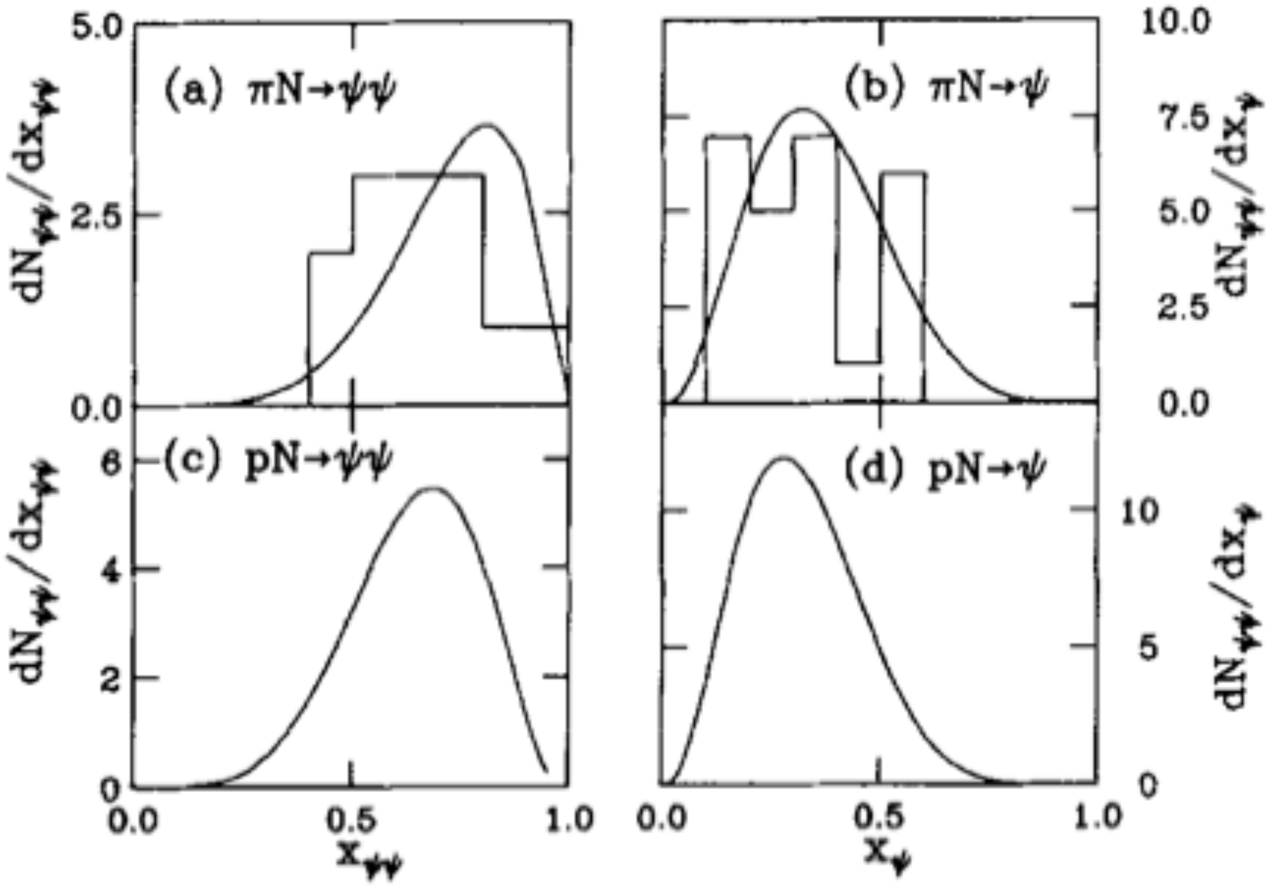


Fig. 3. The $\psi\psi$ pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of J/ψ's from the pairs are shown in (b) and (d). Our calculations are compared with the $\pi^- N$ data at 150 and 280 GeV/c [1]. The $x_{\psi\psi}$ distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single J/ψ's is twice the number of pairs.

NA3 Data

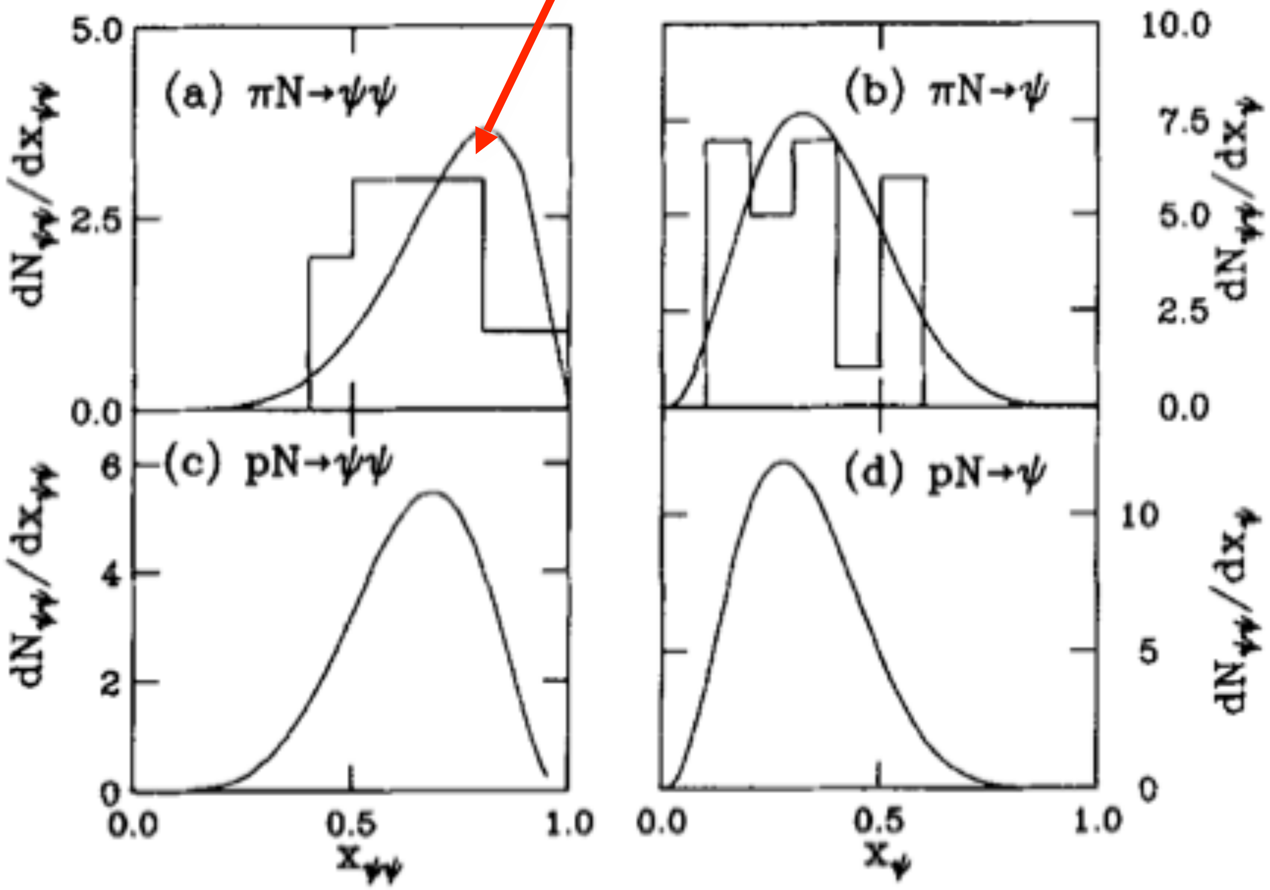
$$\pi A \to J/\psi J/\psi X$$

R. Vogt, sjb

The probability distribution for a general n-particle intrinsic $c\bar{c}$ Fock state as a function of x and $\boldsymbol{k}_T$ is written as

$$\frac{dP_{\rm ic}}{\prod_{i=1}^n dx_i d^2k_{T,i}} = N_n \alpha_s^4(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n \boldsymbol{k}_{T,i})\delta(1-\sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$

All events have $x^F_{\psi\psi} > 0.4$!



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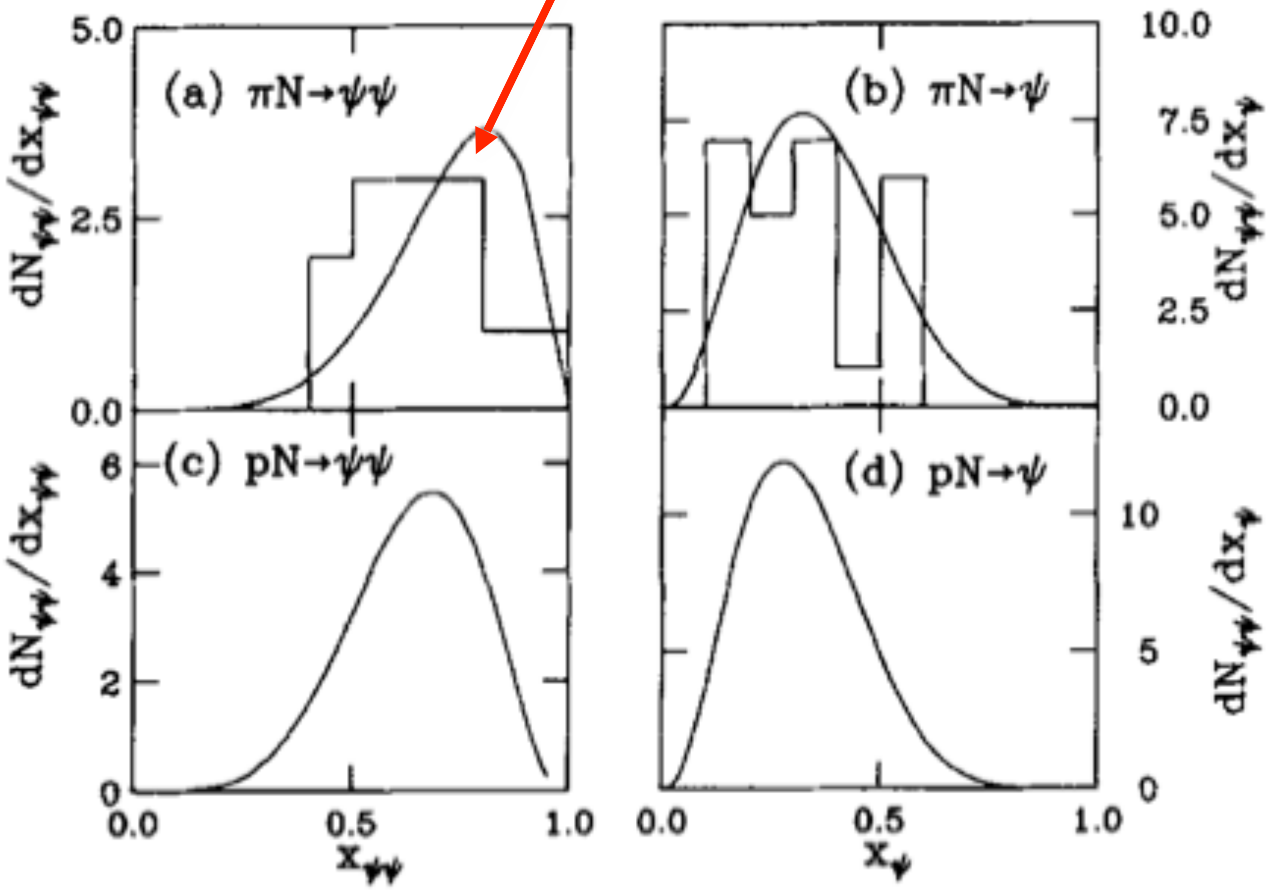
Excludes `color drag' model

$\pi A \to J/\psi J/\psi X$

R. Vogt, sjb



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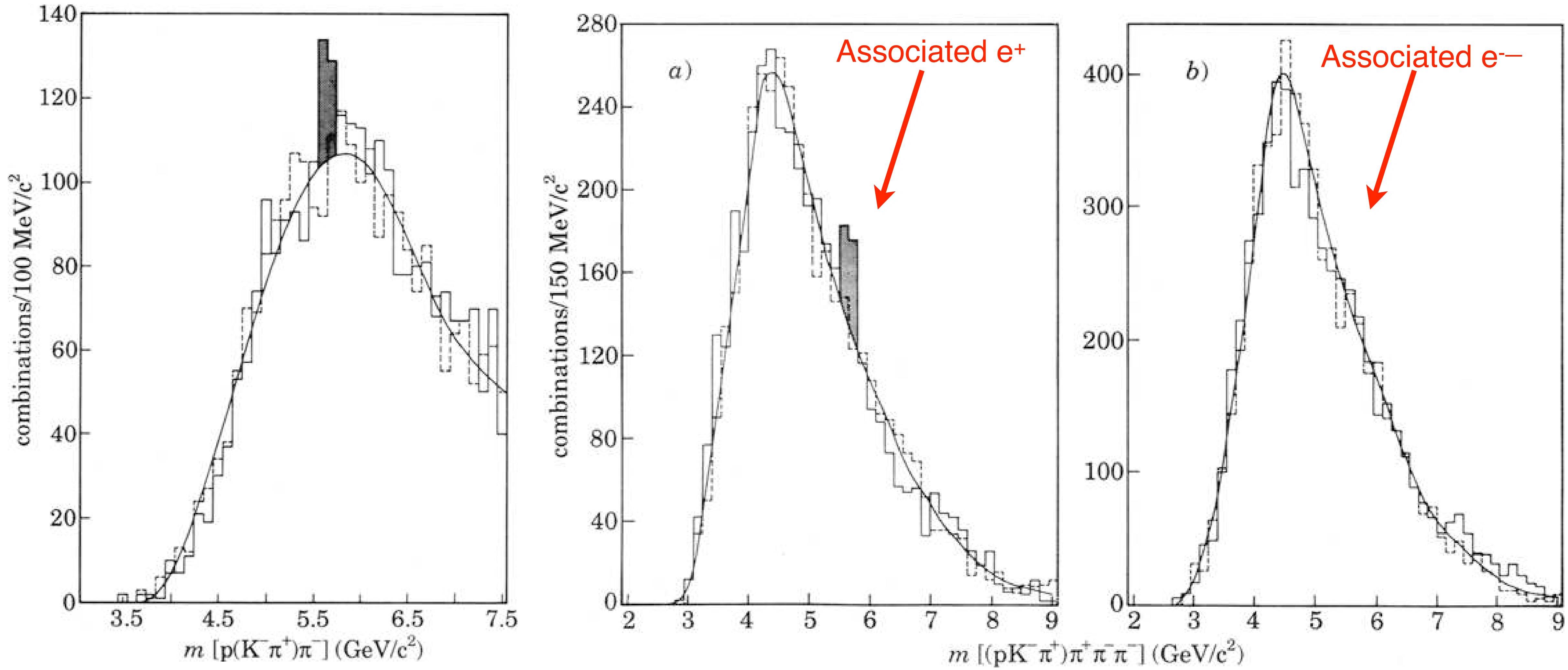
$pp \to \Lambda_b(bud)B(\bar{b}q)X$ at large x_F

CERN-ISR R422 (Split Field Magnet), 1988/1991



$\Lambda_b^0 \to pD^0\pi^-$

$\Lambda_b^0 \to \Lambda_c^+\pi^+\pi^-\pi^-$

Il Nuovo Cimento 104, 1787

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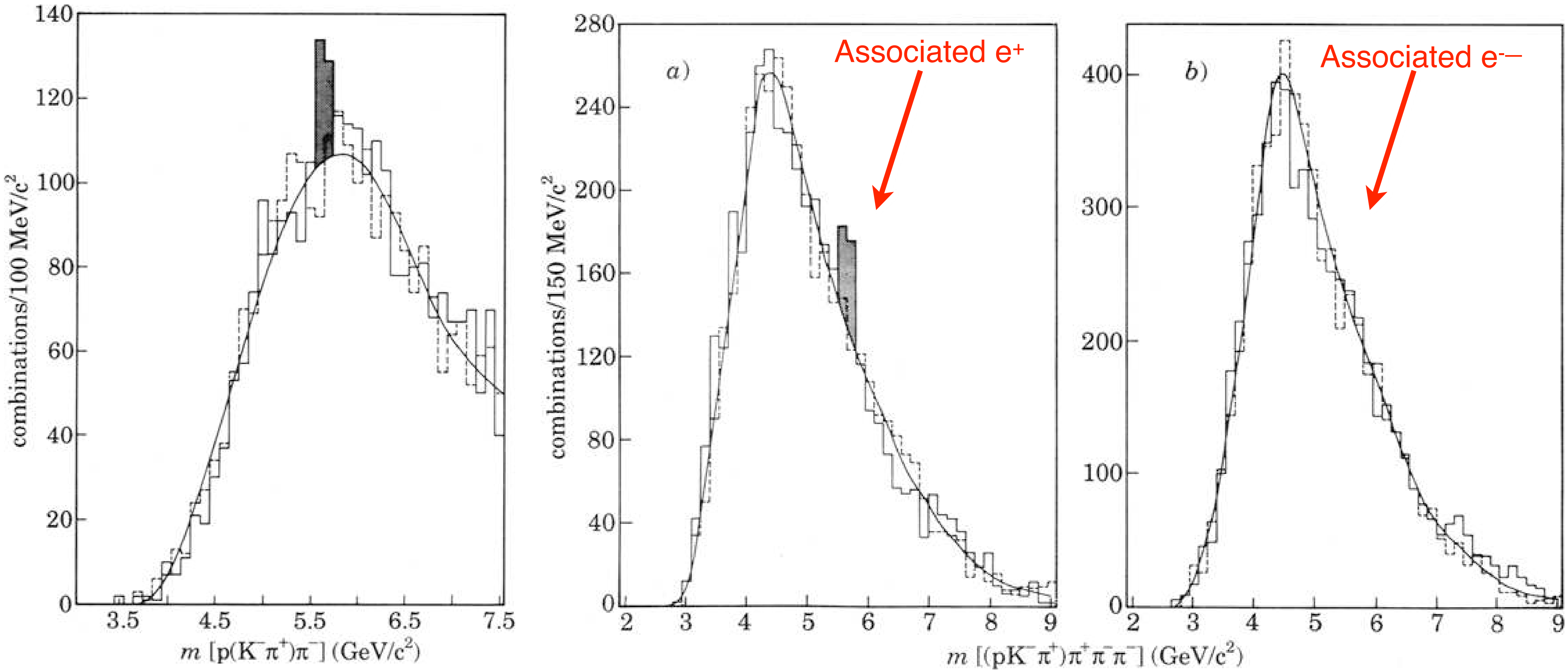


$\Lambda_b^0 \to pD^0\pi^-$ $\qquad$ $\Lambda_b^0 \to \Lambda_c^+\pi^+\pi^-\pi^-$

Il Nuovo Cimento 104, 1787

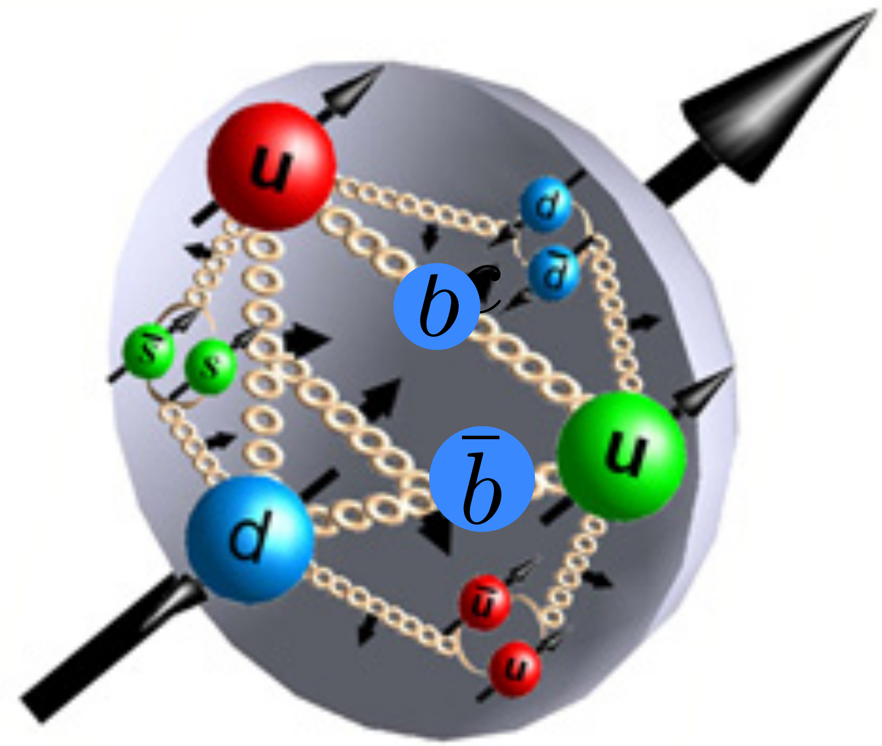
First Evidence for Intrinsic Bottom!

27 May 1991

THE Λ_b^o BEAUTY BARYON PRODUCTION IN PROTON-PROTON INTERACTIONS AT $\sqrt{s}$=62 GeV: A SECOND OBSERVATION

G. Bari, M. Basile, G. Bruni, G. Cara Romeo, R. Casaccia, L. Cifarelli, F. Cindolo, A. Contin, G. D'Alì, C. Del Papa, S. De Pasquale, P. Giusti, G. Iacobucci, G. Maccarrone, T. Massam, R. Nania, F. Palmonari, G. Sartorelli, G. Susinno, L. Votano and A. Zichichi

CERN, Geneva, Switzerland
Dipartimento di Fisica dell'Università, Bologna, Italy
Dipartimento di Fisica dell'Università, Cosenza, Italy
Istituto di Fisica dell'Università, Palermo, Italy
Istituto Nazionale di Fisica Nucleare, Bologna, Italy
Istituto Nazionale di Fisica Nucleare, LNF, Frascati, Italy



Abstract

Another decay mode of the Λ_b^o (open-beauty baryon) state has been observed: $\Lambda_b^o \rightarrow \Lambda_c^+\pi^+\pi^-\pi^-$. In addition, new results on the previously observed decay channel, $\Lambda_b^o \rightarrow pD^o\pi^-$, are reported. These results confirm our previous findings on Λ_b^o production at the ISR. The mass value (5.6 GeV/c^2) is found to be in good agreement with theoretical predictions. The production mechanism is found to be "leading".

27 May 1991

CM-P00063074

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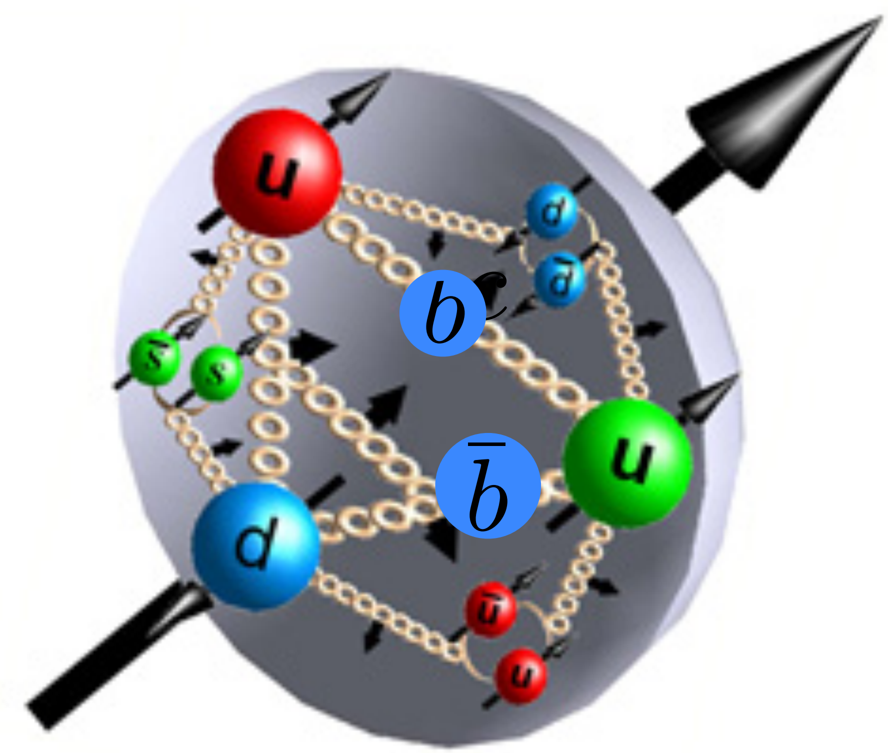

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First Evidence for Intrinsic Bottom!

- IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \to J/\psi X$ (Mueller, Gunion, Tang, SJB)

- Color Octet IC Explains $A^{2/3}$ behavior at high x_F (NA3, Fermilab) (Kopeliovitch, Schmidt, Soffer, SJB)

Color Opaqueness

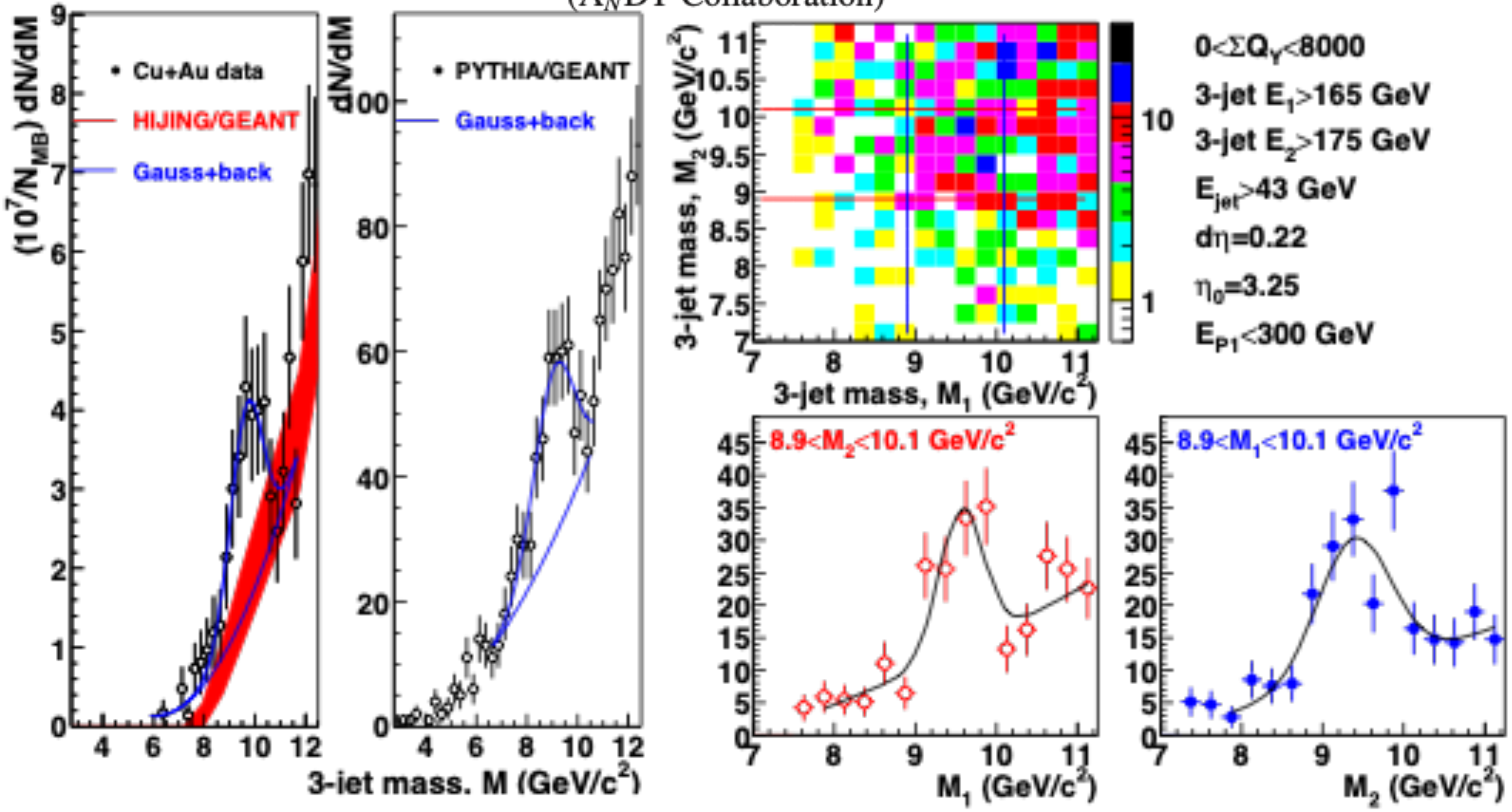
- IC Explains $J/\psi \to \rho\pi$ puzzle (Karliner, SJB)

- IC leads to new effects in B decay (Gardner, SJB)

Observation of Feynman scaling violations and evidence for a new resonance at RHIC

L. C. Bland[a], E. J. Brash[b], H. J. Crawford[c], A.A. Derevschikov[d], K. A. Drees[a], J. Engelage[c], C. Folz[a], E. G. Judd[c], X. Li[e,a], N. G. Minaev[d], R. N. Munroe[b], L. Nogach[d], A. Ogawa[a], C. Perkins[c], M. Planinic[f], A. Quintero[i], G. Schnell[g,h], P. V. Shanmuganathan[j], G. Simatovic[f,a], B. Surrow[i], T. G. Throwe[a], A. N. Vasiliev[d]

(A_NDY Collaboration)*



Evidence for $\Upsilon(1S)$ via its decay to three jets. (left pair) Inclusive forward production from Cu+Au collisions overlayed with HIJING/GEANT simulation. A 5.2σ peak is observed in the data. Comparison is to PYTHIA/GEANT p+p simulations at $\sqrt{s} = 1200$ GeV, using the Perugia 0 tune. (right) $\sim 5\sigma$ evidence for forward pair $\Upsilon(1S)$ production. All Cu+Au distributions have vertical axes scaled as $10^7/N_{MB}$.

AnDY at RHIC: Observe single and double Υ production at high rapidity

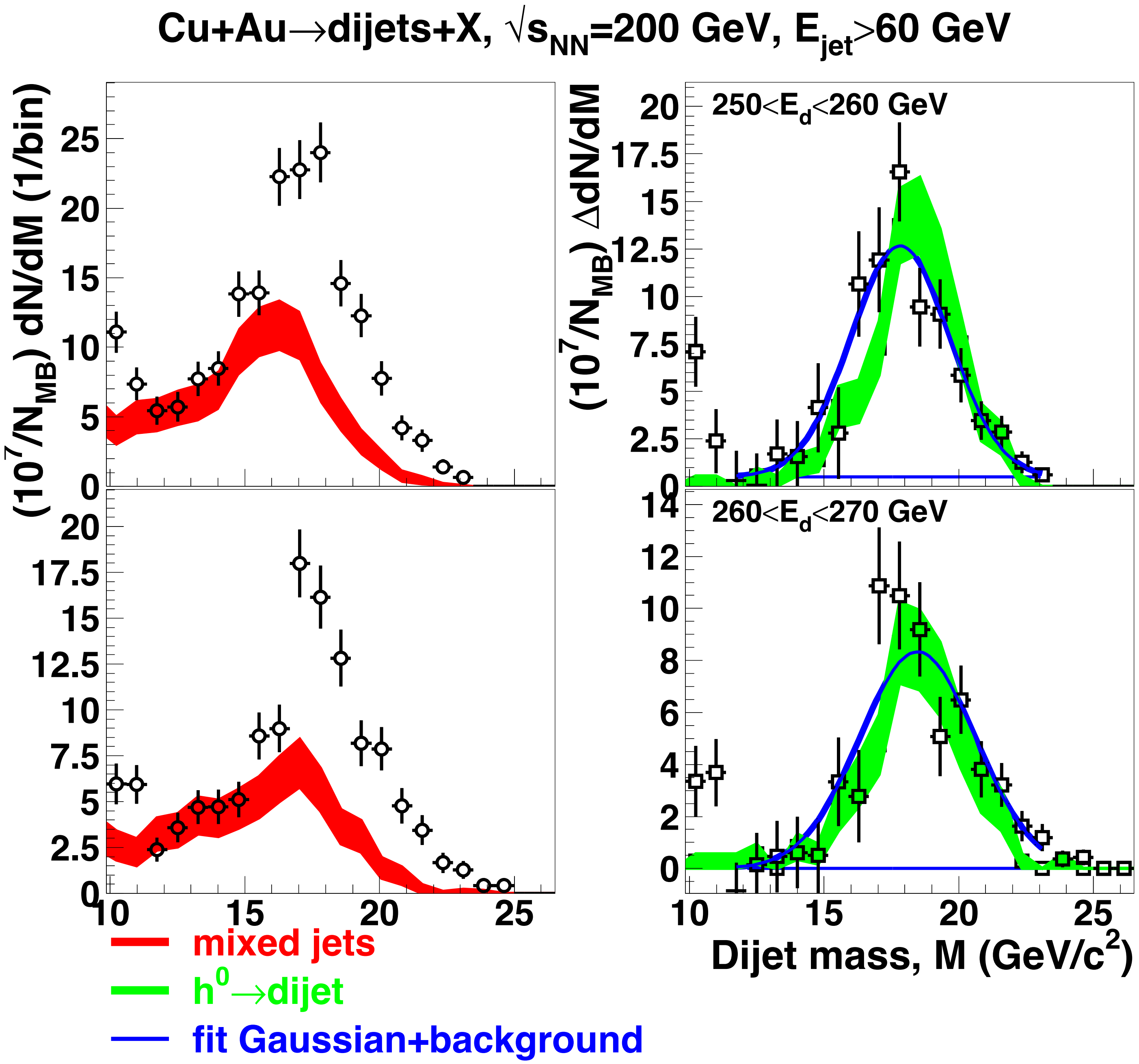
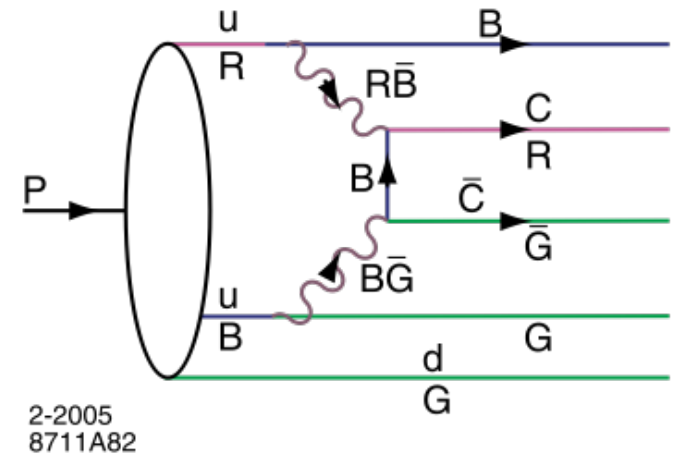


Figure 7: Dijet mass compared to a mixed-event analysis in the left column. The right column forms the difference between data and mixed events, and compares that difference to a simulation of the production of a resonance that decays to jet pairs. All Cu+Au distributions have vertical axes scaled as $10^7/N_{MB}$.

AnDY at RHIC: Observe $bb\bar{b}\bar{b}$ production at high rapidity

Hoyer, Peterson, Sakai, sjb
M. Polyakov, et. al

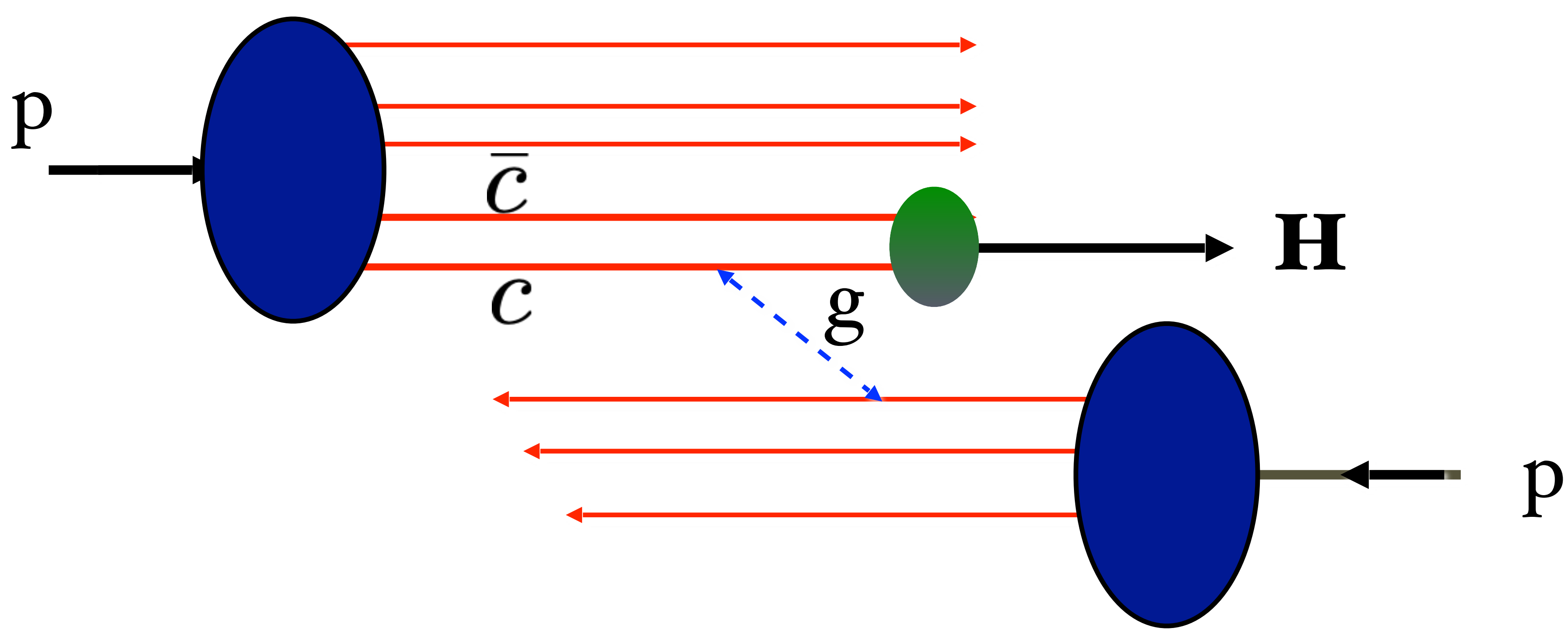
Intrinsic Heavy-Quark Fock States



- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production at high x_F (Kopeliovich, Schmidt, Soffer, Goldhaber, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests (Gardener, Karliner, ..)

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



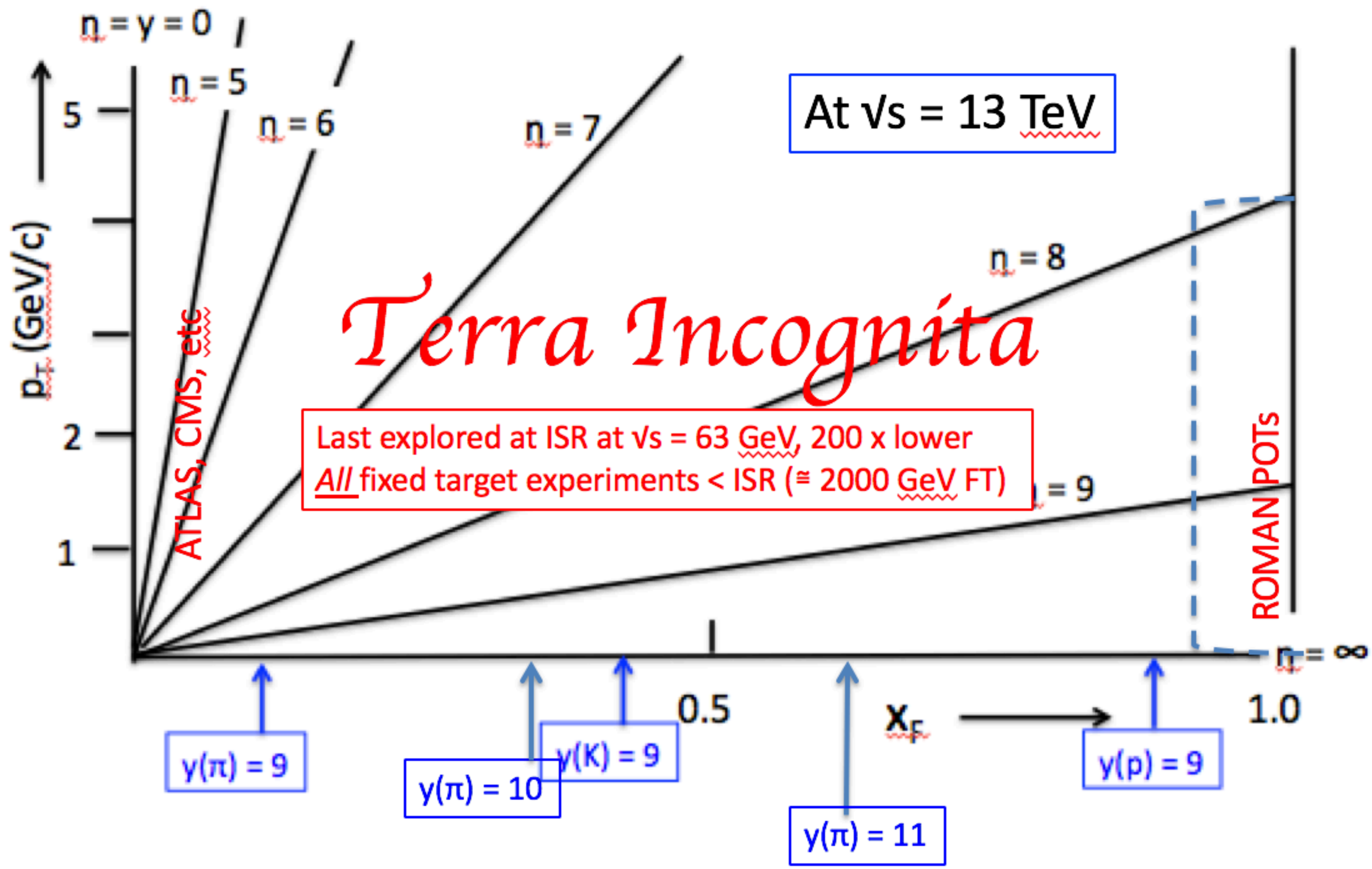
$$pp \rightarrow HX$$

Also: intrinsic strangeness, bottom, top

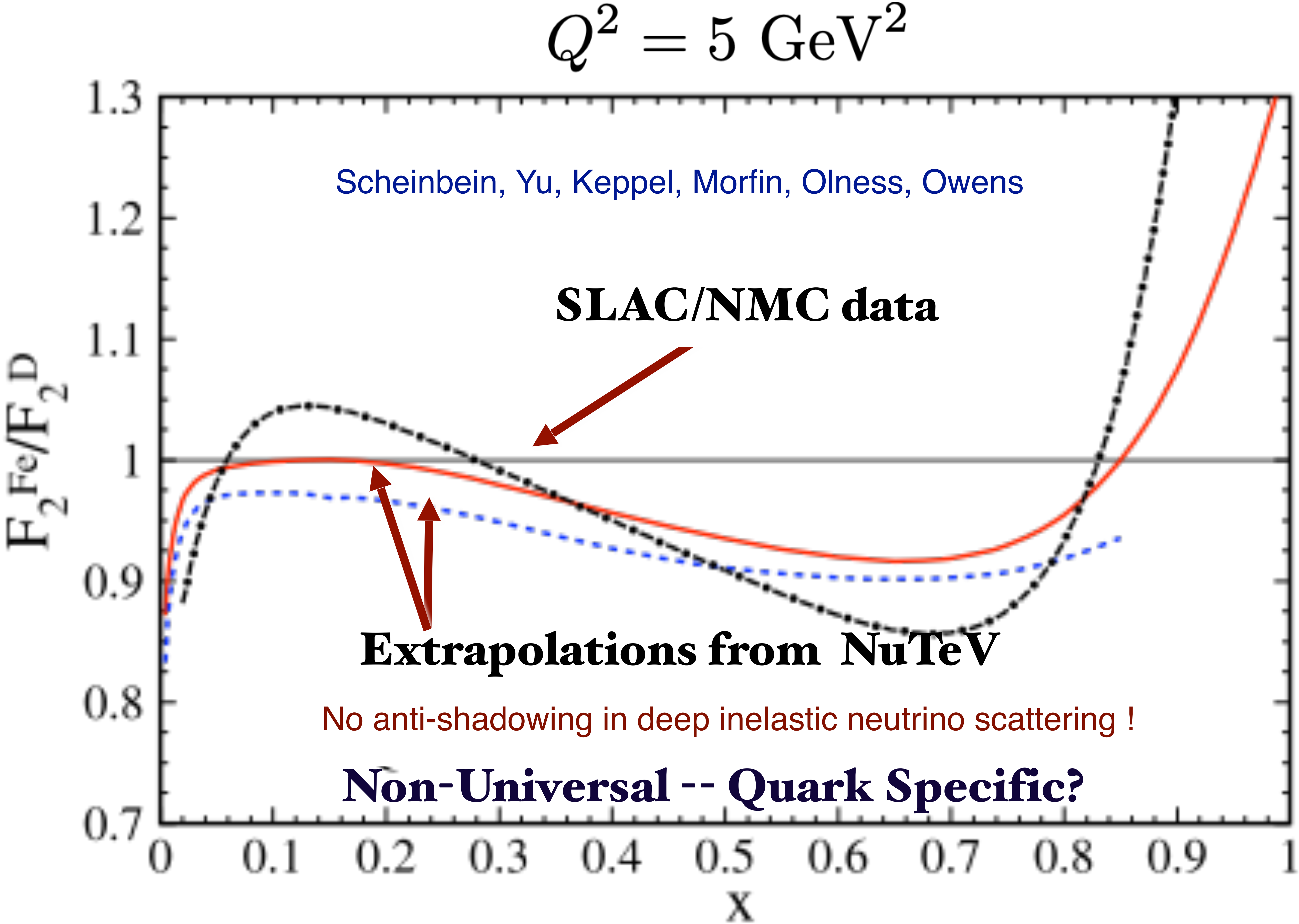
Higgs can have > 80% of Proton Momentum!

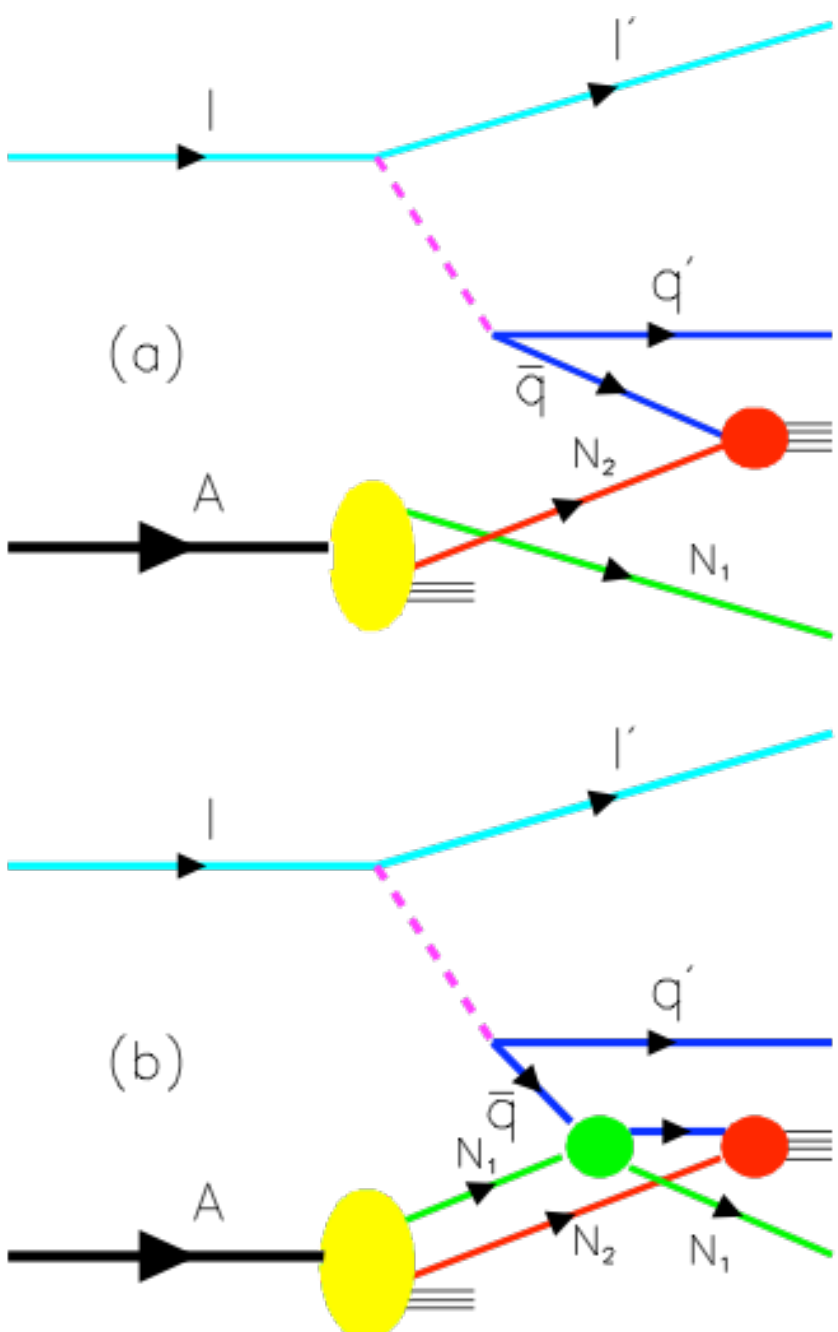
New production mechanism for Higgs at the LHC

Mike Albrow: Forward Hadron Spectrometer



For pp collisions at $\sqrt{s} = 13$ TeV, regions of low transverse momentum p_T and all Feynman-x, x_F, showing lines of constant pseudorapidity η. Protons with $x_F > 0.9$ are measured in Roman pots, and neutral particles in calorimeters around 0°. Identified charged hadrons have not been measured except at $\eta < 4$ at LHCb, so most of this phase space is Terra Incognita.


$Q^2 = 5\ \mathrm{GeV}^2$
Scheinbein, Yu, Keppel, Morfin, Olness, Owens
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
Non-Universal -- Quark Specific?
F_2^{Fe}/F_2^D
x
1.3
1.2
1.1
1
0.9
0.8
0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



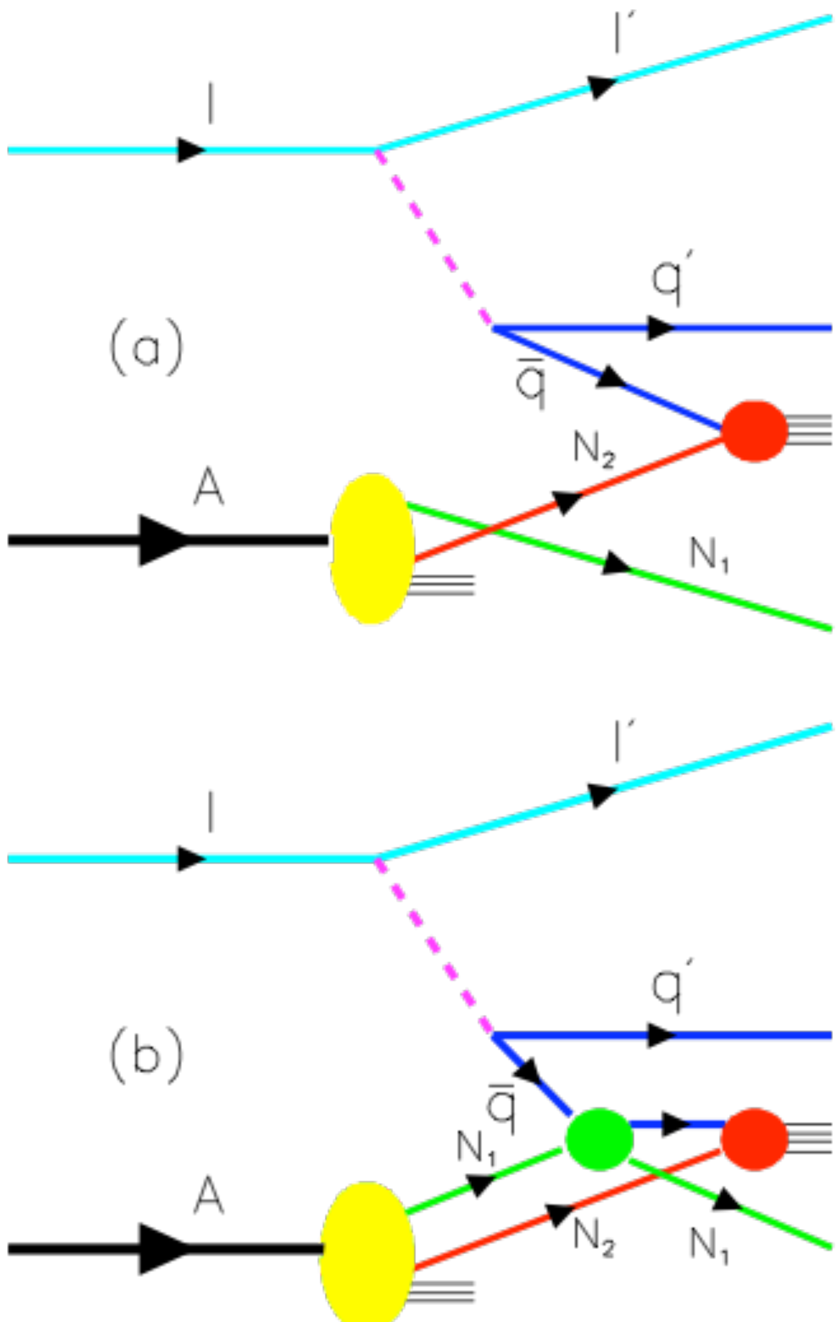
The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\bar{q}$ flux reaching N_2.

Diffraction via Pomeron gives destructive interference!

Shadowing



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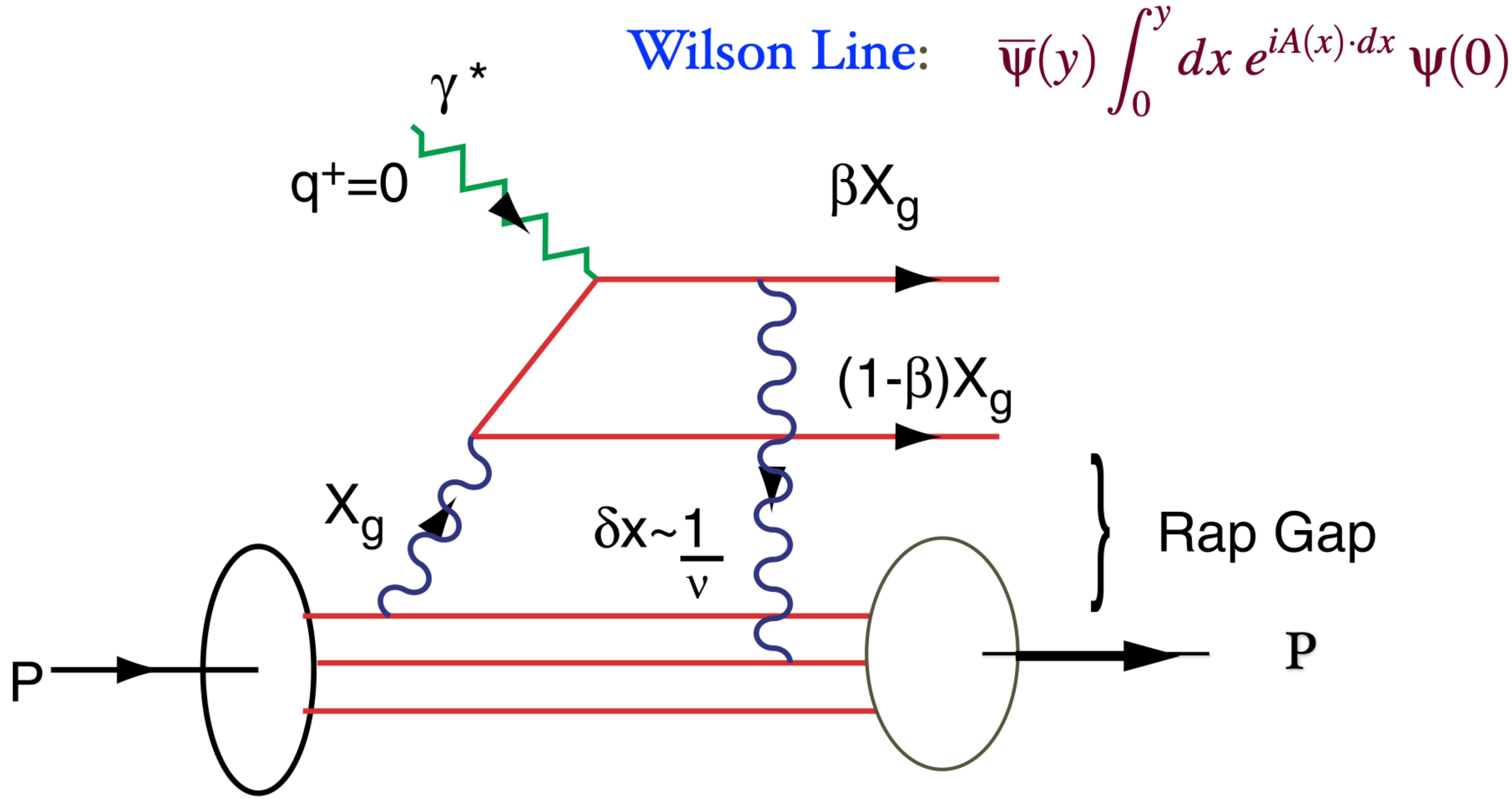
$\rightarrow$ Shadowing of the DIS nuclear structure functions.

Diffraction via Pomeron gives destructive interference!

Shadowing

QCD Mechanism for Rapidity Gaps



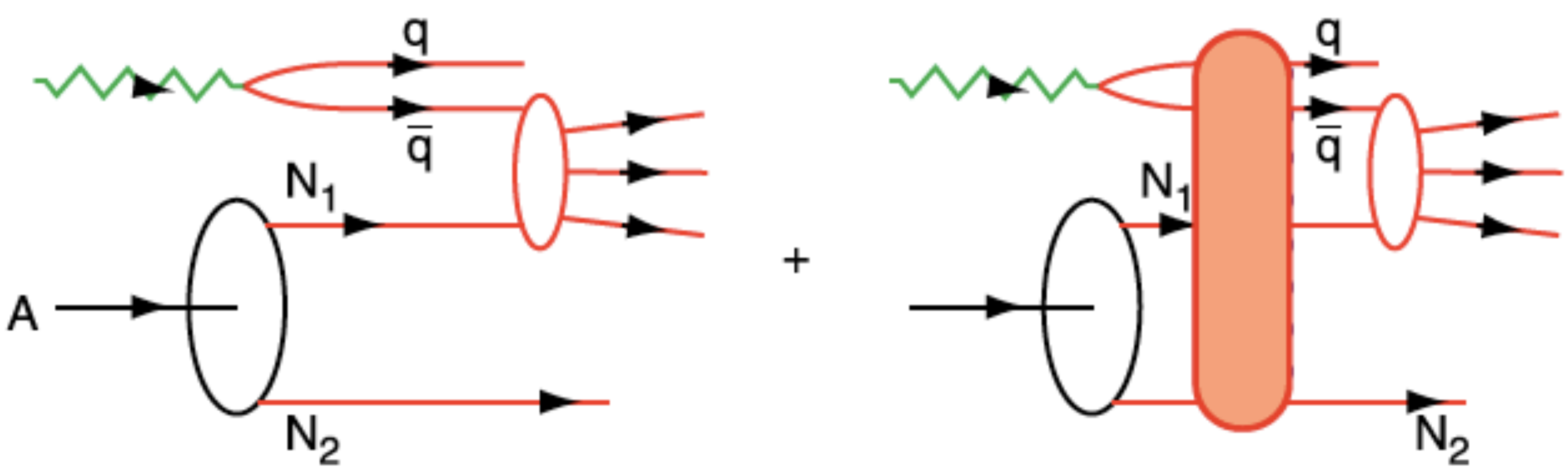
Reproduces lab-frame color dipole approach
DDIS: Input for leading twist nuclear shadowing

H. J. Lu, sjb

Schmidt, Yang, sjb

Nuclear Anti-shadowing in QCD

Constructive Interference **Flavor-Specific!**



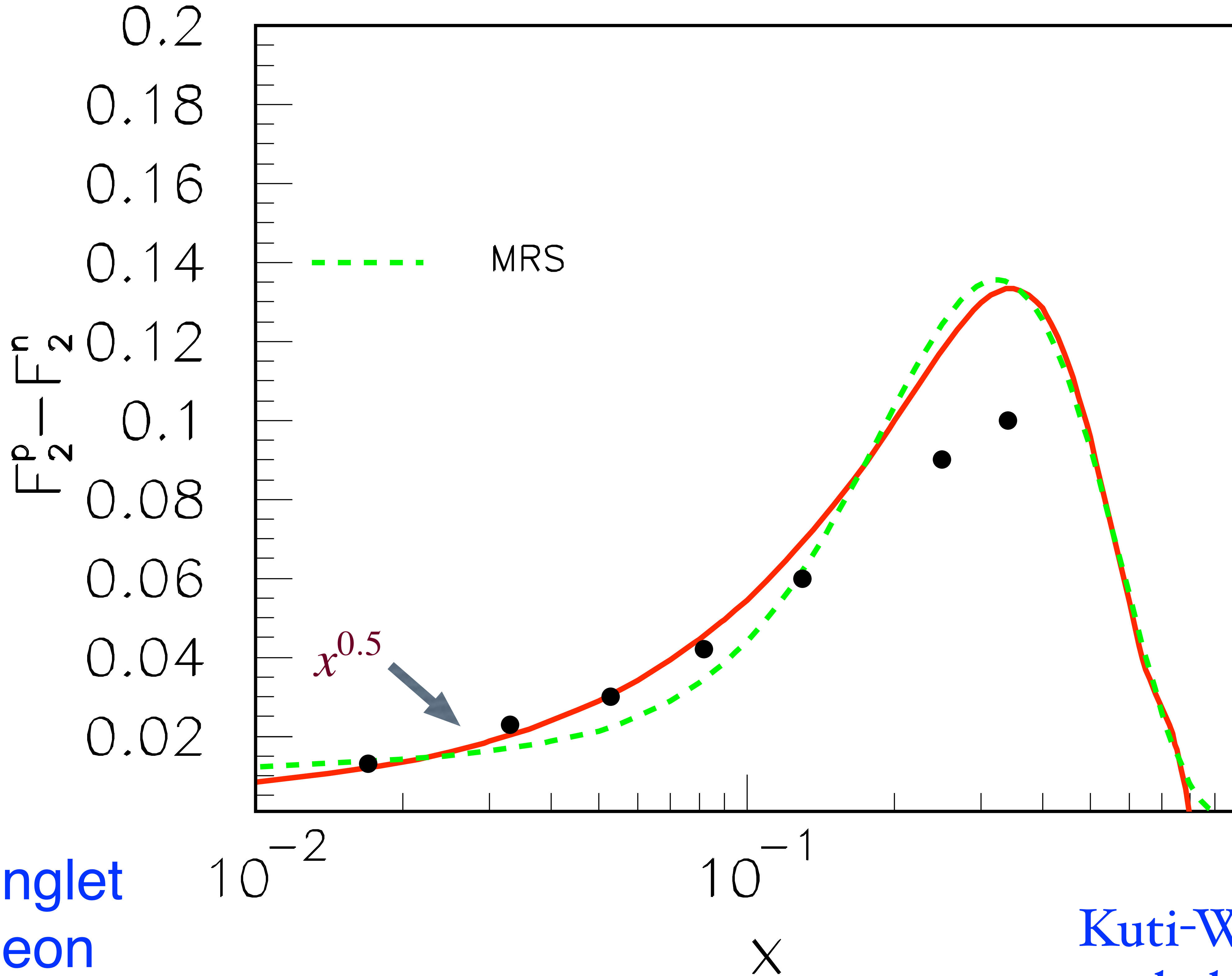
Diffractive Reggeon Exchange

Antishadowing (Reggeon exchange) is not universal!

Reggeon coupling fixed from Kuti-Weisskopf: $F_{2p}(x) - F_{2n}(x) \simeq Cx^{1/2}$

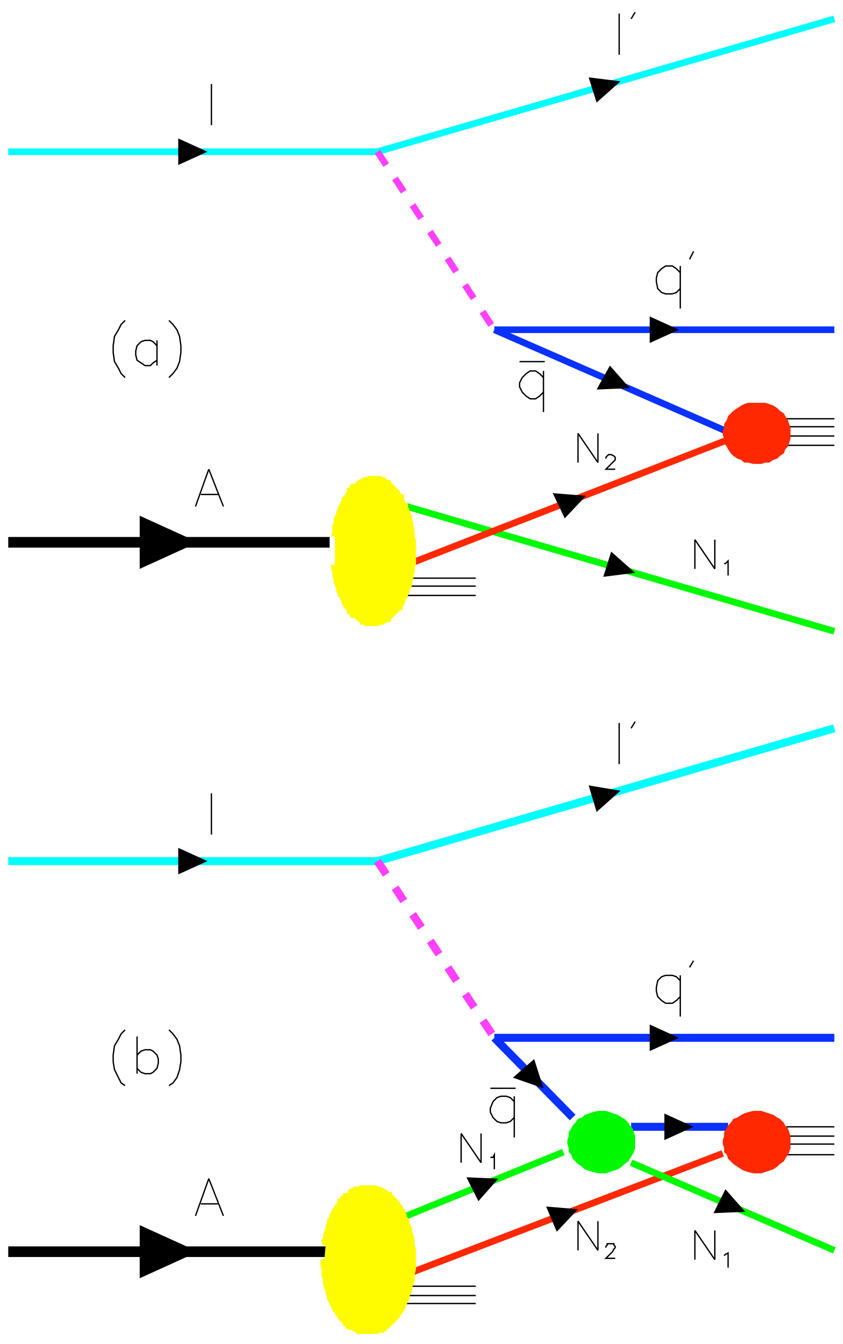
Nuclear Anti-shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus

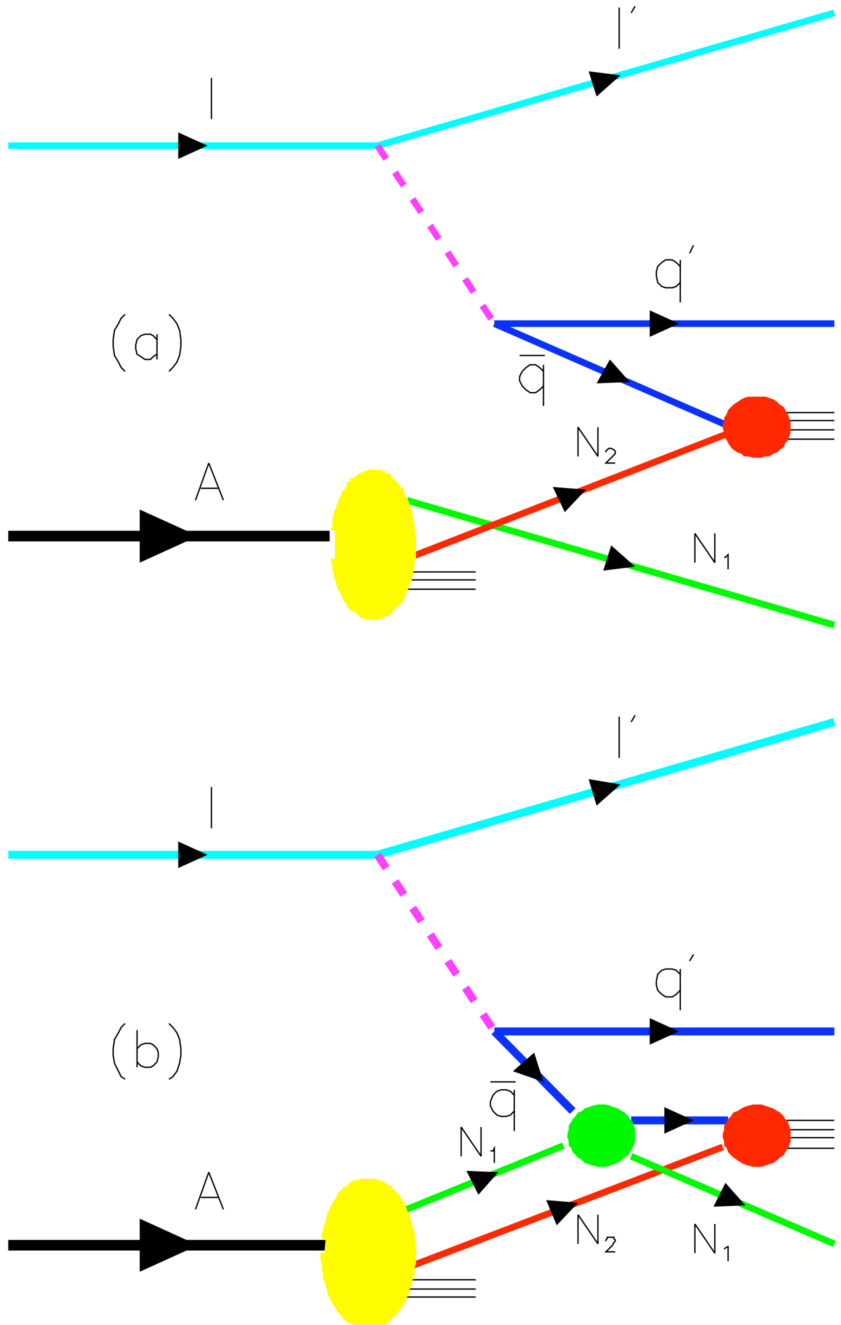


Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior



Anti-shadowing



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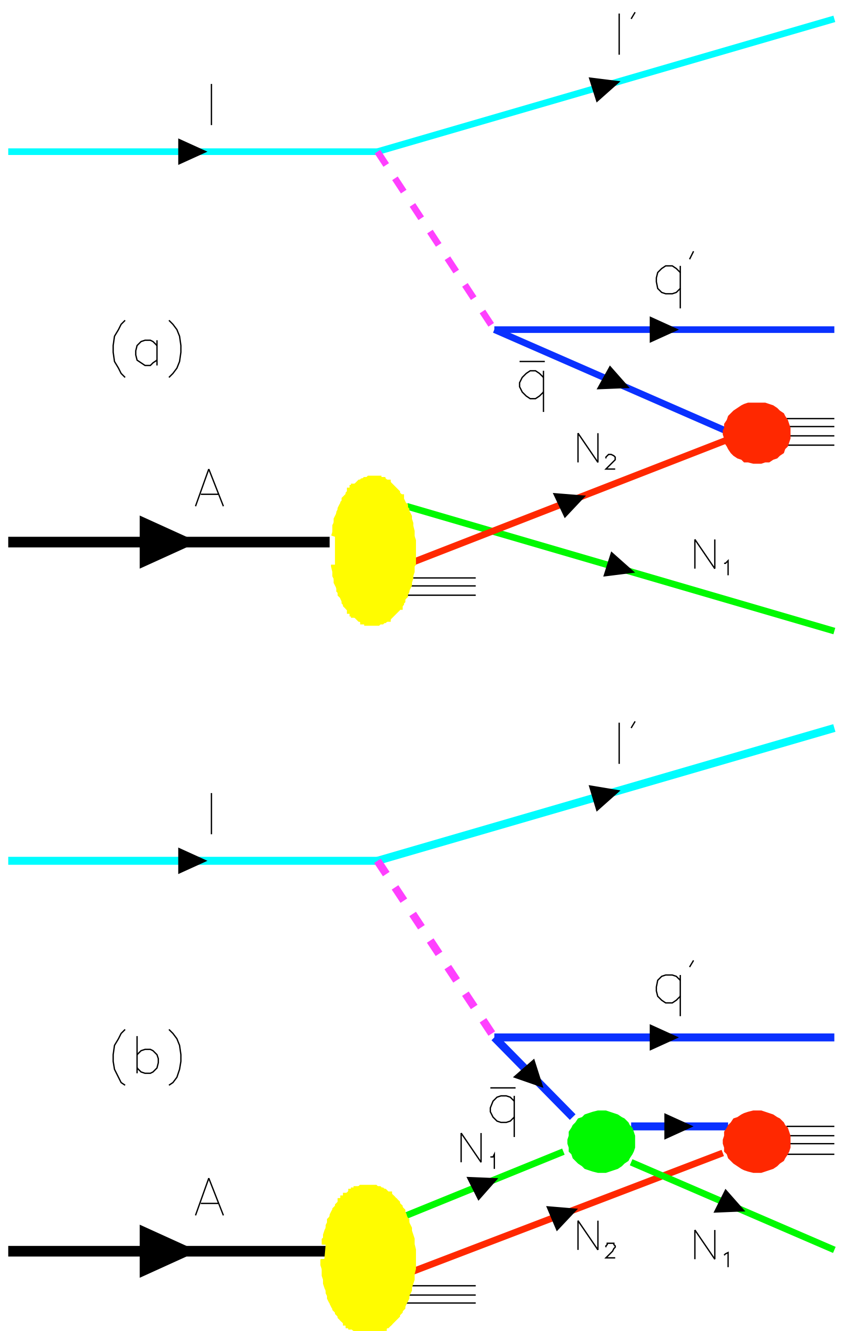

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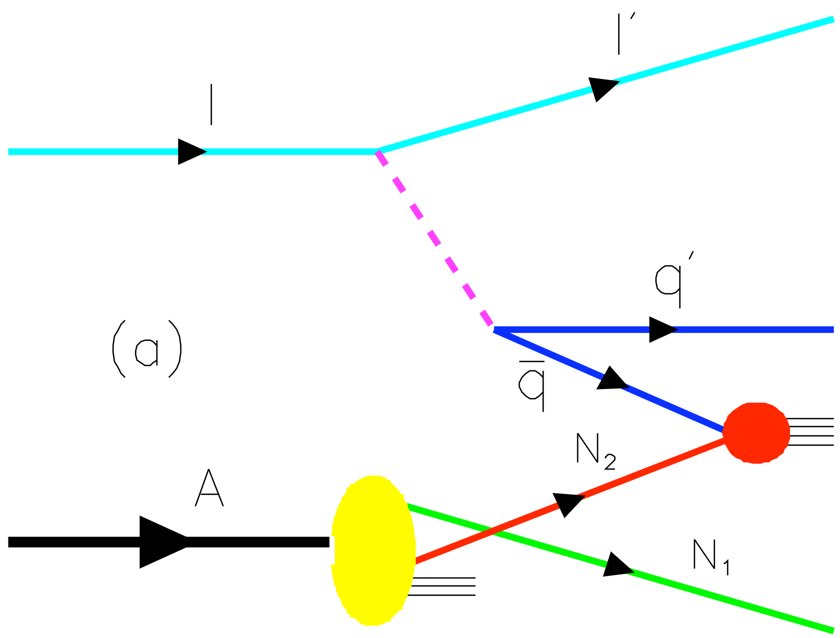

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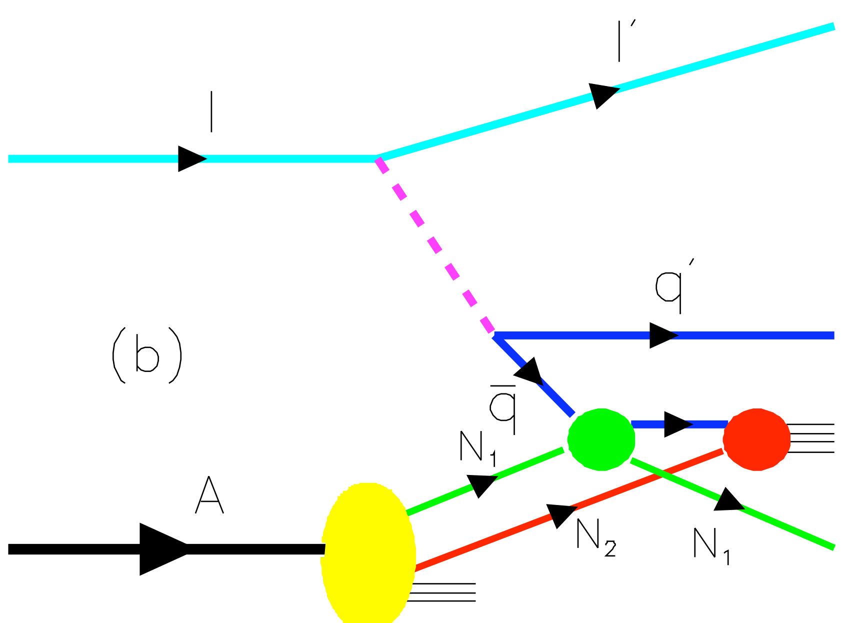


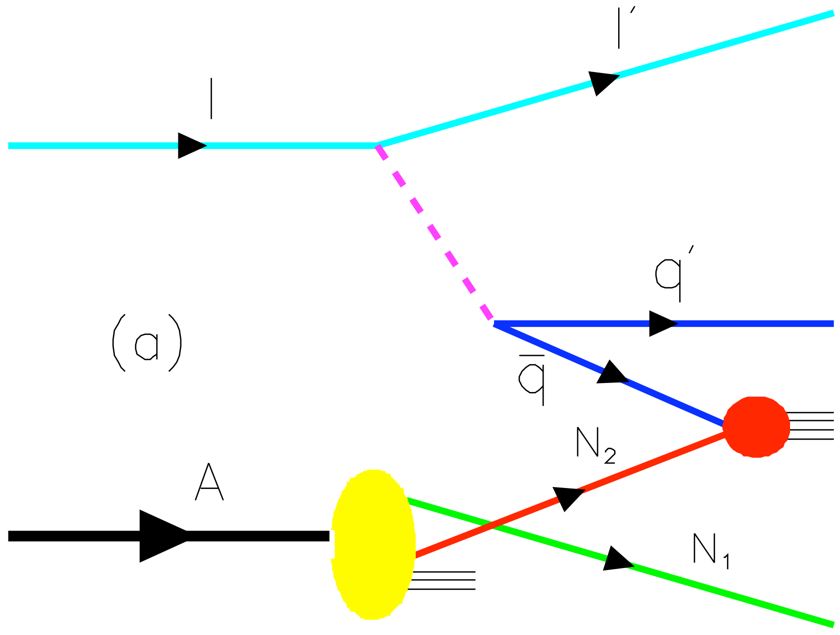
Regge

If the scattering on nucleon N_1 is via ~~pomeron~~ exchange, the one-step and two-step amplitudes are ~~opposite in phase, thus diminishing the $\bar{q}$ flux reaching N_2.~~

constructive in phase

thus increasing the flux reaching N_2

Anti-shadowing



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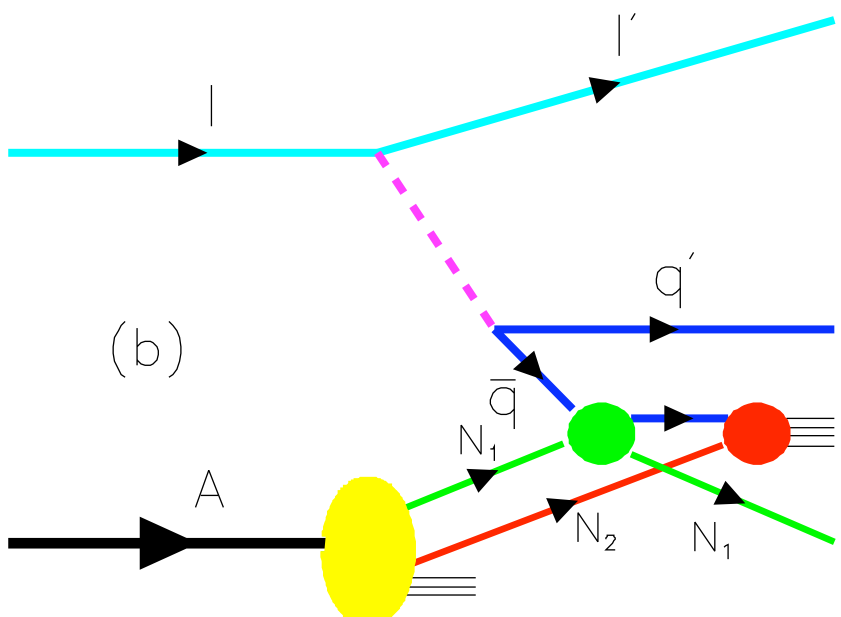

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constructive in phase

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Reggeon DDIS produces nuclear flavor-dependent anti-shadowing

Anti-shadowing

Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1-i) \times i = \frac{1}{\sqrt{2}}(i+1)$$

Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^\pm$

Critical test: Tagged Drell-Yan

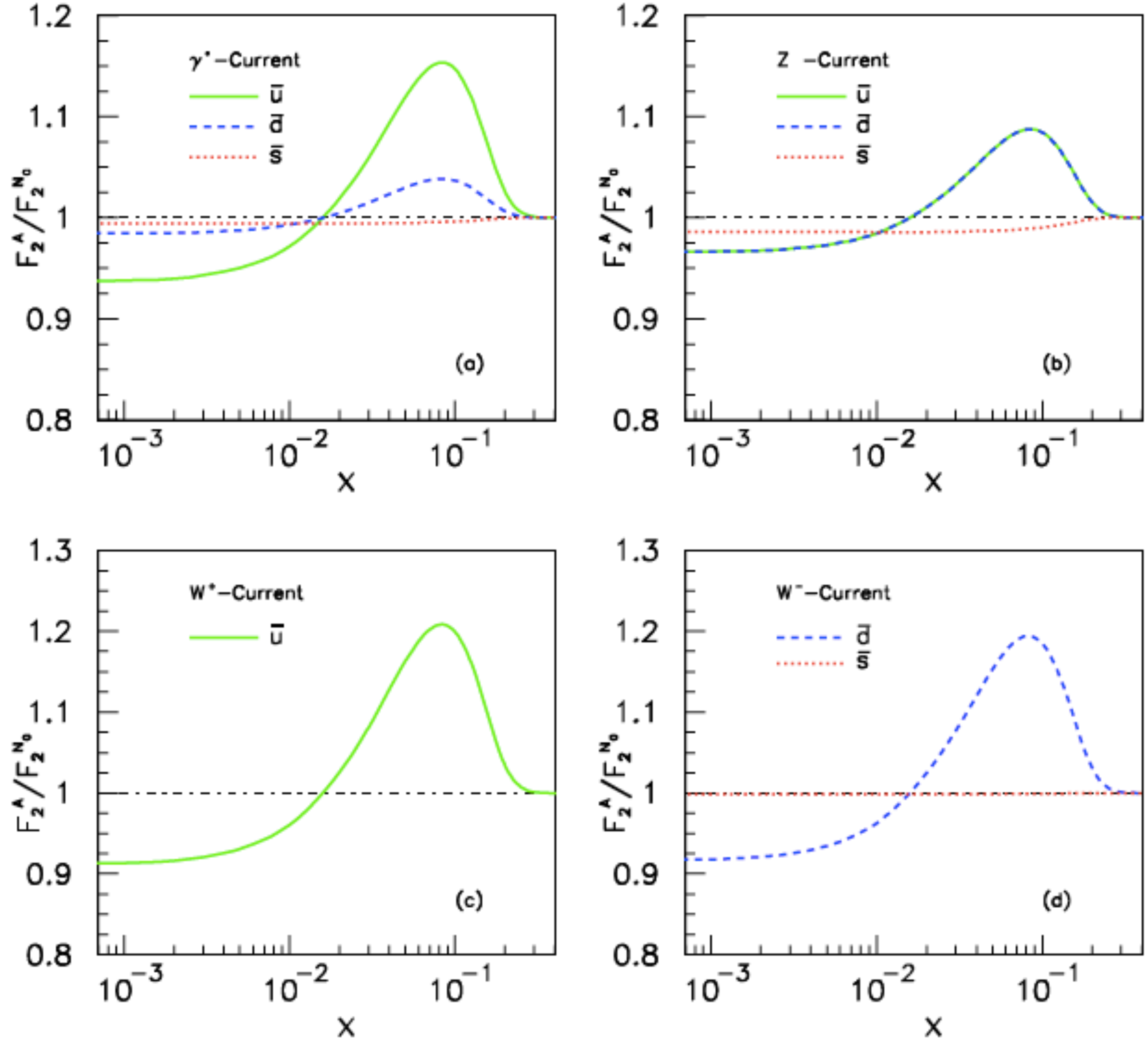
Shadowing and Antishadowing in Lepton-Nucleus Scattering

- Shadowing: Destructive Interference
of Two-Step and One-Step Processes
Pomeron Exchange

- Antishadowing: Constructive Interference
of Two-Step and One-Step Processes!
Reggeon and Odderon Exchange

- Antishadowing is Not Universal!
Electromagnetic and weak currents:
different nuclear effects !
Potentially significant for NuTeV Anomaly}

Jian-Jun Yang
Ivan Schmidt
Hung Jung Lu
sjb



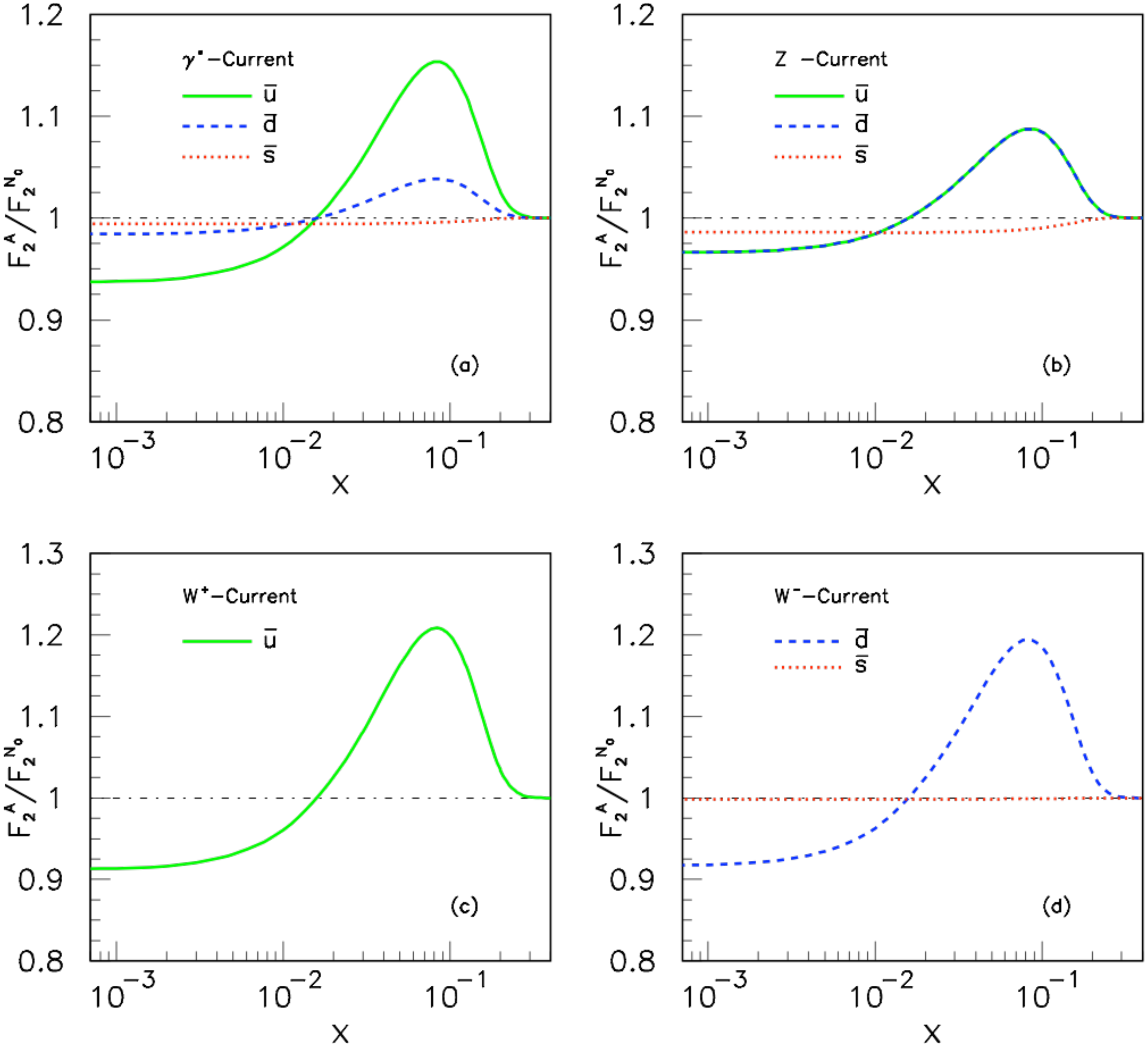
Schmidt, Yang; sjb

Reggeon Contribution to DDIS

Constructive Interference!

Phase from signature factor

Nuclear Antishadowing not universal !



Schmidt, Yang; sjb

Modifies NuTeV extraction of

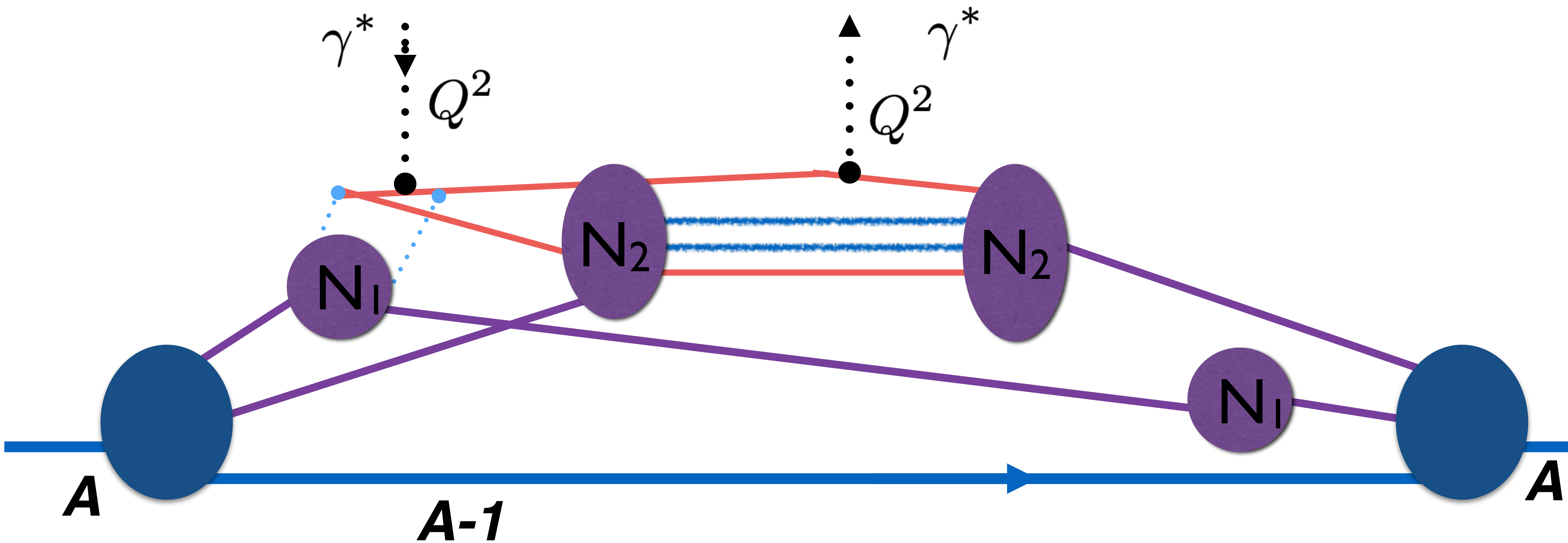
$$\sin^2 \theta_W$$

Test in flavor-tagged DIS at the EIC

Nuclear Antishadowing not universal !

$$q^+ = 0 \quad q_\perp^2 = Q^2 = -q^2$$

Illustrates the LF time sequence



Front-Face Nucleon N_1 struck | *Front-Face Nucleon N_1 not struck*

One-Step / Two-Step Interference

Study Double Virtual Compton Scattering $\gamma^* A \to \gamma^* A$

Cannot reduce to matrix element of local operator! No Sum Rules!

Liuti, Schmidt sjb

S. Liuti, I. Schmidt, sjb

- Unlike shadowing, anti-shadowing from Reggeon exchange is flavor specific;
- Each quark and anti-quark will have distinctly different constructive interference patterns.
- The flavor dependence of antishadowing explains why anti- shadowing is different for electron (neutral electro- magnetic current) vs. neutrino (charged weak current) DIS reactions.
- Test of the explanation of antishadowing: Bjorken-scaling leading-twist charge exchange DDIS reaction $\gamma^* p \to n X^+$ with a rapidity gap due to I=1 Reggeon exchange

The finite path length due to the on-shell propagation of V_0 between N_1 and N_2 contributes a finite distance $(\Delta z)^2$ between the two virtual photons in the DVCS amplitude.

The usual "handbag" diagram where the two $J_\mu(x)$ and $J_\nu(0)$ currents acting on an uninterrupted quark propagator are replaced by a local operator $T_{\mu\nu}(0)$ as $Q^2 \to \infty$, is inapplicable in deeply virtual Compton scattering from a nucleus since the currents act on different nucleons.

$$\Delta z^2 \text{ does not vanish as } \frac{1}{Q^2}.$$

OPE and Sum Rules invalid for nuclear pdfs

QCD Myths

- **Anti-Shadowing is Universal**
- **ISI and FSI are higher twist effects and universal**
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **Heavy quarks only from gluon splitting**
- **Renormalization scale cannot be fixed**
- **QCD condensates are vacuum effects**
- **QCD gives 10^{42} to the cosmological constant**
- **Colliding Pancakes**

Invariance Principles of Quantum Field Theory

- *Polncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*
- *Causality: Information within causal horizon: Front Form*
- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (BLM-PMC)*
- *Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)*

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Similarly for m_ρ.

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

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de Tèramond, Dosch, Lorce, sjb

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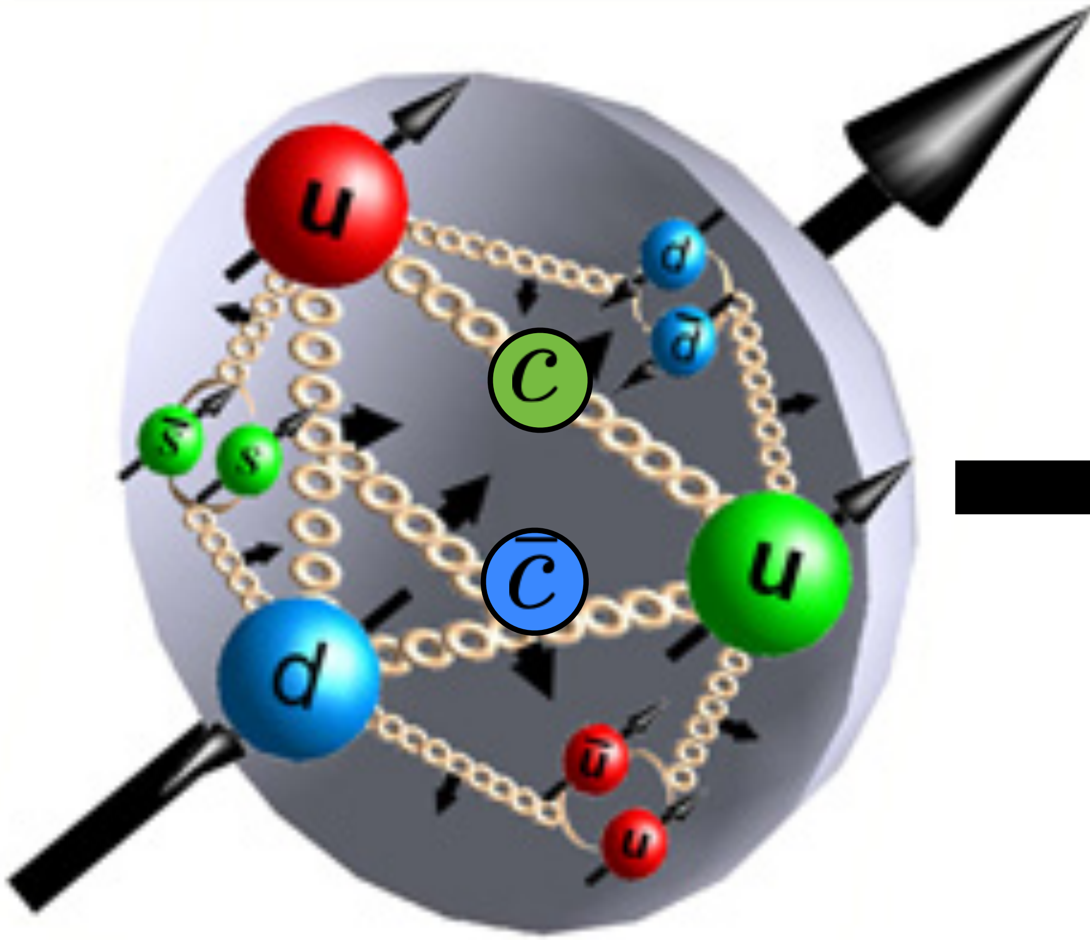
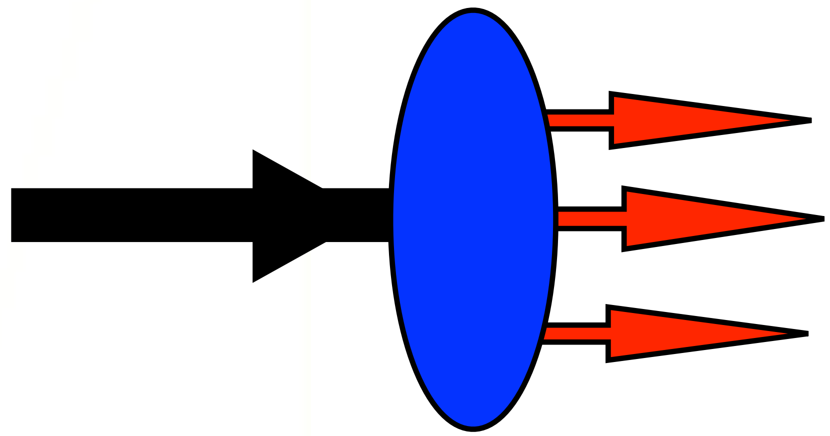
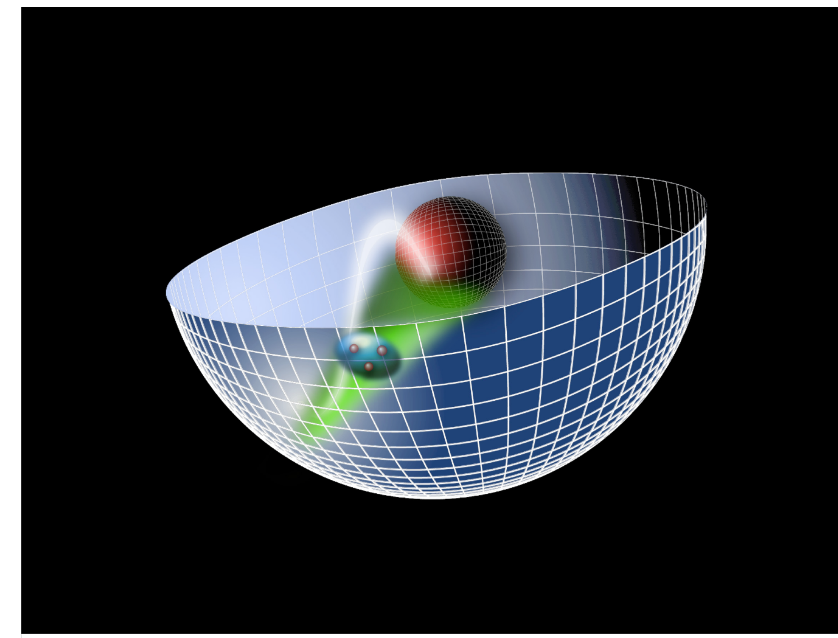
Supersymmetric Properties of Hadron Physics and Other Remarkable Features of Hadron Physics







Congratulations to Paul Mackenzie!

Fermilab

Colloquium
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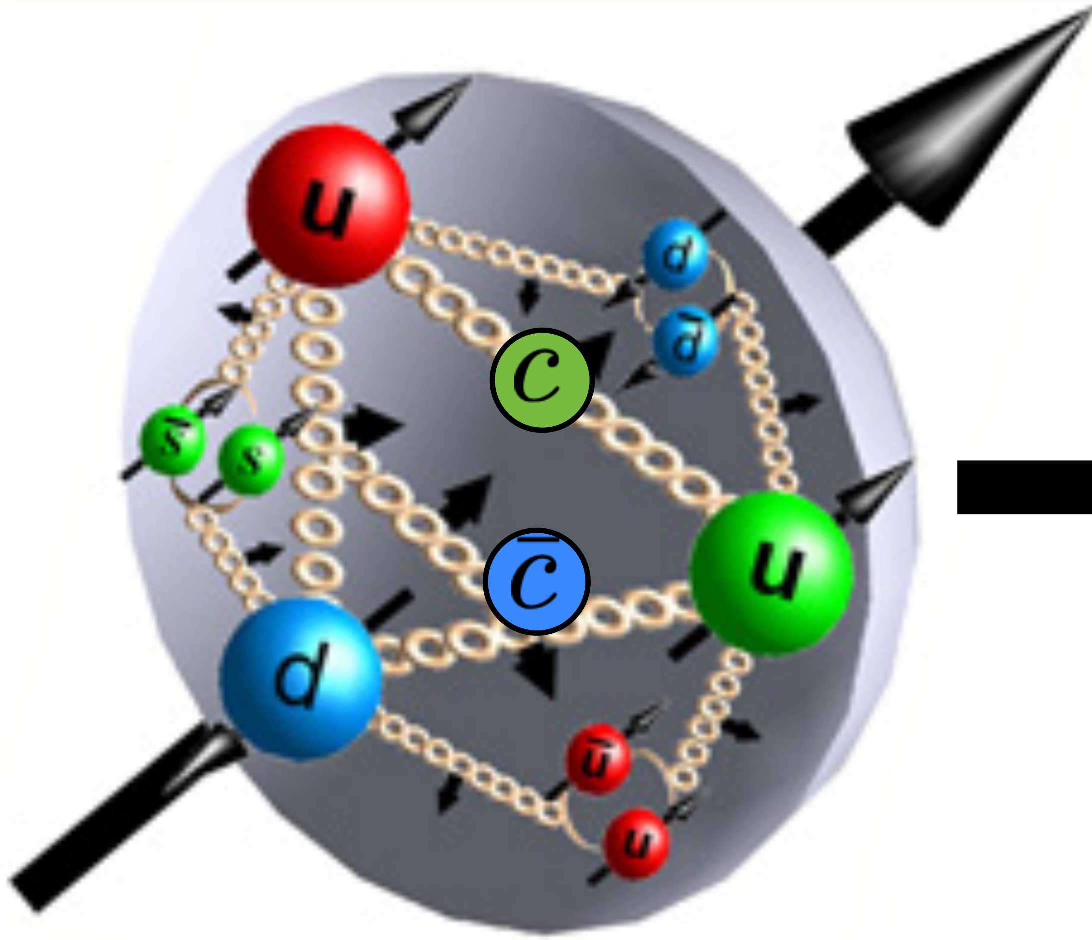

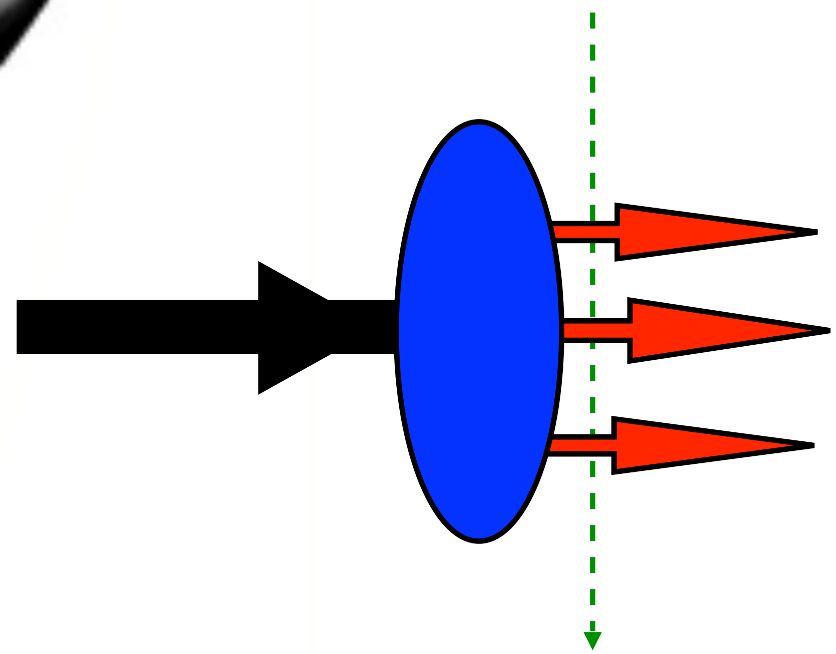

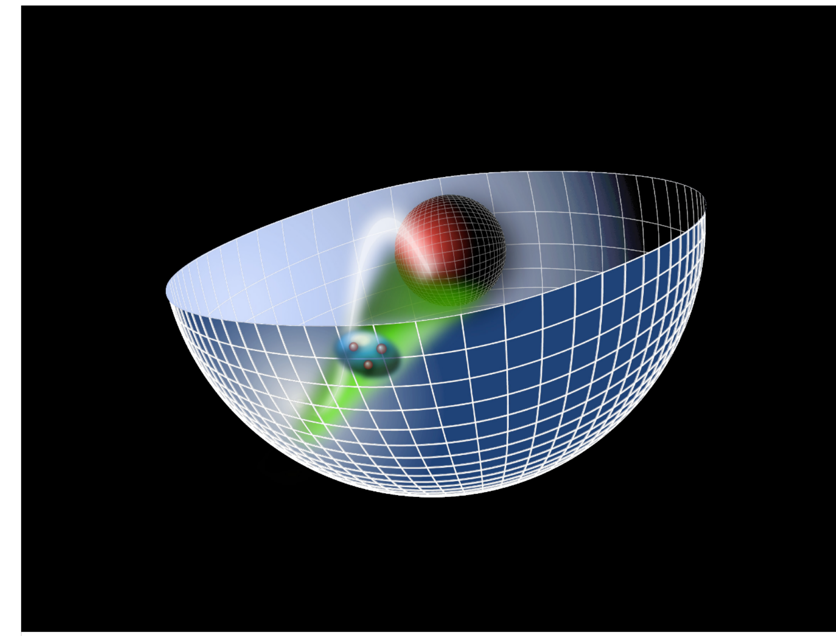

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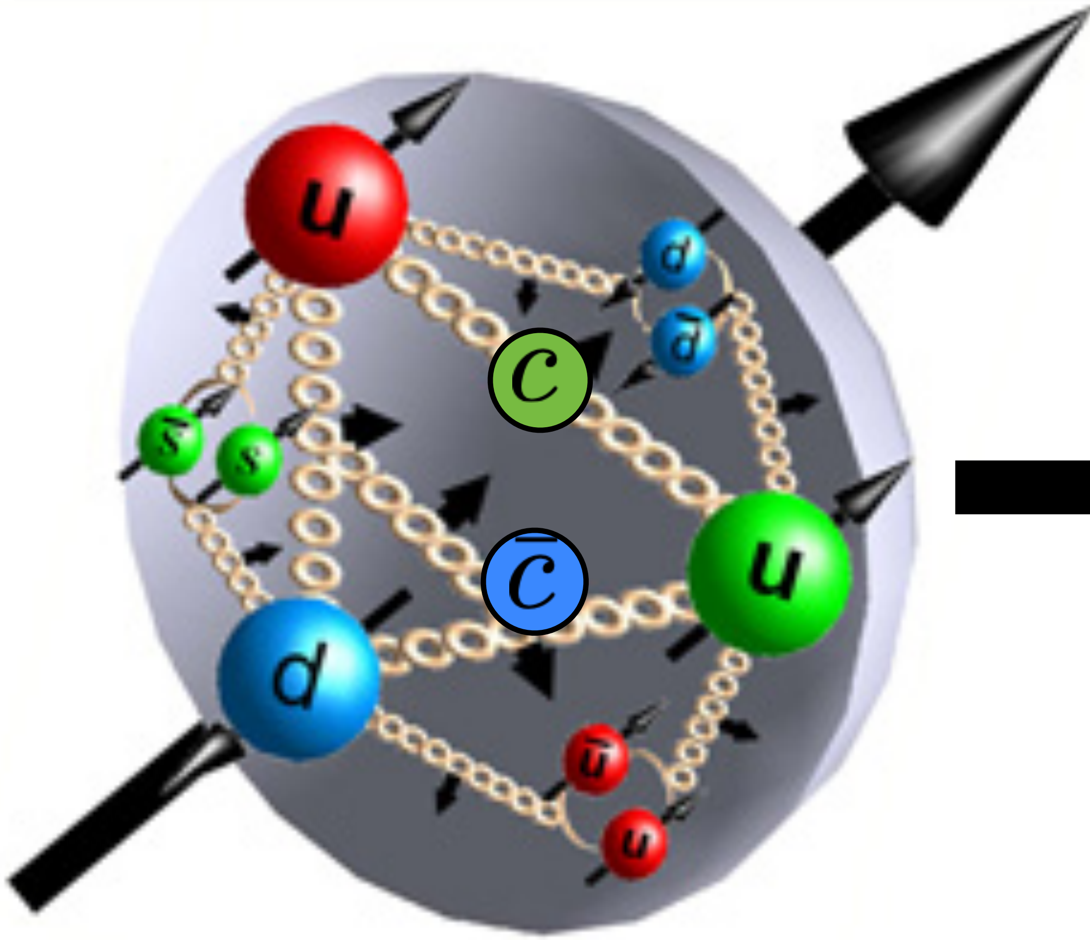

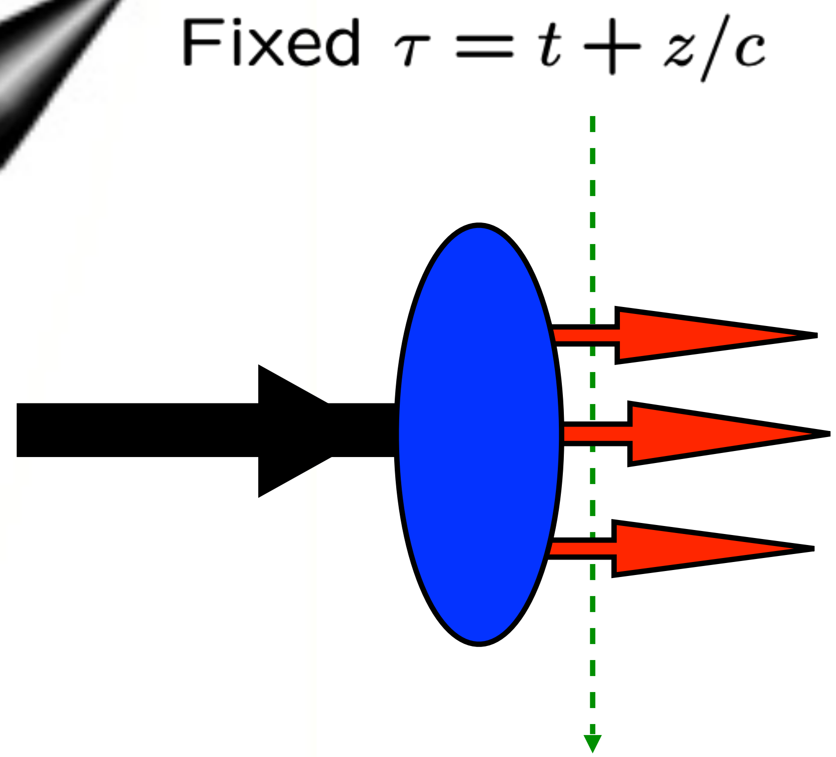

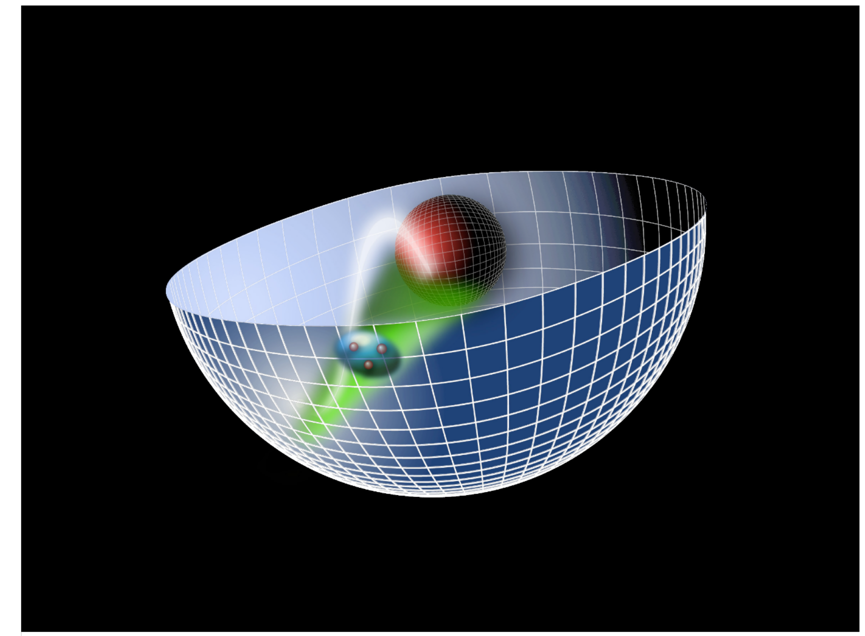

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