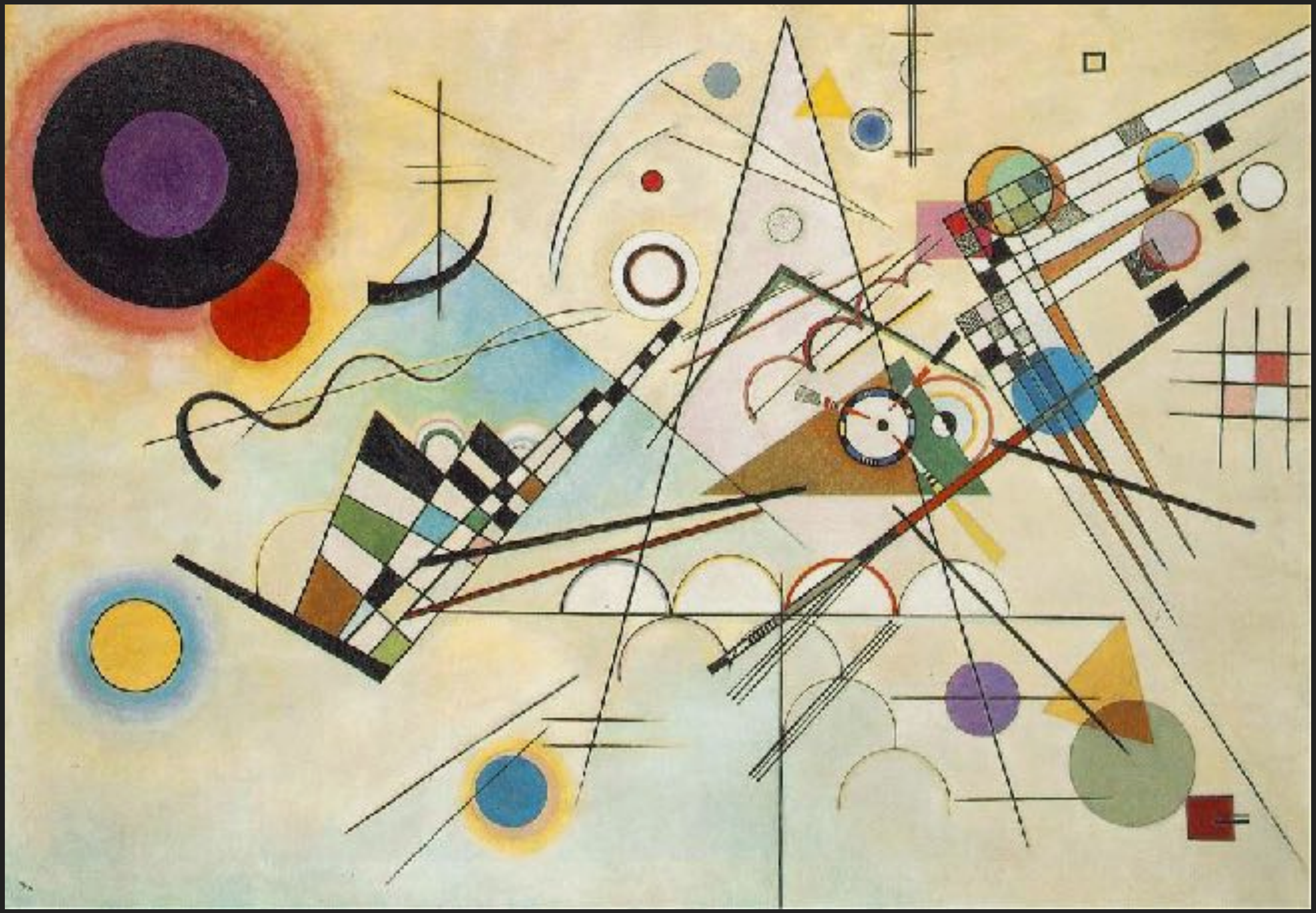

Physics and Mathematics in Music

Jason Bono, Fermilab

Fermilab Colloquium
May 30, 2018

- Art exists in every known culture
- Music may be the most common form of art
    - And the most common form of human entertainment
    - It's a major feature of humanity!

mu·sic (myo͞o′zĭk)

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n.

1. The art of arranging sounds in time so as to produce a continuous, unified, and evocative composition, as through melody, harmony, rhythm, and timbre.
2. Vocal or instrumental sounds possessing a degree of melody, harmony, or rhythm.
3.
  a. A musical composition.
  b. The written or printed score for such a composition.
  c. Such scores considered as a group: *We keep our music in a stack near the piano.*
4. A musical accompaniment.
5. A particular category or kind of music.
6. An aesthetically pleasing or harmonious sound or combination of sounds: *the music of the wind in the pines.*

And yet music escapes a satisfactory definition

Music:

- What is it?
- When did it start, and why?
- Was it inspired from the environment, or from something inside us?
- What did it sound like around the world and throughout its history?
- Where did the musical elements in our time and culture come from?
  - Are they well preserved as they pass through the ages?
  - Do they, and will they, inevitably reemerge?
  - Are they arbitrary?
- Where is music going?
  - Will music in a thousand years, resemble anything in our age?
- Why is it so important to so many of us?

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But…

The uncovering of truth is, for many, one of the primary functions of art.

Exploring basic questions can inspire the artist.

Answering basic questions can empower the artist.

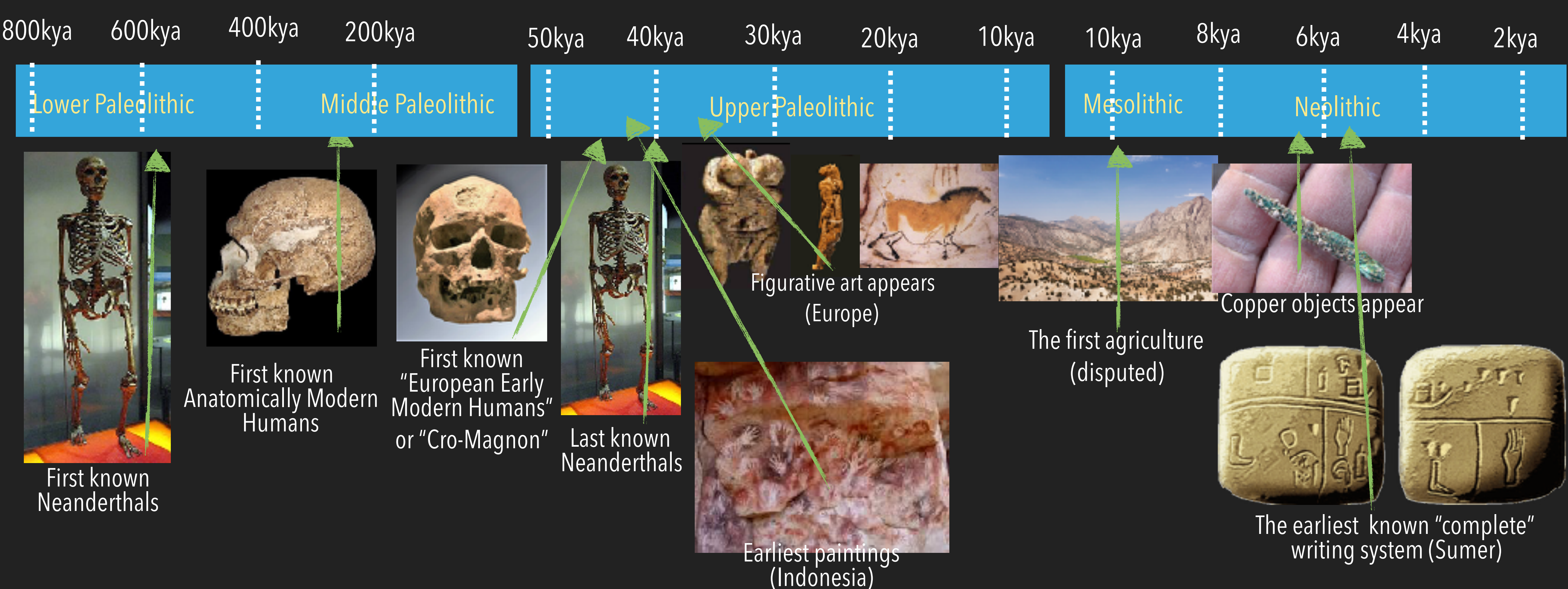
One's aesthetic sense is influenced by knowledge.
Art is often best appreciated in context!

Where are the clues?

- Look at the archeological record
- Look at early history
- Physical and mathematical characterization/analysis
- Biology
- Sociology
- Psychology
- Physiology
- Neurology
- Philosophy
- …

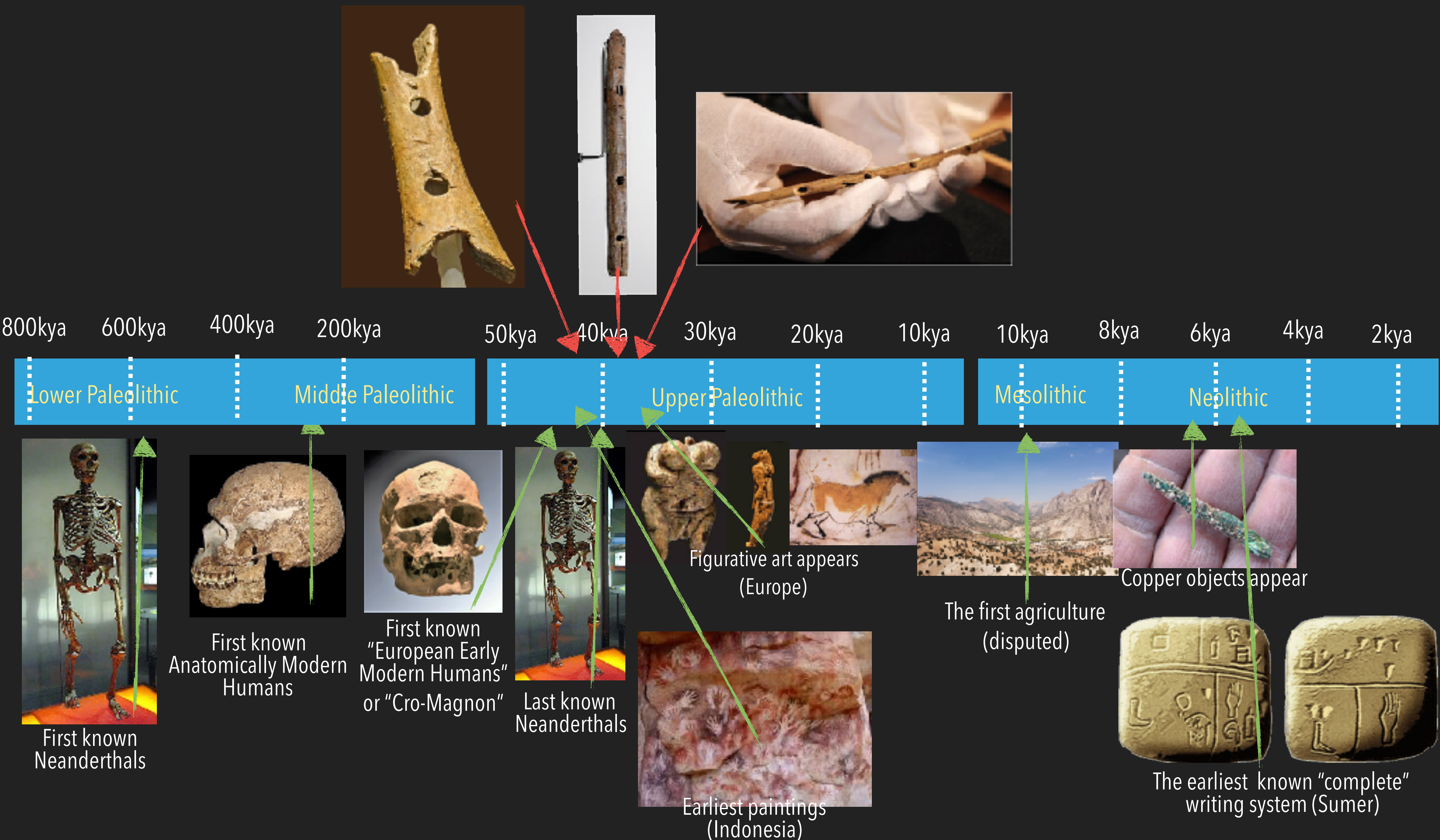
What are the oldest known musical artifacts?

What are the oldest known musical artifacts?



What are the oldest known musical artifacts?

The earliest unequivocally musical artifacts are flutes, which appear numerously in Europe, near the dawn of the upper Paleolithic era (~40kya).

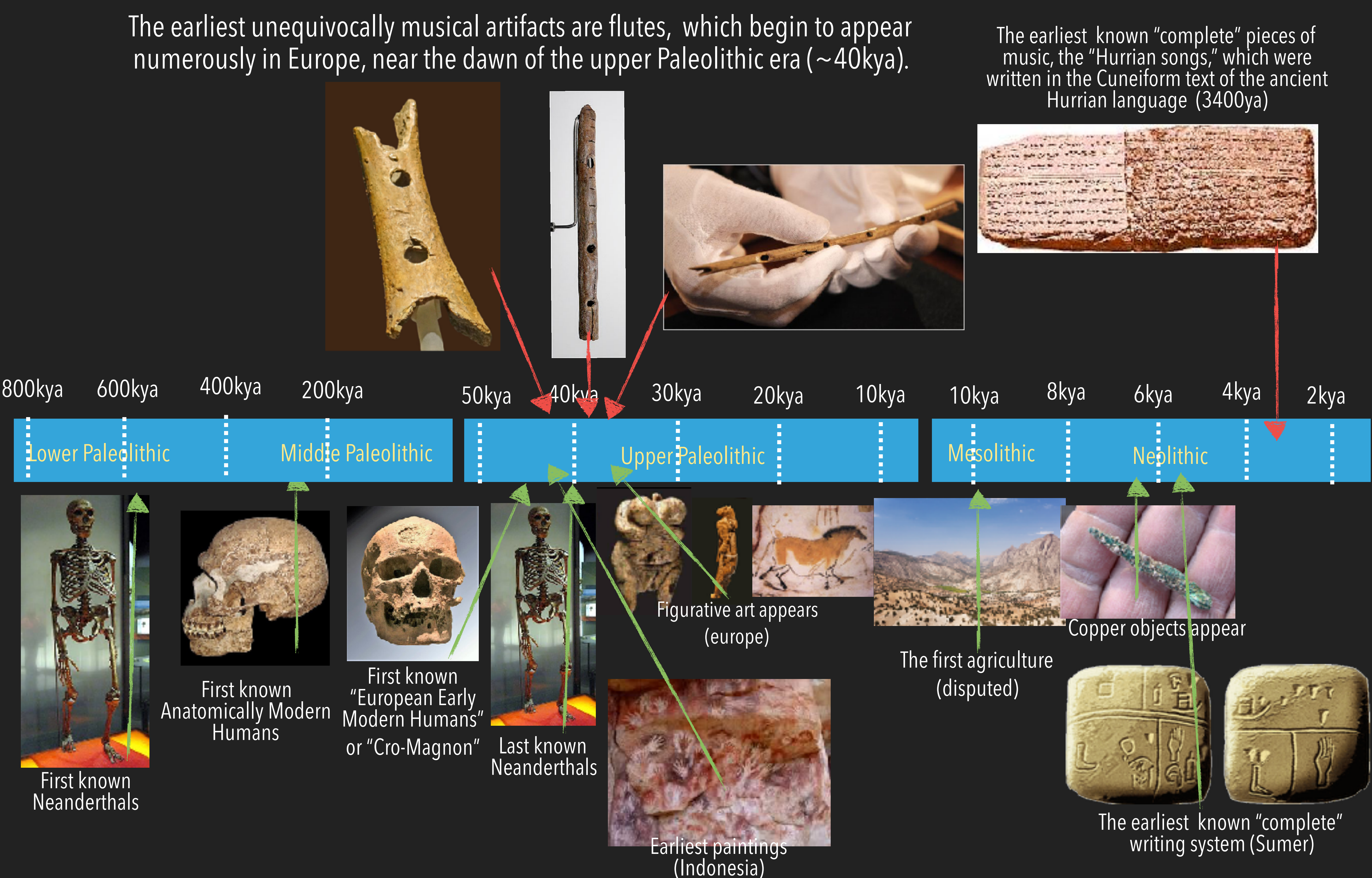


Several flutes have been recovered dating back to the arrival of early modern humans in Europe (~40,000 BC), suggesting a still older musical culture

Interestingly, many of these flutes are constructed in the same “tonal language” that is most often expressed today!

Paleolithic flutes tend to be pentatonic, some are argued to be diatonic

We’ll have to fast forward past the advent of writing before we can hope to sample specific melodic/harmonic/rhythmic content



The earliest unequivocally musical artifacts are flutes, which begin to appear numerously in Europe, near the dawn of the upper Paleolithic era (~40kya).
The earliest known "complete" pieces of music, the "Hurrian songs," which were written in the Cuneiform text of the ancient Hurrian language (3400ya)
800kya
600kya
400kya
200kya
50kya
40kya
30kya
20kya
10kya
10kya
8kya
6kya
4kya
2kya
Lower Paleolithic
Middle Paleolithic
Upper Paleolithic
Mesolithic
Neolithic
First known Neanderthals
First known Anatomically Modern Humans
First known "European Early Modern Humans" or "Cro-Magnon"
Last known Neanderthals
Figurative art appears (europe)
Earliest paintings (Indonesia)
The first agriculture (disputed)
Copper objects appear
The earliest known "complete" writing system (Sumer)

Hurrian Hymn 6, the “oldest known melody,” on a lyre, in Just Intonation

The term “Just Intonation,” will be part of our physical and mathematical descriptions of music, but first, listen…

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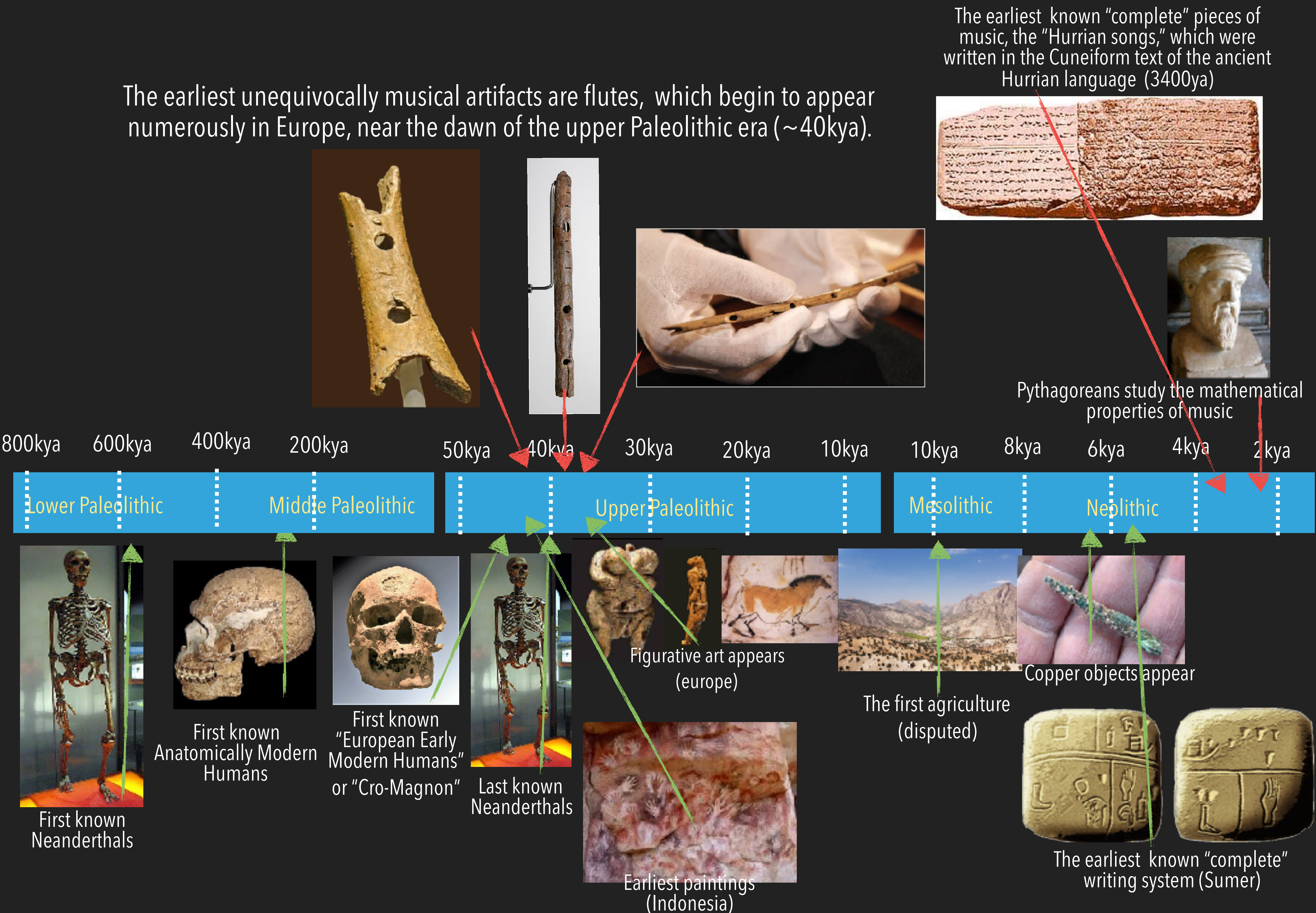

Pythagorus

- The Ancient Indians, Chinese, Egyptians, Mesopotamians, and Greeks studied the mathematical principles of sound
- The Ancient Greeks, in particular the Pythagoreans, were the first to express musical scales as ratios of small whole numbers

Pythagorus found that length ratios of small whole numbers gave the simple consonant intervals in music
This suggested a deep connection between abstract mathematics and human experience

Interval	↑ Octave	↑ Perfect Fifth	↑ Perfect Fourth	Unison	↓ Perfect Fourth	↓ Perfect Fifth	↓ Octave
L_1	1	1	1	1	1	1	1
L_2	1/2	2/3	3/4	1	4/3	3/2	2
L_{Ratio} (L_2/L_1)	1/2	2/3	3/4	1	4/3	3/2	2

Following his intuitive understanding of "Occam's razor" (~two millennia before the phrase), he used only 1,2,3, and 4, since with their ratios, he was able to generate all the intervals in Greek music…

Interval	↑ Octave	↑ Perfect Fifth	↑ Perfect Fourth	Unison	↓ Perfect Fourth	↓ Perfect Fifth	↓ Octave
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- Example, the "pythagorean third" is found by fifths and octaves
  - $L_2 = (2/3)^4 \times 2^2 = 64/81$
  - Using only 2 and 3… sort of…

Interval	↑ Octave	↑ Perfect Fifth	↑ Perfect Fourth	↑ major third	Unison	↓ Perfect Fourth	↓ Perfect Fifth	↓ Octave
L_1	1	1	1	1	1	1	1	1
L_2	1/2	2/3	3/4	64/81	1	4/3	3/2	2
L_{Ratio} (L_2/L_1)	1/2	2/3	3/4	64/81	1	4/3	3/2	2

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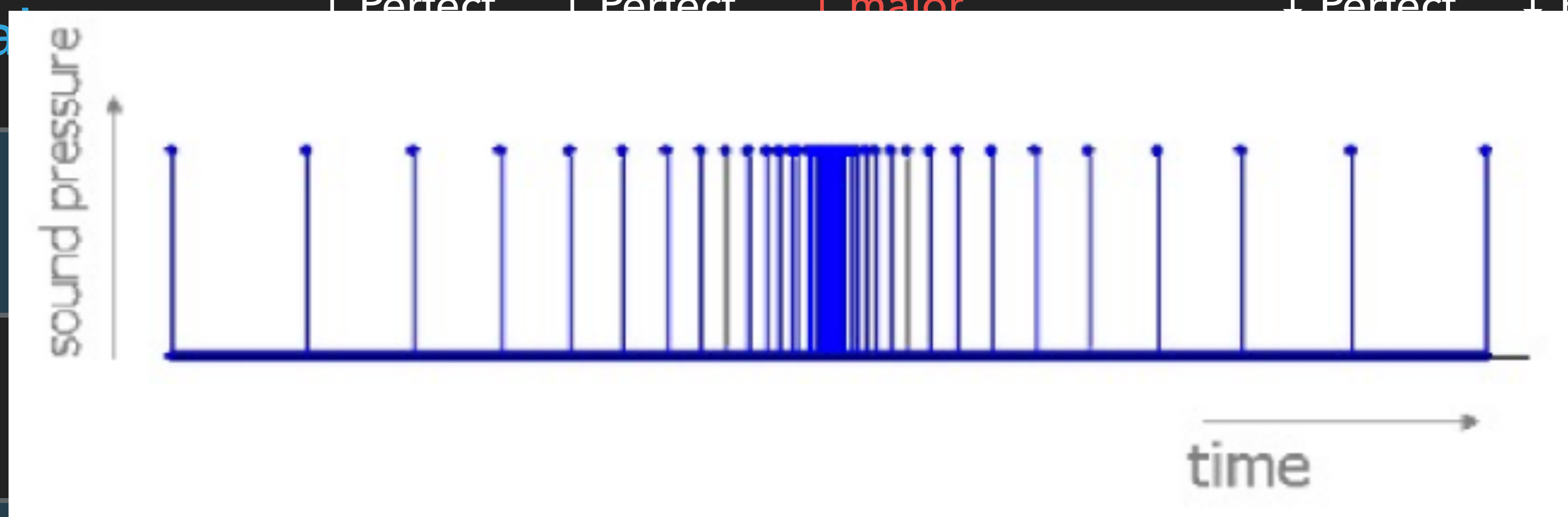

- Something that Pythagorus didn't discover was the relationship between tone and frequency

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L_{Ratio} (L_2/L_1)	1/2	2/3	3/4	64/81	1	4/3	3/2	2
$L_{frequency}$ (f_2/f_1)	2	3/2	4/3	81/64	1	3/4	2/2	1/2

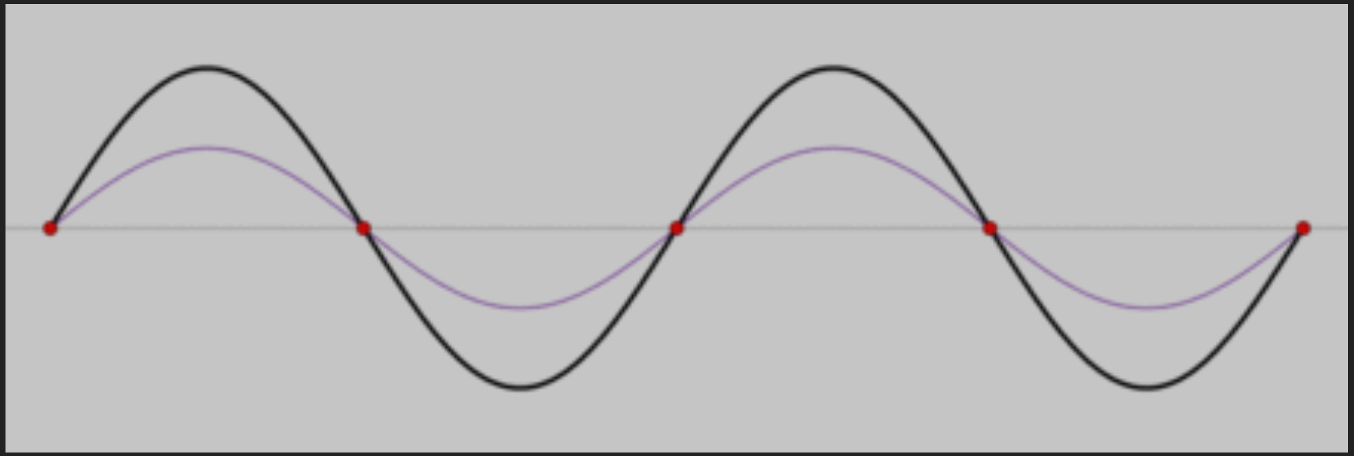
- Something that Pythagorus didn't discover was the relationship between tone and frequency

The Basis of Tone on a String: The Fundamental Frequency

$$s = \sqrt{\frac{T}{\mu}}$$

$$\boxed{f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}}$$

$$\lambda = 2L$$

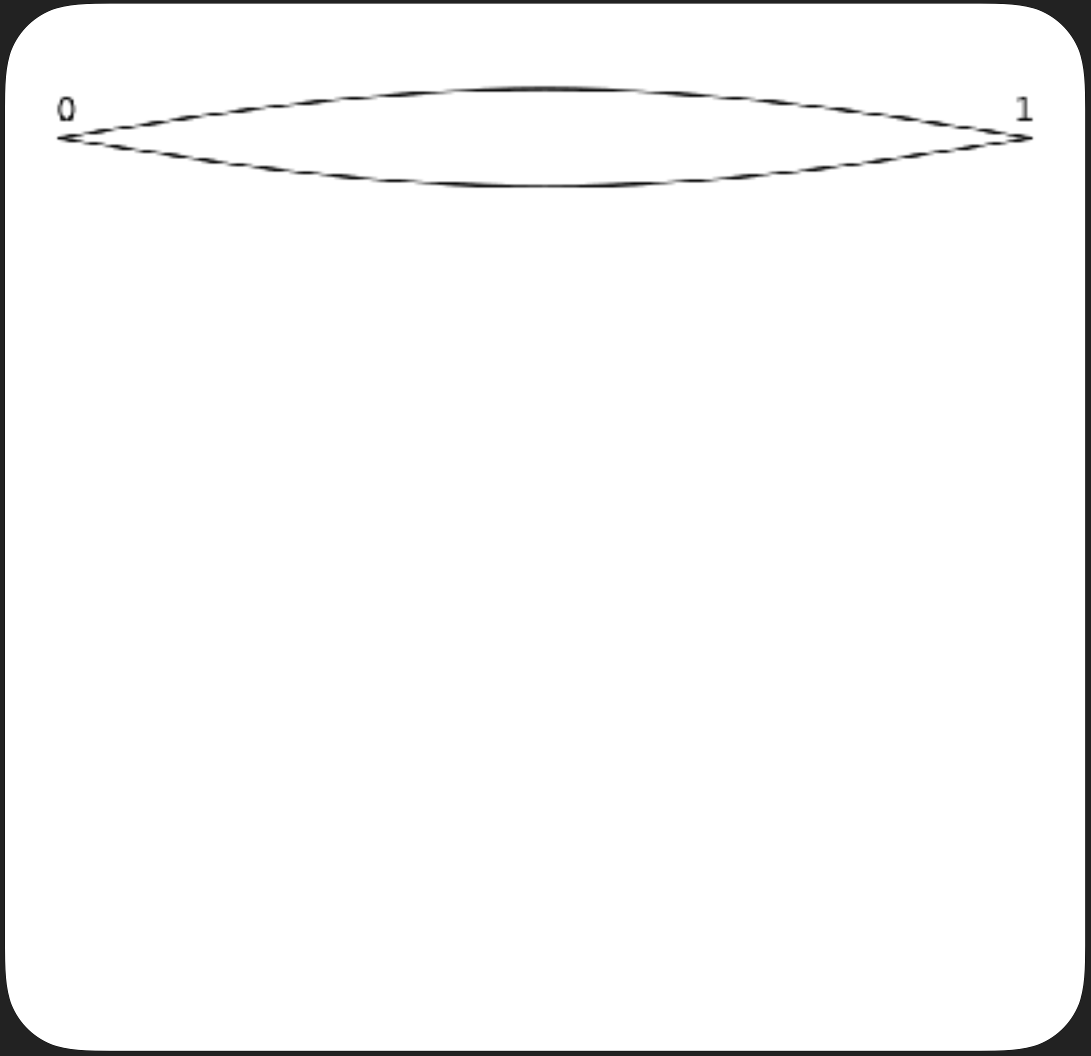


The Basis of Tone on a String: The Fundamental Frequency

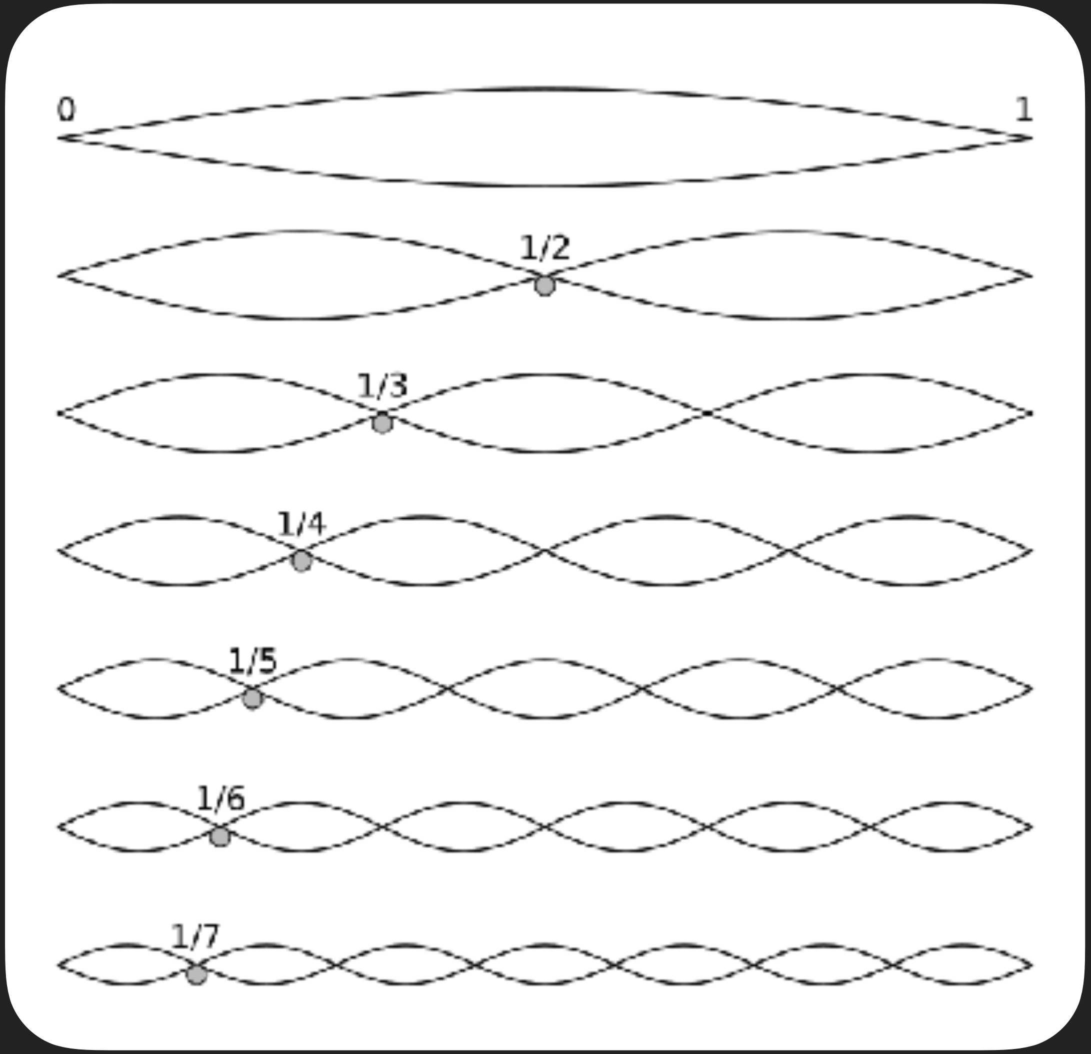
$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

No wonder why Pythagorus was able to elegantly link tone to length, but not to tension or string thickness!

The Fundamental Frequency



Beyond The Fundamental Frequency

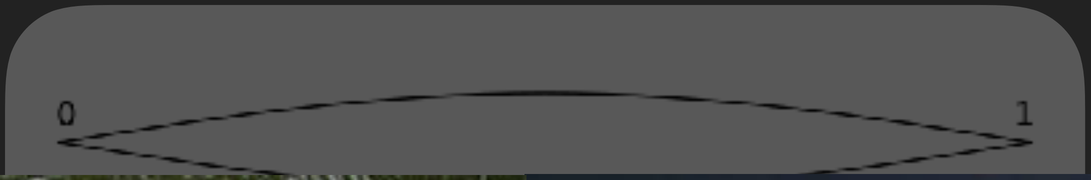


The "harmonic series"

We'll be speaking in terms of frequency rather than wavelength, so our harmonic series is:

1, 2, 3, 4, 5, 6, 7…

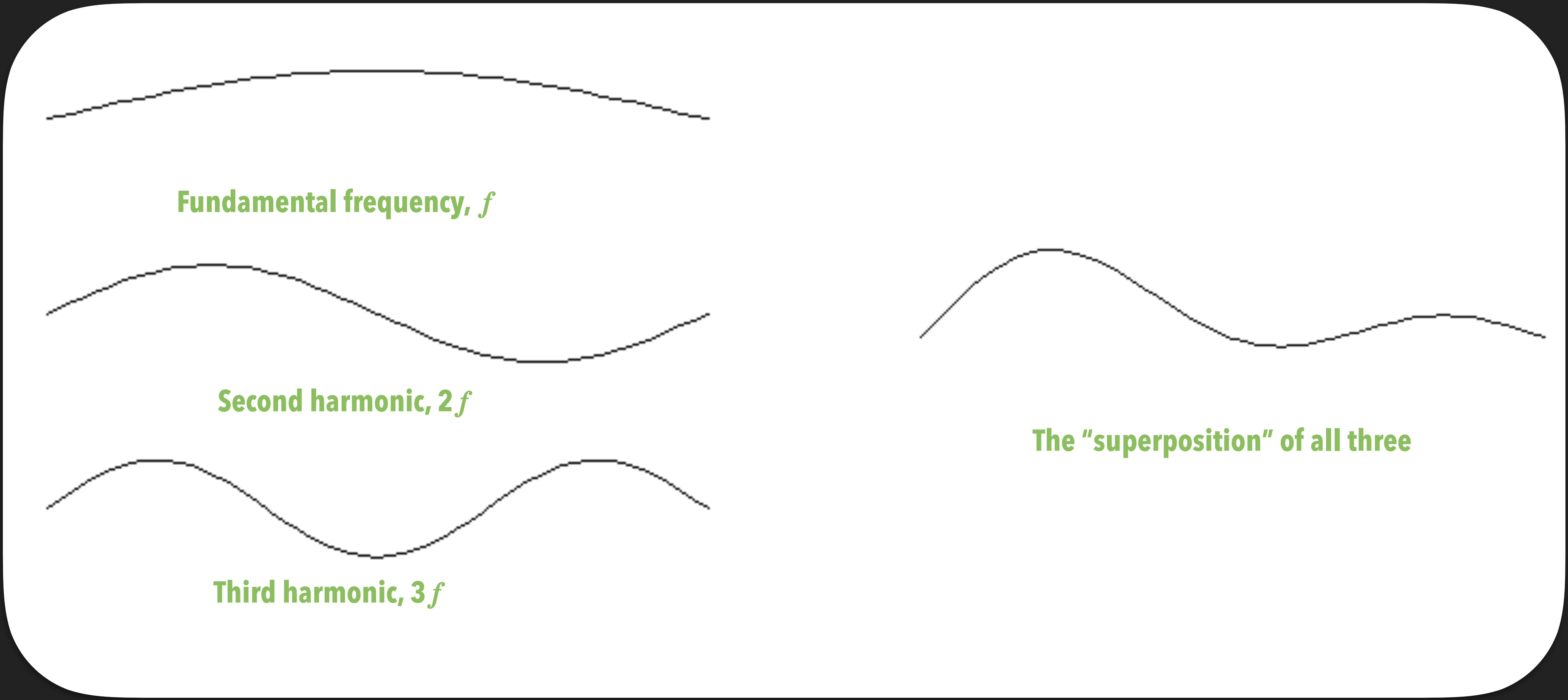
Superposition



Linearity

Broken Linearity

A simple example of harmonic overtones



The relative volume of each overtone largely distinguishes instruments

Timbre

- The timbre of an instrument is more than just the relative amplitudes (volume) of each overtone
  - An initial and momentary “non musical” noise
    - This largely distinguishes a banjo, from a guitar, from a piano etc

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  - Envelope
    - "attack," "sustain," and "decay"
    - each overtone, in general, has its own envelope
  - Inharmonicity of the overtones
    - The frequencies don't exactly follow the harmonic series

Long strings (low inharmonicity)
Short strings (large inharmonicity)

- So the fundamental determines pitch, and the overtones help determine timbre?
  - Not exactly!
  - Sometimes the fundamental frequency can be removed or greatly reduced
    - For example, some rooms naturally filter out certain frequencies
  - With the fundamental removed you still tend to hear the correct pitch
  - <u>The spacing of the harmonics must play a role in pitch perception</u>

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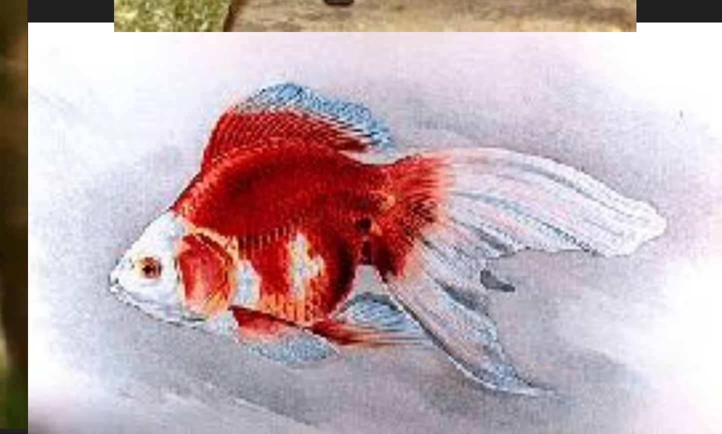
    - Many species perceive the missing fundamental
    - May arise in some combination of cochlear, cortical, subcortical areas!





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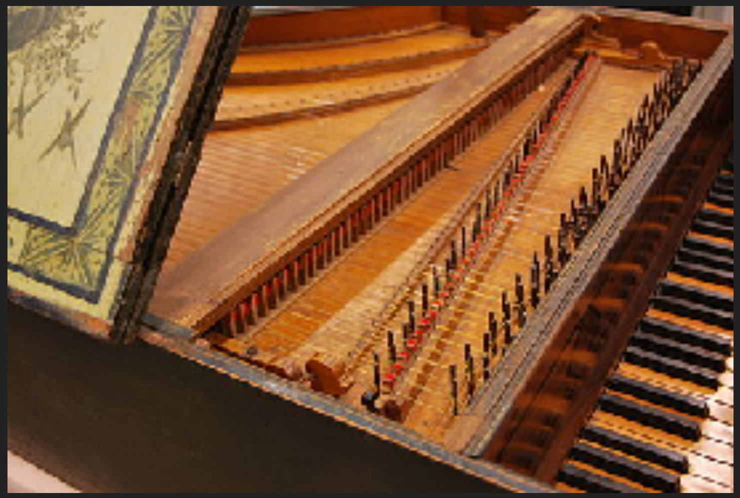
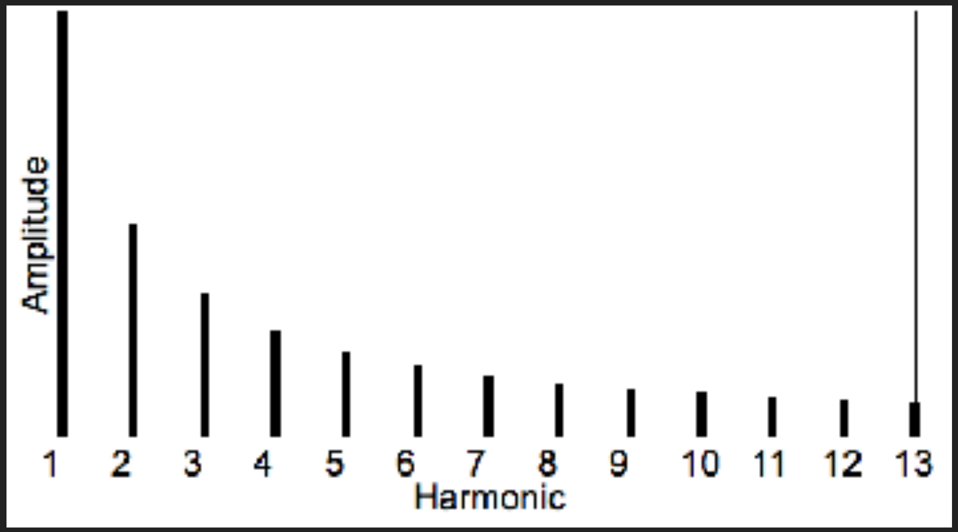
What if the overtone spacing is irregular, i.e. not harmonic?

"Inharmonic" overtones are the general case after all!

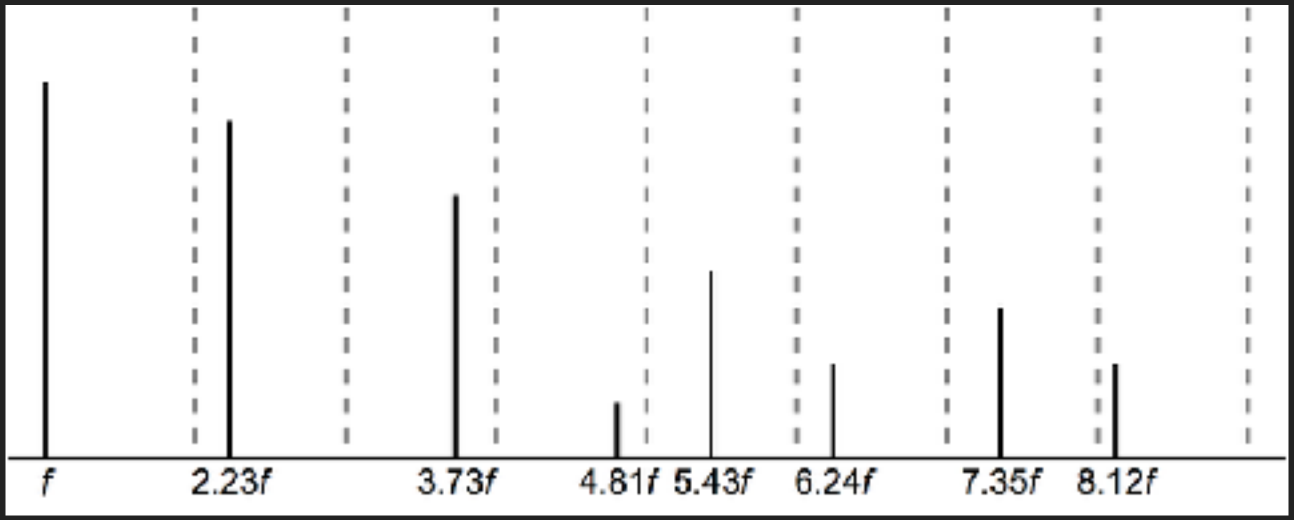
One-dimension, fixed ends





Harmonic overtones

Two-dimensions, free ends



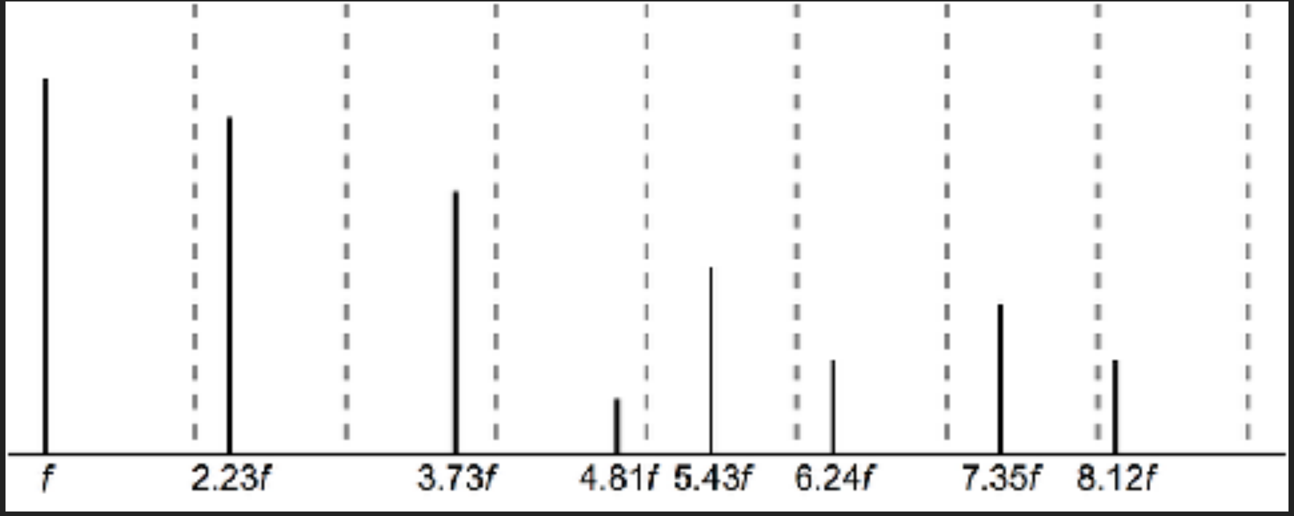
Inharmonic overtones

Why is there such a difference in clarity?

Two-dimensions, free ends

Extremely Inharmonic overtones

Two-dimensions, free ends



Inharmonic overtones

Given sufficient inharmonicity, like in a gong for example, the pitch of the note is entirely lost

"Un-pitched" Instruments

Inharmonicity in Music

- Example: inharmonicity is prominent in Javanese gamelan music
  - which has been present in the native cultures of Java and Bali for at least 1.4 millennia
- Most of the instruments are 2-dimensional, and thus possess additional/ inharmonic vibrational modes
  - Still plenty of harmonic overtones!
  - The instruments tend to be “detuned” with one another for greater effect

A gamelan set

It's easiest to make inharmonic instruments, but:

We've been bothering to make harmonic instruments for at least 40,000 years.

Why?

It's easiest to make inharmonic instruments, but:

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Why?

<u>The answer may in lie the deep role that symmetry plays in perception</u>

Symmetry

- There is a strong relationship between aesthetics and symmetry

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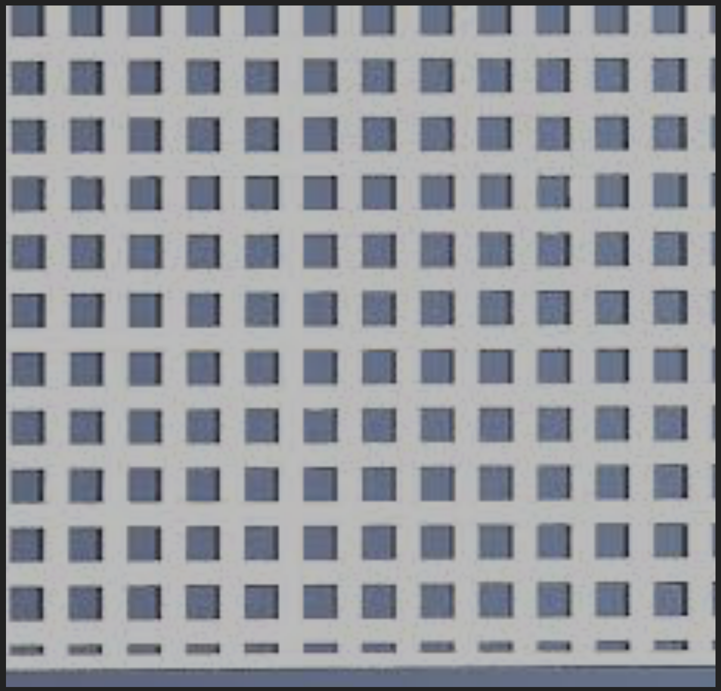
  - Simple and direct symmetries tend to feel dull



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  - Simple and direct symmetries tend to feel dull
  - Overly complex or inscrutable patterns are often unpleasant
  - But there are common, if not universal "sweet spots" that balance coherence and complexity
    - In art, symmetry is often presented partially, or even only suggested
    - You could say that we like the presented data to be abundant, but highly compressible

Symmetry in Music

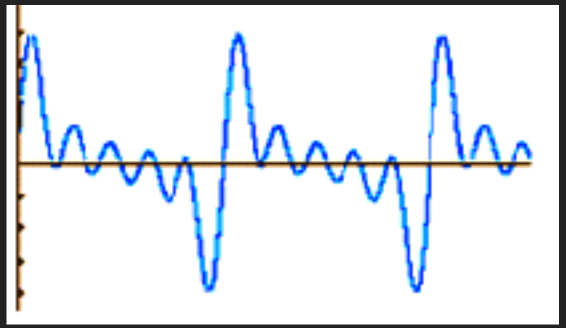
- A “dynamic art,” like dance, can exhibit repetition in space and time

Symmetry in Music

- A “dynamic art,” like dance, can exhibit repetition in space and time
- Music and its symmetries are mostly confined to time
  - Though some musical performances use “space” and symmetries therein (e.g. perception of where sound is coming from)
  - All musicality involves periodic sound
    - periodicity is inherent to rhythm, melody, harmony, and often, the compositional structure
    - periodicity is deeply imbedded in musical tone itself

Symmetry within Tone

- Like rhythm, harmony, and melody, the salient attribute to nearly all musical tones themselves is that they are, to a large degree, periodic
  - Purely aperiodic sound is awful



Complex, but repeating

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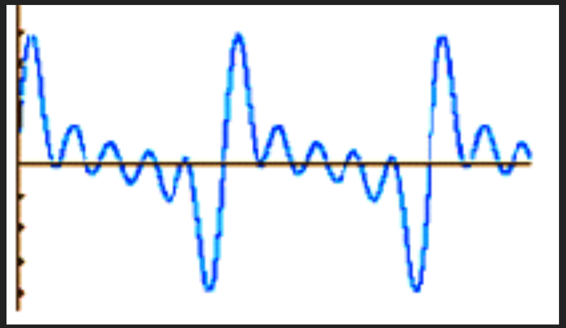

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Symmetry within Tone

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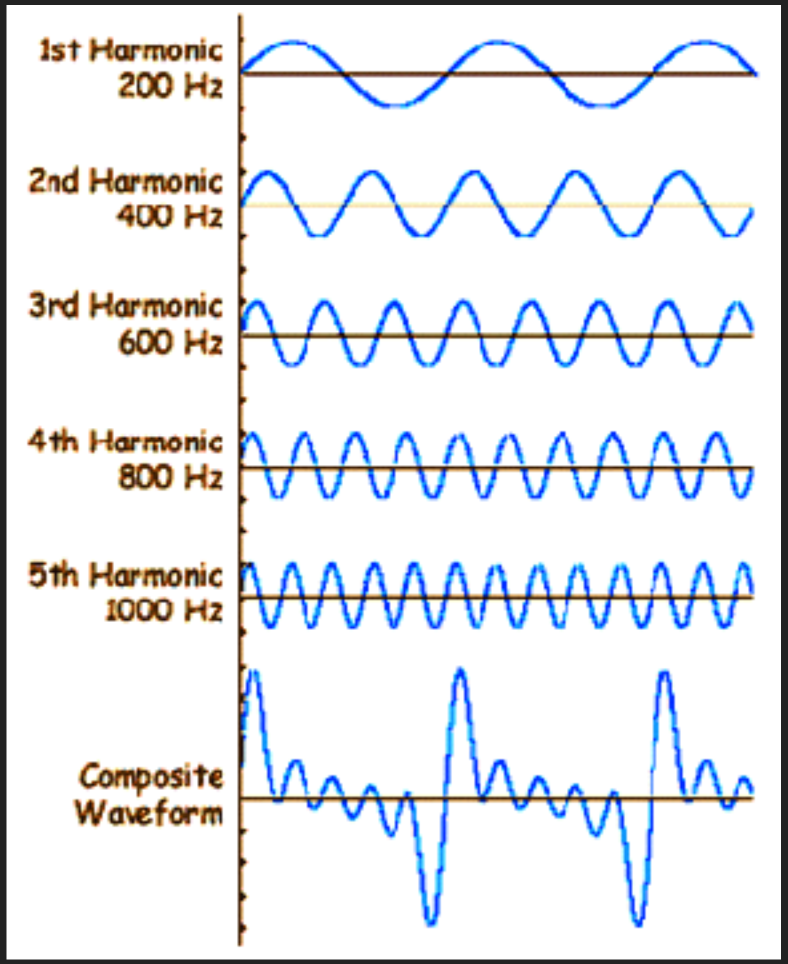
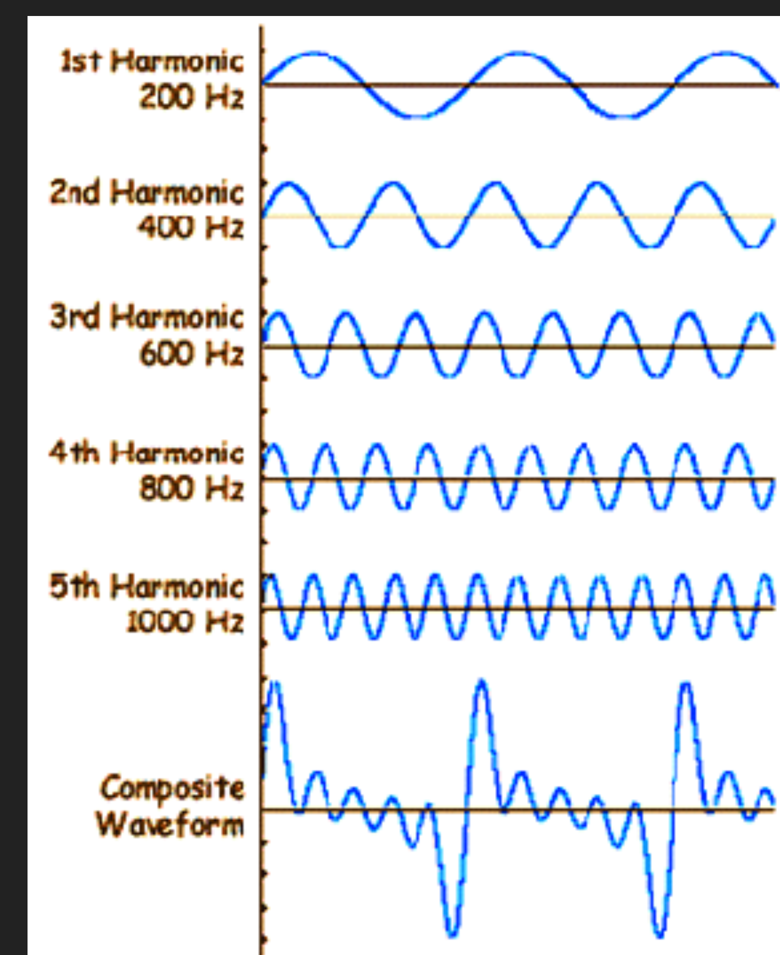
- There is something common to all periodic tone
  - The harmonic series!



Symmetry within Tone: The Fourier Theorem

- All periodic (and continuous) waveforms, e.g. musical tones, can be expressed as the sum of harmonic sine waves
- What does this mean?
  - <u>If periodicity is an aesthetic requirement, so too are harmonic overtones!</u>

Jean-Baptiste Joseph Fourier
(1768 - 1830)



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Another consequence of superposition!

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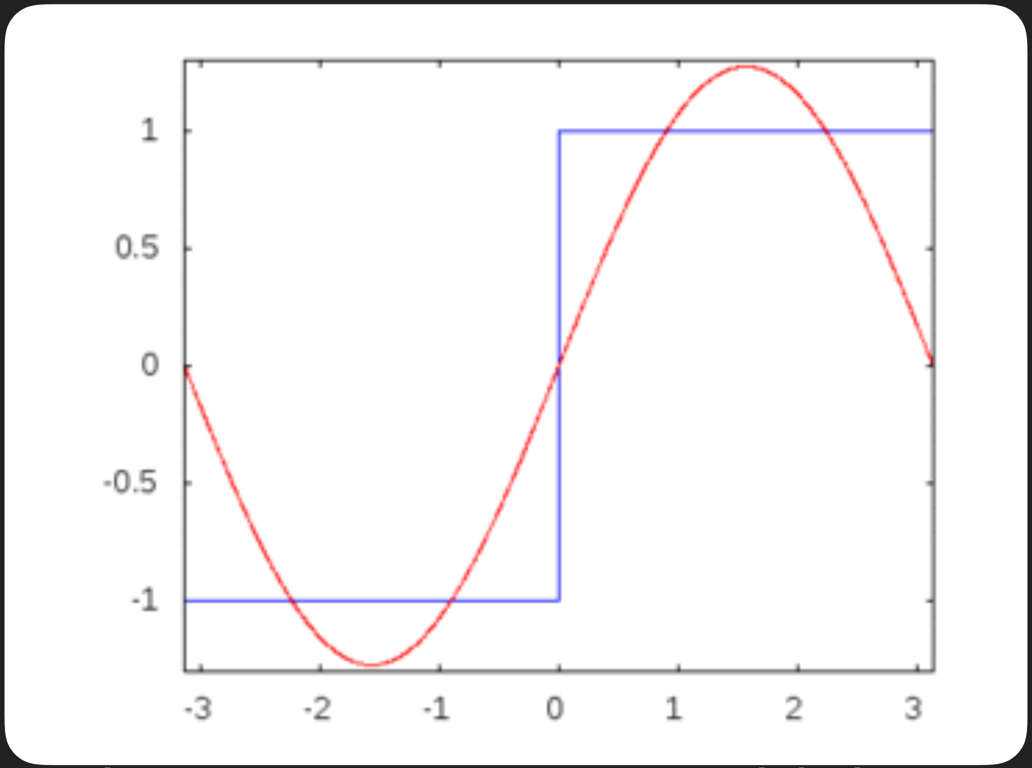

The square wave's decomposition into (odd) harmonic sine waves
f, 3f, 5f, 7f....

Symmetry within Tone: The Fourier Theorem

Let's try a "real" instrument

Symmetry within Tone: The Fourier Theorem

Let's try a "real" instrument

The alto saxophone playing a C4 (fundamental~262 Hz)

Symmetry within Tone: The Fourier Theorem

Let’s try a “real” instrument

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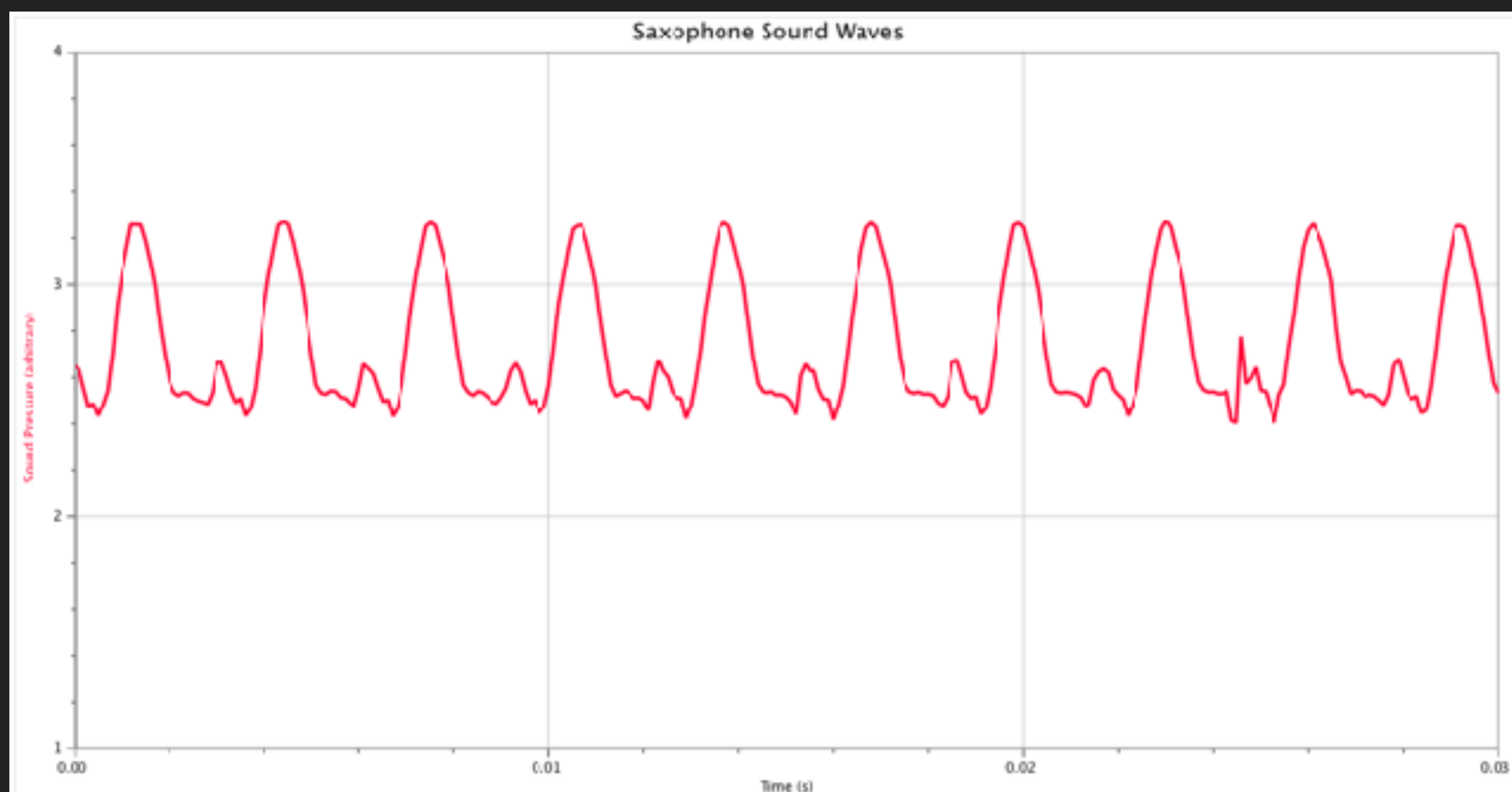
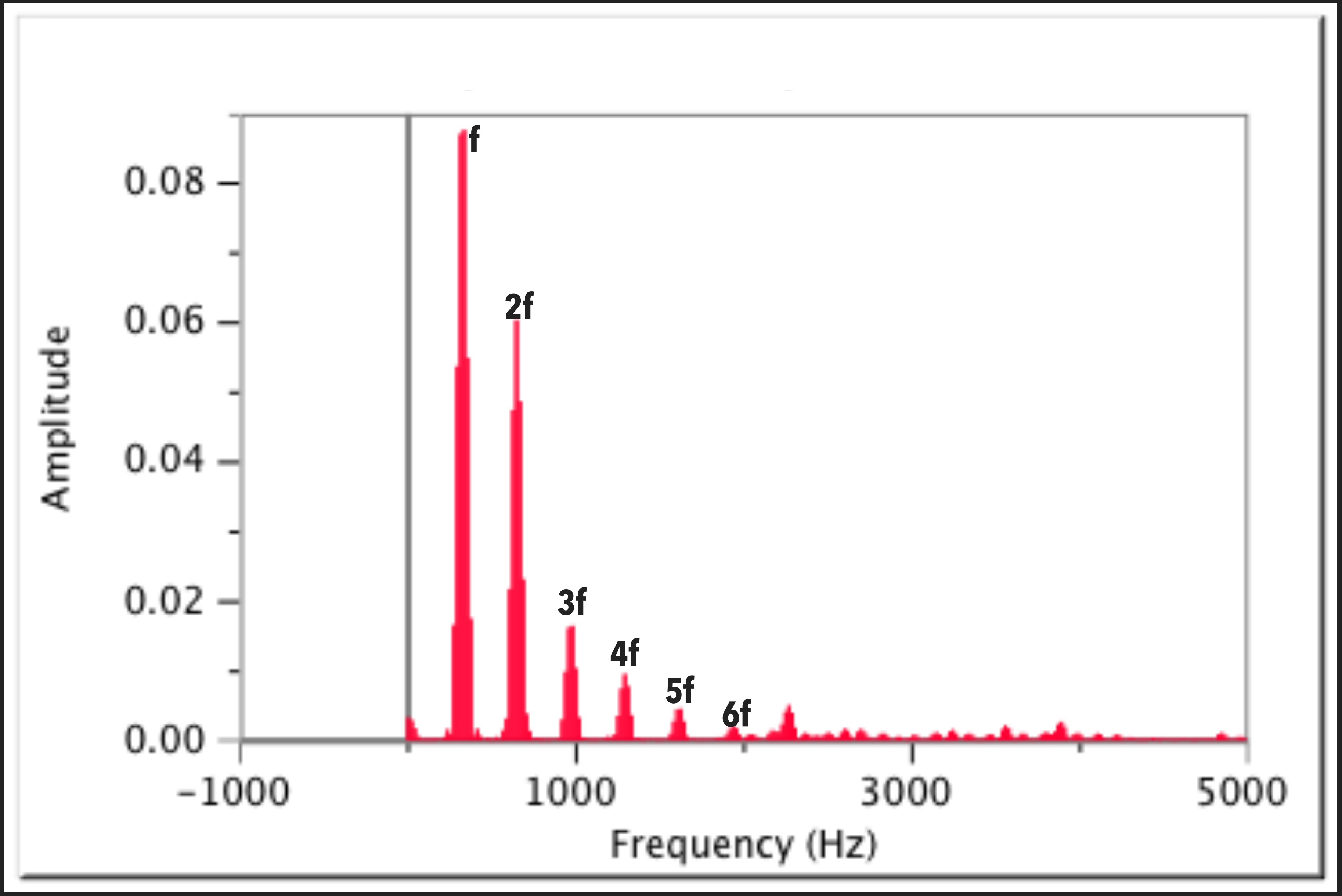


Looks periodic, so it’s mostly harmonic

Symmetry within Tone: The Fourier Theorem

Let's try a "real" instrument

The alto saxophone playing a C4 (fundamental~262 Hz)



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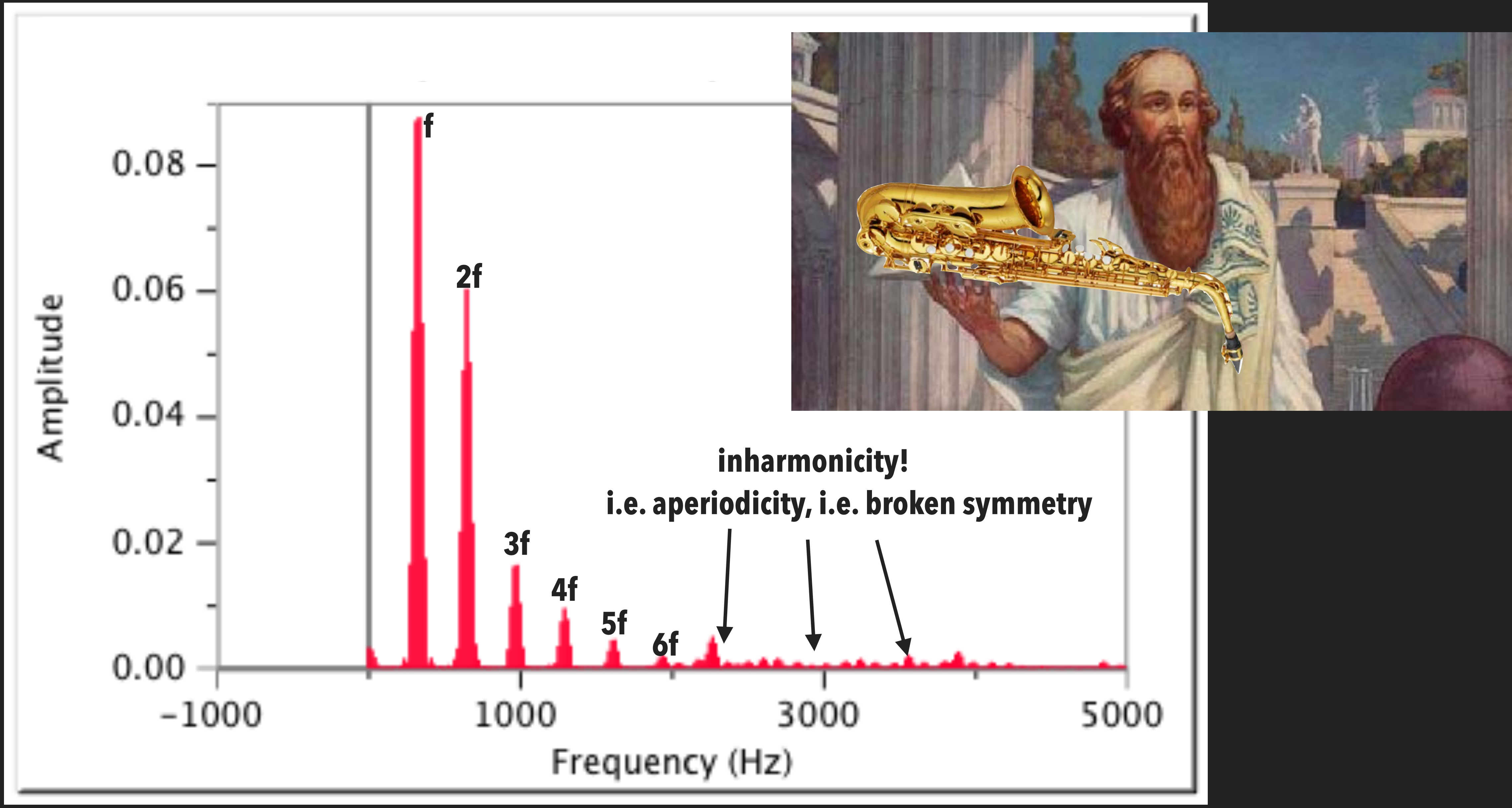
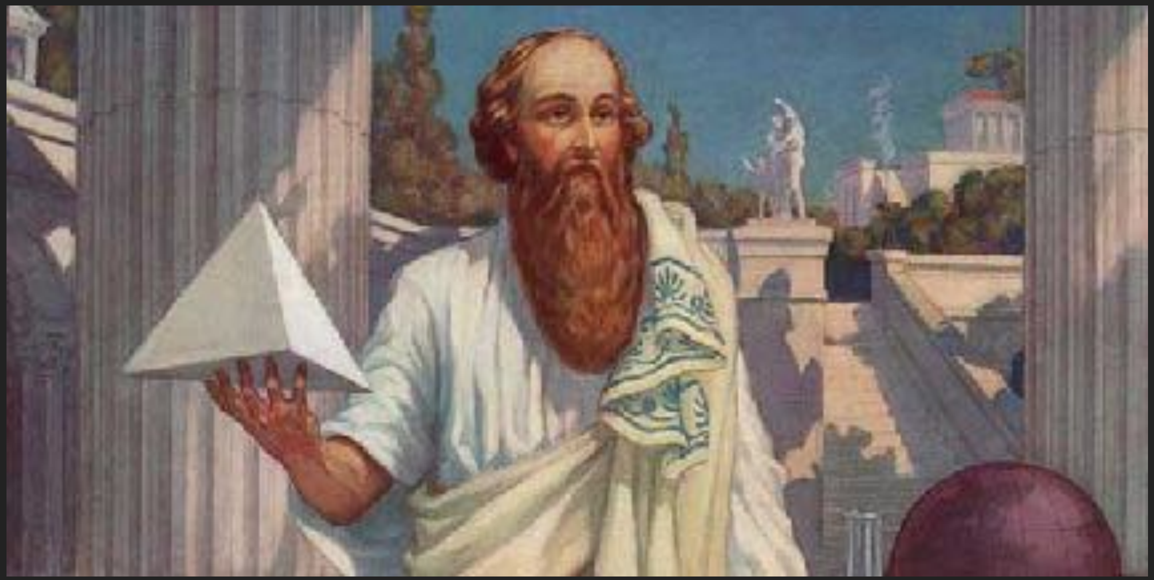


Symmetry within Tone: The Fourier Theorem

- A Pythagorean might have been disturbed by knowledge of inharmonicity in musical tone
- But slightly broken symmetries are what bring color and life to music
  - Example: perfectly timed computerized drum machines can sound “cold” compared to a human performance

Symmetry within Tone

- We have a partial answer to our question:
  - For millennia, people have been crafting harmonic instruments, because the are moved when they unconsciously perceive symmetry



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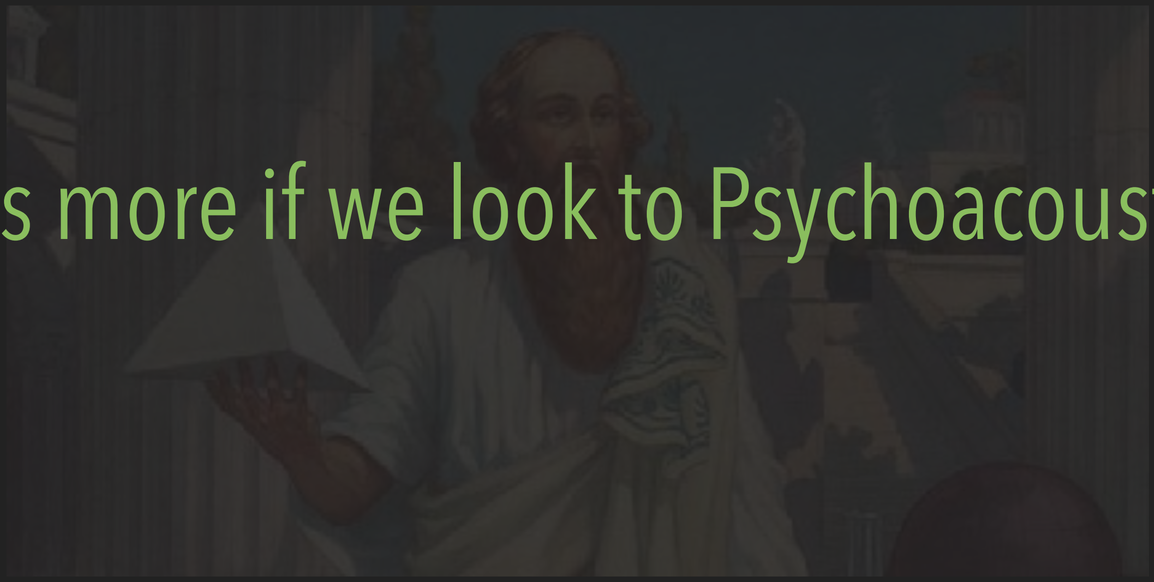

There is more if we look to Psychoacoustics

Psychoacoustic Phenomena: Combination Tones

- Nonlinearities in the cochlear response give rise to "combination tones"
- "Addition tones" can be perceived from two or more tones played together
  - $A = f_1 + f_2$
- "Subtraction tones" can also be perceived
  - $S = |f_1 - f_2|$

Psychoacoustics: Combination tones in a harmonic series

- Let's just look at the subtraction tones
  - Addition tones give a similar result!

$$s = | f_2 - f_1 |$$

Harmonic frequencies are $f, 2f, 3f, 4f \ldots$

Call the subtraction frequency between the i_{th} and j_{th} overtone, S_{ij}

$$S_{11} = 0, S_{12} = f, S_{13} = 2f, S_{14} = 3f \ldots$$

$$S_{21} = f, \; S_{22} = 0, S_{23} = f, S_{24} = 2f \ldots$$

$$\vdots$$

All the non-zero subtraction frequencies in a harmonic series are themselves harmonics

Psychoacoustics: Combination tones in an inharmonic series

Cause a small disturbance so that the frequencies can be called f+a, 2f+b, 3f+c, 4f+d …

$S_{11} = 0, S_{12} = f+b-a, S_{13} = 2f+c-a, S_{14} = 3f+d-a\ldots$

$S_{21} = f+a-b, S_{22} = 0, S_{23} = f+c-b, S_{24} = 2f+d-c\ldots$

$\vdots$

The original overtone series does not contain the subtraction frequencies!

Harmonic frequencies are f, 2f, 3f, 4f …

$S_{11} = 0, S_{12} = f, S_{13} = 2f, S_{14} = 3f\ldots$

$S_{21} = f, S_{22} = 0, S_{23} = f, S_{24} = 2f\ldots$

$\vdots$

All the non-zero subtraction frequencies in a harmonic series are themselves harmonics

The harmonic series, combination tones, and the "missing fundamental"

Cause a small disturbance so that the frequencies can be called $f+a, 2f+b, 3f+c, 4f+d \ldots$

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$S_{21} = f+a-b, S_{22} = 0, S_{23} = f+c-b, S_{24} = 2f+d-b \ldots$

$\vdots$

The original overtone series does not contain the subtraction frequencies!

<u>We've got another unique aspect</u>

harmonicity begets harmonicity

noise begets noise

Harmonic frequencies are $f, 2f, 3f, 4f \ldots$

$S_{11} = 0, S_{12} = f, S_{13} = 2f, S_{14} = 3f \ldots$

$S_{21} = f, S_{22} = 0, S_{23} = f, S_{24} = 2f \ldots$

$\vdots$

All the non-zero subtraction frequencies in a harmonic series are themselves harmonics

The harmonic series, combination tones, and the "missing fundamental"

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We mentioned earlier that if the fundamental frequency is removed from a musical tone, it's still somehow perceived if the rest of the harmonics are there…

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Let's look at the subtraction tones from: ~~f~~, 2f, 3f, 4f …

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$$S_{11} = 0, S_{12} = \underline{f}, S_{13} = 2f...$$

The fundamental reemerges from the subtraction tones

Recent research suggests some redundancy in cochlear, cortical, subcortical areas!

Undesired Asymmetry within Tone

- Technology for electronic music has given almost unlimited control over the waveform
- But many physical instruments have perceptible inharmonicity
  - E.g, the piano heard earlier with short strings

Undesired Asymmetry within Tone: The Piano

A similar inharmonic overtone series as before can be adopted: f + a, 2f+b, 3f+c, 4f+d …

$S_{11} = 0$, $S_{12} = f+b-a$, $S_{13} = 2f+c-a$, $S_{14} = 3f+d-a$…

$S_{21} = f+a-b$, $S_{22} = 0$, $S_{23} = f+c-b$, $S_{24} = 2f+d-c$…

$\vdots$

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$$\vdots$$

Now let the inharmonicity be linear in overtone number, i.e. $b=2a$, $c=3a$, $d=4a$

$$\boxed{f + a, 2(f+a), 3(f+a), 4(f+a) \ldots}$$

$$\boxed{\begin{array}{l} S_{11} = 0, S_{12} = f+a, S_{13} = 2(f+a), S_{14} = 3(f+a) \ldots \\ S_{21} = f - a, S_{22} = 0, S_{23} = f+a, S_{24} = 2f+a \ldots \\ \vdots \end{array}}$$

Undesired Asymmetry within Tone: The Piano

A similar inharmonic overtone series as before can be adopted: $f + a, 2f+b, 3f+c, 4f+d \dots$

Because of the approximate linearity with overtone number in the degree of inharmonicity, many of the subtraction frequencies are effectively absorbed into the physical overtone spectrum

$S_{11} = 0, S_{12} = f+b\text{-}a, S_{13} = 2f+c\text{-}a, S_{14} = 3f+d\text{-}a \dots$

$S_{21} = f+a\text{-}b, S_{22} = 0, S_{23} = f+c\text{-}b, S_{24} = 2f+d\text{-}c \dots$

Regardless of the linearity, we see that new unphysical tones are produced from the small deviations from perfect harmonics!

Now let the inharmonicity be linear in overtone number, i.e. b=2a, c=3a, d=4a

$$\boxed{f + a, 2(f+a), 3(f+a), 4(f+a) \dots}$$

$$S_{11} = 0, S_{12} = f+a, S_{13} = 2(f+a), S_{14} = 3(f+a) \dots$$

$$S_{21} = f - a, S_{22} = 0, S_{23} = f+a, S_{24} = 2f+a \dots$$

$$\vdots$$

We've addressed harmonic overtones, but…

Why are we partial to scales that follow the harmonic series?

scales that follow the series = each note or interval is taken from the low harmonics

Harmonic Scales

- As it turns out, the vast majority of melodic content worldwide is based on the harmonic series

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A Java Sparrow

A Pigeon

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- The North American Hermit Thrush seems to exhibit a conscious preference, meaning the tendency is not physiologically or mechanically imposed, for melodies based on the harmonic series
    - But their vocalizations are very close to a single sine wave, i.e. no harmonic spectrum!

What is going on?

Harmonic Scales

What is going on?

Start here:

- Instruments with large inharmonicities tend not to be used for melody
- Pure sine waves are virtually never used in human music (and only recently possible)
  - ie melodic instruments almost always possess harmonic overtones
    - However, the hermit thrush produces nearly pure sine waves
    - But we know from the "missing fundamental" that overtone spacing maps to frequency
    - Might the mapping go the other way? Let's assume so!
    - So we're assuming that perception of purely "f" is in some way associated with 2f,3f…

Considering the above facts and assumptions, let's explore spectral similarity

Harmonic Scales

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

This is our set of notes

f, 2f, 3f, 4f, 5f, 6f....

For any f,
this is the overtone spectrum

Harmonic Scales

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

This is our set of notes

f, 2f, 3f, 4f, 5f, 6f....

For any f,
this is the overtone spectrum

What does the set of harmonic overtones for each note look like?

Multiply every number by 1,2,3,4...

Harmonic Scales

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

This is our set of notes

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

This is the set of all overtones

Nothing new is created by the overtones!

Harmonic Scales

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

This is our set of notes

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

This is the set of all overtones

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

This is our set of combination tones

<u>We have a sonic landscape with arbitrary complexity and unique order</u>

Art and Science — Complexity with Order

- Animals experience enormous sensory data streams
  - The world is *really* complex
- One needs filtering, storage, and compression (e.g. the conscious or unconscious recognition of symmetry)
  - Information compression helps us predict and make sense of the world; it's generally advantageous for animal species, and it's clearly gratifying for humans
    - The experience of beauty in science occurs when complex physical phenomena are objectively understood by the conscious realization of simple explanatory principles
    - The experience of beauty in art occurs when complex features, internal or external to the work itself, are consciously or unconsciously tied together

Harmony and Rhythm: identical, but on different timescales

- Worldwide musical forms also follow these frequency relationships in rhythm, to good approximation
- Slowed down by $\sim\mathcal{O}(1)$ to $\sim\mathcal{O}(4)$, we can interpret harmony as complex polyrhythm
  - Even when considering the blending of the all overtone spectra!

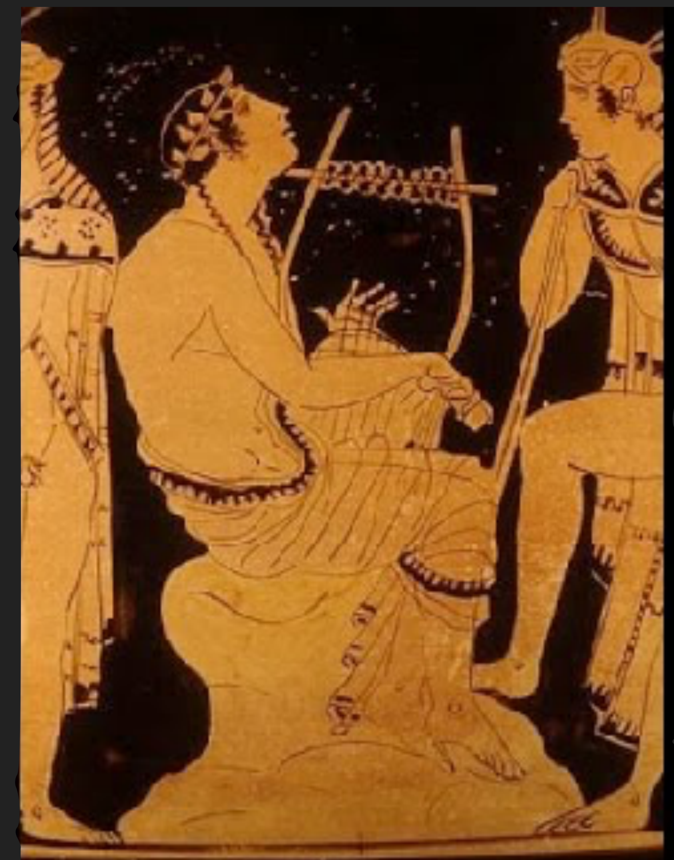
1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

Approximate Symmetries

There are issues, rooted in mathematics, technology, and musical convention, that make precise adherence to small whole number ratios difficult, and, depending on the desired effect, counterproductive

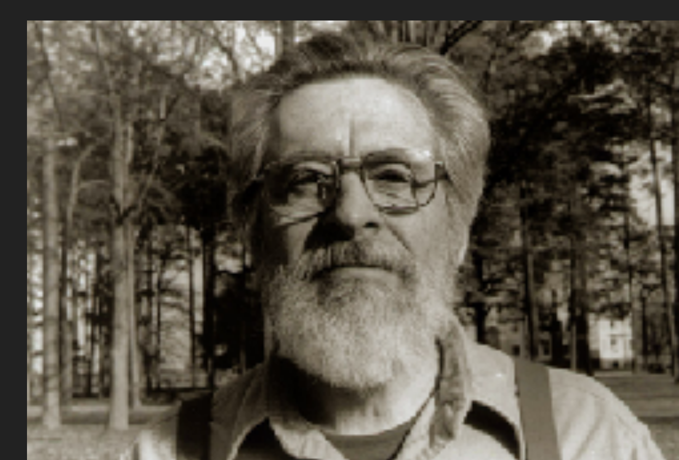
1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮

Just Intonation



Exact subsets of the below intervals were used in the music of ancient Greece.
Music theorists from the Renaissance advocated their use in acapella singing.
They started coming into use again in the 20th century.

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮



Worldwide Tunings









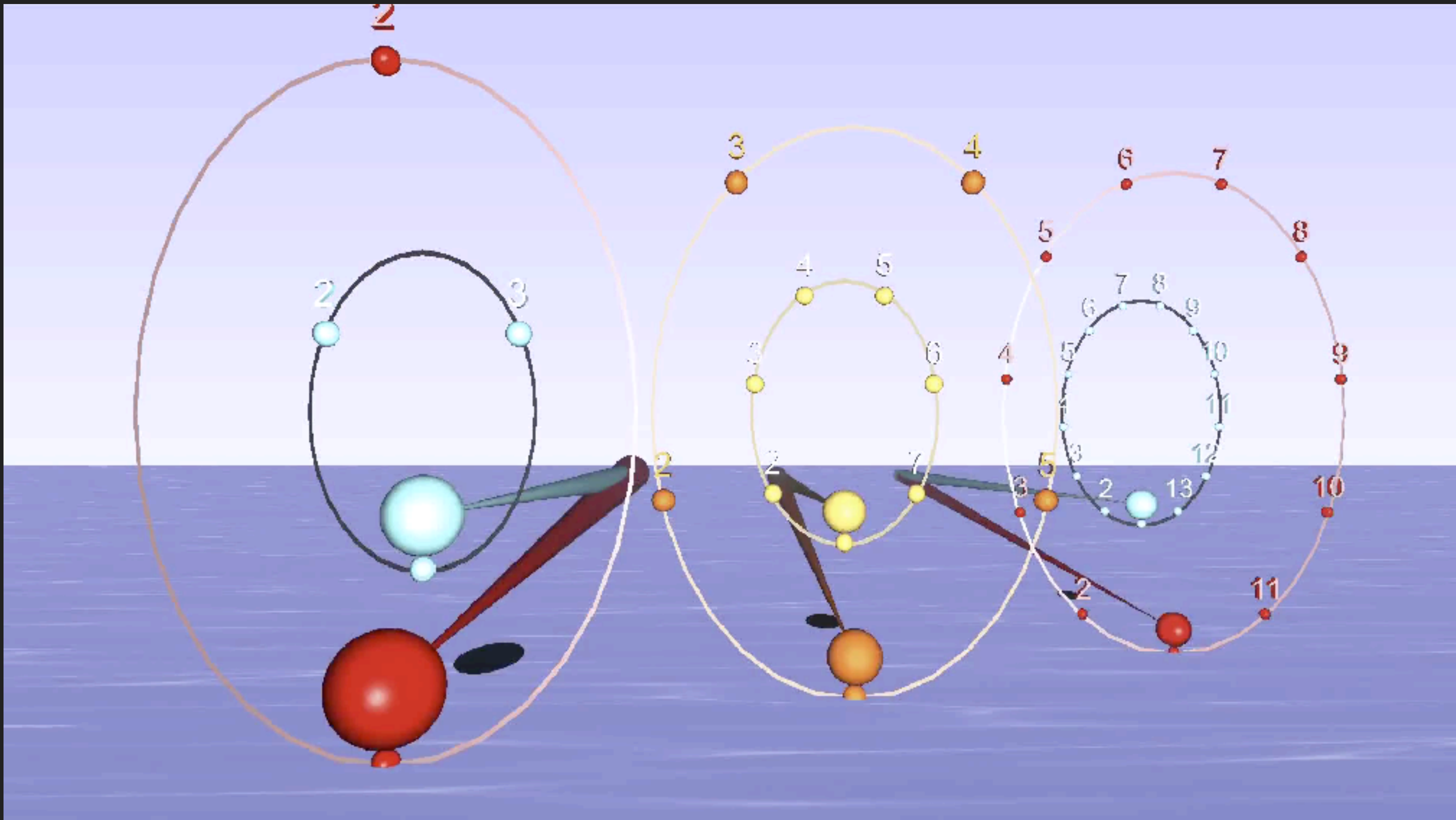




Approximations of small whole number ratios are nearly universal in melody (and rhythm)

The particular approximation scheme and subset of intervals depend on cultural and technological factors

1/1, 2/1, 3/1, 4/1, 5/1 ....
1/2, 2/2, 3/2, 4/2, 5/2 ....
1/3, 2/3, 3/3, 4/3, 5/3 ....
1/4, 2/4, 3/4, 4/4, 5/4 ....
⋮



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