Ultracold Atoms

How Quantum Field Theory invaded Atomic Physics

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support
Department of Energy
National Science Foundation
Simons Foundation

Ultracold Atoms
How QFT Invaded Atomic Physics

- Ultracold Atoms

aside: *X(3872)* meson

- Quantum Field Theory

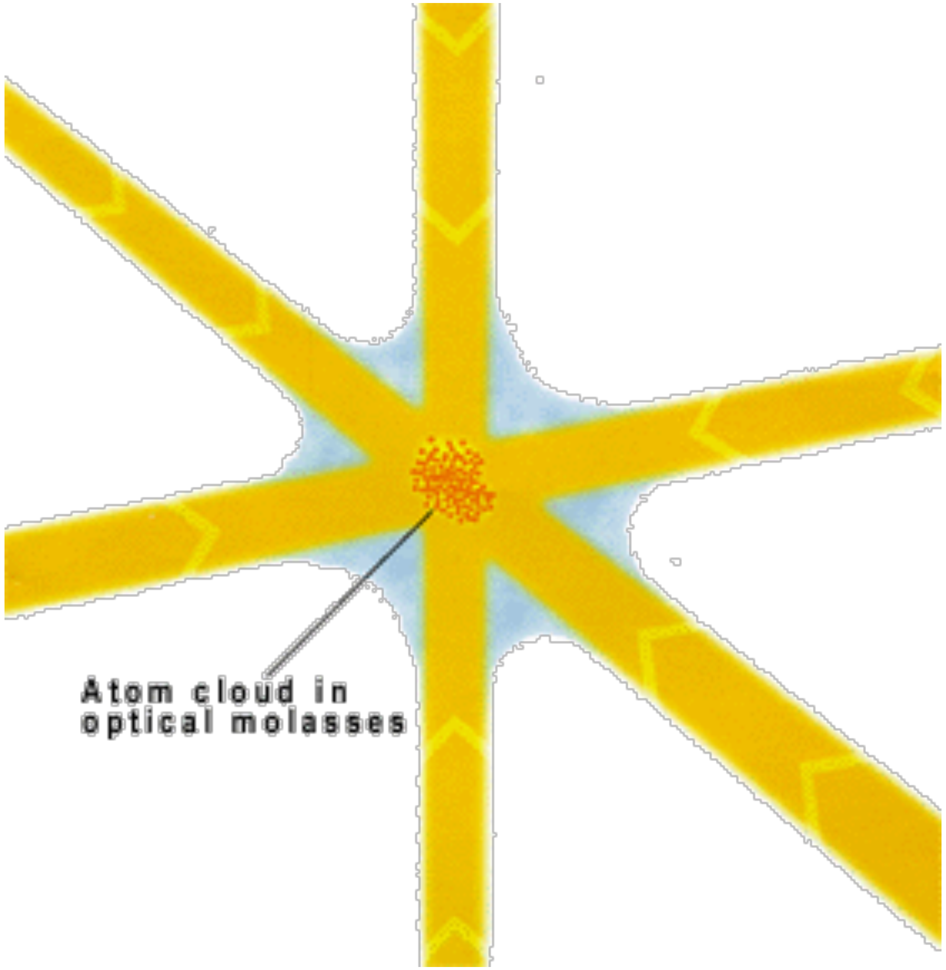
- Fermions with two spin states
  phase diagram, contact

- Identical bosons
  trimer spectrum, unitary Bose gas

aside: quark-mass dependence in nuclear physics

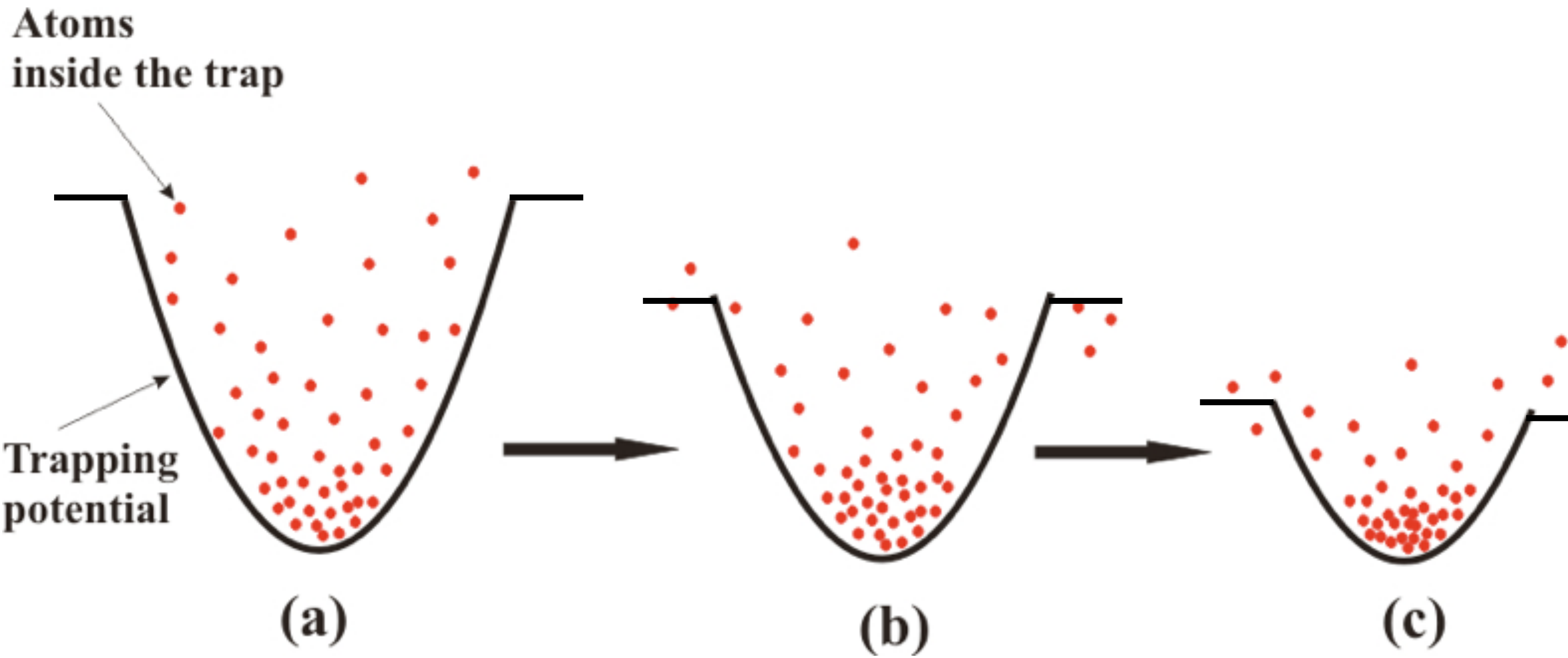
Cold Atom Physics

Atoms trapped and cooled using lasers



Nobel Prize 1997: Chu, Cohen-Tannoudji, Phillips

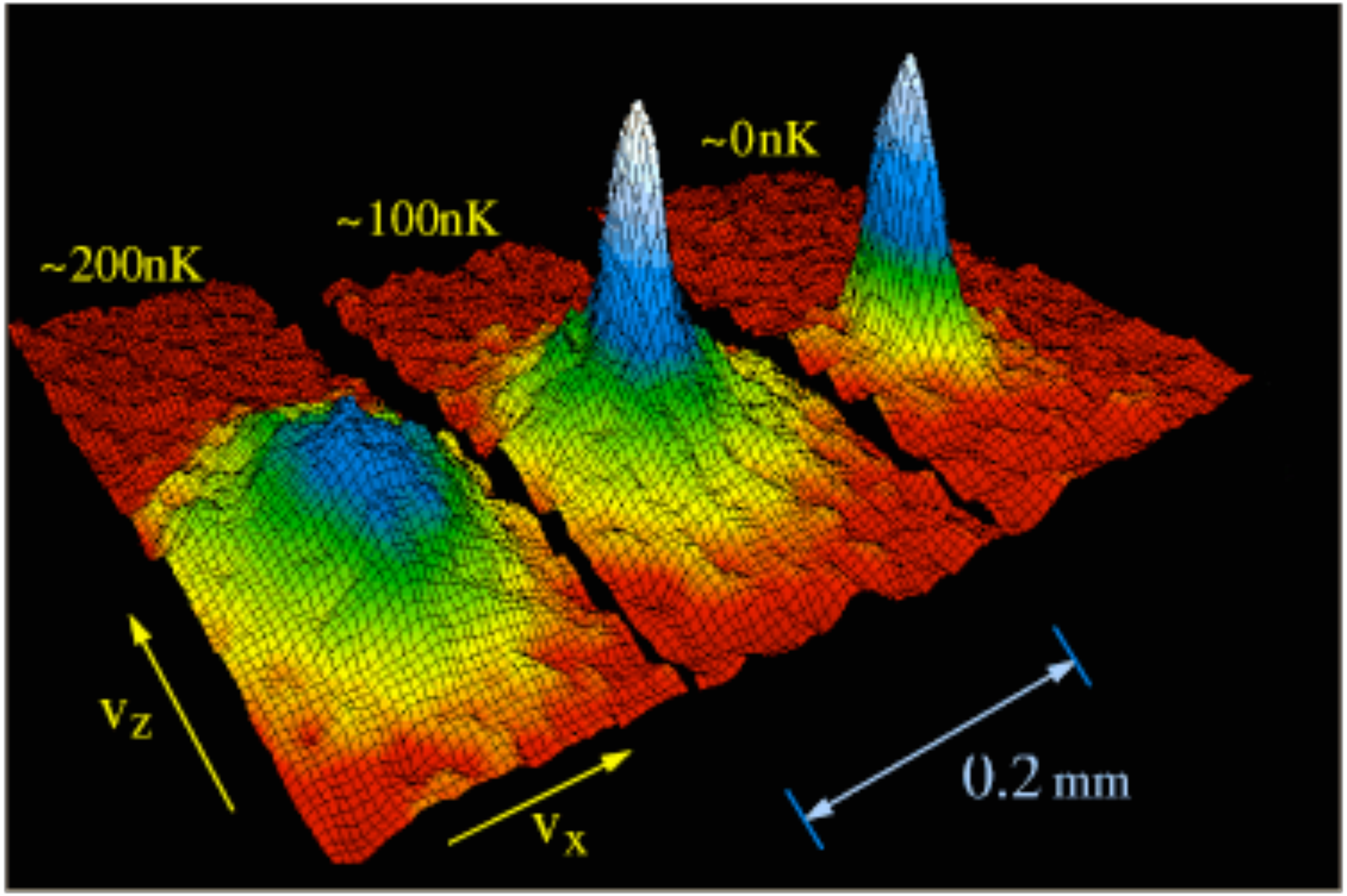
Temperature of trapped atoms decreased further by evaporative cooling



Bose-Einstein condensation of atoms!

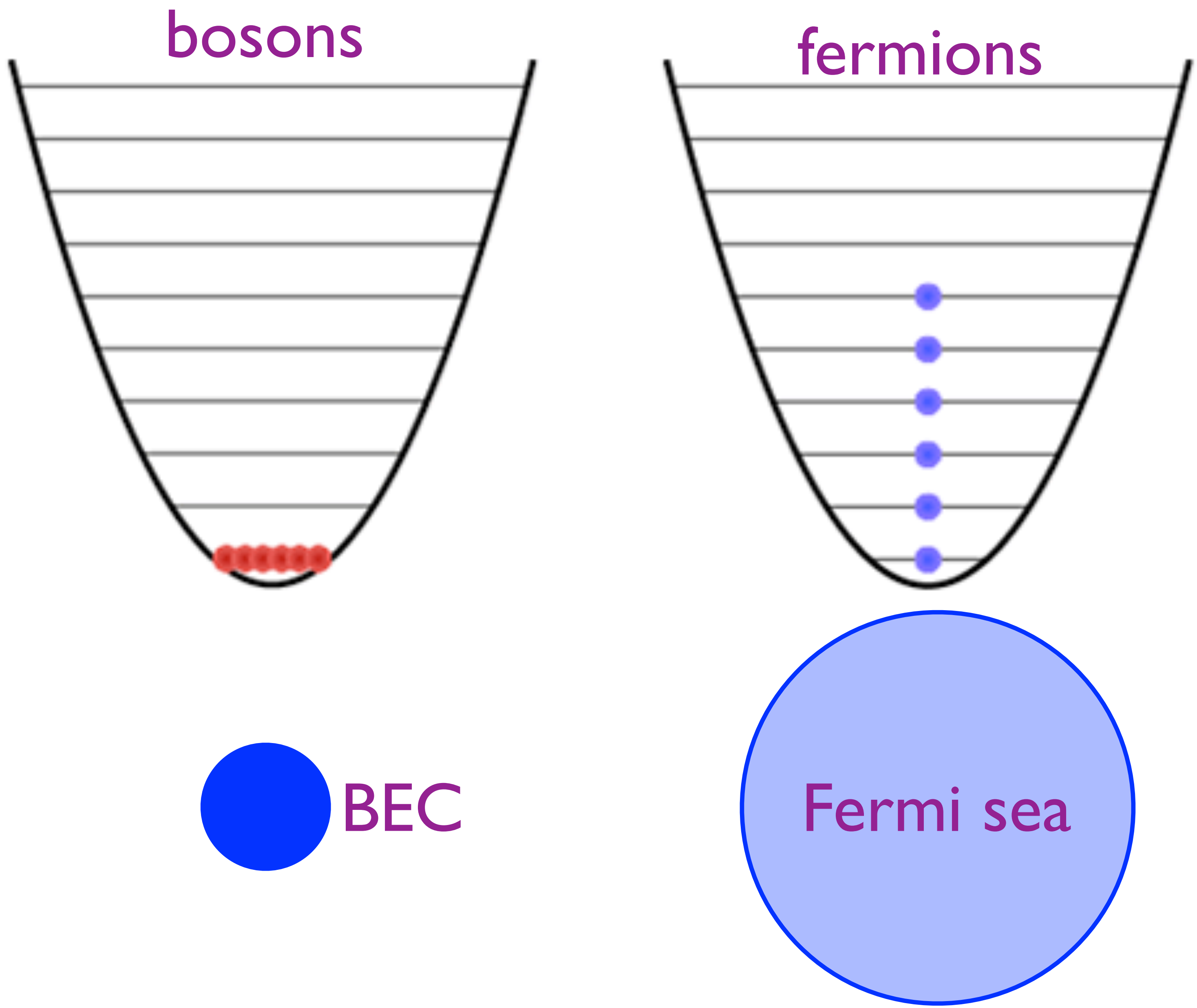
^{87}Rb atoms	JILA (Cornell, Wieman)	1995
^{7}Li atoms	Rice (Hulet)	1995
^{23}Na atoms	MIT (Ketterle)	1995

2 D velocity distributions



Nobel Prize 2001: Cornell, Wieman, Ketterle

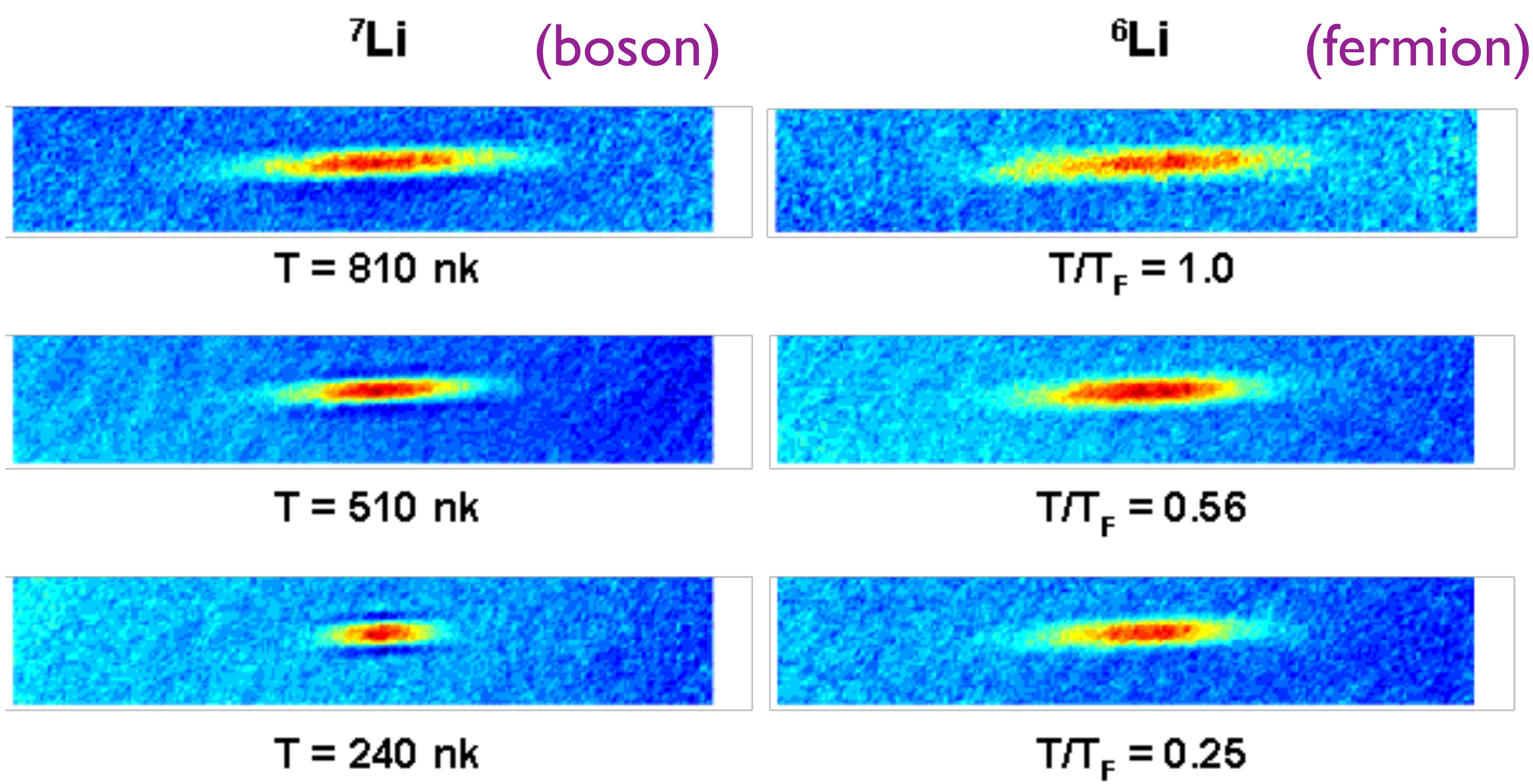
ground state of many-atom system



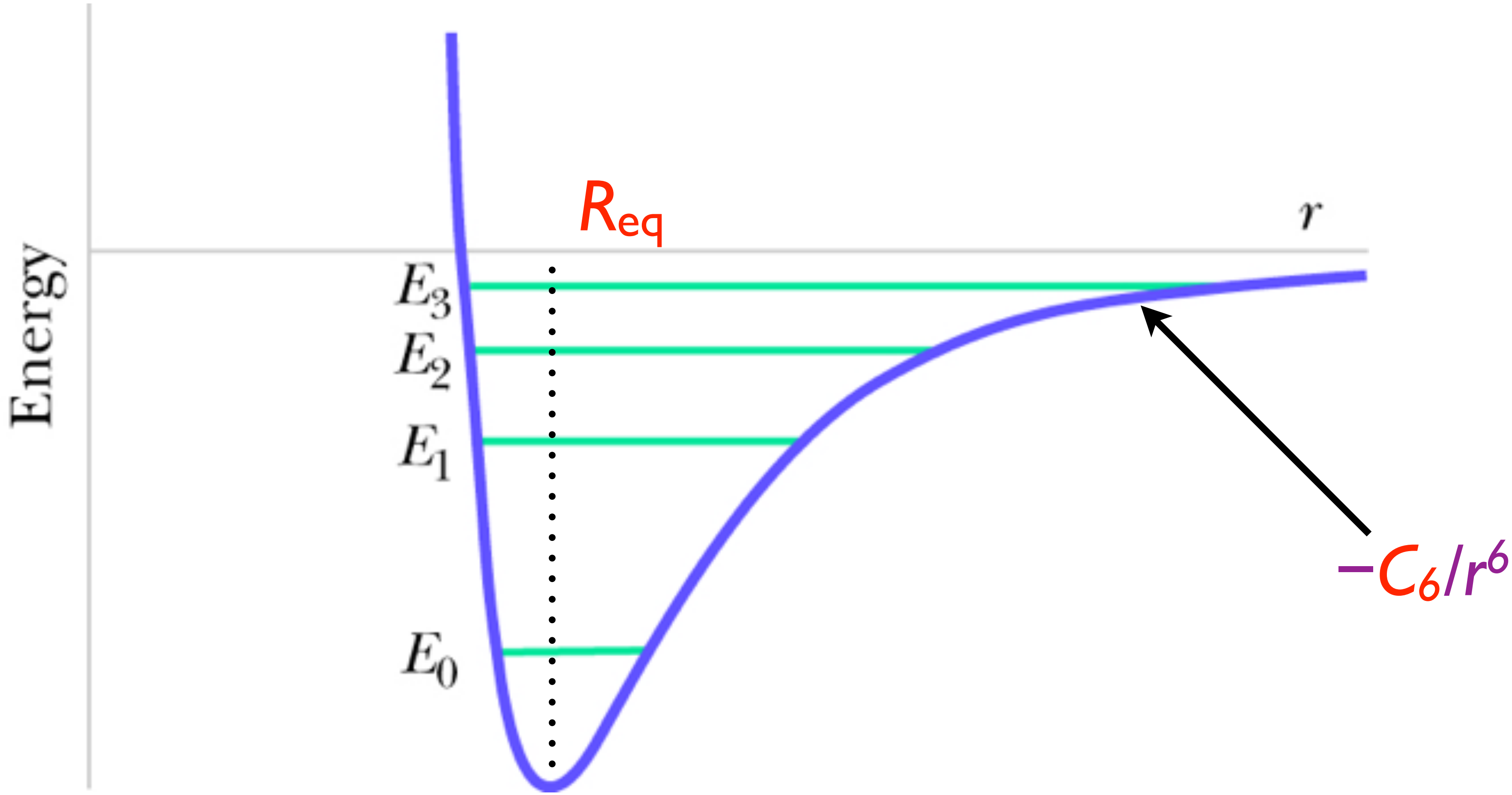
Cold Atom Physics

Cooling of fermions to quantum degeneracy!

^{40}K atoms	JILA (Jin)	Jan 2001
^{6}Li atoms	Ecole Normale (Salomon)	July 2001
^{6}Li atoms	Rice (Hulet)	Aug 2001



Interactions between Atoms



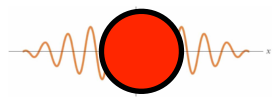
size of atoms: $R_{eq} \sim 0.4$ nm (for Rb)

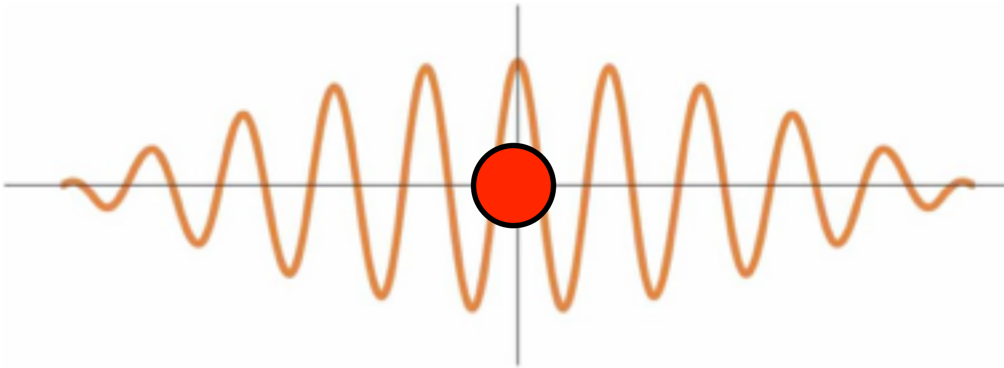
interaction range: $R_6 = (mC_6/\hbar^2)^{1/4} \sim 8$ nm (for Rb)

Interactions between Atoms

thermal de Broglie wavelength: $\lambda_{th} = (2\pi\hbar^2/mkT)^{1/2}$

	T	λ_{th} (for Rb)
cold	1 K	0.2 nm
	1 mK	6 nm
ultracold	1 μK	20 nm
	1 nK	600 nm





size of atoms: $R_{eq} \sim 0.4$ nm (for Rb)

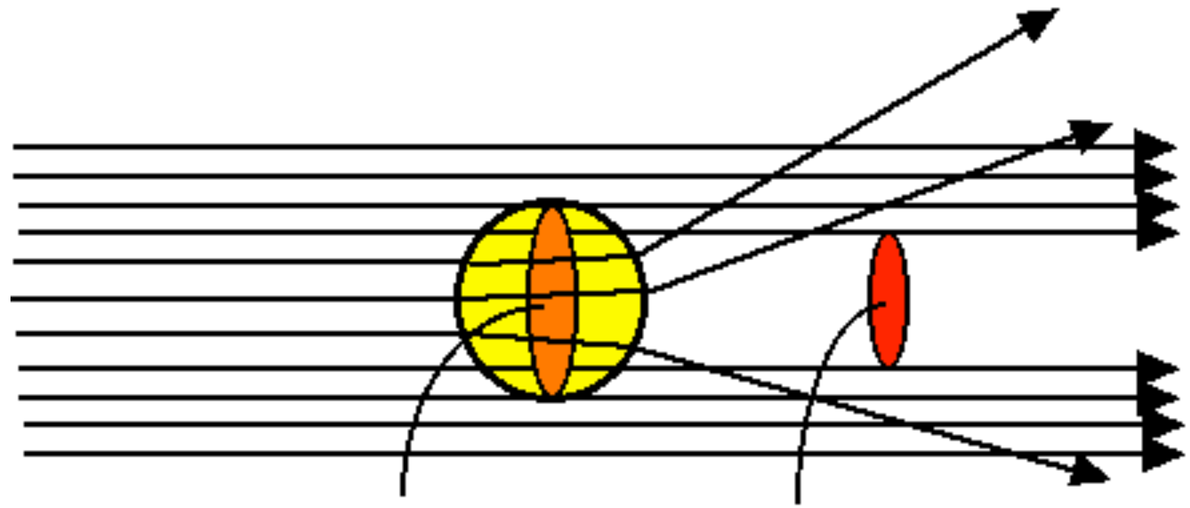
interaction range: $R_6 \sim 8$ nm

$T < 1$ K: atoms behave like point particles.

$T < 1$ mK: atoms behave as if they had zero-range interactions.

scattering cross section:

area of beam that intercepts
as many particles as are scattered



geometrical
cross section

scattering
cross section

generically,
scattering cross section is comparable to (range)^2

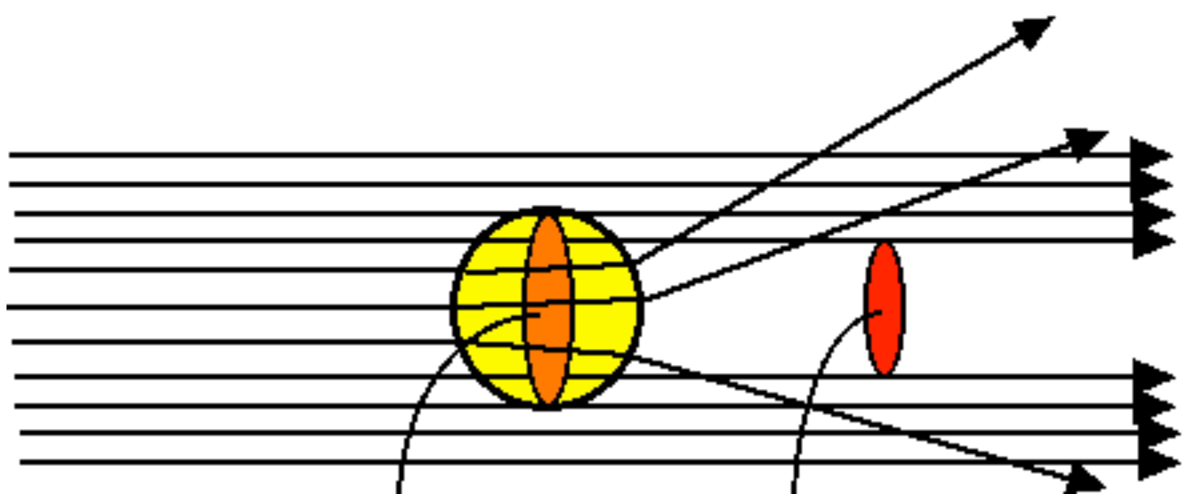
convenient measure of interaction strength
for low-energy atoms:

scattering cross section at zero energy

$\sigma = 4\pi a^2$

OR scattering length a

generically, a is comparable to interaction range



Large Scattering Length

But quantum mechanics allows scattering of particles far beyond the interaction range!

Helium atoms (^4He)

range: 0.7 nm

scattering length:

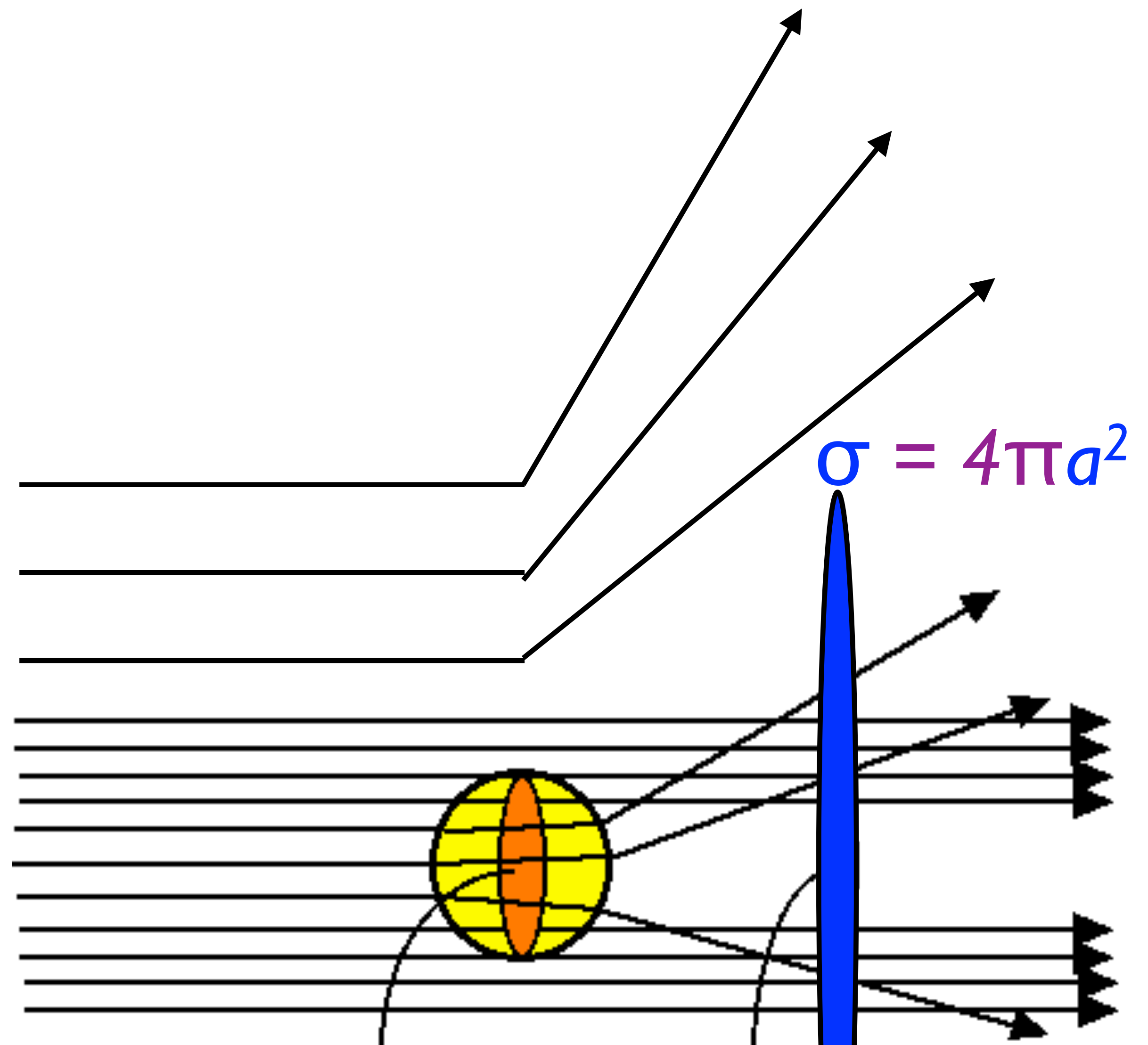
a = +8 nm

Neutrons

range: 3 fm

scattering length:

a = −20 fm



Large Scattering Length

Quantum mechanics allows bound states
whose constituents spend most of their time
beyond their interaction range!

Deuteron

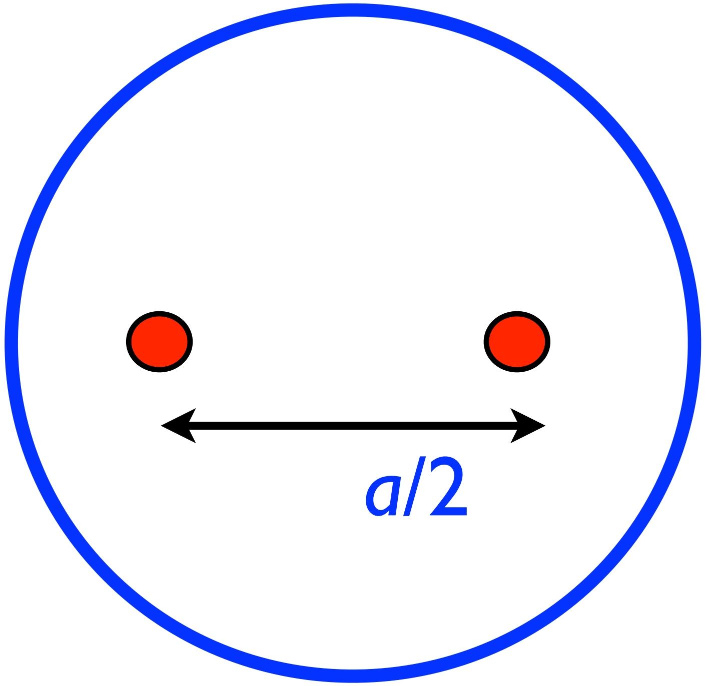
p n range: 1.8 fm

mean separation: $\langle r \rangle$ = 2.7 fm

^{4}He dimer

range: 0.7 nm

mean separation: $\langle r \rangle$ = 4 nm



Universal properties determined by *a*

binding energy: $\hbar^2/(m\, a^2)$

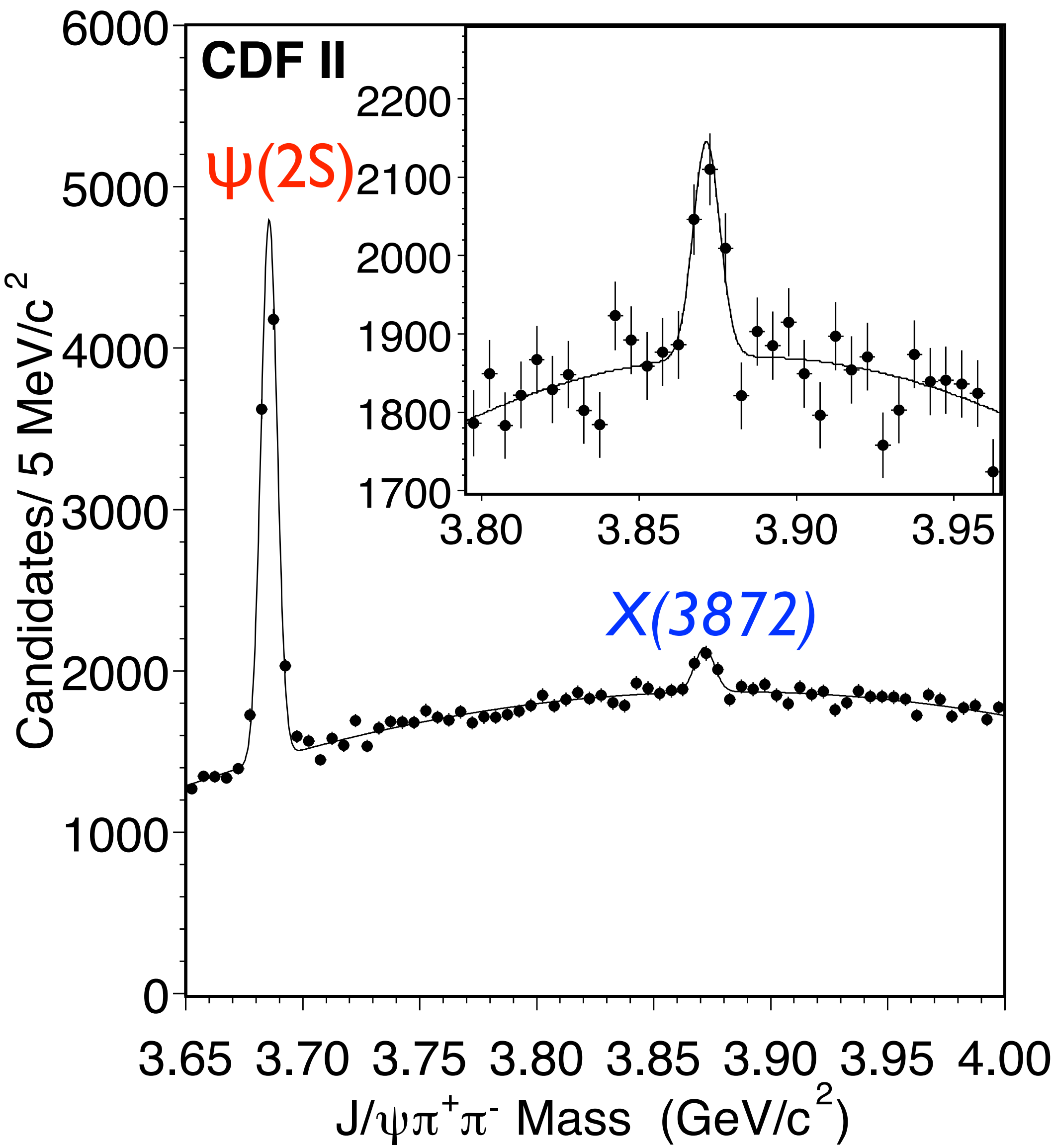
mean separation: $a/2$

X(3872) Meson

discovered in B^+ decay — Belle (September 2003)
confirmed in $p\bar{p}$ collisions — CDFII (December 2003)

- decays into $J/\psi\ \pi^+\pi^-$ like $\psi(2S) = c\ \bar{c}$ meson
- decay is into $J/\psi\ \rho^*$ which has isospin 1

$\Longrightarrow$ cannot be $c\ \bar{c}$ meson which has isospin 0



What is the *X(3872)*?

$X(3872)$ Meson

- quantum numbers 1^{++} LHCb 2014

$\Longrightarrow$ S-wave coupling to charm mesons $D^{*0}\ \bar{D}^0$

- mass is extremely close to the threshold for the charm mesons $D^{*0}\ \bar{D}^0$

mass measured most accurately by CDF2, Belle, LHCb, Babar, BES3

threshold measured most accurately by Babar, CLEO, LHCb, KEDR

$\Longrightarrow$ binding energy is only 0.2+0.3 MeV

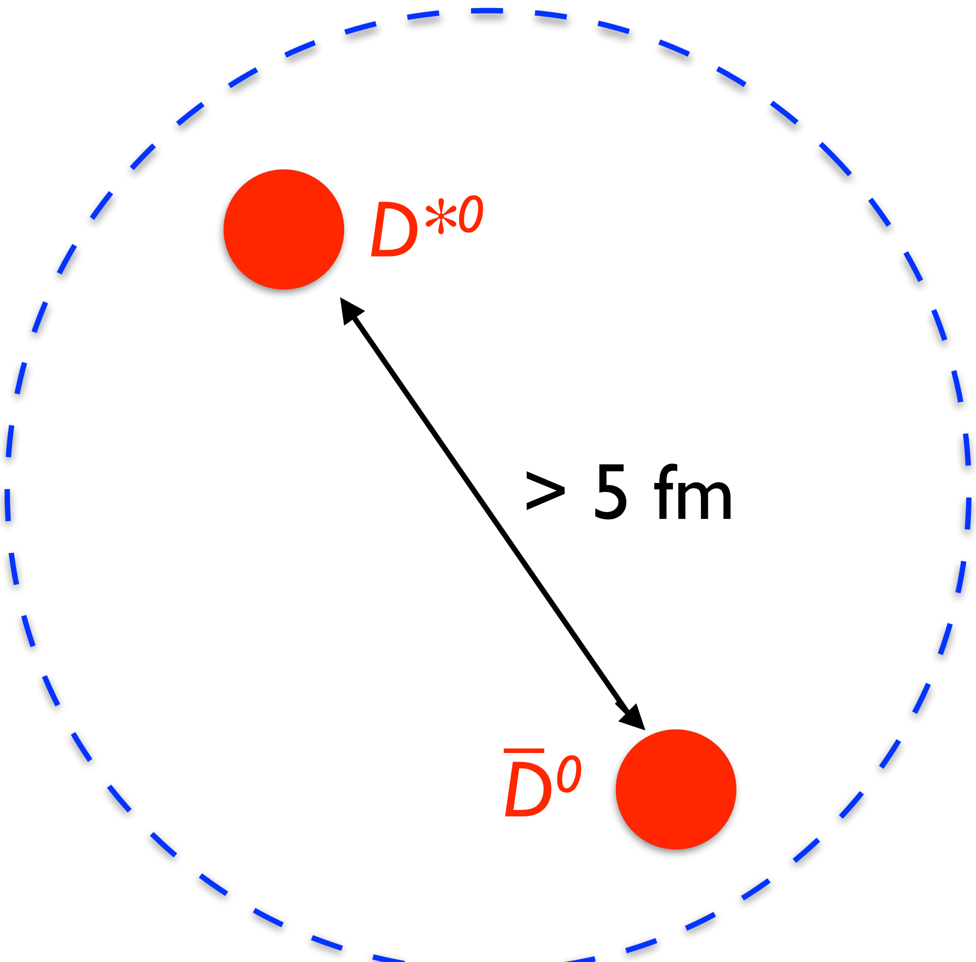
$\Longrightarrow$ must be weakly bound molecule of $D^{*0}\ \bar{D}^0$

with universal properties
determined by binding energy

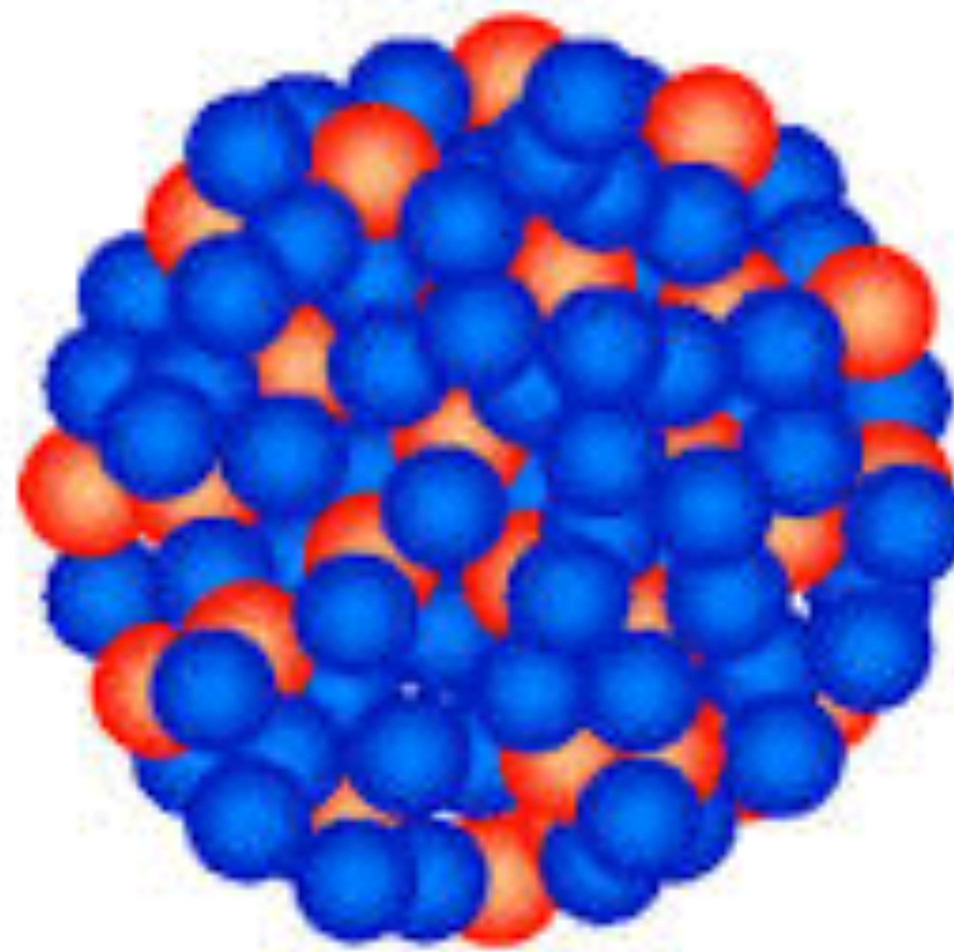
X(3872) Meson

loosely bound charm meson molecule
comparable in size to the largest nuclei!

X(3872)



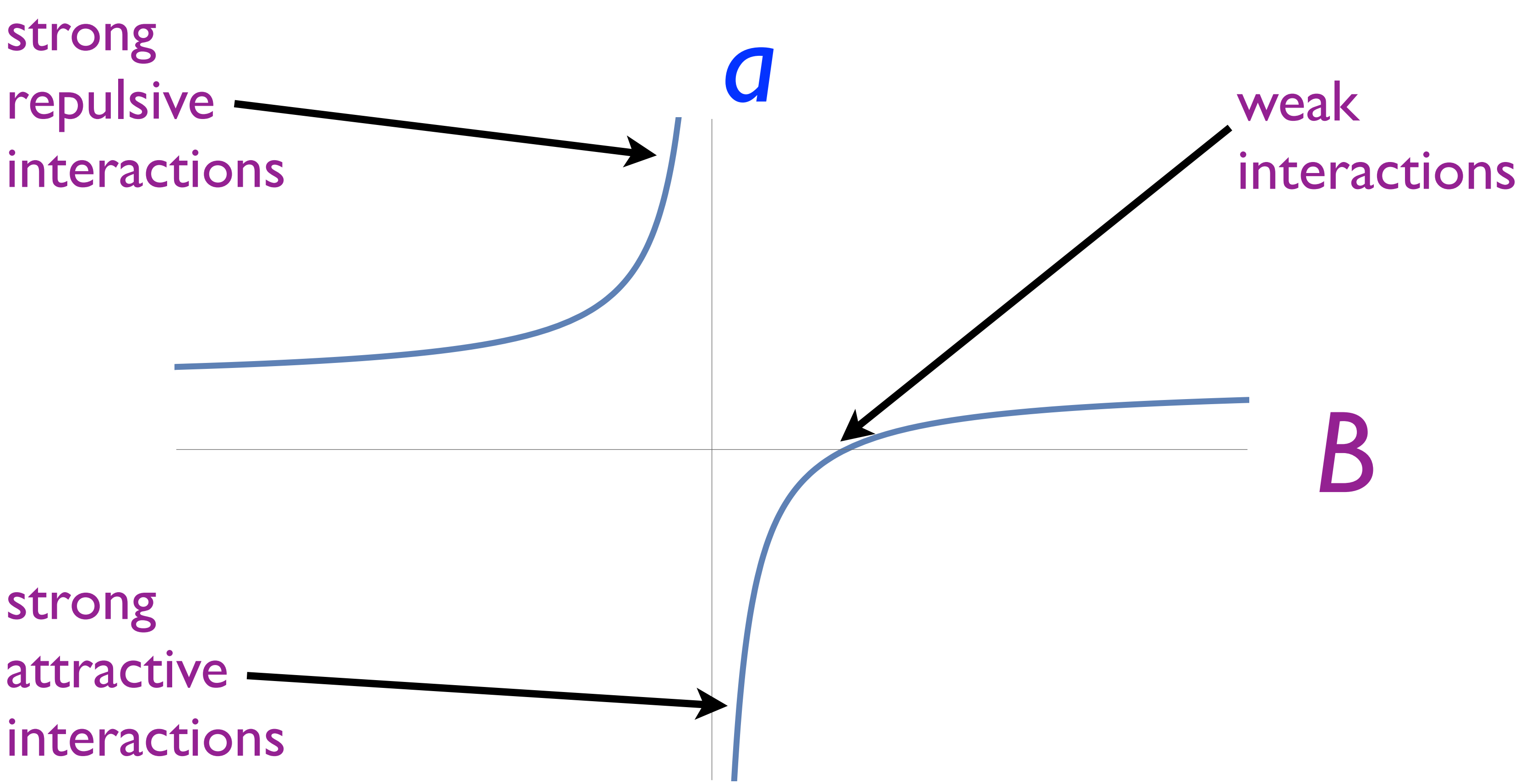
Uranium nucleus



Scattering length *a*

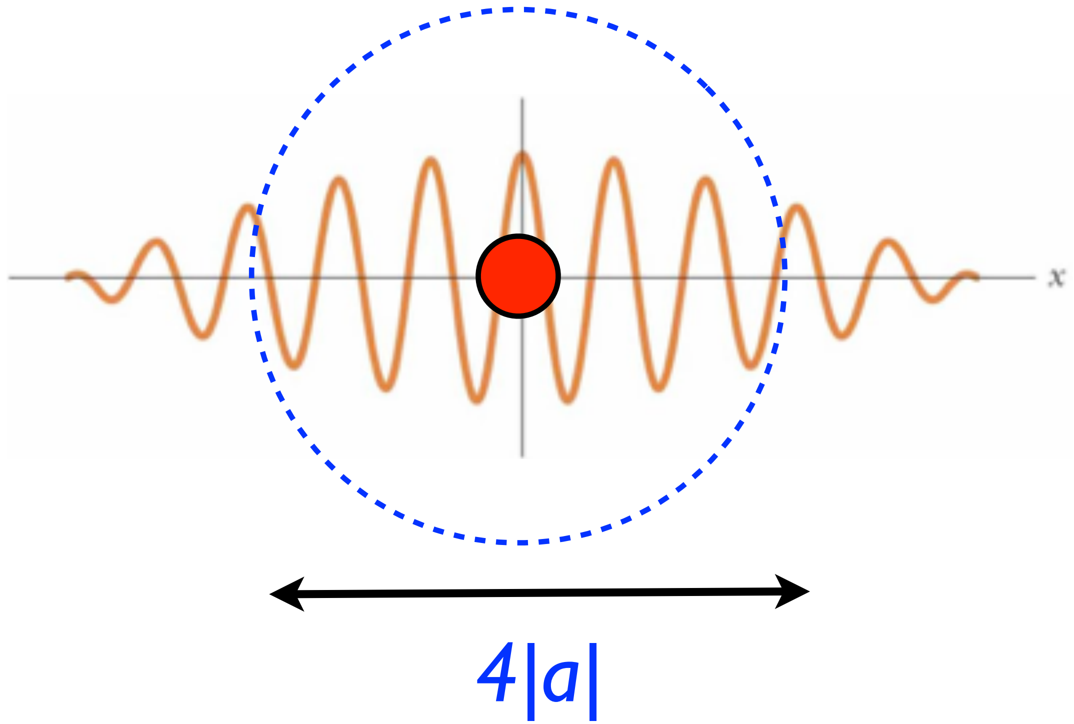
for ultracold atoms, can be controlled experimentally!

a changes slowly with magnetic field *B*
except near Feshbach resonance where *a* diverges to $\pm\infty$



Large Scattering Length

scattering length *a* can be controlled by magnetic field
can be made much larger than range

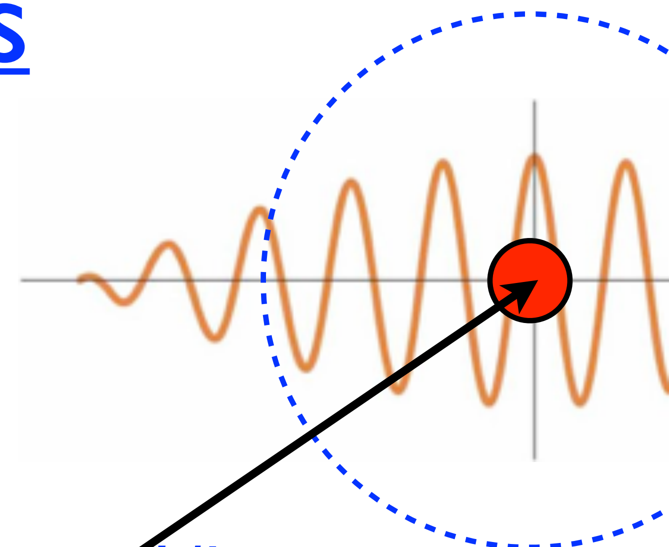


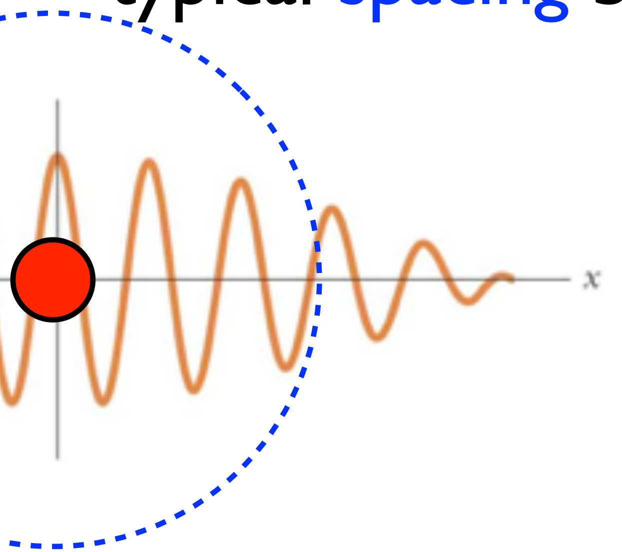
Trapped Atoms

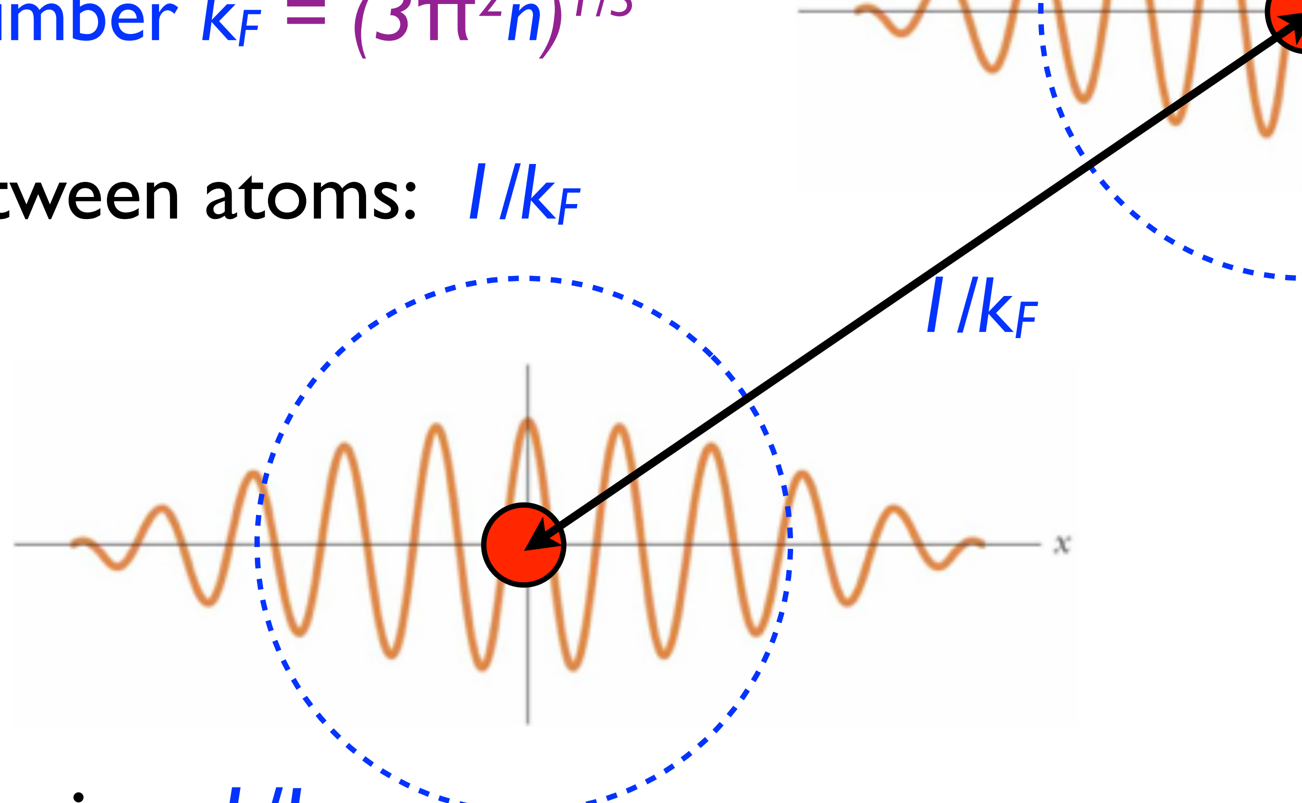
number density of atoms n

OR Fermi wavenumber $k_F = (3\pi^2 n)^{1/3}$

typical spacing between atoms: $1/k_F$





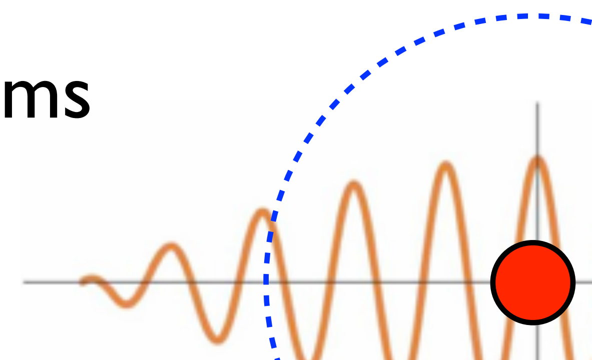


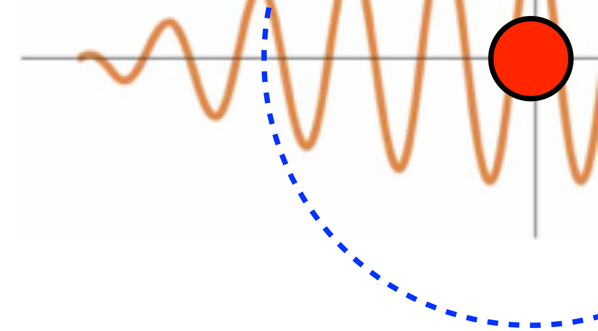
inter-atom spacing $1/k_F$:

controlled by number of trapped atoms

and by trapping potential

(even at center of trap, $1/k_F \gg$ range)





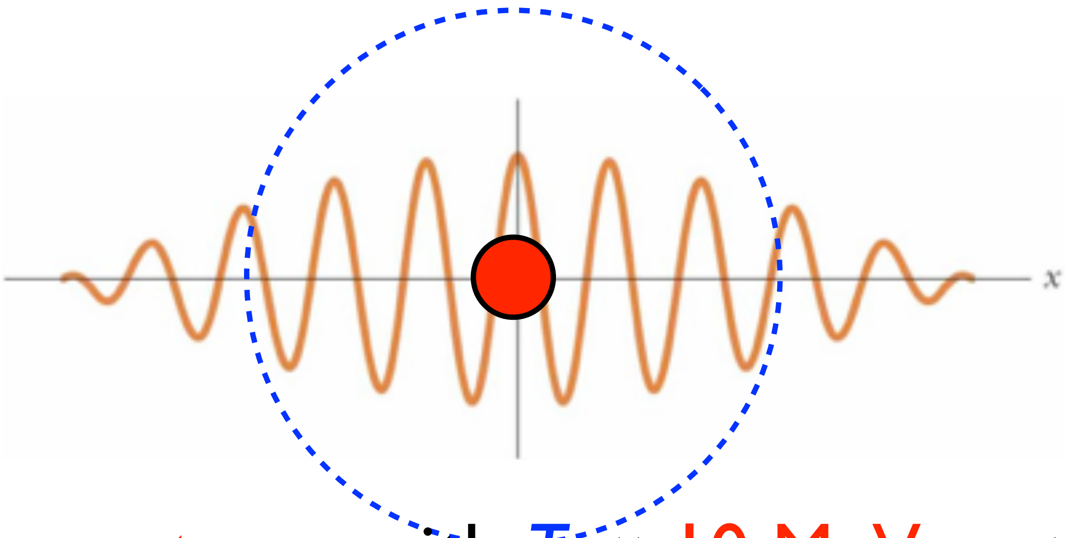
Universality

particles with short-range interactions

and large scattering length $|a| \gg$ range

have identical behavior at low temperature, low density

(if expressed in terms of dimensionless variables $k_F\, a$, $k_F\, \lambda_{th}$)

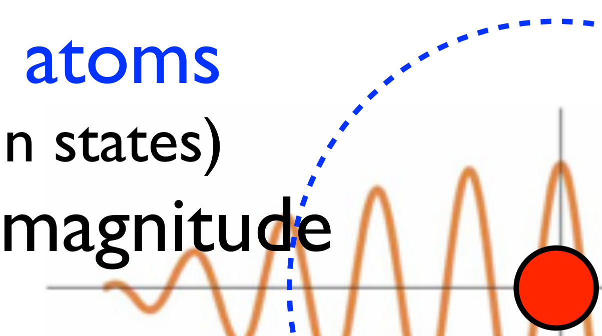


neutrons with $T \ll$ 10 MeV, $n \ll 10^{-3}$/fm^3

can be studied experimentally using ultracold atoms

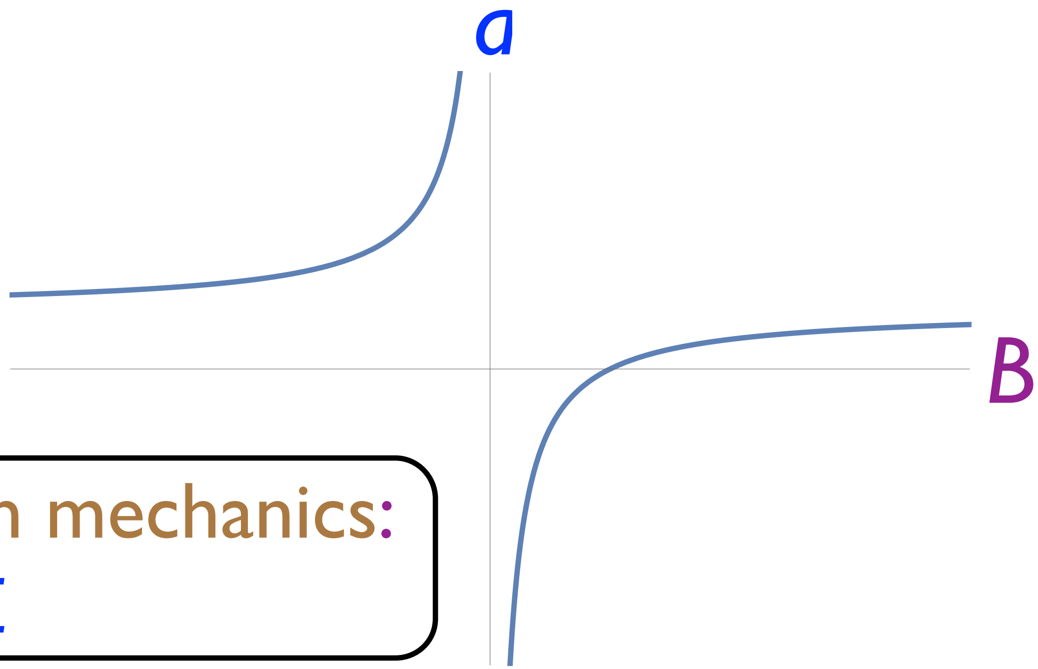
(^{6}Li atoms in lowest two hyperfine spin states)

even though length scales differ by orders of magnitude



Scale Invariance

If $a = \pm\infty$ (unitary limit),
scattering cross section
at zero energy is infinite!



Unitarity bound from quantum mechanics:

$$\sigma \leq 4\pi\hbar^2/mE$$

At nonzero energy,
scattering cross section saturates unitarity bound:

$$\sigma(E) = 4\pi\hbar^2/mE$$

no length scale $\Longrightarrow$ scale-invariant interactions!

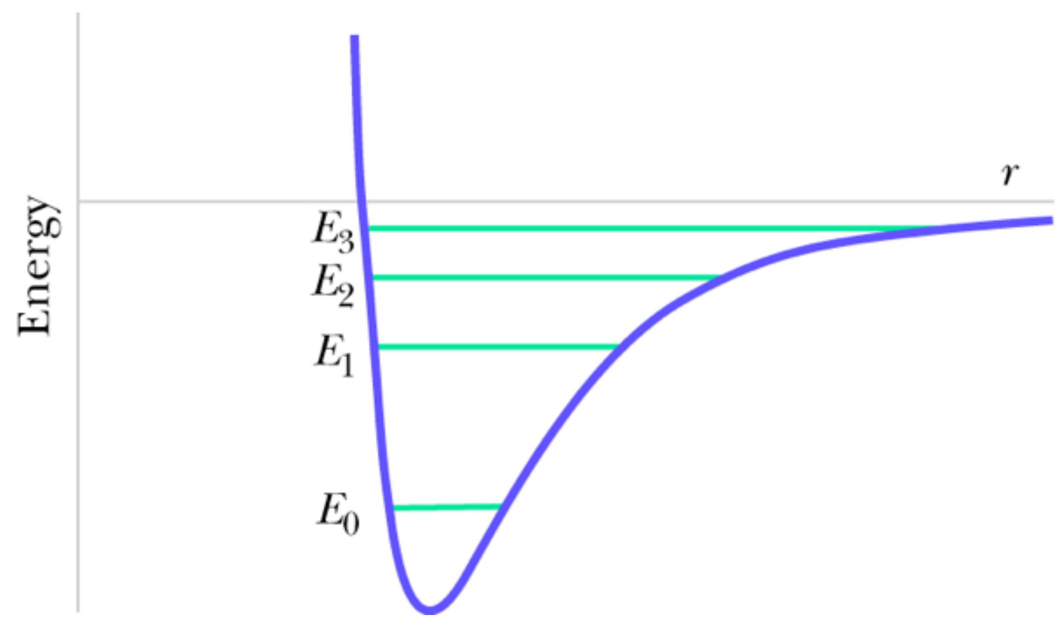
QFT for Ultracold Atoms

(sufficiently) Fundamental Theory

many-body Schroedinger equation

for atoms in a trapping potential *V(r)*

interacting through interatomic potential *U(r-r')*



(atoms may have multiple spin states)

equivalent formulation:

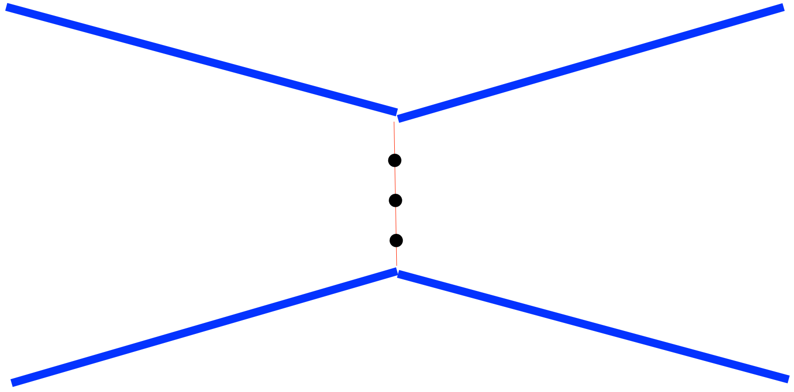
Nonlocal Quantum Field Theory

for atoms in a trapping potential *V(r)*

interacting through potential *U(r-r')*

particles: atoms

interaction at a distance!

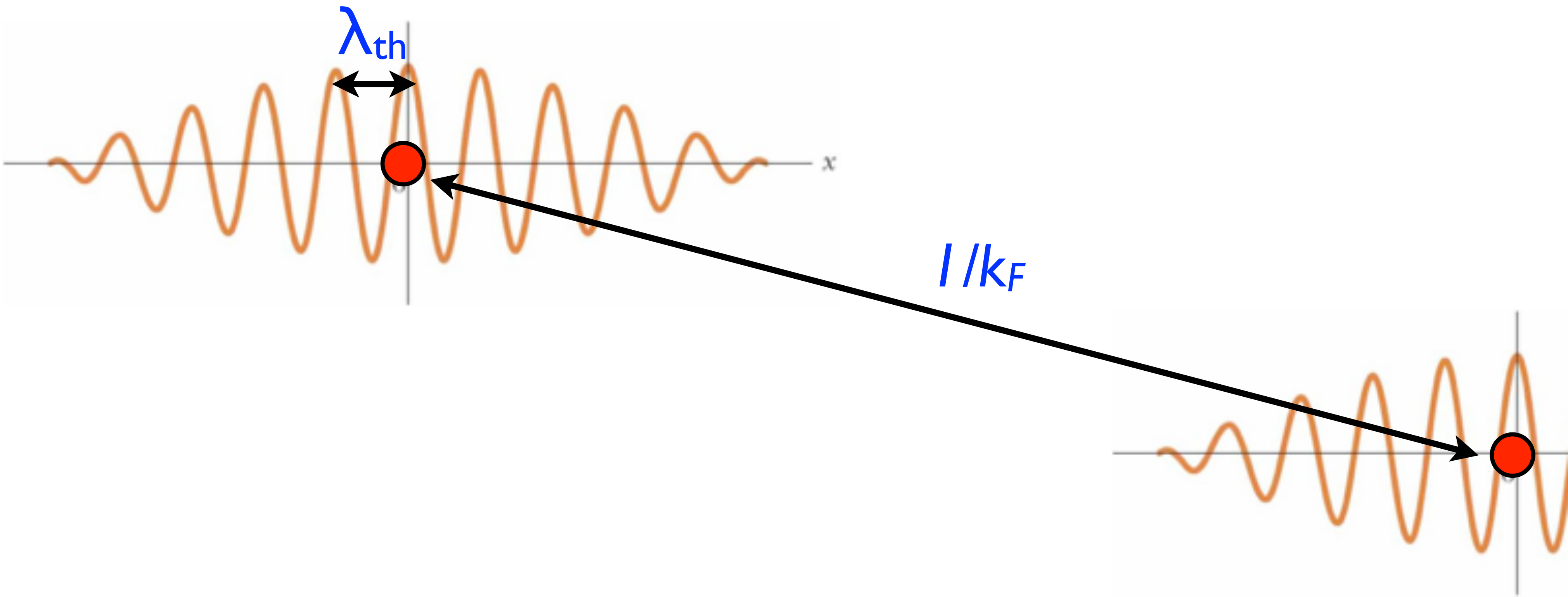


However ultracold atoms behave like point particles
with zero-range interactions

thermal wavelength λ_{th}
interatom spacing $1/k_F$

much larger than

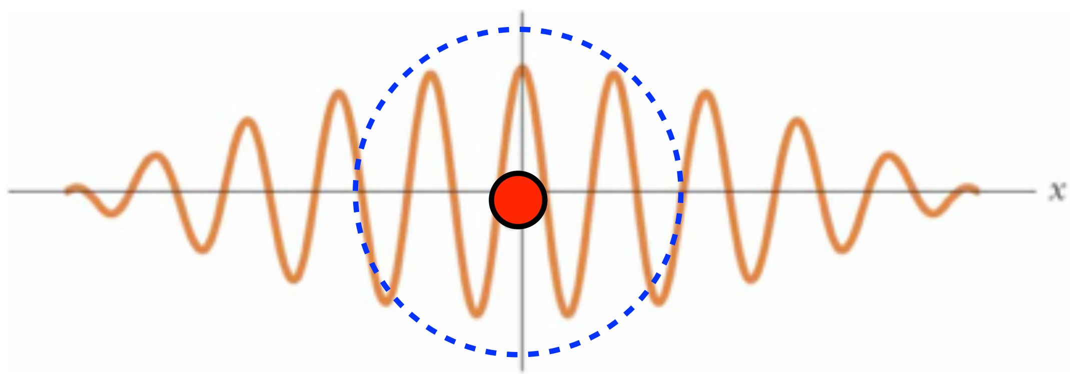
size of atoms
interaction range



can be described by local quantum field theory

Ultracold Atoms

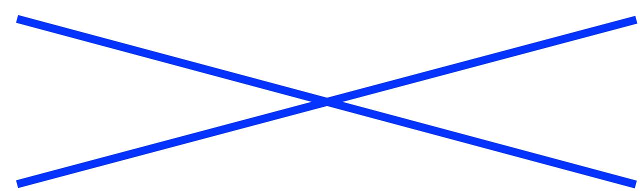
Local Quantum Field Theory (zero-range interactions)



particles: atoms

(perhaps with multiple spin states)

point interaction



interaction strength: scattering length a

(perhaps different scattering length for each pair of spin states)

Ultracold Atoms can be described by Local Quantum Field Theory

Advantages

- zero-range limit is taken from beginning
- allows different calculational methods
  - integral equations
  - lattice Monte Carlo
  - operator product expansion

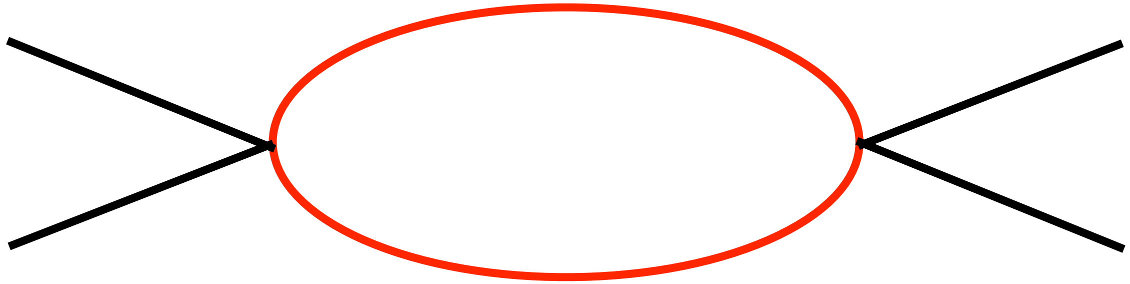
interactions can be weak: $k_F\,|a| \ll 1$

or strong: $k_F\,|a| \sim 1$

or infinitely strong: $a = \pm\infty$ (unitary limit)

Local Quantum Field Theory

loop diagrams involve integrals
over momenta of virtual particles
that are often ultraviolet divergent



divergences can be controlled by "renormalization"

Local Quantum Field Theory

Stephen Weinberg
"What is quantum field theory and what did we think it is?"
hep-th/9702027

general framework for interacting particles

consistent with
- quantum mechanics
- ~~Lorentz invariance~~
  Galilean invariance
- cluster decomposition

weakly interacting QFT
can be defined in terms of Feynman diagrams

Strongly-coupled Quantum Field Theory

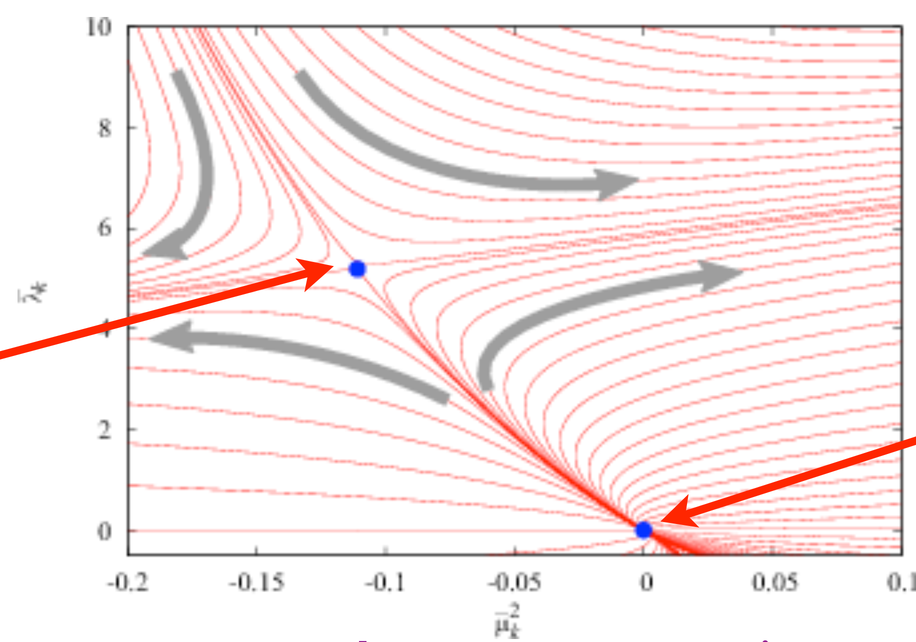
can be defined by

Renormalization Group flow to ultraviolet fixed point

Ken Wilson

RG defines flow in abstract theory space

of equivalent theories at increasingly shorter distances



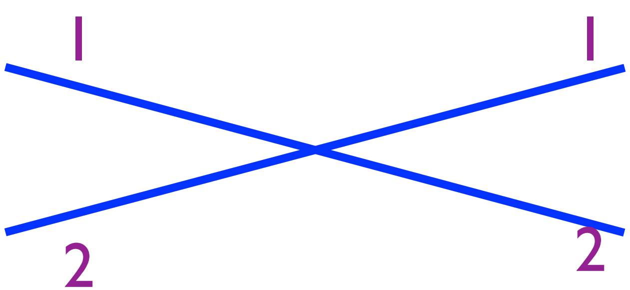
RG fixed point $\Longrightarrow$ scale invariance!

Fermion with Two Spin States

simplest QFT for ultracold atoms

particles
fermionic atoms (spin states 1 and 2)

point interaction



interaction strength: scattering length *a*

Fermion with Two Spin States

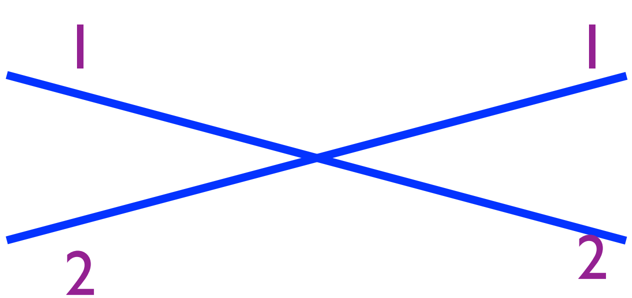
fermionic quantum fields: ψ_1, ψ_2

Lagrangian density

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} - \mathcal{H}_{\text{int}} - \mathcal{H}_{\text{trap}}$$

$$\mathcal{L}_{\text{kinetic}} = \sum_{i=1,2} \psi_i^\dagger \left(i\frac{\partial}{\partial t} + \frac{1}{2m}\nabla^2 \right) \psi_i$$

$$\mathcal{H}_{\text{int}} = \frac{g_0}{m} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$$



$$\mathcal{H}_{\text{trap}} = V(\vec{r}) \left(\psi_1^\dagger \psi_1 + \psi_2^\dagger \psi_2 \right)$$

Weak coupling

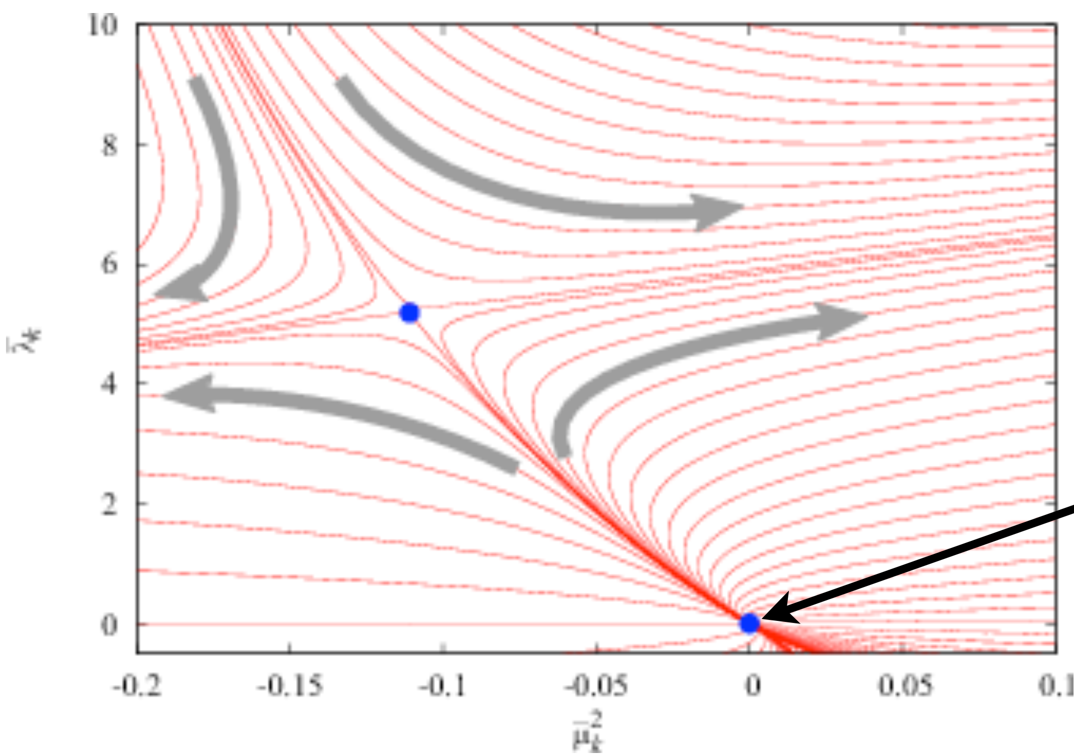
quantum fields: ψ_i scaling dimension 3/2

interaction operator: $\psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$ scaling dimension 6

($>5 \implies$ irrelevant)

perturbatively nonrenormalizable!

$$g_0 = 4\,\pi\, a \text{ (+ counterterms)}$$



RG fixed point:
free field theory

Strong coupling

quantum fields: ψ_i — scaling dimension 3/2

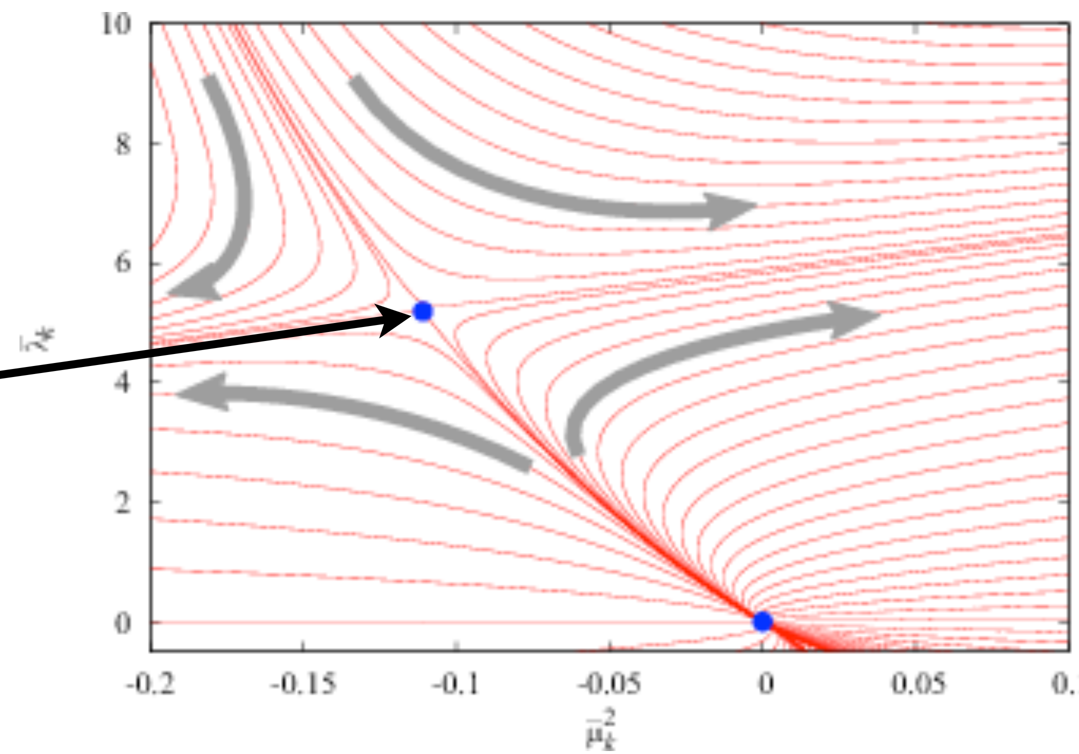
interaction operator: $\psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$ — scaling dimension 4

($<5 \Longrightarrow$ relevant)

nonperturbatively renormalizable!

anomalous scaling dimensions!

RG fixed point:
scale-invariant
interacting theory
(unitary limit!)



2-Body Problem

can be solved analytically

Cross section

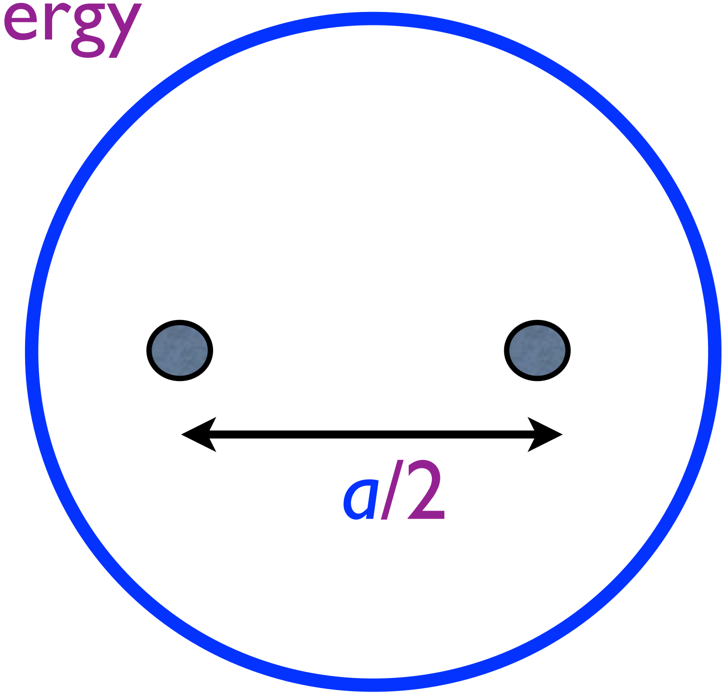
$\sigma \rightarrow 4\pi a^2$ at low energy

$\rightarrow 4\pi \hbar^2/(m E)$ at high energy

Diatomic molecule if $a > 0$

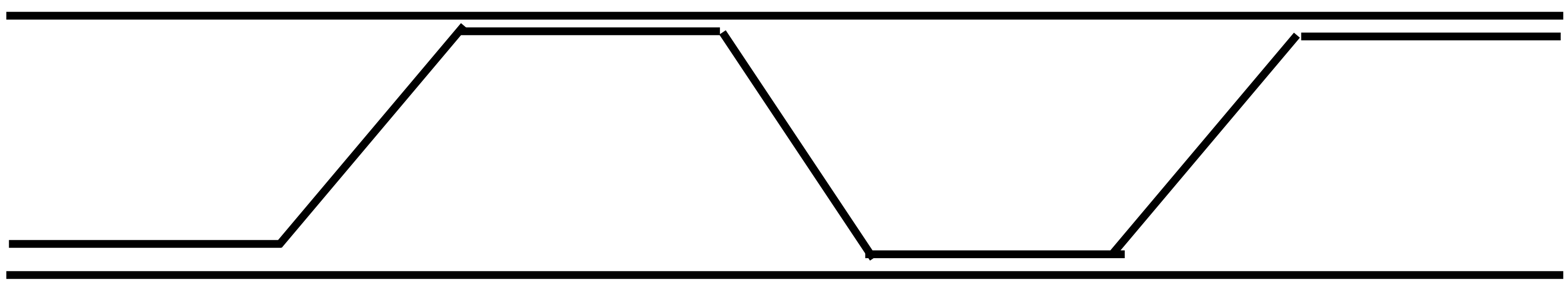
binding energy: $\hbar^2/(m a^2)$

mean radius: $a/2$



3-Body Problem

can be solved exactly numerically



where  = 

4-Body Problem

can be solved exactly numerically

5-Body Problem

frontier of few-body physics

Fermi Gas with Two Spin States

balanced gas ($n_1 = n_2$)

weak interactions: $k_F\,|a| \ll 1$

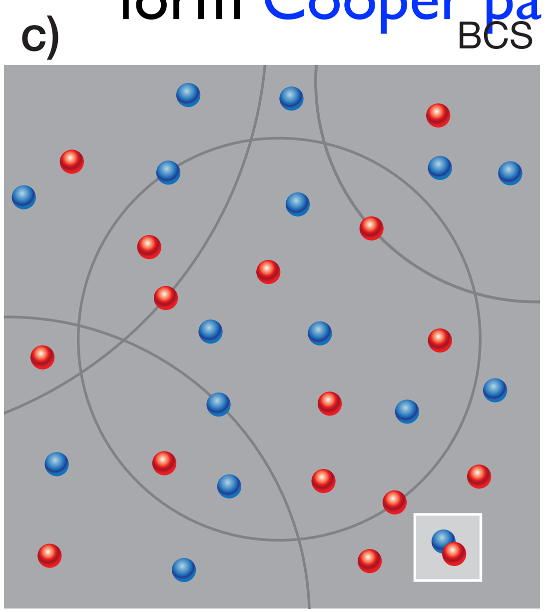
Ground state ($T=0$) is a Superfluid!

<u>attractive interaction</u> $a < 0$

pairs of fermions
with momenta near Fermi surface
form Cooper pairs, which condense

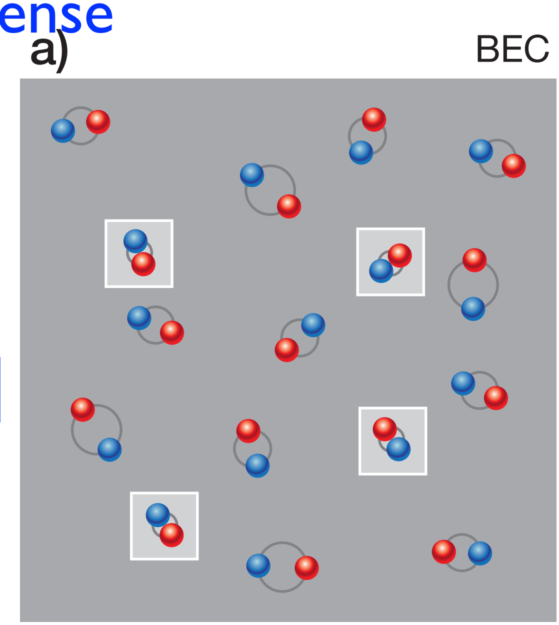
<u>repulsive interaction</u> $a > 0$

pairs of fermions bind
to form diatomic molecules,
which condense



BCS superfluid

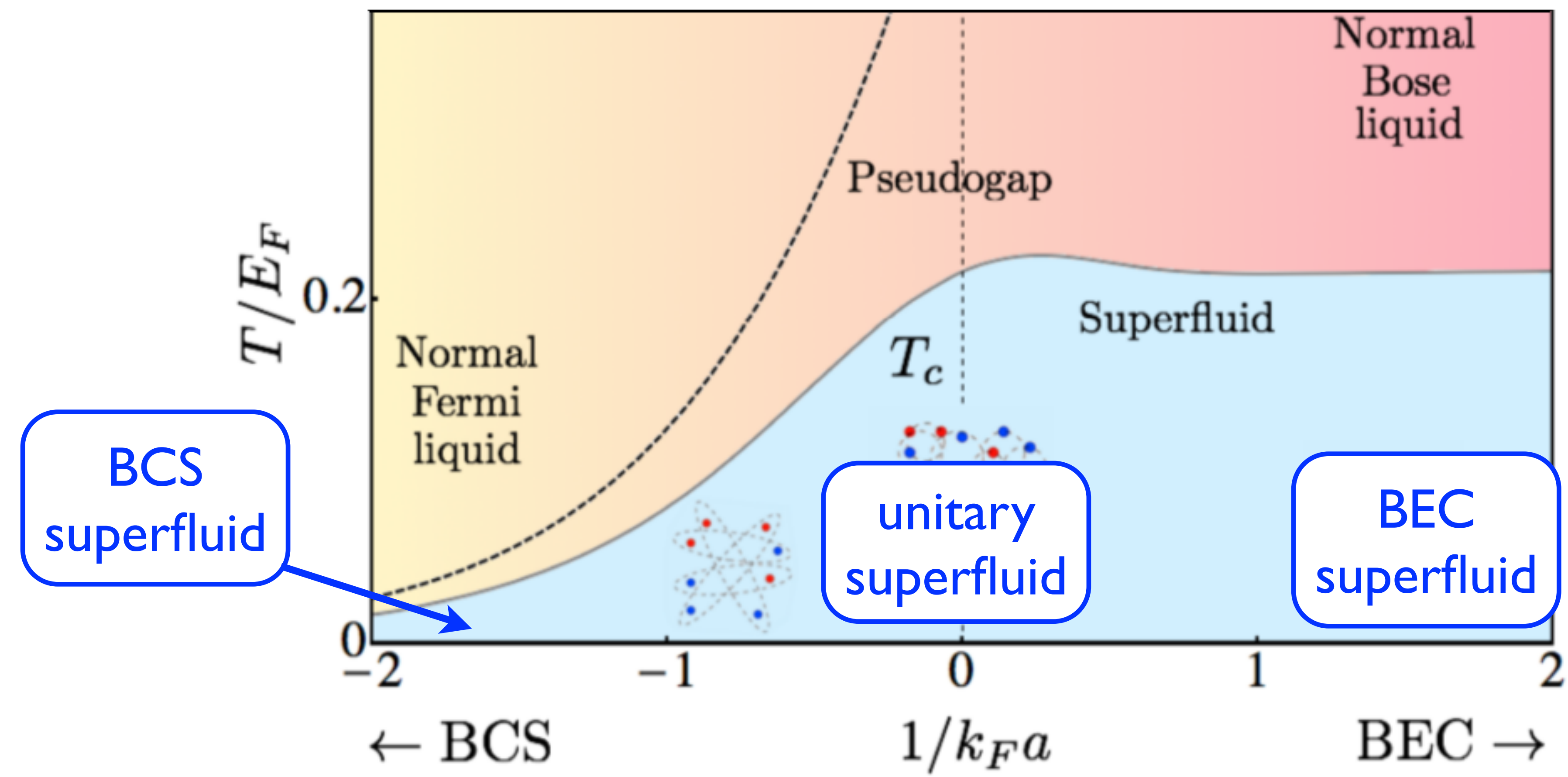
BEC superfluid



What happens in the unitary limit?

Fermi Gas with Two Spin States

phase diagram for homogeneous balanced gas ($n_1 = n_2$)

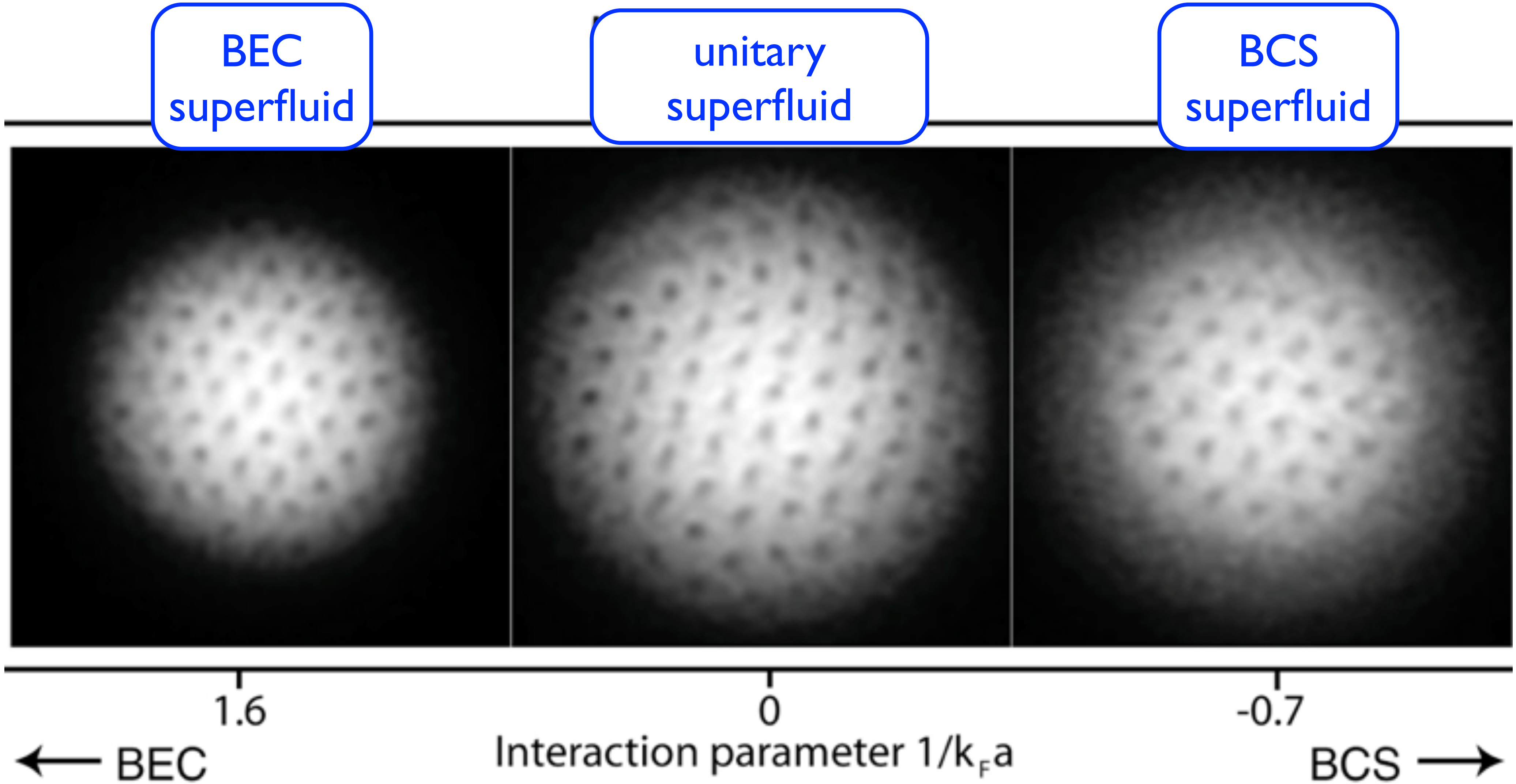


smooth crossover through unitary limit! Leggett 1980

Fermi Gas with Two Spin States

signature of superfluidity: vortices!

Ketterle group (MIT) using ^6Li atoms 2005



Fermi Gas with Two Spin States

spin-imbalanced gas ($n_1 > n_2$)

Phase diagram?

dimensionless variables: $1/k_F a$, T/E_F, and $n_2/(n_1+n_2)$

<u>homogeneous phases</u>

normal

superfluid with gapped fermions

superfluid with gapless fermions?

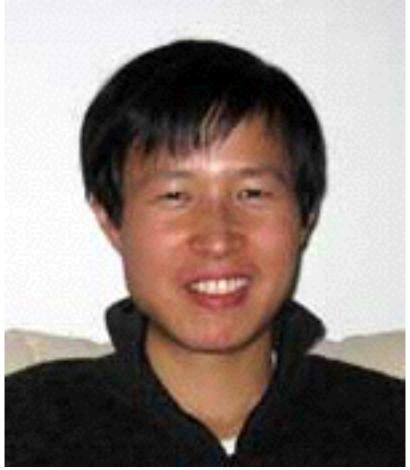
Sarma?

<u>nonhomogeneous phases</u>

Fulde-Ferrel?

Larkin-Ovchinnikov?

Contact



In 2005, a graduate student
at the University of Chicago named Shina Tan
introduced a new concept into many-body physics
called the "Contact"

The contact appears in many "Universal Relations"
that hold for any state of the system
(few-body or many-body, trapped or homogeneous, normal or superfluid, ...)

The contact relates the thermodynamics
to the tails of correlation functions.

The contact plays a central role in many of the
most important probes of ultracold atoms
(photoassociation, rf spectroscopy, photoemission spectroscopy...)

What is the Contact?

- contact is the thermodynamic variable conjugate to *1/a*

- the contact *C* is extensive:
  integral over space of the contact density $\mathcal{C}(\vec{R})$

$$C = \int d^3R \, \mathcal{C}(\vec{R})$$

- contact has dimensions 1/(length)
  contact density has dimensions $1/(\text{length})^4$

- contact density measures the number of 1-2 pairs per $(\text{volume})^{4/3}$

Tail of the momentum distribution

Shina Tan cond-mat/0505200

momentum distribution has a power-law tail that falls like $1/k^4$

$$n_\sigma(k) \longrightarrow \frac{1}{k^4} C$$

same coefficient C for both spins: $\sigma = 1,2$

C is the contact

normalization: $\int \frac{d^3k}{(2\pi)^3} n_\sigma(k) = N_\sigma$

"Adiabatic relation"

Shina Tan cond-mat/0508320

change in free energy
from small change in scattering length *a*

$$\frac{d}{da}F = \frac{\hbar^2}{4\pi m a^2}C$$

If *C* is known as a function of *a*,
it can be integrated to get *F*.
C determines all other thermodynamic functions!

Contact

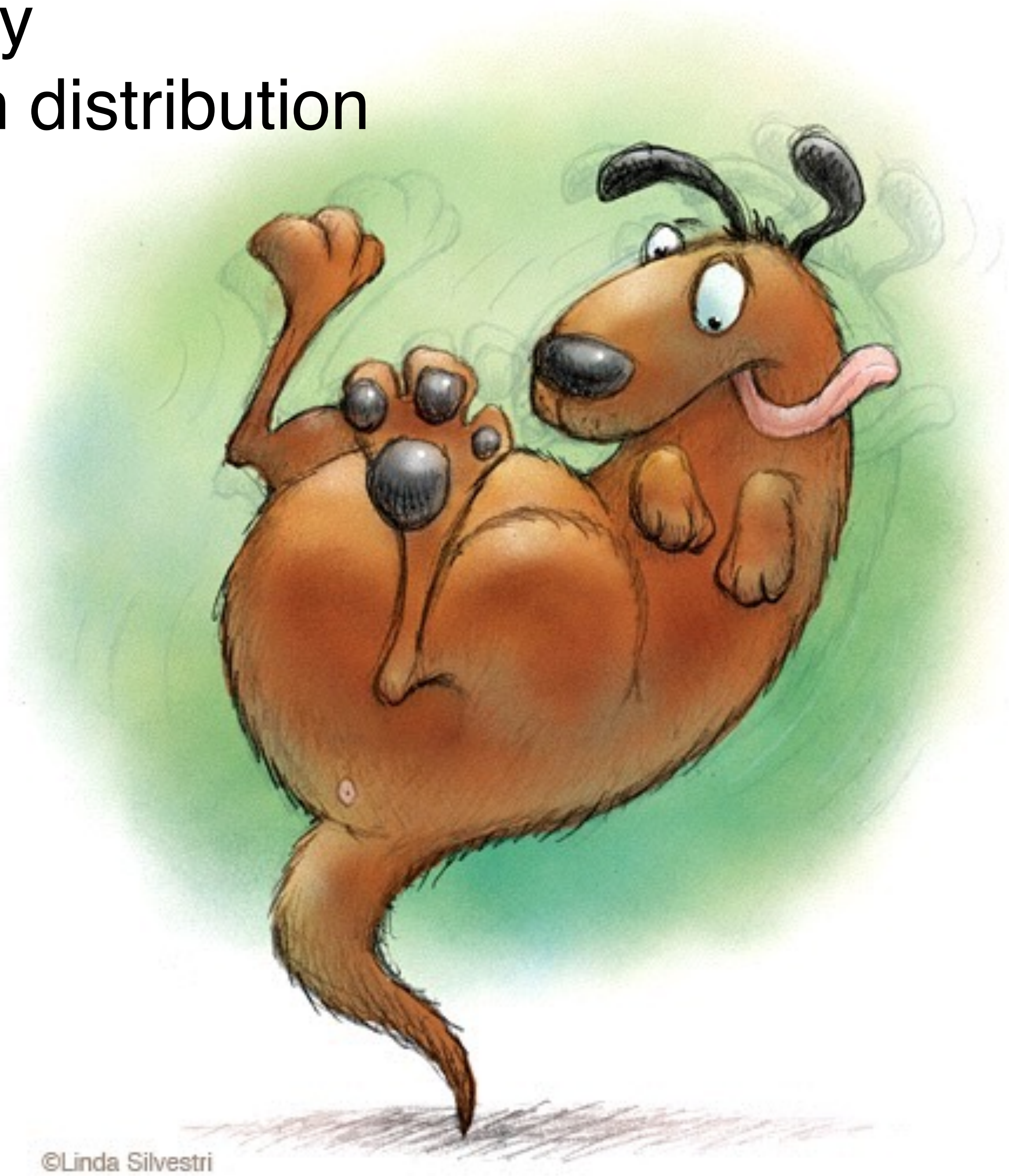
Contact can be defined by
tail of the momentum distribution

$$n_\sigma(k) \longrightarrow \frac{1}{k^4} C$$

Contact determines
thermodynamics

$$\frac{d}{da} F = \frac{\hbar^2}{4\pi m a^2} C$$

Tail wagging the dog?



Contact

QFT Derivation

Braaten and Platter 2008

interaction energy density $\mathcal{H}_{\rm int} = \frac{g_0}{m}\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1$

contact density operator $\mathcal{C} = g_0^2\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1$

- Adiabatic relation

$$\frac{d}{da}F = \frac{\hbar^2}{4\pi m a^2}C$$

from renormalization of effective field theory

- Tail of the momentum distribution

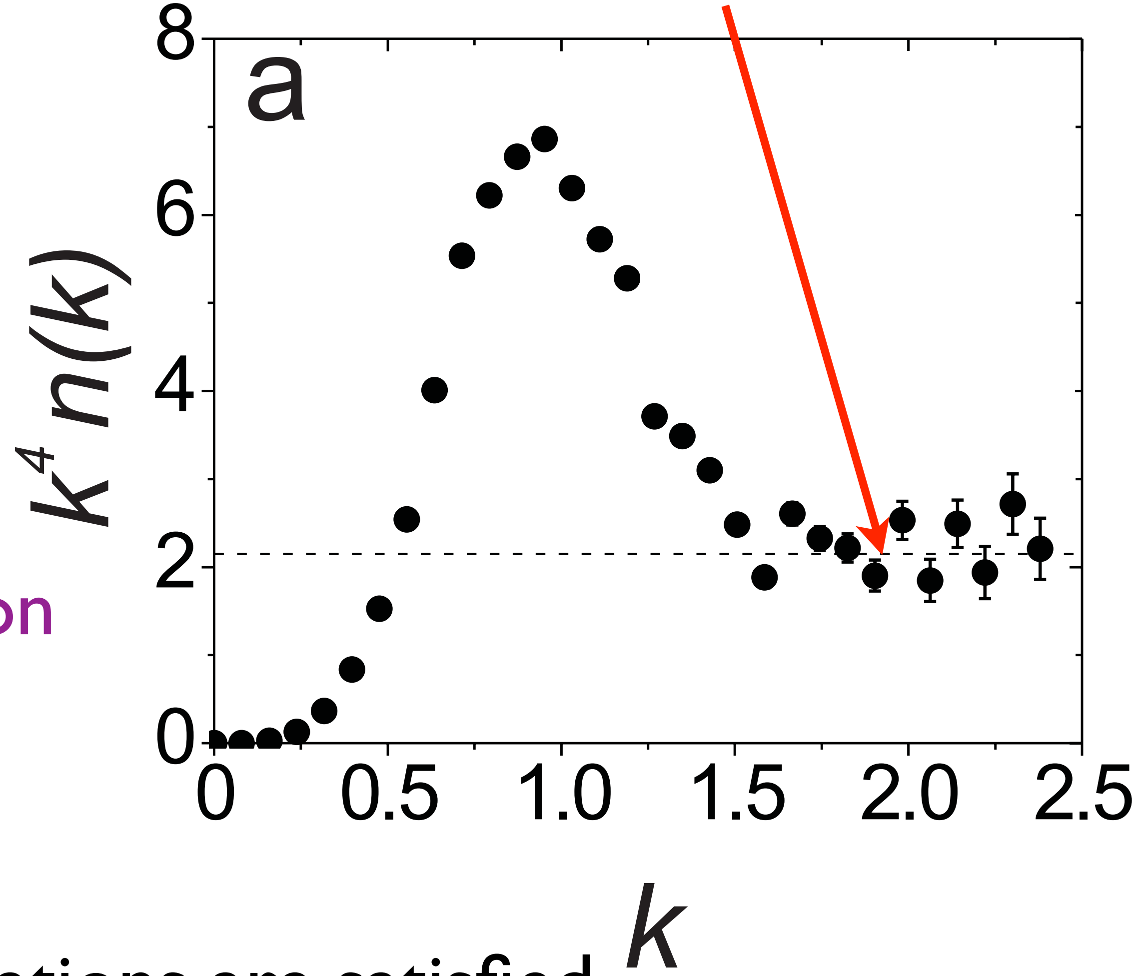
$$n_\sigma(k) \longrightarrow \frac{1}{k^4}C$$

from operator product expansion

Experimental Validation

Jin group (JILA) using ^{40}K atoms 2010

- verified that momentum distribution has $1/k^4$ tail!



- measured contact using
  - -- momentum distribution
  - -- rf spectroscopy
  - -- virial theorem

- verified that universal relations are satisfied

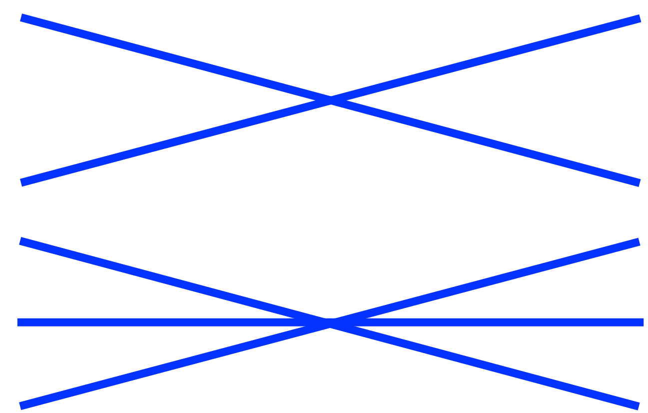
Identical Bosons

<u>not</u> the simplest QFT for ultracold atoms!

<u>point interactions</u>

$2 \rightarrow 2$

$3 \rightarrow 3$



<u>interaction parameters</u>

scattering length a

Efimov parameter κ_* momentum scale on which dependence can only be log-periodic

Weak coupling

quantum field: ψ — scaling dimension 3/2

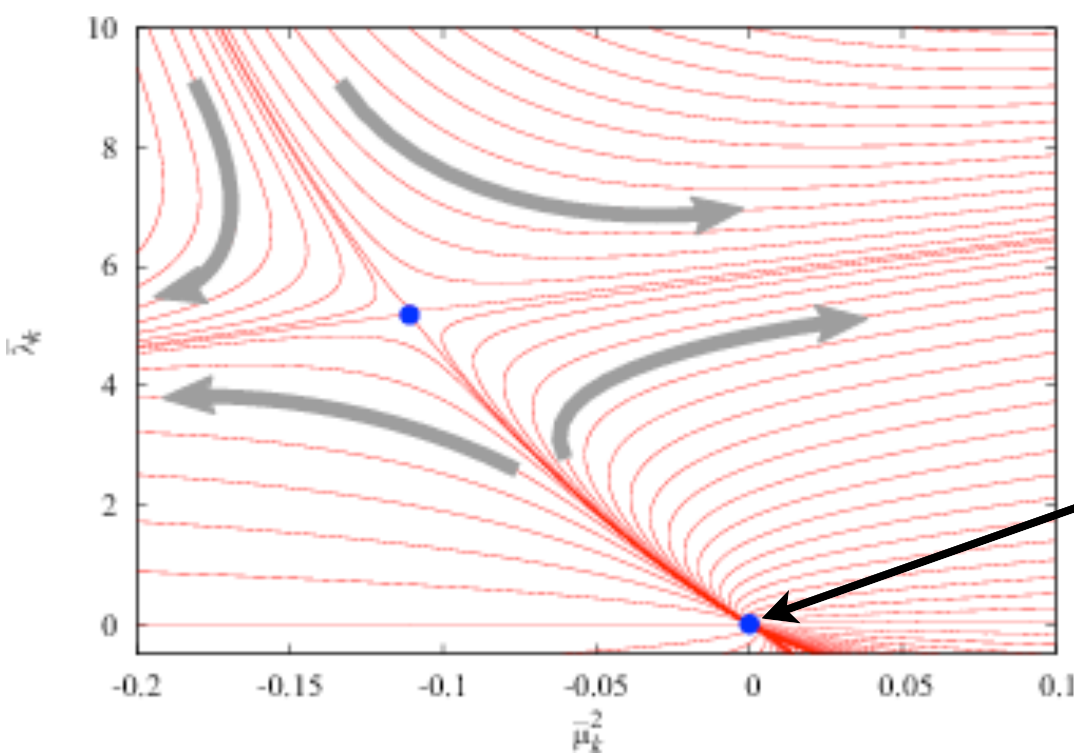
interaction operator: $\psi^\dagger\psi^\dagger\psi\psi$ — 6

$\psi^\dagger\psi^\dagger\psi^\dagger\psi\psi\psi$ — 9

(>5 $\Longrightarrow$ irrelevant)

perturbatively non-renormalizable!

$$g_0 = 8\,\pi\, a \text{ (+ counterterms)}$$



RG fixed point:
free field theory

Strong coupling

quantum field: ψ scaling dimension 3/2

interaction operators: $\psi^\dagger\psi^\dagger\psi\psi$ scaling dimension 4

(<5 $\Longrightarrow$ relevant)

$\psi^\dagger\psi^\dagger\psi^\dagger\psi\psi\psi$ scaling dimension 5

(=5 $\Longrightarrow$ marginal)

nonperturbatively renormalizable!

anomalous scaling dimensions!

two interaction parameters

scattering length *a*

3-body parameter

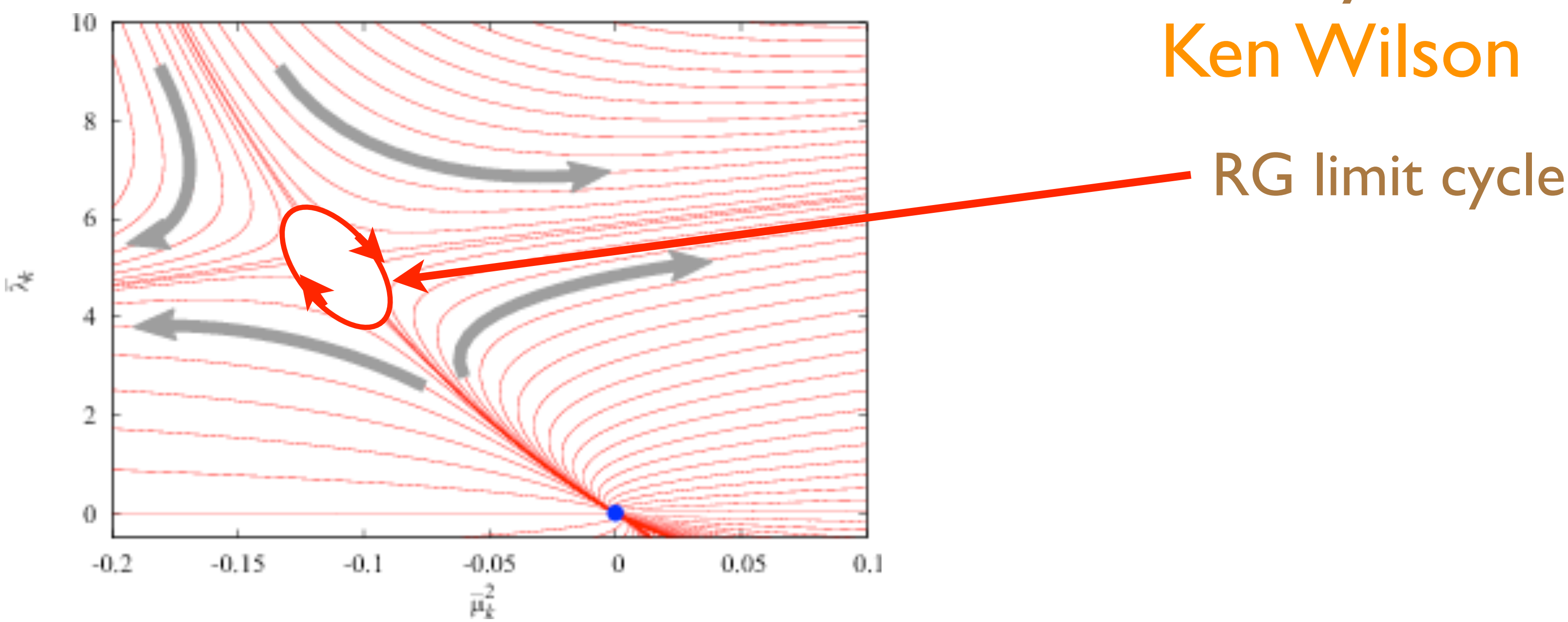
Strongly-coupled Quantum Field Theory

can be defined by

Renormalization Group flow to ultraviolet fixed point

or limit cycle or ...

Ken Wilson



complete flow around the RG limit cycle

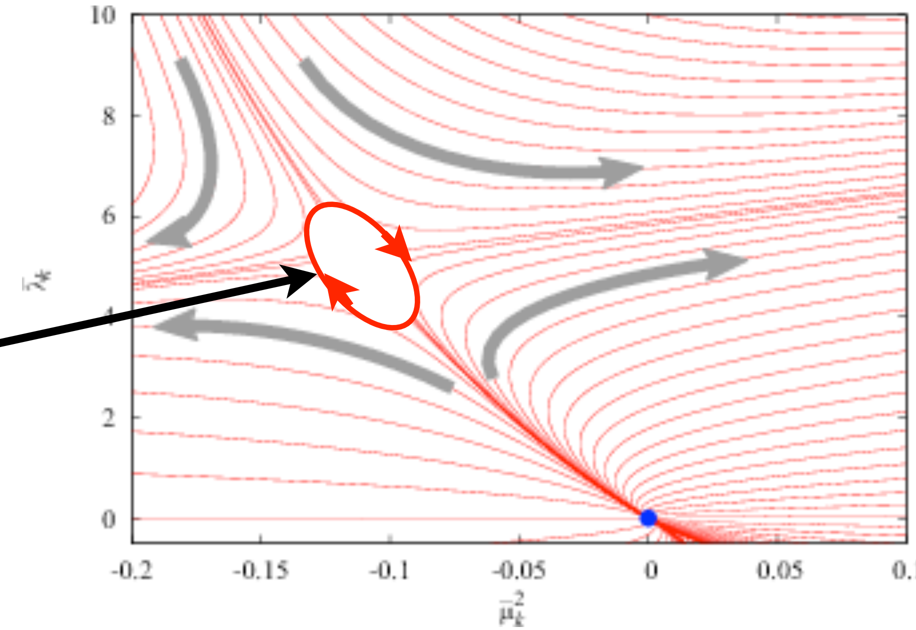
changes scale by a discrete scaling factor λ_0

but returns to the same system

$\Longrightarrow$ discrete scale invariance!

Identical Bosons

strongly interacting QFT
can be defined
by RG limit cycle



Renormalization of local QFT for identical bosons
involves RG limit cycle
with discrete scaling factor 22.7
Bedaque, Hammer, and van Kolck 1999

implies the existence of a physical momentum scale κ_*
that is equivalent to $\lambda_0\, \kappa_*$

$\Longrightarrow$ dependence on κ_* can only be log-periodic
(such as $\sin[s_0 \log(k/\kappa_*)]$, where $\lambda_0 = e^{\pi/s_0}$)

2-Body Problem

can be solved analytically

<u>Cross section</u>

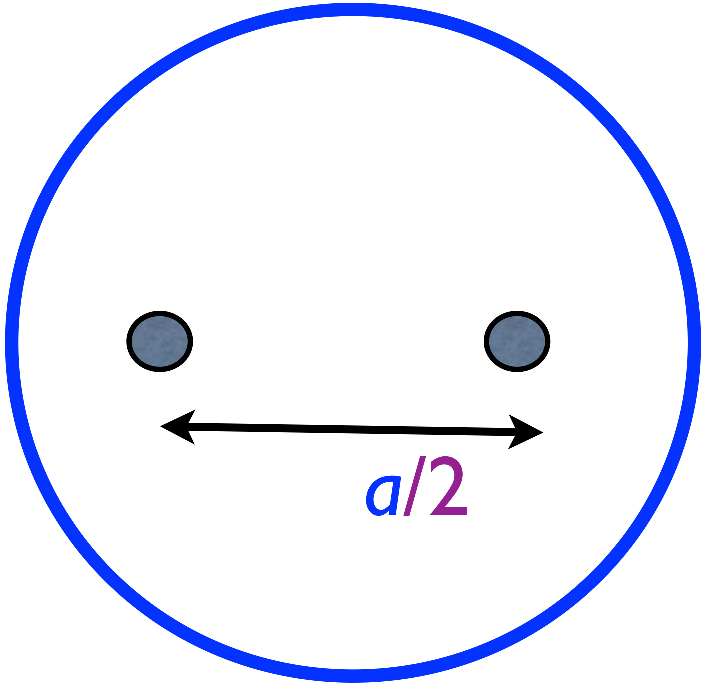
$\sigma \rightarrow 8\pi a^2$ at low energy

$\rightarrow 8\pi \hbar^2/(m E)$ at high energy

<u>Diatomic molecule</u> if $a > 0$

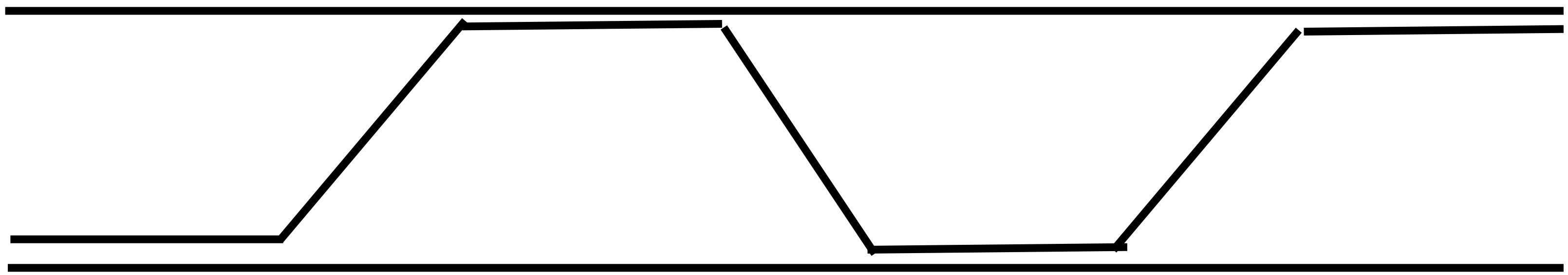
binding energy: $\hbar^2/(m a^2)$

mean radius: $a/2$



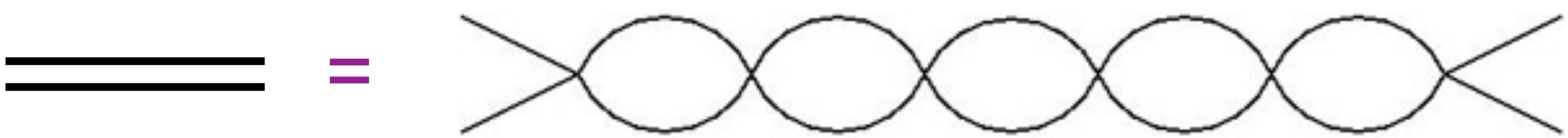
3-Body Problem

can be solved exactly numerically



+ 3→3 interactions

where



4-Body Problem

can be solved exactly numerically

5-Body Problem

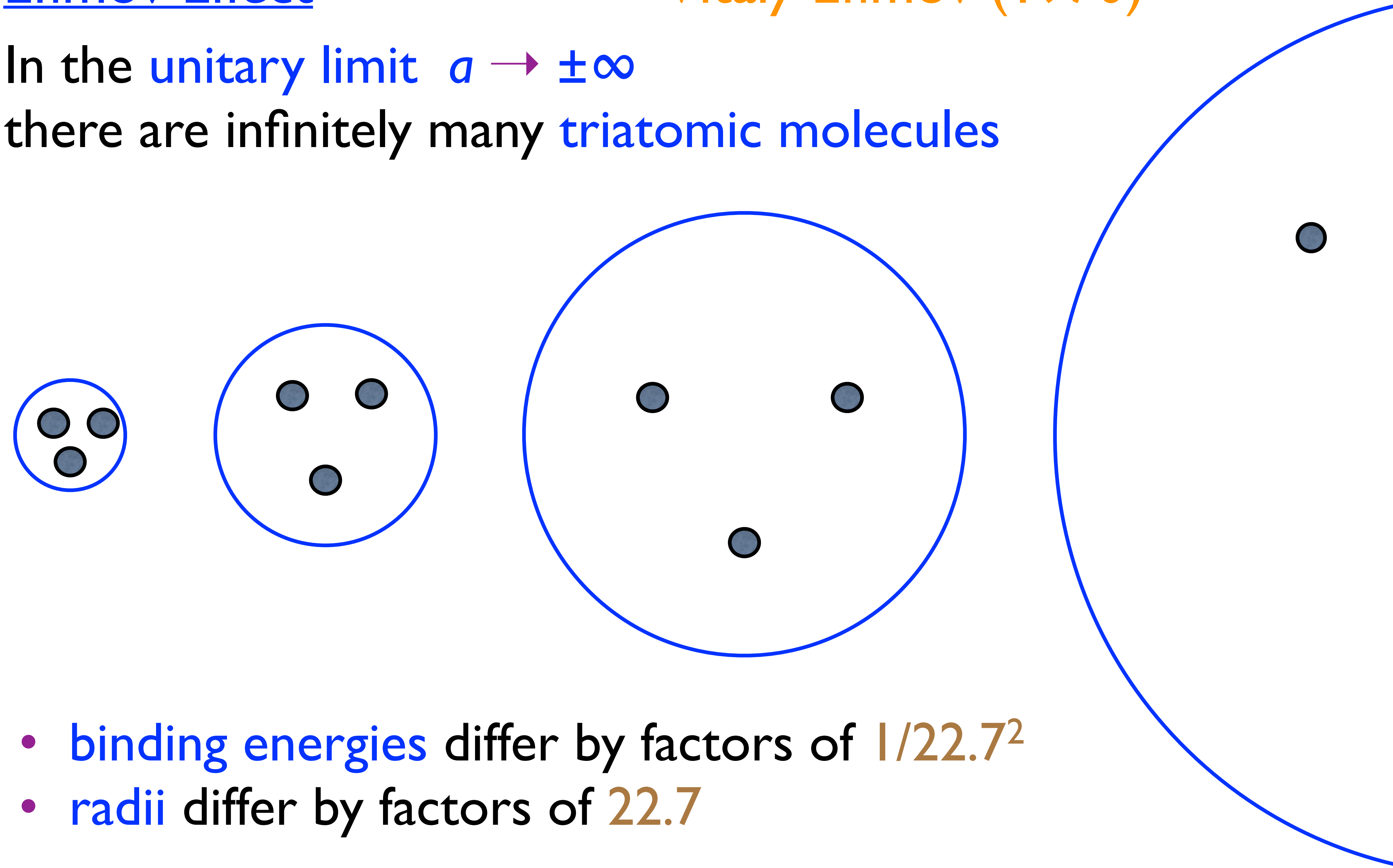
frontier of few-body physics

3-Body Problem

Efimov Effect

Vitaly Efimov (1970)

In the unitary limit $a \to \pm\infty$
there are infinitely many triatomic molecules



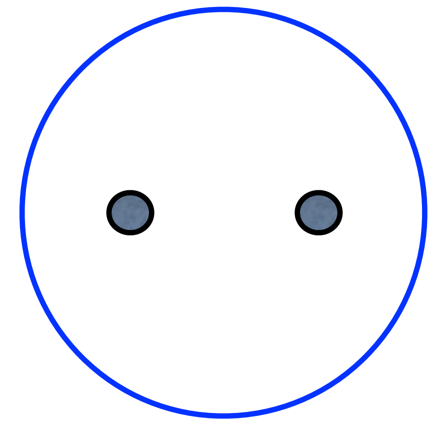
- binding energies differ by factors of $1/22.7^2$
- radii differ by factors of 22.7

Low-energy Nuclear Physics

Both *NN* scattering lengths
are large compared to the range
$a_{i=1} = -21$ fm $\quad a_{i=0} = +5.4$ fm

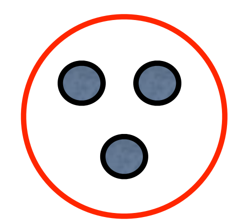
<u>2-nucleon bound states</u>

deuteron (pn): binding energy ≈ 2.2 MeV
[dineutron (nn): almost bound]



<u>3-nucleon bound states</u>

triton (pnn): binding energy ≈ 7.6 MeV
^{3}He (ppn): ≈ 7.7 MeV

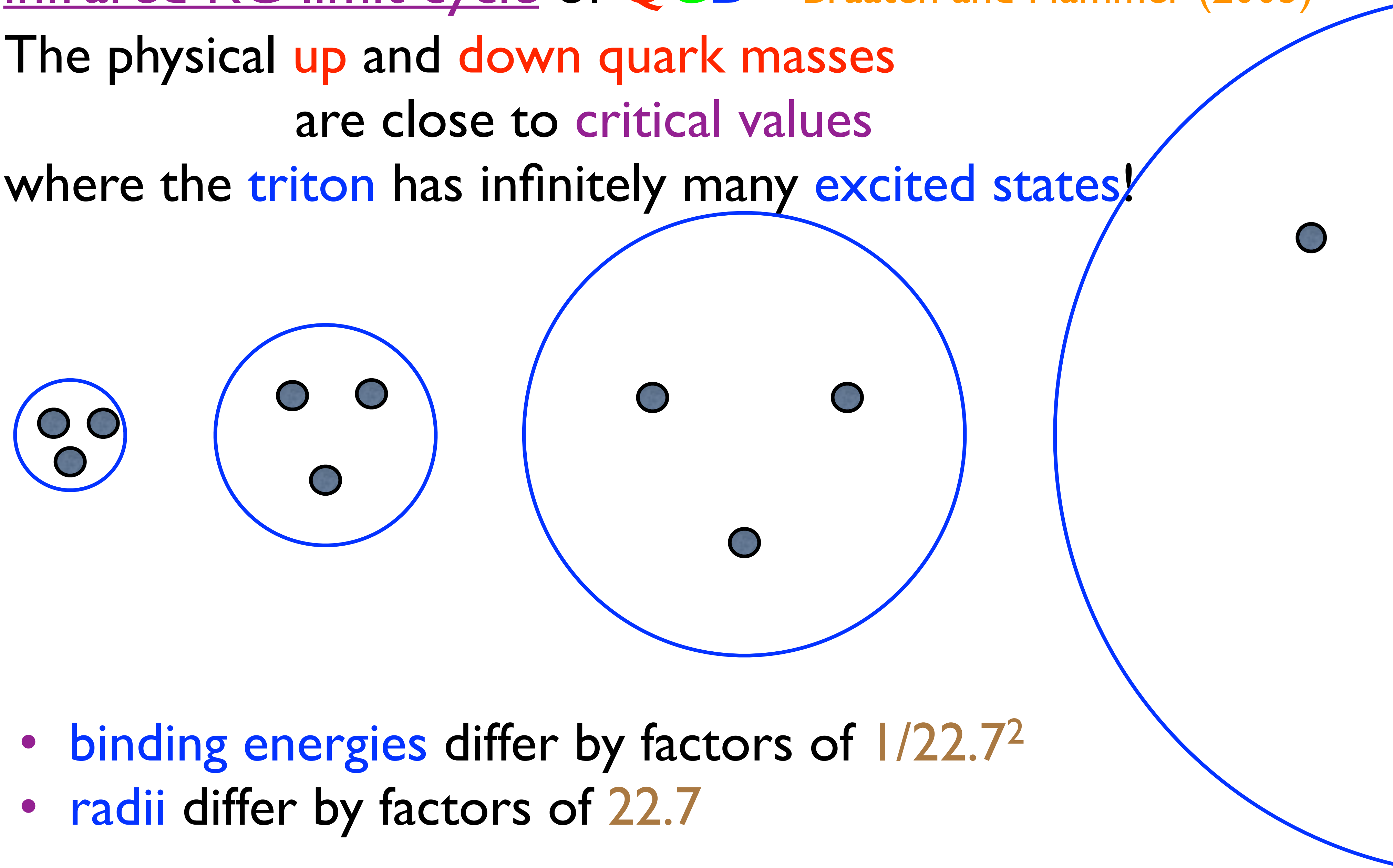


What would happen if you changed
the up and down quark masses?

Low-energy Nuclear Physics

<u>infrared RG limit cycle</u> of QCD   Braaten and Hammer (2003)

The physical up and down quark masses are close to critical values where the triton has infinitely many excited states!

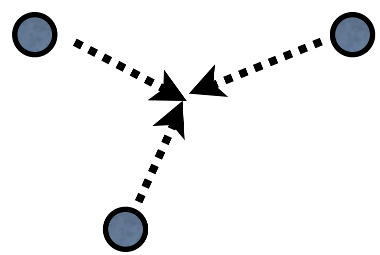


- binding energies differ by factors of $1/22.7^2$
- radii differ by factors of 22.7

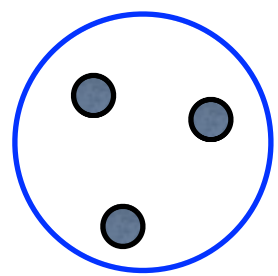
3-Body Recombination

resonant enhancement from Efimov trimer
near 3-atom threshold

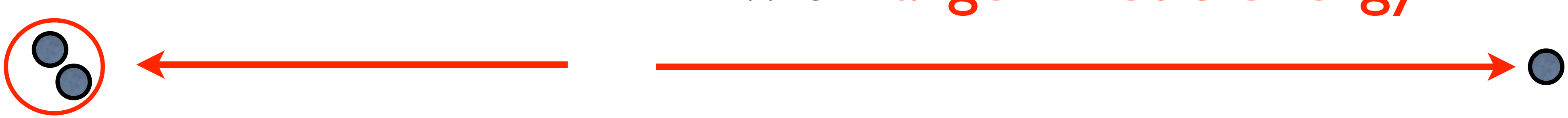
- three low-energy atoms collide



- they form a virtual Efimov trimer

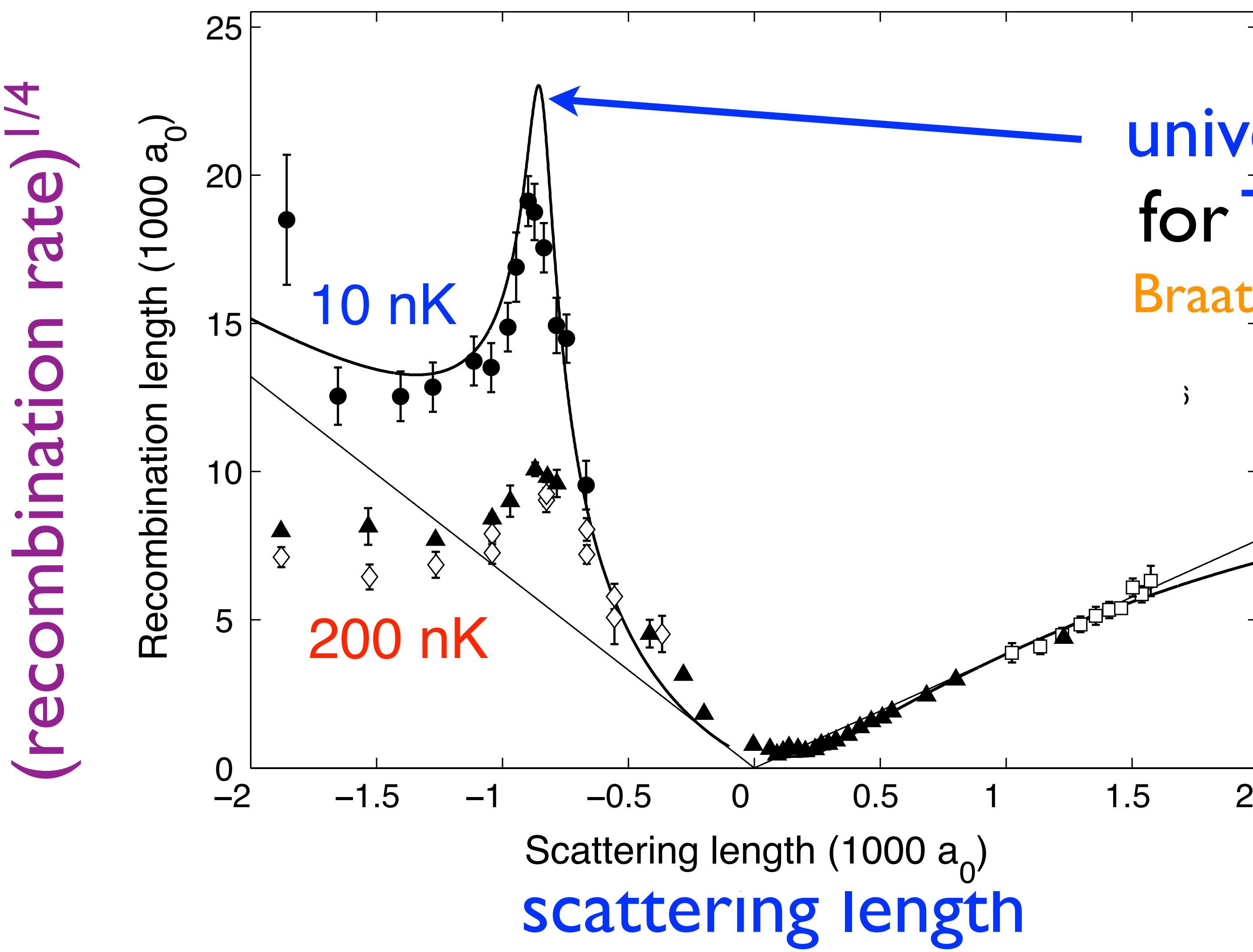


- trimer decays into atom and dimer
with large kinetic energy



discovery of Efimov trimer in ^{133}Cs atoms through atom loss resonance

Grimm group (Innsbruck) Nov 2005



universal line shape for T=0 from QFT

Braaten & Hammer 2003

Universal Relations for Identical Bosons

Braaten, Kang, Platter 2011

- derived from Operator Product Expansion
- involve 2-body contact C_2 and 3-body contact C_3!

Tail of the momentum distribution

$1/k^4$ tail plus log-periodic $1/k^5$ tail

$$n(k) \longrightarrow \frac{1}{k^4} C_2 + \frac{F(k)}{k^5} C_3$$

$$F(k) = 89.3 \sin[2 s_0 \log(k/\kappa_*) - 1.34]$$

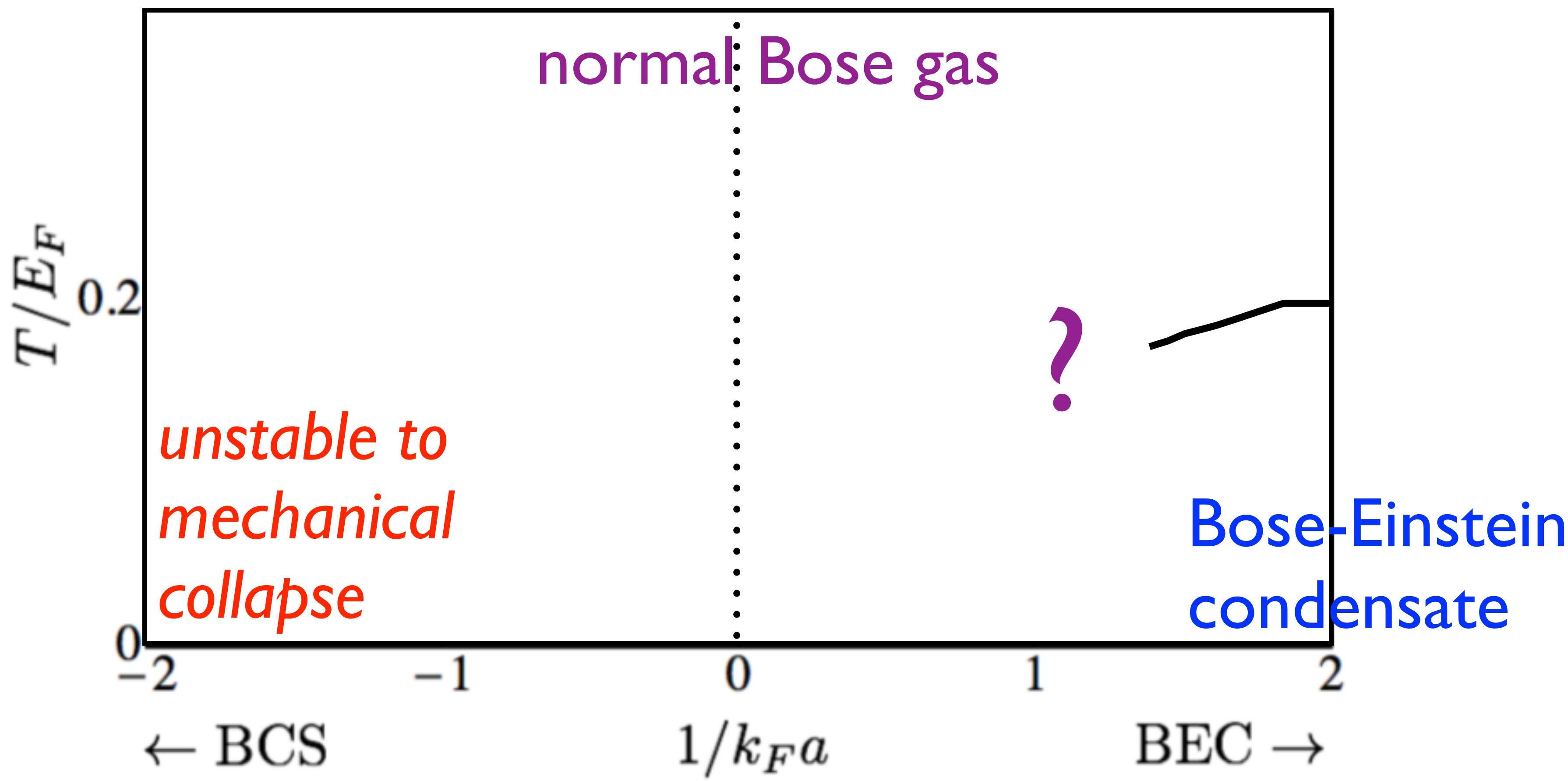
where s_0 = 1.00624

κ_* = binding momentum of Efimov trimer at $a = \pm\infty$

(determined by position of Efimov loss resonance)

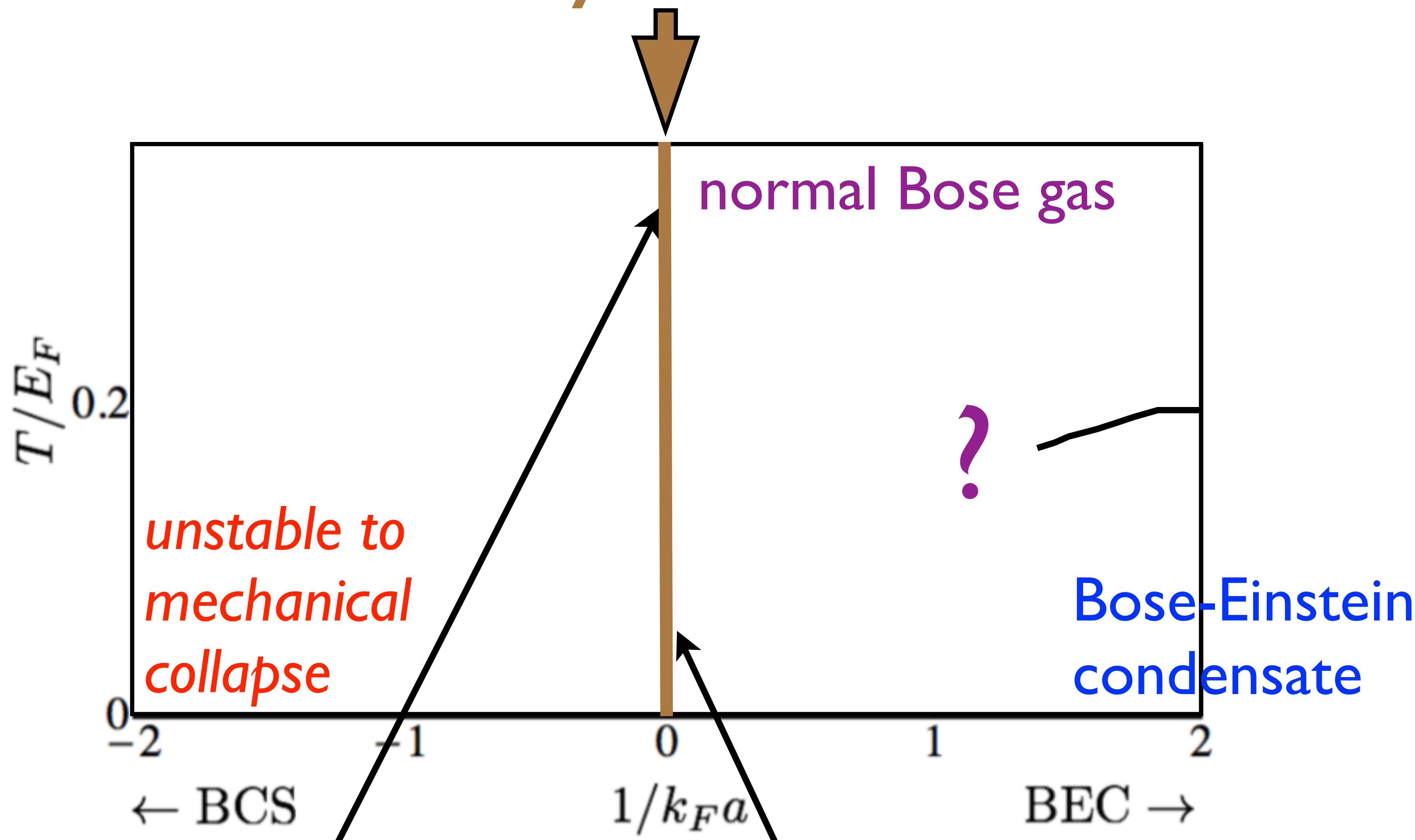
Bose Gas

phase diagram for homogeneous gas



Where is boundary of BEC superfluid phase?
Does it extend to unitary limit?

Unitary Bose Gas



1st experiments in 2013

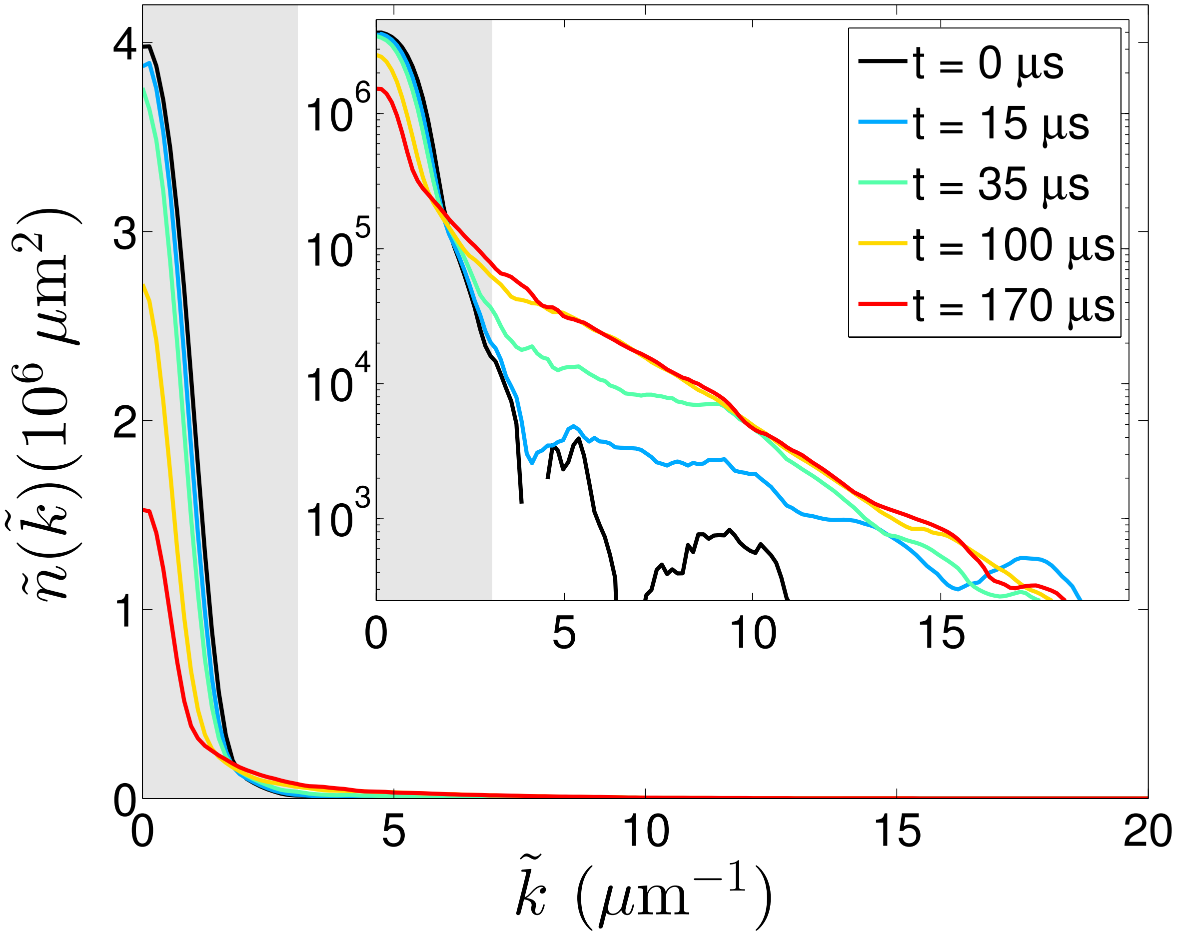
Salomon group (Ecole Normale)

Jin group (JILA)

Jin group (JILA) 2013

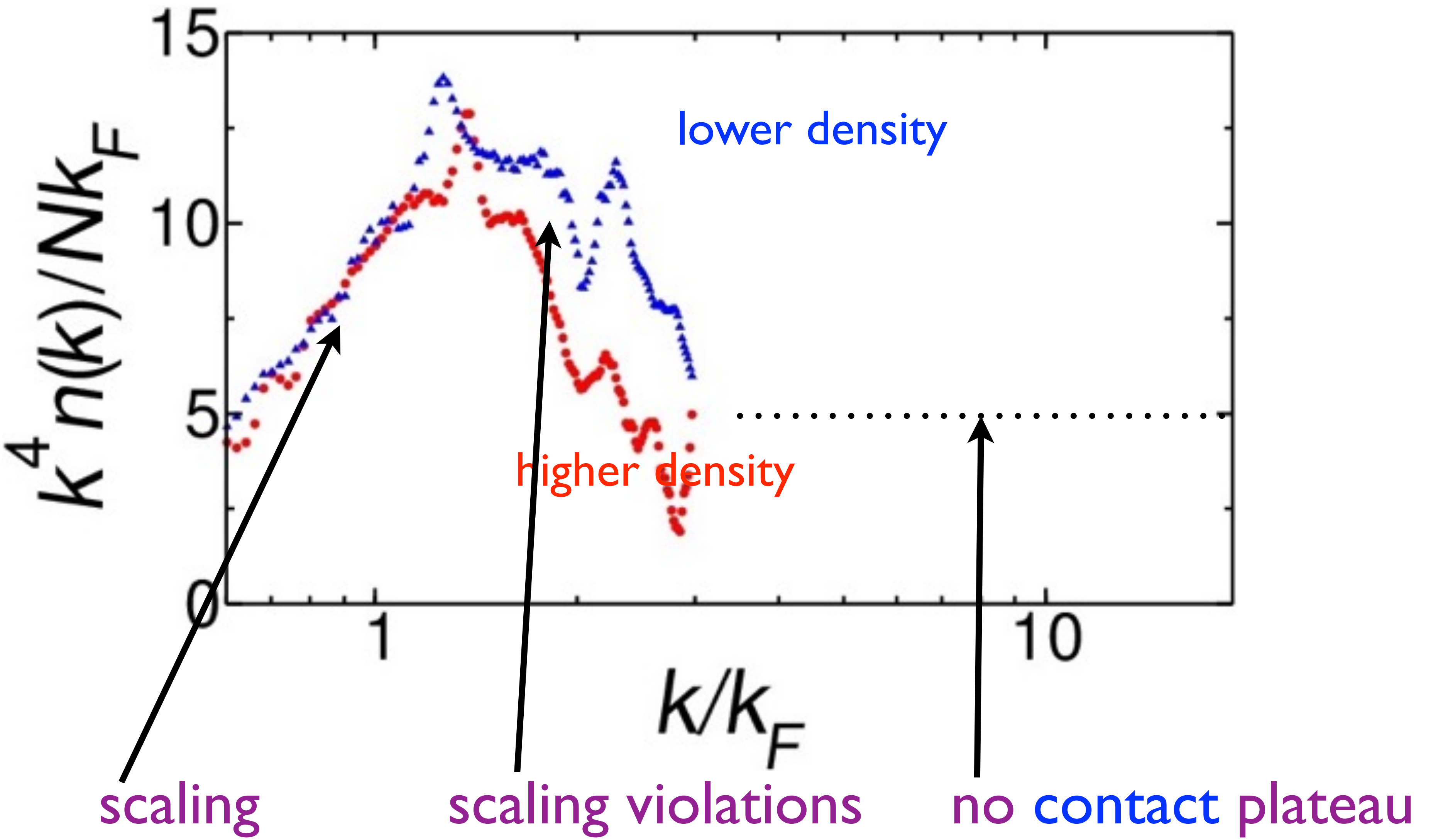
Weakly interacting BEC of ^{85}Rb atoms
Ramped suddenly to unitary limit ($a = \pm\infty$)
Wait for a variable holding time t
Measure momentum distribution

High-momentum tail:
grows, then
saturates after 100 μs



JILA momentum distributions

- multiply by k^4
- scale by $k_F = (6\pi^2\langle n\rangle)^{1/3}$



Universal Relations for Identical Bosons

Braaten, Kang, Platter 2011

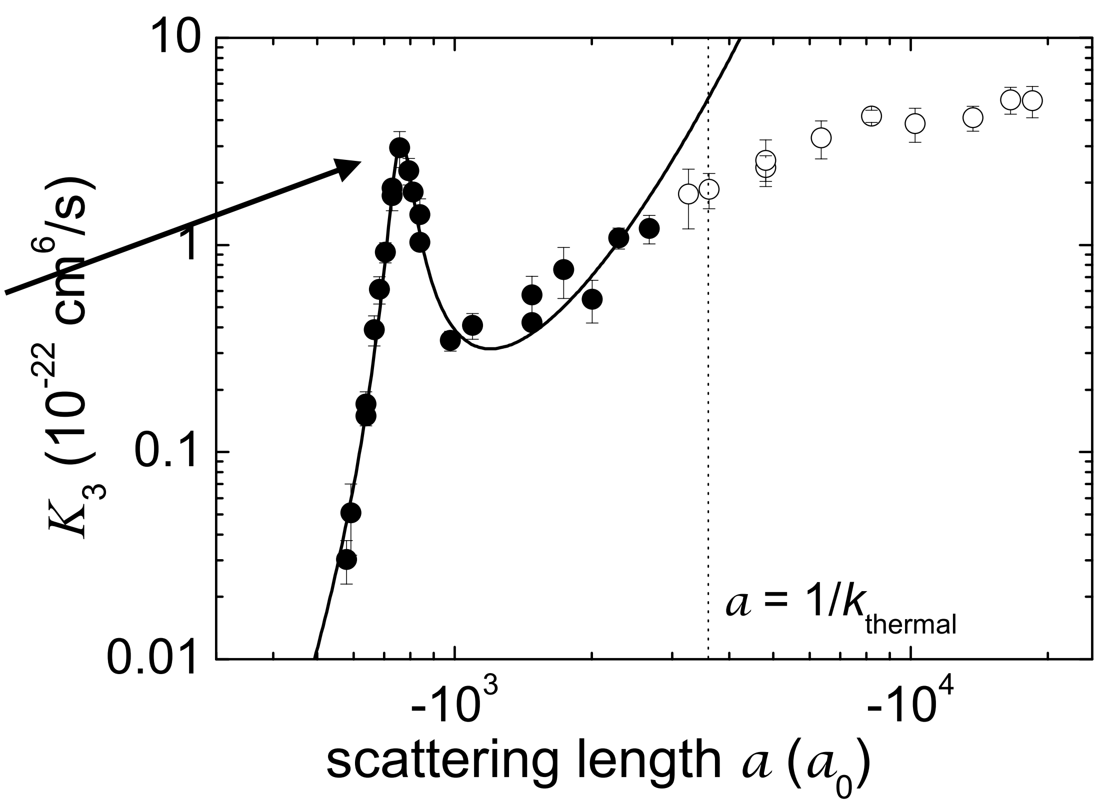
Tail of the momentum distribution

$1/k^4$ tail plus log-periodic $1/k^5$ tail

$$n(k) \longrightarrow \frac{1}{k^4} C_2 + \frac{F(k)}{k^5} C_3 \qquad F(k) = 89.3 \sin[2 s_0 \log(k/\kappa_*) - 1.34]$$

where s_0 = 1.00624

κ_* determined by position of Efimov loss resonance measured by JILA 2011



only unknowns are C_2 and C_3

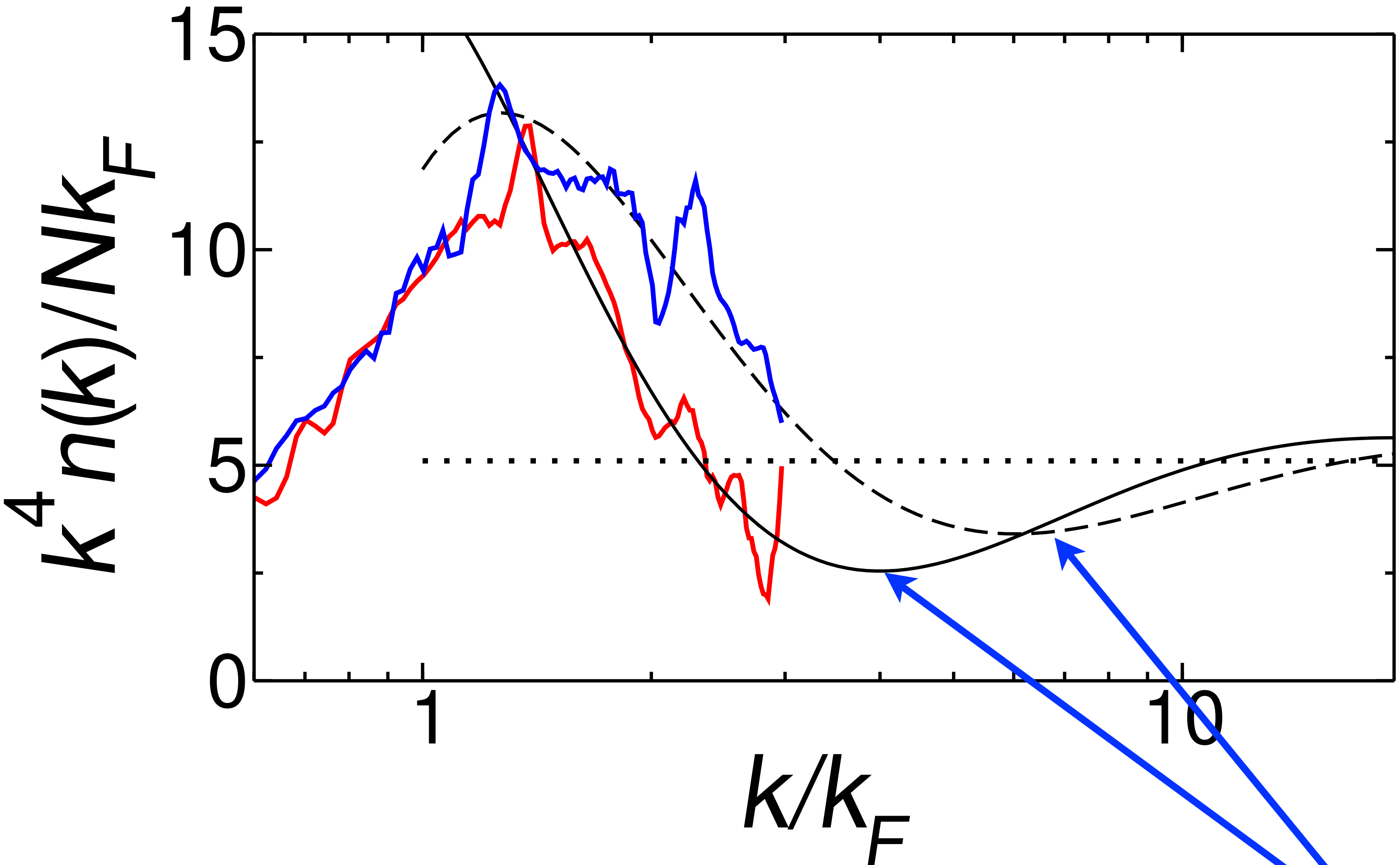
Unitary Bose gas

JILA momentum distributions

can be fit very well by $1/k^4$ tail from 2-body contact

plus log-periodic $1/k^5$ tail from 3-body contact

Smith, Kang, Platter, Braaten 2014



- 2 adjustable parameters: 2-body and 3-body contacts
- positions of minima determined by JILA observation of Efimov trimer

Summary

Ultracold atoms
can be approximated by point particles
with zero-range interactions
can be described by a local quantum field theory

The relevant strongly coupled QFT's can be defined
by an RG fixed point (fermions with 2 spin states)
or by an RG limit cycle (identical bosons)

Methods of QFT developed in particle physics
have powerful applications to ultracold atoms
(e.g. renormalization,
operator product expansion, ...)

Applications of QFT to ultracold atoms
can also provide insights into particle physics.