

# Measurement of antineutrino scattering on free protons in MINER $\nu$ A

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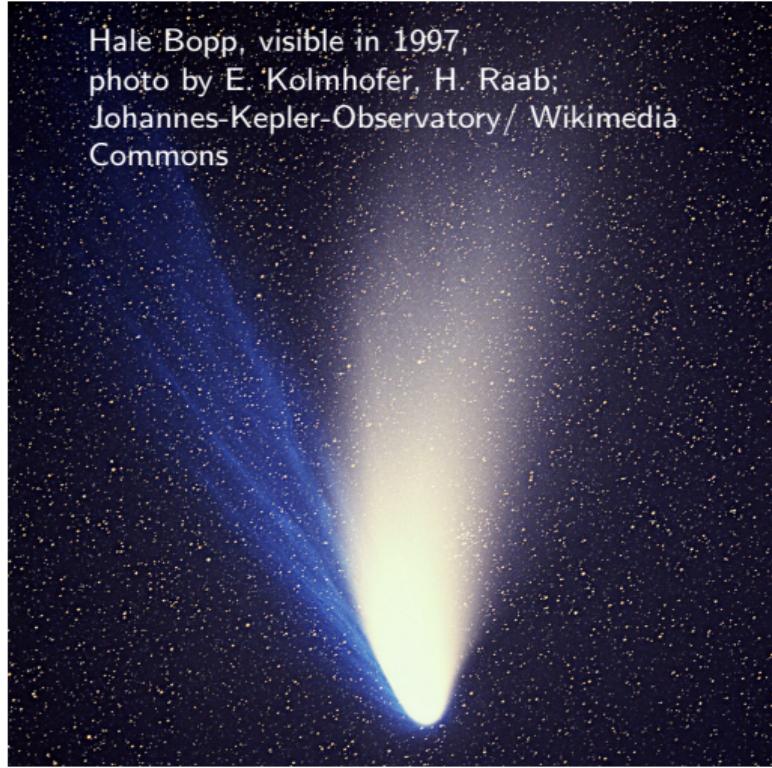




## High resolution image of Andromeda galaxy

Credit:NASA, ESA, J. Dalcanton (University of Washington, USA), B. F. Williams (University of Washington, USA), L. C. Johnson (University of Washington, USA), the PHAT team, and R. Gendler.

Hale Bopp, visible in 1997,  
photo by E. Kolmhofer, H. Raab;  
Johannes-Kepler-Observatory/ Wikimedia  
Commons



Comet C/2022 E3 (ZTF), photo by Steven Bellavia. Should be visible today!



## Neutrinos are interesting because ...

Super-K and SNO confirmed neutrino oscillate in 1998 and 2001. Neutrinos are produced in one of the three flavor eigenstates :  $e, \mu, \tau$ , but travels as a mixture of mass eigenstates :

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

with the PMNS mixing matrix  $U$ :

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta_{CP}} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta_{CP}} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

where  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$ , and  $\delta_{CP}$  is the CP-violating phase.

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$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

$$|\nu_j(t)\rangle = e^{-i(E_j t - \vec{p}\vec{x})} |\nu_j(0)\rangle$$

$$\approx e^{-i\left(\frac{m_j^2 L}{2E}\right)} |\nu_j(0)\rangle$$

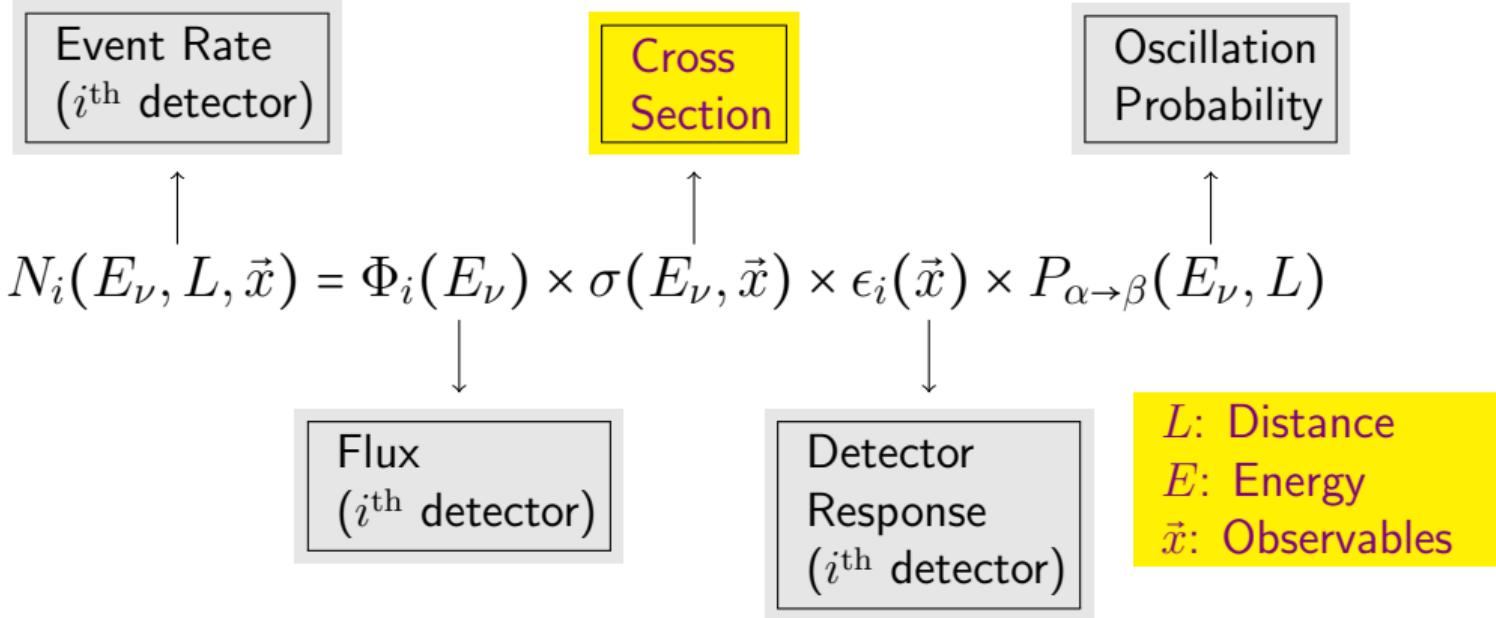
$$P_{\alpha \rightarrow \beta} = \langle \nu_\beta | \nu_\alpha \rangle$$

$$= \delta_{\alpha\beta} - 4 \sum \mathcal{R}(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) + \dots$$

$L$ : Distance

$E$ : Energy

$\Delta m_{ij}^2 = m_i^2 - m_j^2$

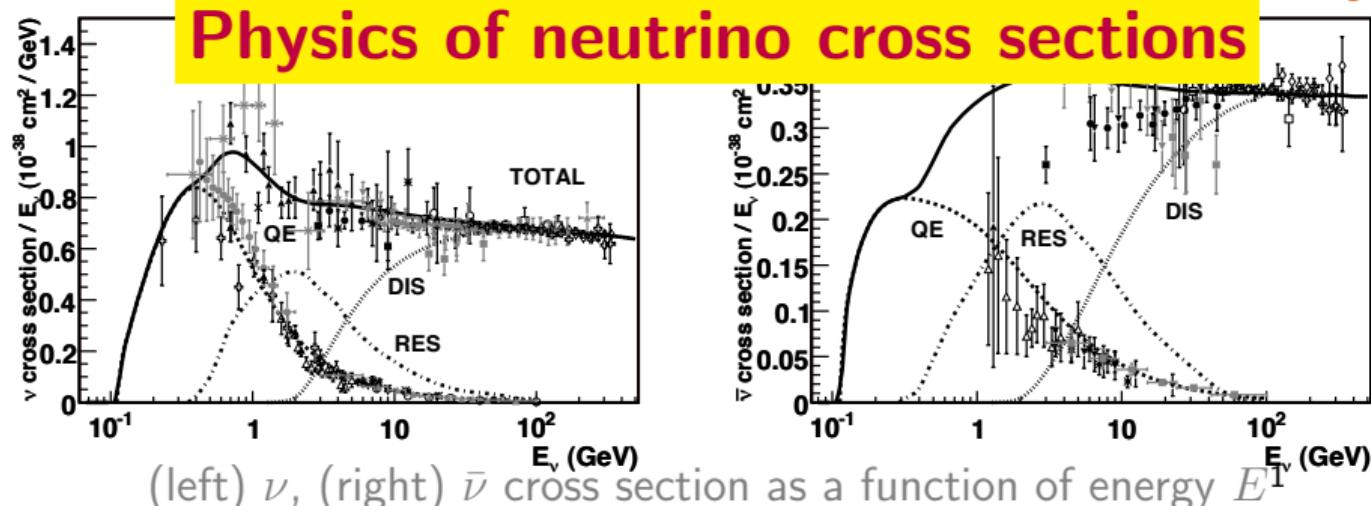


- Near/far detector scheme means potentially different  $\Phi$  and  $\epsilon$ .
- Systematics do not fully cancel.
- $\sigma$  contains nucleon and nuclear physics. Difficult to model.

An active areas of research ...



Important for oscillation measurements



## Challenges with neutrino measurements

- No direct knowledge of initial neutrino kinematics
- Rely on reconstructed observables of interactions to infer the initial kinematics
- Technologies that may not be sensitive to all observables
- Inference requires nuclear physics models
- Many nuclear targets exist ( $\text{CH}$ ,  $\text{H}_2\text{O}$ ,  $\text{Ar}$ ,  $\text{Fe}$ ,  $\text{Pb}$ , etc..)

Difficulty: No direct knowledge of the neutrino kinematics

- Neutrinos are neutral – do not ionize atoms
- Energy and momentum reconstruction depends on
  - ▶ visible calorimetric energy and/or
  - ▶ final state particle kinematics
- Don't see everything – need models.

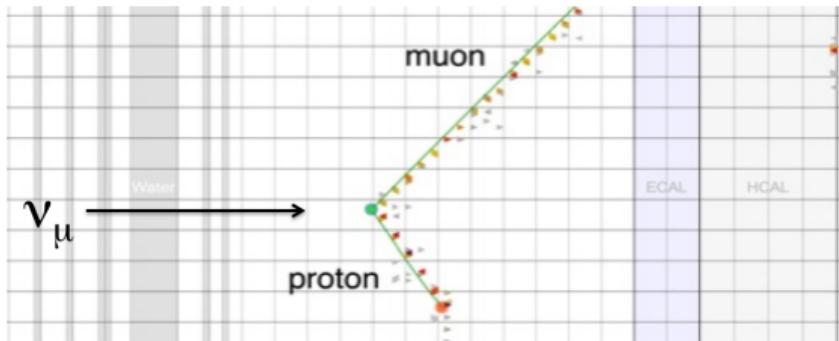


Figure: MINERvA event display, a muon and a proton in the final states

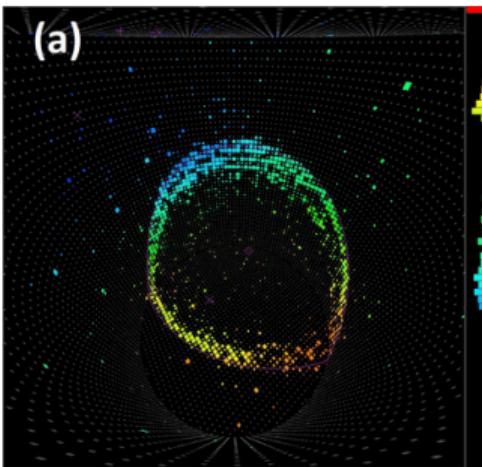


Figure: A muon-like ring in Super-K

# MINER $\nu$ A

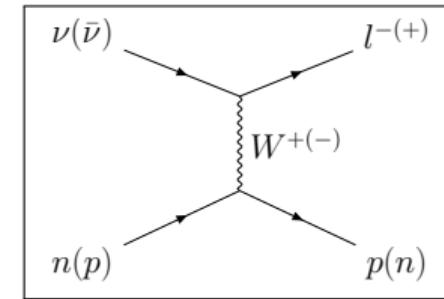
**Main Injector Neutrino ExpeRiment  $\nu$ -A** studies neutrino-nucleus scatterings at the few-GeV region:

- Precise measurements of the signal and background processes relevant to oscillation experiments.
- Improves understanding of nuclear effects that affect the reconstruction of neutrino energy.
- Demonstrates experimental techniques that benefit current and future oscillation experiments.

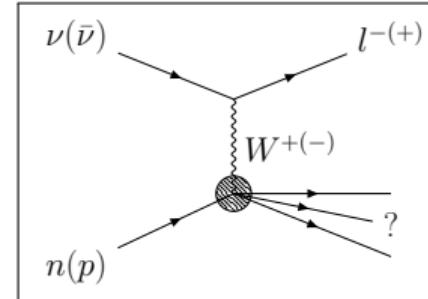


## Nucleon and nuclear physics

# Free nucleon (proton or neutron) response to charged current (CC) interactions

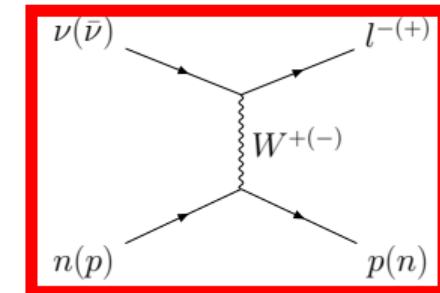


elastic

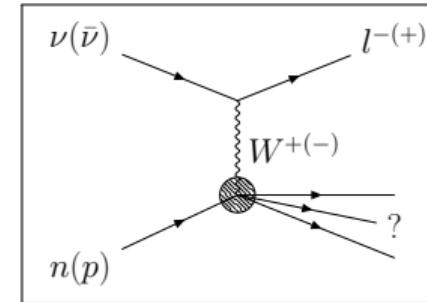


inelastic

# Folding in nuclear effects from multiple bound nucleons

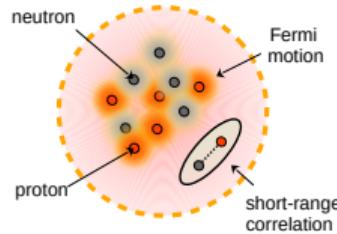


elastic

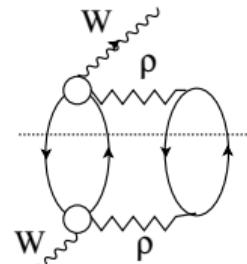


inelastic

**Quasi-elastic (QE):** inseparable nucleon and nuclear effects.

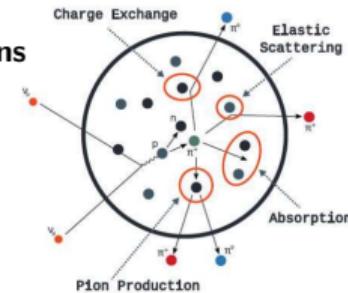


**Initial States:**  
Fermi motion  
short-range  
correlation  
binding energy, etc.



2p2h/mutli-nucleon effects

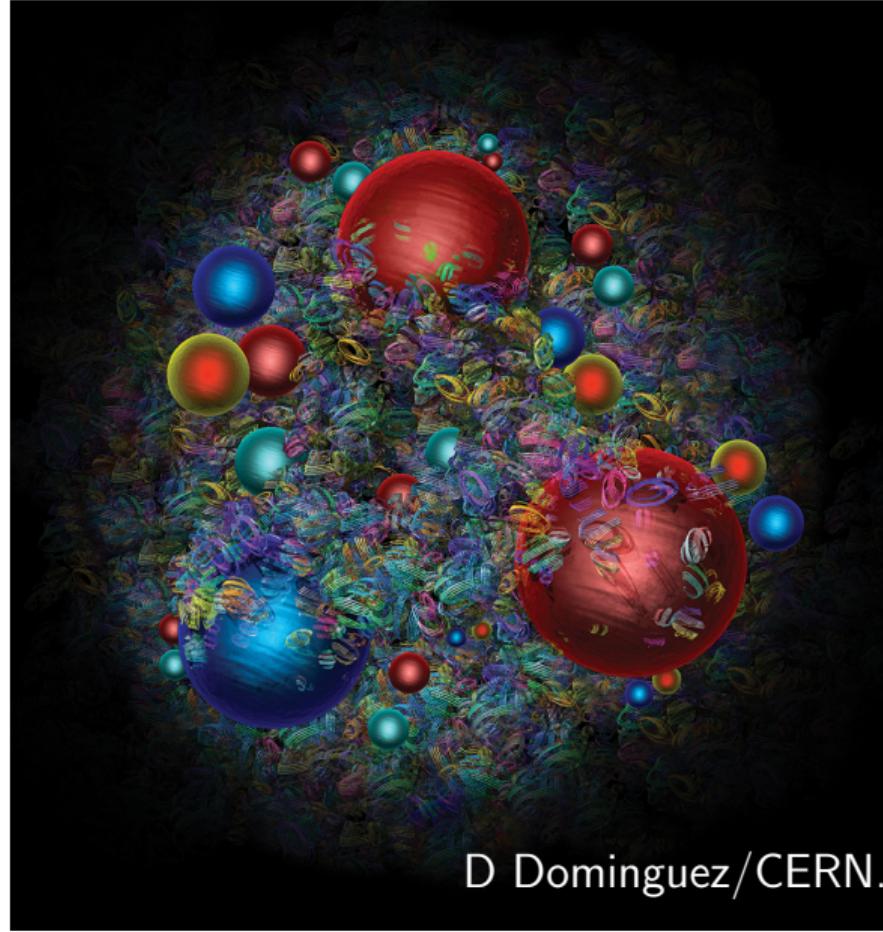
**Final State Interactions (FSI):**  
elastic, inelastic,  
charge-exchange,pion  
production, pion  
absorption, etc.



**Quasi-elastic-like (QELike):** only nucleons in the final states.

# Nucleon physics

Form factors parameterize QCD processes.



D Dominguez/CERN.

# Nucleon physics

Form factors parameterize QCD processes.

**Low 4-momentum squared ( $Q^2$ ):**

Fourier transform of charge distribution  
 $\sim$  charge radius as  $Q^2 \rightarrow 0$ .

i.e. exponential charge distribution  $\rightarrow$   
 dipole form factor  $F^D$

Experiments can access **much higher**  
 $Q^2 \rightarrow$  more theory guidance needed  
 appreciated!

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{point}} |F(\vec{q})|^2$$

$$\rho(r) = \frac{8\pi}{m^3} e^{-mr} \rightarrow F(\vec{q}) = \left( 1 - \frac{\vec{q}^2}{m^2} \right)^{-2}$$

$$F^D(Q^2) = F^D(0) \left( 1 + \frac{Q^2}{M^2} \right)^{-2}$$

D Dominguez/CERN.

# Free nucleon cross section: Llewellyn Smith, 1971<sup>2</sup>

$$\frac{d\sigma}{dQ^2} \begin{pmatrix} \nu n \rightarrow l^- p \\ \bar{\nu} p \rightarrow l^+ n \end{pmatrix} = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[ A(Q^2) \mp B(Q^2) \frac{(s-u)}{M^2} + C(Q^2) \frac{(s-u)^2}{M^4} \right], \quad (1)$$

$$\begin{aligned} A(Q^2) &= \frac{m^2 + Q^2}{4M^2} \left[ \left( 4 + \frac{Q^2}{M^2} \right) |F_A|^2 - \left( 4 - \frac{Q^2}{M^2} \right) |F_V^1|^2 \right. \\ &\quad \left. + \frac{Q^2}{M^2} \left( 1 - \frac{Q^2}{4M^2} \right) |\xi F_V^2|^2 + \frac{4Q^2}{M^2} \operatorname{Re} F_V^{1*} \xi F_V^2 + \mathcal{O}\left(\frac{m^2}{M^2}\right) \right], \\ B(Q^2) &= \frac{Q^2}{M^2} \operatorname{Re} F_A^*(F_V^1 + \xi F_V^2), \\ C(Q^2) &= \frac{1}{4} \left( |F_A|^2 + |F_V^1|^2 + \frac{Q^2}{4M^2} |\xi F_V^2|^2 \right) \end{aligned} \quad (2)$$

Depends on the “form factors”,  $F_V^1$ ,  $\xi F_V^2$ , and  $F_A$ .

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$$\begin{aligned} A(Q^2) &= \frac{m^2 + Q^2}{4M^2} \left[ \left( 4 + \frac{Q^2}{M^2} \right) |\textcolor{blue}{F}_A|^2 - \left( 4 - \frac{Q^2}{M^2} \right) |\textcolor{blue}{F}_V^1|^2 \right. \\ &\quad \left. + \frac{Q^2}{M^2} \left( 1 - \frac{Q^2}{4M^2} \right) |\xi F_V^2|^2 + \frac{4Q^2}{M^2} \text{Re} \textcolor{blue}{F}_V^{1*} \xi F_V^2 + \mathcal{O}\left(\frac{m^2}{M^2}\right) \right], \\ B(Q^2) &= \frac{Q^2}{M^2} \text{Re} \textcolor{blue}{F}_A^* (\textcolor{blue}{F}_V^1 + \xi F_V^2), \\ C(Q^2) &= \frac{1}{4} \left( |\textcolor{blue}{F}_A|^2 + |\textcolor{blue}{F}_V^1|^2 + \frac{Q^2}{4M^2} |\xi F_V^2|^2 \right) \end{aligned} \quad (2)$$

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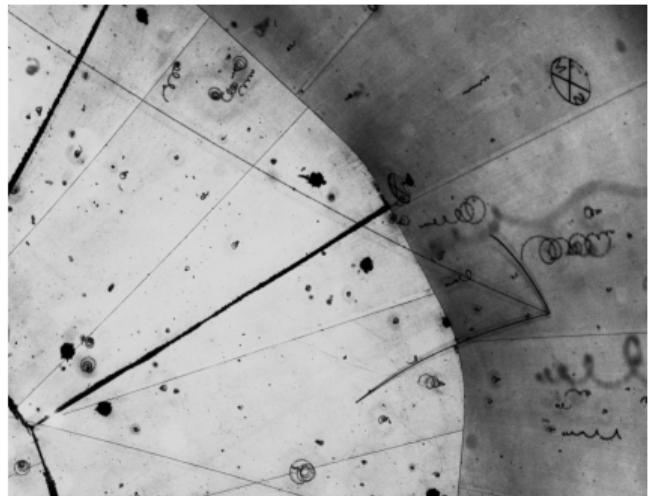
Neutrino-nucleon cross section depends on

- $F_V^1, \xi F_V^2$ : electric and magnetic charge, measured by electron scattering experiments – lots of data.
- $F_A$ : axial vector “charge”. Accessible only through neutrino scattering – very little data exists.
  - ▶ likelihood of  $u \leftrightarrow d$  conversion.
  - ▶ Historically assumed a dipole form:

$$F_A(Q^2) = F_A(0) \left(1 + \frac{Q^2}{M_A^2}\right)^{-2}$$

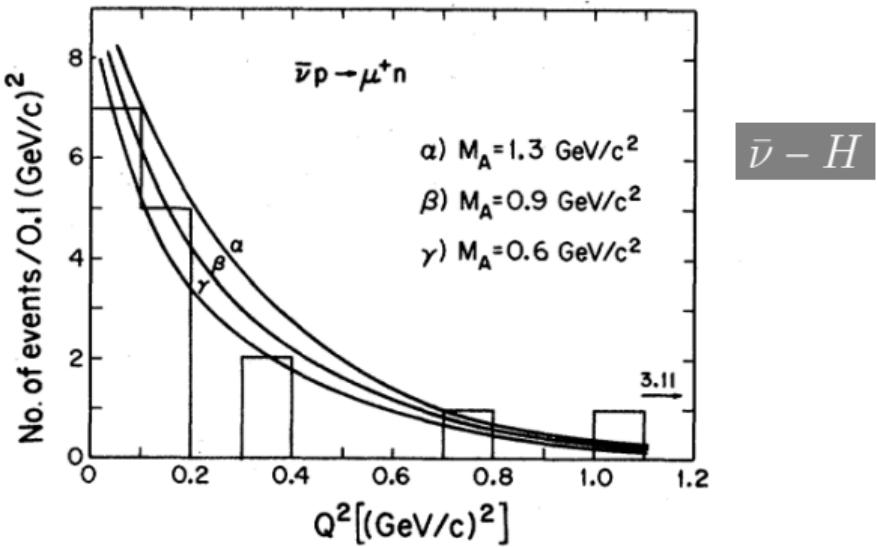
- ▶  $F_A(0) = -1.2723 \pm 0.0023$  from neutron decay measurements<sup>3</sup>.
- ▶  $M_A$  is the axial mass.

# $D_2(H)$ bubble chambers



First neutrino detected by ANL 12-ft D2 bubble chamber.

Past bubble chamber experiments (ANL 12-ft, BNL 7-ft, FNAL 15-ft, 1973-1983) primarily measured  $\nu D \rightarrow \mu^- + p + p^S$  reactions.  
BNL measured  $\bar{\nu} H$  reaction using 13 events.

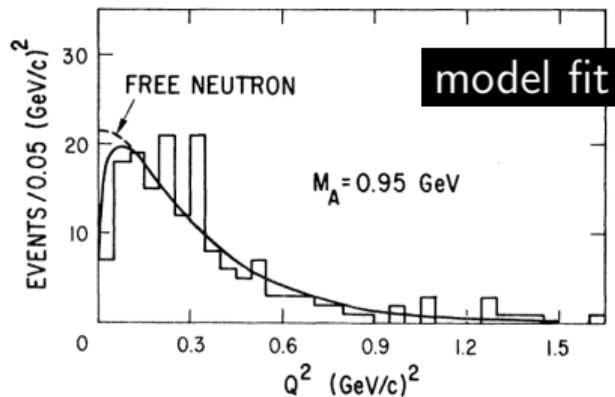
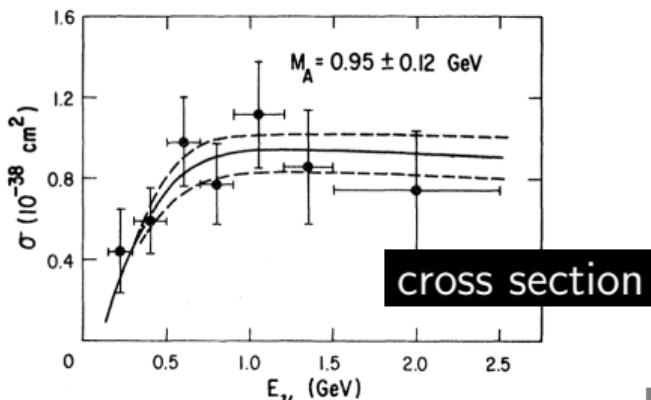


Fanourakis et al., 1980<sup>4</sup>

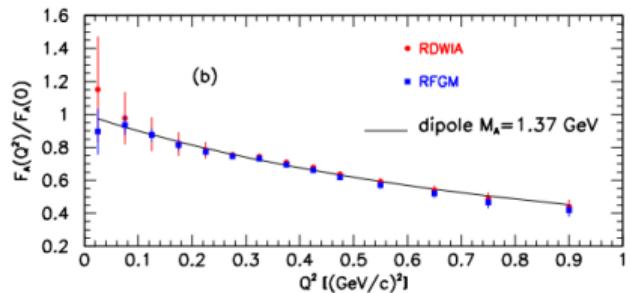
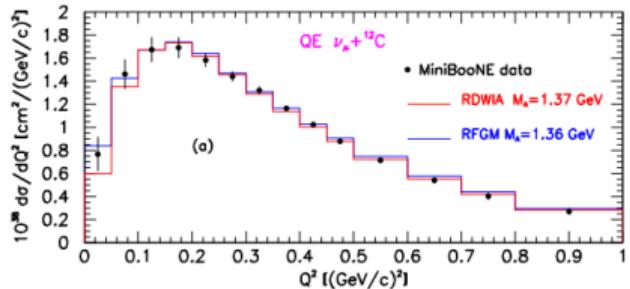
# First neutrino - deuterium measurement

Reported exactly 50 years ago:

- Flux uncertainty was 15%
- Extracting form factor needs model-dependent deuterium corrections
  - ▶ Fermi motion
  - ▶ Pauli exclusion principle



# Measurements with heavier targets



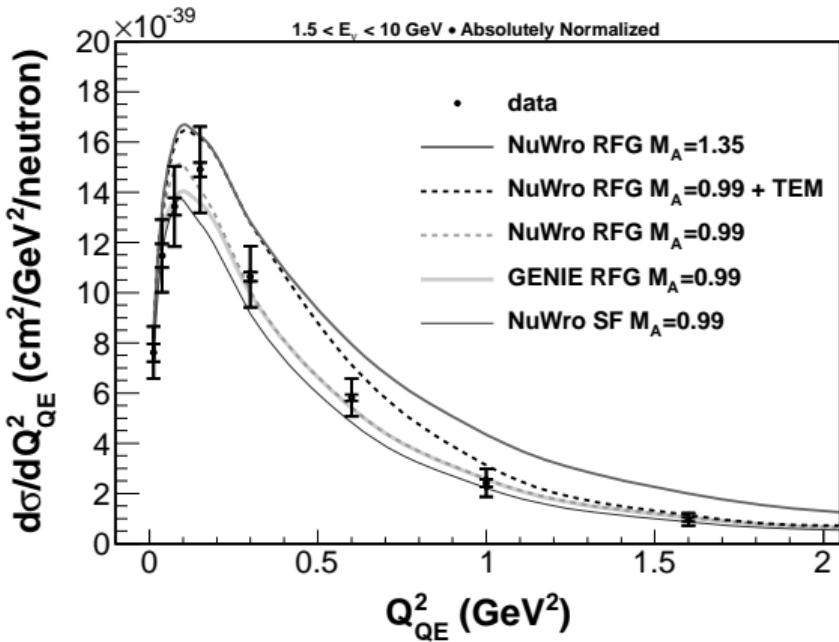
MiniBooNE QE measurement  
 Aguilar-Arevalo et al., 2010<sup>6</sup>  
 Butkevich and Perevalov, 2013<sup>7</sup>

- MiniBooNE – inclusive, lepton-only measurement – interpreted as  $F_A$  on carbon, insensitive to lepton-hadron correlation.

# Measurements with heavier targets

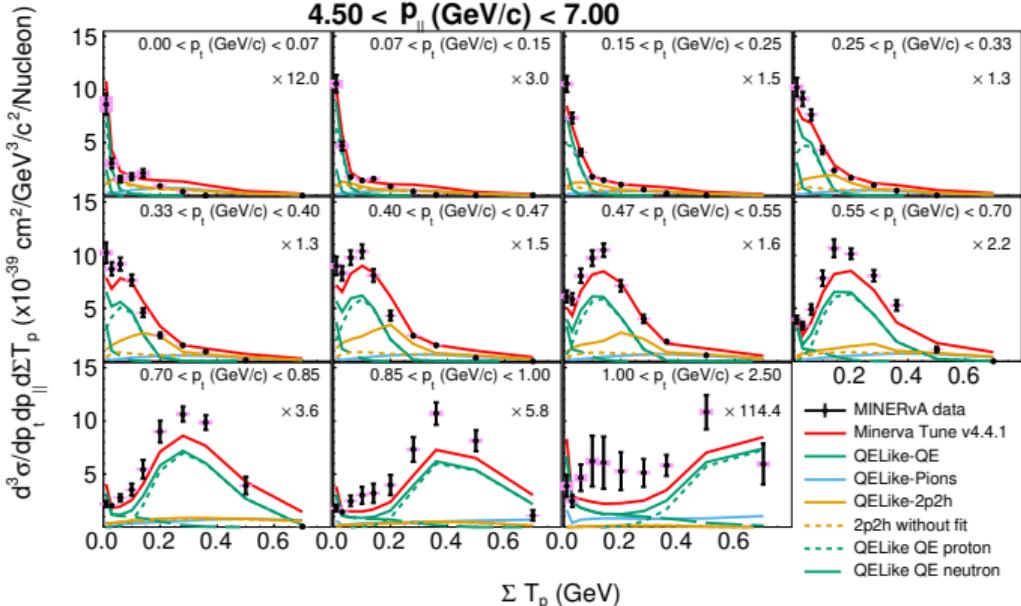
MINER $\nu$ A  $\nu_\mu$  QE measurement

Fiorentini et al., 2013<sup>8</sup>



- MiniBooNE – inclusive, lepton-only measurement – interpreted as  $F_A$  on carbon, insensitive to lepton-hadron correlation.
- MINER $\nu$ A QE results: disagree with  $M_A = 1.35 \text{ GeV}/c^2$ 
  - ▶ Tension points to nuclear effects.

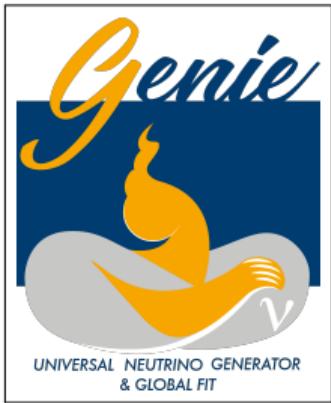
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- MINERνA QE results: disagree with  $M_A = 1.35 \text{ GeV}/c^2$ 
  - ▶ Tension points to nuclear effects.
- Better able to identify nuclear effects with lepton-hadron correlations.

Muon  $p_{\parallel}$  vs muon  $p_t$  vs total proton kinetic energy  
MINERνA Ruterbories et al., 2022<sup>9</sup>

# Nucleon and nuclear effects are inseparable



Neutrino measurements depend on models.  
Most factor out nucleon and nuclear effects:

- 1 Neutrino-nucleon interactions
- 2 Fermi motion, binding energy
- 3 Final state interactions
- 4 Event kinematics → combine uncertainty from previous steps!



A model-independent nucleon  $F_A$  measurement is beneficial

How – in the future we could build a new bubble chamber.

## Objectives for Building a MMBC

Current goals of the newly funded project!

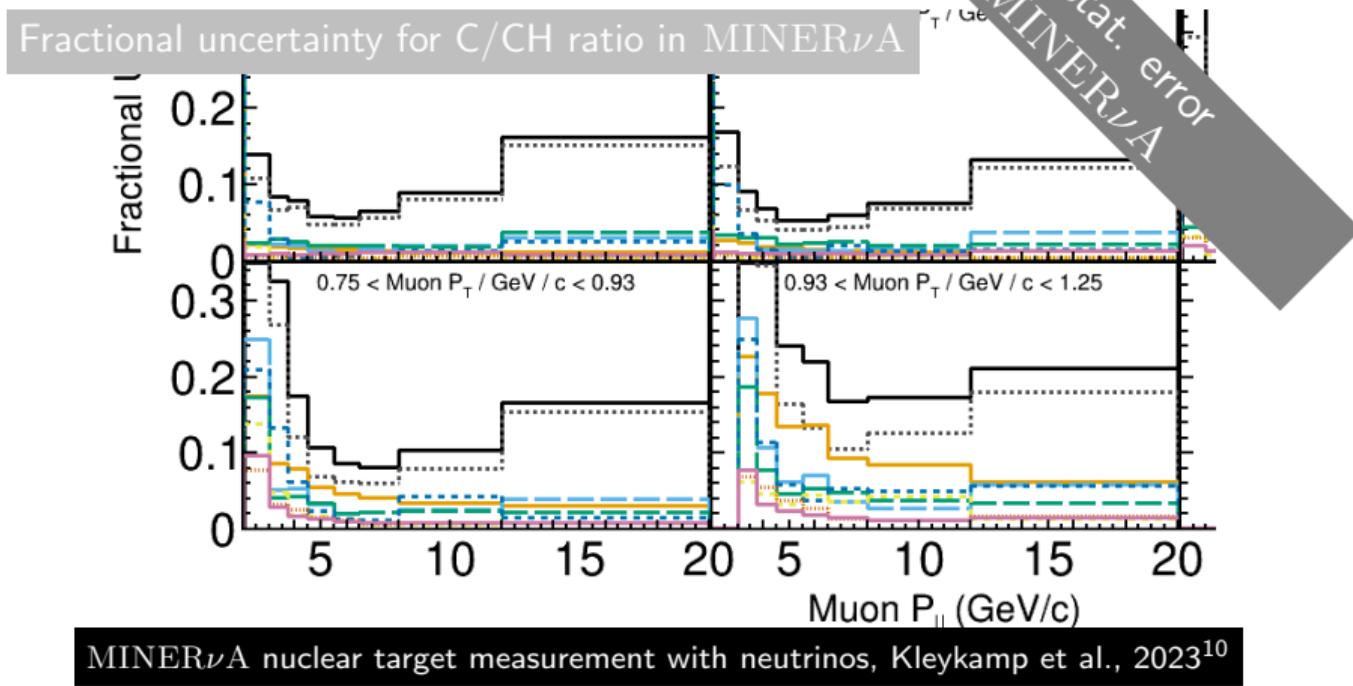
In the process of finalizing the project scope!

takes time

- **First Objective (FY23.5)** A fully leak checked, pressure ready, and vacuum ready device in MiniBooNE Hall.
- **Second Objective (FY23.5)** Device active time long enough to capture an entire spill from the LBNF beam (at minimum 10 microseconds).
- **Third Objective (FY24.0)** 1 Hz cycling time.
- **Fourth Objective (FY24.0)** Minimum possible cycling time.
- **Fifth Objective (FY24.0/FY24.5)** Maximum active time without interior changes. Polish, coating, or plating and retest maximum active time.
- **Sixth Objective (FY25/FY25.5)** Precision track reconstruction on cosmic ray muons.
- **Seventh objective (FY25/FY25.5)** sync to the Fermilab Testbeam clock and observe hadron decays.

Bryan Ramson, Snowmass 2022

# How – could perform C/CH ratio



# How – could perform C/CH ratio

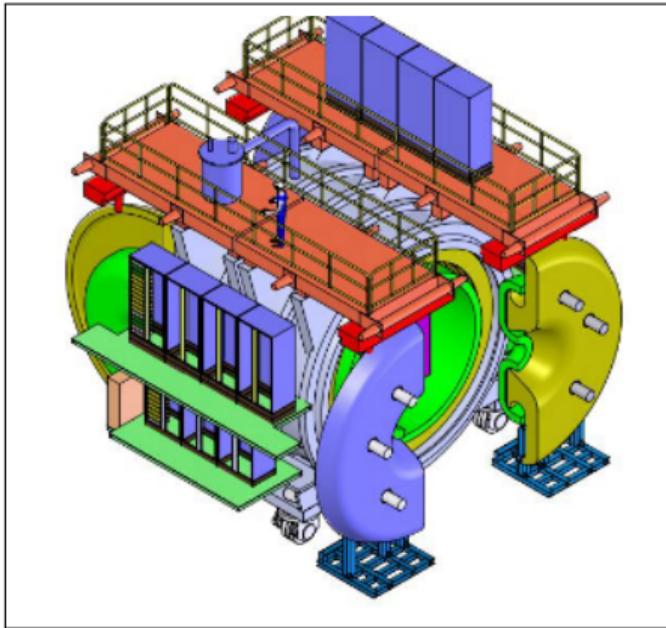


Figure from Vicenzi, 2022<sup>11</sup>

DUNE's near detector SAND is planning a C/CH ratio measurement in the next decade.

# How – directly measure H from a CH target!

Measure the charged current elastic (CCE) scattering

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n,$$

with the Medium Energy NuMI antineutrino beam, and the scintillators in MINER $\nu$ A.

- No nuclear effect on hydrogen.
- Large carbon background.

# Extracting the cross section – a road map

$$\left( \frac{d\sigma}{dQ^2} \right)_i = \frac{\sum_j U_{ji} (N_j^{\text{data}} - N_j^{\text{bkg-pred}})}{\Phi T \epsilon_i (\Delta Q^2)_i}$$

# Extracting the cross section – a road map

$$\left( \frac{d\sigma}{dQ^2} \right)_i = \frac{\sum_j U_{ji} (N_j^{\text{data}} - N_j^{\text{bkg-pred}})}{\Phi T \epsilon_i (\Delta Q^2)_i}$$

- $Q^2$ : square of the 4-momentum transfer.
- Reconstructed quantity:
  - ▶  $E_{\bar{\nu}} = \frac{M_n^2 - M_p^2 - m_\mu^2 + 2M_p E_\mu}{2(M_p - E_\mu + p_\mu \cos \theta_\mu)}$
  - ▶  $Q_{\text{QE}}^2 = 2E_{\bar{\nu}}(E_\mu - p_\mu \cos \theta_\mu) - m_\mu^2$
- $(d\sigma/dQ^2)_i$ : cross section in the  $i^{\text{th}}$   $Q^2$  bin.
- $(\Delta Q^2)_i$ : width of the  $i^{\text{th}}$  bin.

# Extracting the cross section – a road map

$$\left( \frac{d\sigma}{dQ^2} \right)_i = \frac{\sum_j \mathbf{U}_{ji} (N_j^{\text{data}} - N_j^{\text{bkg-pred}})}{\Phi \mathbf{T} \epsilon_i (\Delta Q^2)_i}$$

- $\mathbf{U}_{ji}$ : Unfolding matrix – correcting detector smearing.
- $\Phi$ : (Anti)neutrino flux (Zazueta et al., 2022<sup>12</sup>).
- $\mathbf{T}$ : Number of hydrogen atoms in the scintillator.
- $\epsilon_i$ : Detector efficiency in the  $i^{\text{th}}$  bin.

# Extracting the cross section – a road map

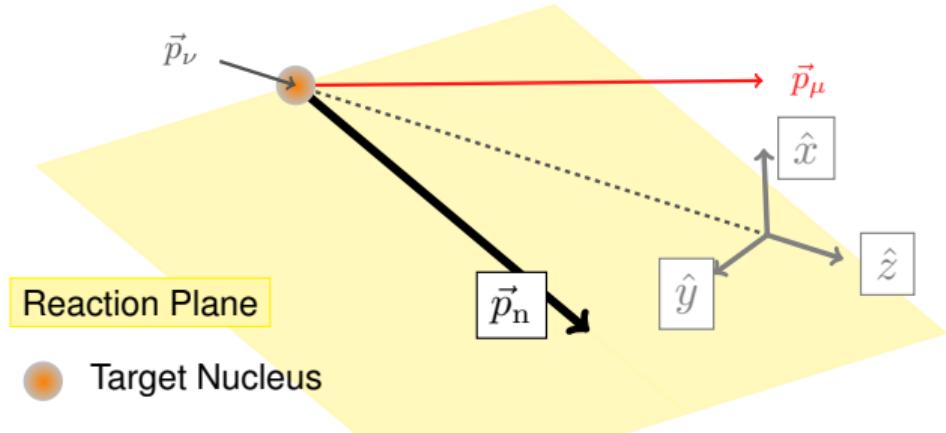
$$\left( \frac{d\sigma}{dQ^2} \right)_i = \frac{\sum_j U_{ji} (N_j^{\text{data}} - N_j^{\text{bkg-pred}})}{\Phi T \epsilon_i (\Delta Q^2)_i}$$

- Measured  $Q_{\text{QE}}^2$ :
  - ▶  $N_j^{\text{data}}$ : detected event rate in the  $j^{\text{th}}$   $Q_{\text{QE}}^2$  bin.
  - ▶  $N_j^{\text{bkg-pred}}$ : reconstructed background prediction in the same bin.
- Everything sounds simple, except
  - ▶ signal channel is not dominant, and
  - ▶ background channels are not necessarily well-modeled.

Needs to get clever with data selection!

# Using nuclear effects to advantage

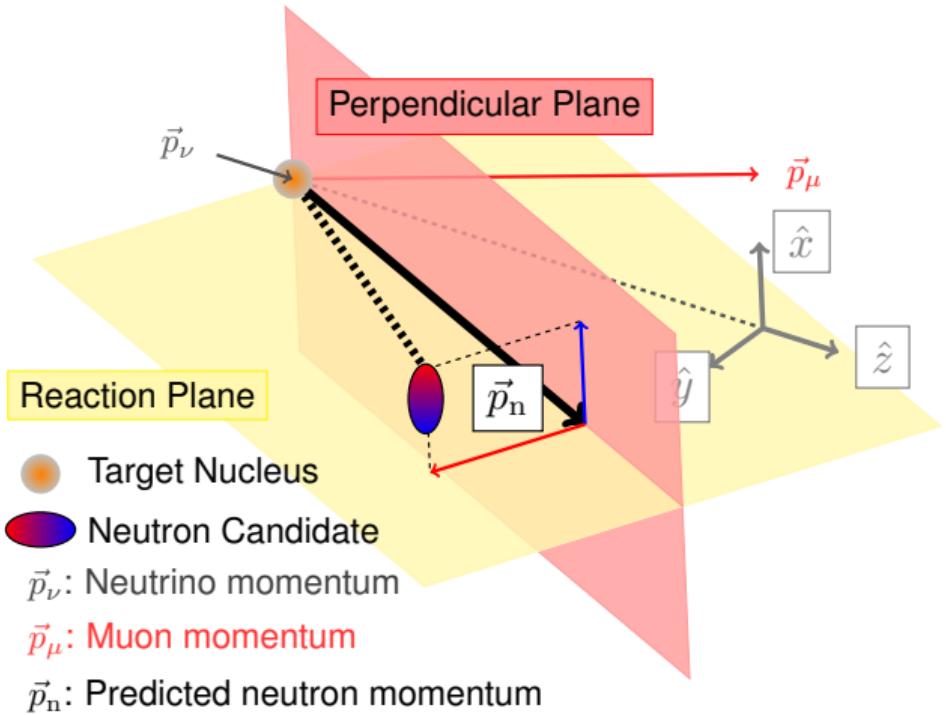
$\bar{\nu}_\mu H$  is a 2-body process.  
 Real, elastic, flavor-changing,  
 billiard balls. We know exactly  
 where the neutron would go.



# Using nuclear effects to advantage

$\bar{\nu}_\mu H$  is a 2-body process.  
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Carbon doesn't play by the same  
 rules. Neutrons happily deviate.



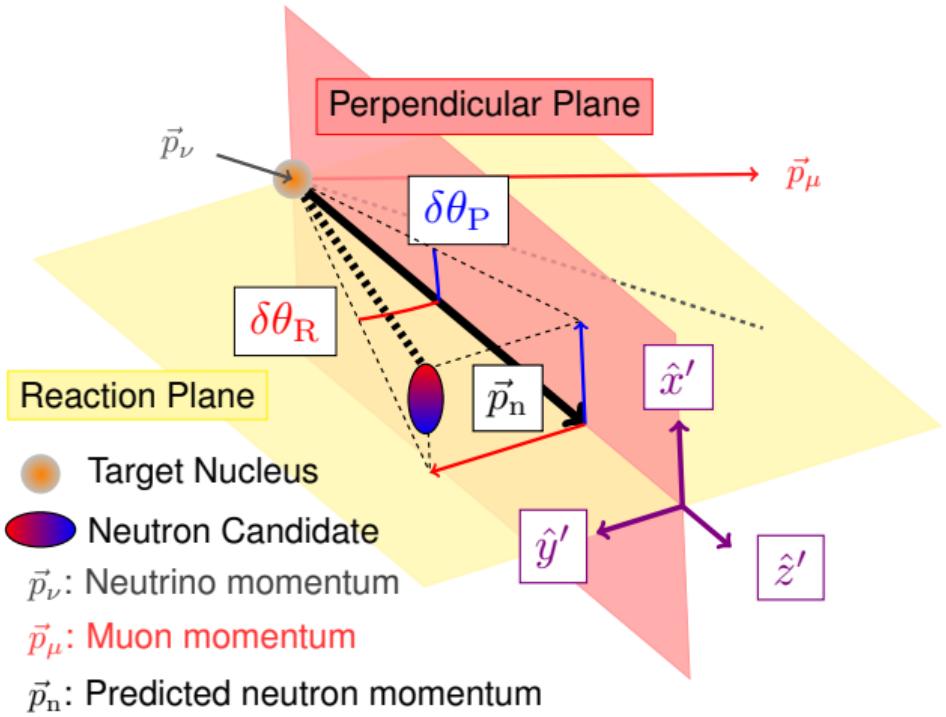
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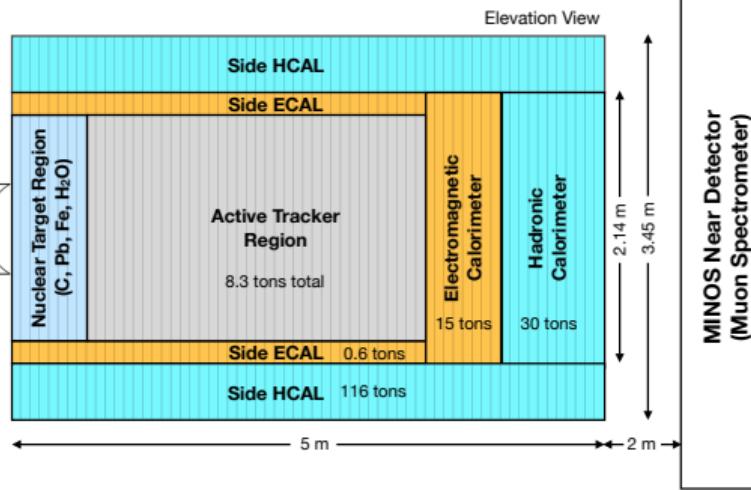
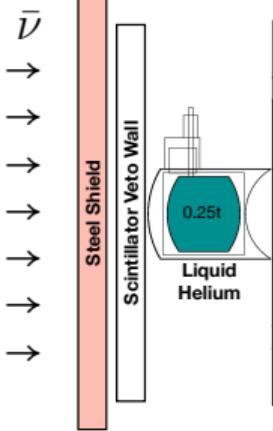
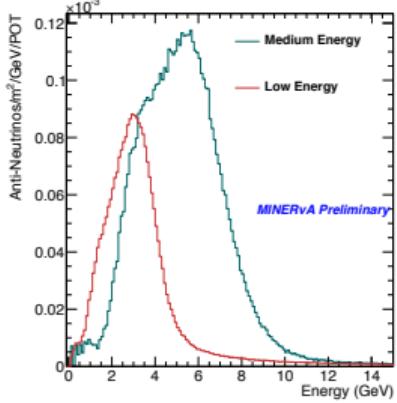
Carbon doesn't play by the same  
rules. Neutrons happily deviate.

Capture this deviation in neutron  
direction, using  $\delta\theta_P$  and  $\delta\theta_R$

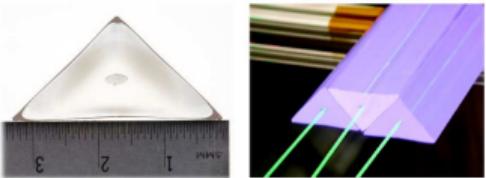


Wait, you need neutrons?

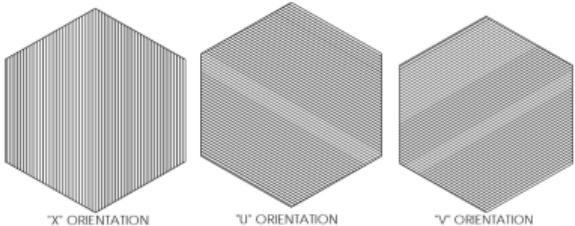
# MINER $\nu$ A detector



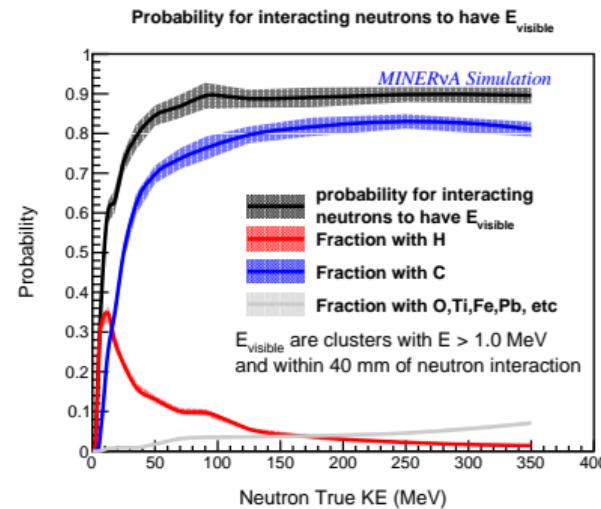
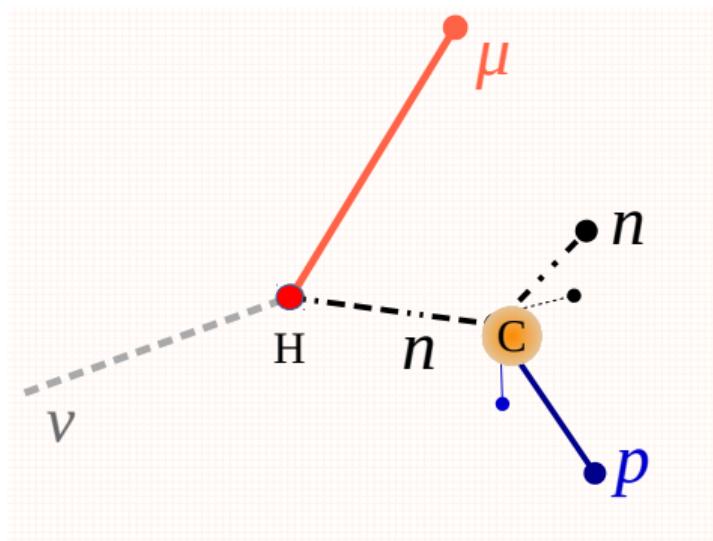
Nucl. Inst. and Meth. A743 (2014) 130.



Element	# of Targets
H	$2.61 \times 10^{29}$
C	$2.38 \times 10^{29}$

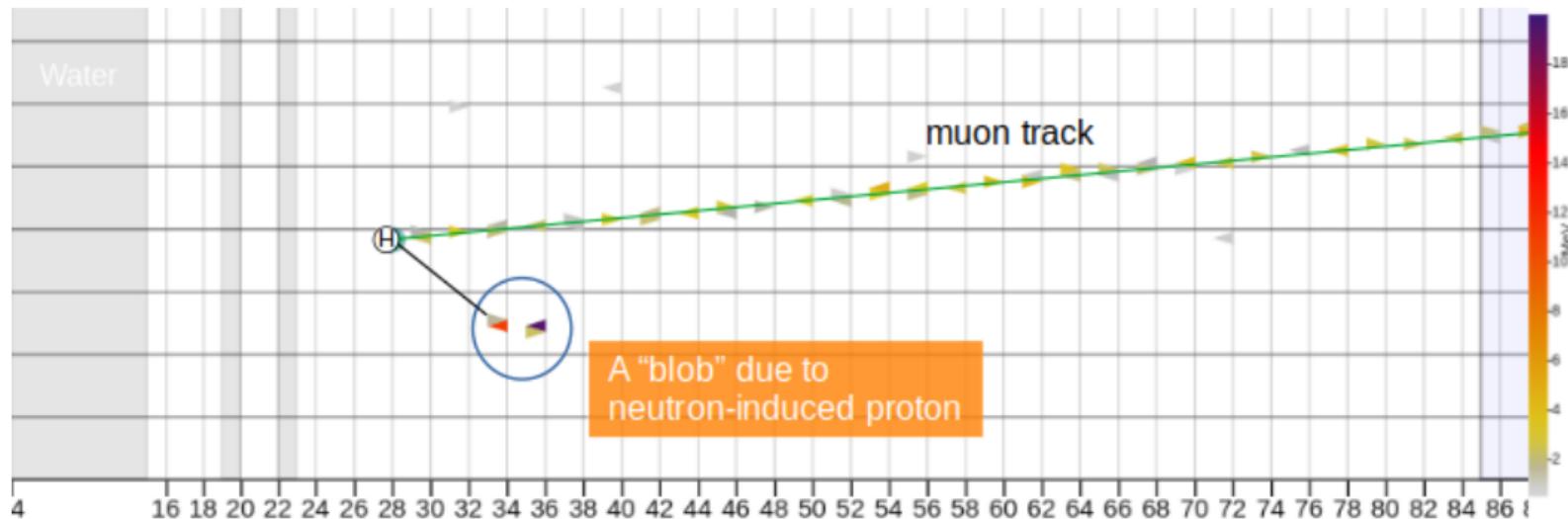


Neutrons inside the detector interact with hydrogen or carbon to produce charged secondary particles.



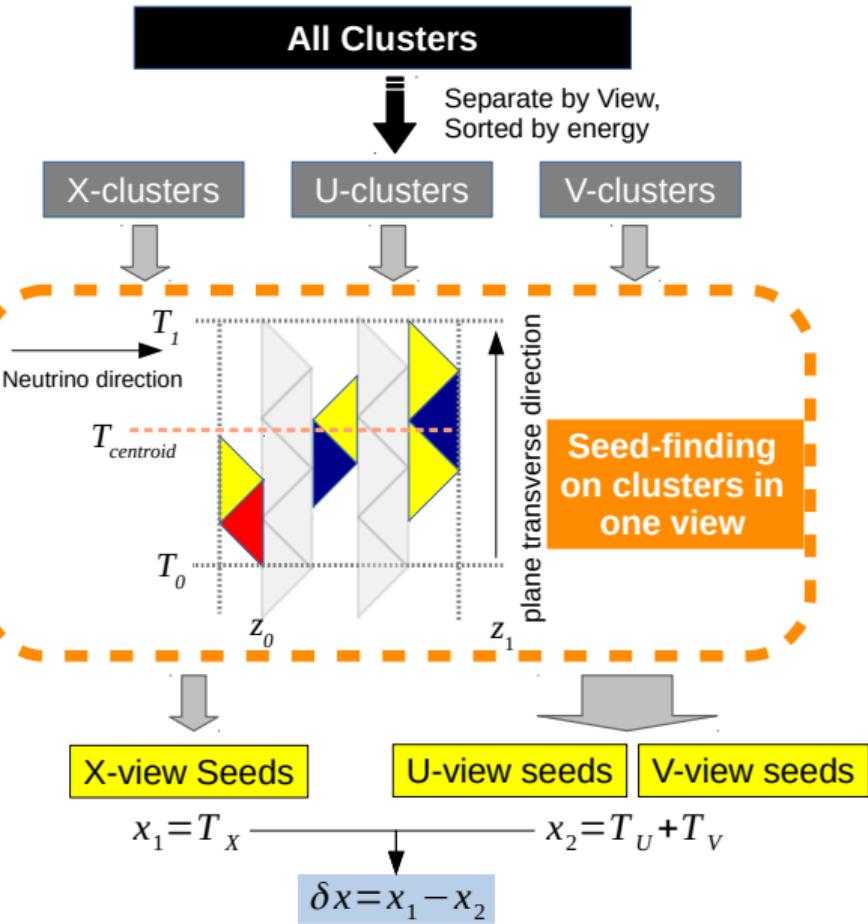
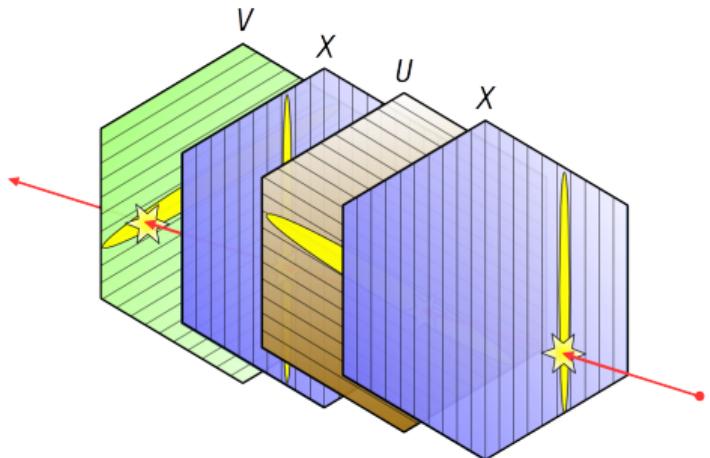
Most prompt neutron energy deposits due to knockout protons.

# Neutron signature

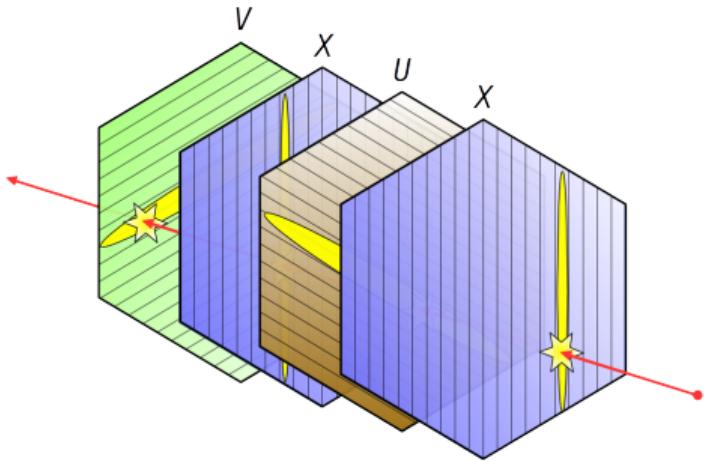


An incoming anti-neutrino scatters off a hydrogen producing neutron. The neutron undergoes secondary interactions to produce visible proton.

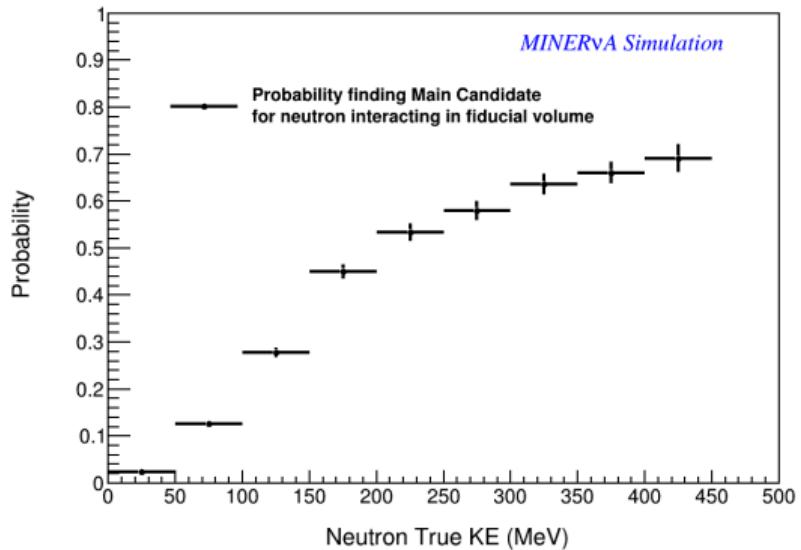
# Blobbing algorithm



# Blobbing algorithm



Probability for interacting neutron to have main candidate



# Forming event selection

Event topology:

- 1  $\mu^+$  and no reconstructed hadronic track

Neutron selection:

- $\geq 1$  three-dimensional neutron candidate
- Leading candidate energy deposited 10 cm away from the muon axis.

Muon acceptance due to detector shape:

- $1.5 \text{ GeV} < E_\mu < 20 \text{ GeV}$
- $\theta_\mu < 20^\circ$  w.r.t. to neutrino beam

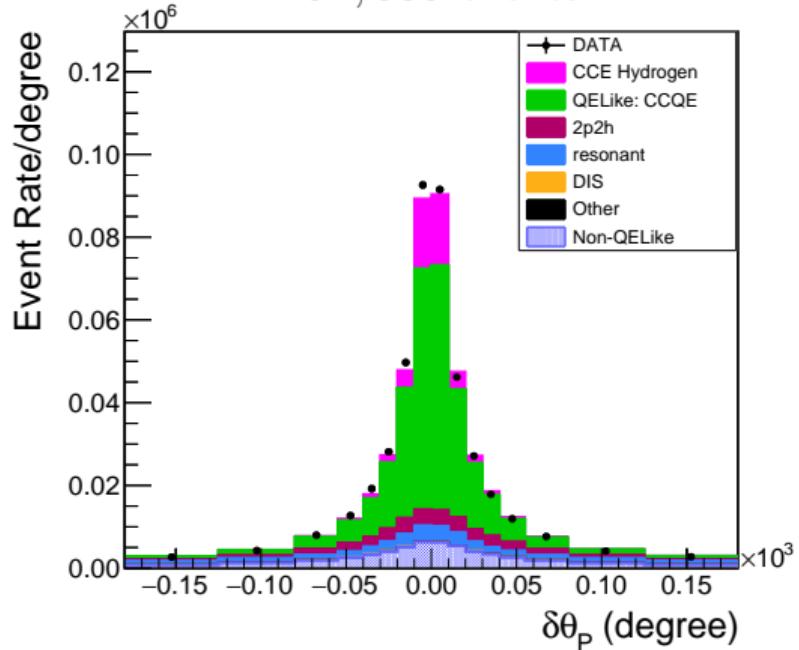
NuMI Antineutrino Mode:  $1.12 \times 10^{21}$   
Protons on Target

Total recoil energy cut:

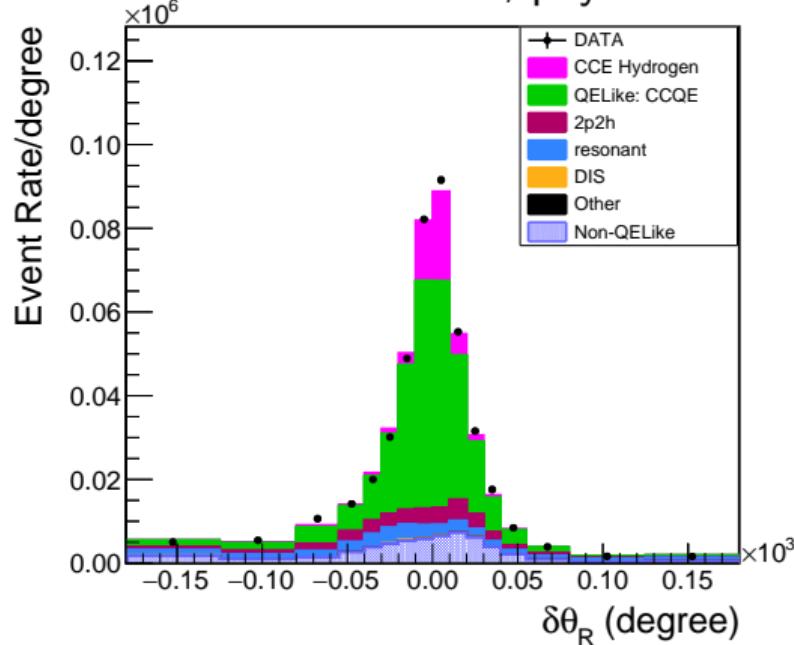
Condition	$E_{\max}^{\text{recoil}}$ (GeV)
$Q_{\text{QE}}^2 < 0.3 \text{ (GeV}/c)^2$	$0.04 + 0.43Q_{\text{QE}}^2/(\text{GeV}/c)^2$
$Q_{\text{QE}}^2 < 1.4 \text{ (GeV}/c)^2$	$0.08 + 0.3Q_{\text{QE}}^2/(\text{GeV}/c)^2$
$Q_{\text{QE}}^2 > 1.4 \text{ (GeV}/c)^2$	0.50

# A sneak peak at the fun part

~ 94,000 events.



Nuclear effects are real, physics works!



CCE hydrogen

QELike CCQE carbon

2p2h

# Model prediction

MnvGENIE-v2.5.1

- 1 Nieves 2p2h<sup>13</sup> and low-recoil tune<sup>14</sup>
- 2 Random phase approximation<sup>15</sup> (RPA) tune
- 3 Non-resonant pion reduction<sup>16</sup>
- 4 Low- $Q^2$  resonant pion suppression<sup>17</sup>
- 5 Carbon elastic FSI reweight<sup>18</sup>
- 6 CCQE carbon NuWro spectral function (SF)<sup>19,20</sup> reweight on GENIE

A GEANT4 neutron reweight<sup>21,22</sup>



# Model prediction

MnvGENIE-v2.5.1

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reweight on GENIE

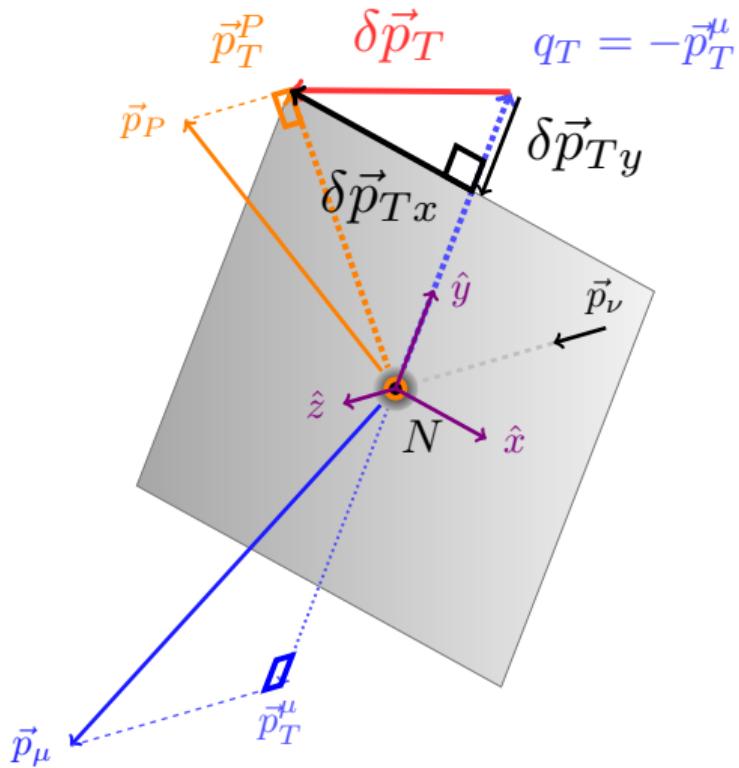
A GEANT4 neutron reweight<sup>21,22</sup>



Rationale for reweighting GENIE relativistic Fermi gas (RFG) model to NuWro's spectral function (SF) model.

Low energy era measurement: MINER $\nu$ A developed a set of *transverse kinematics imbalance* techniques for the neutrino sample:

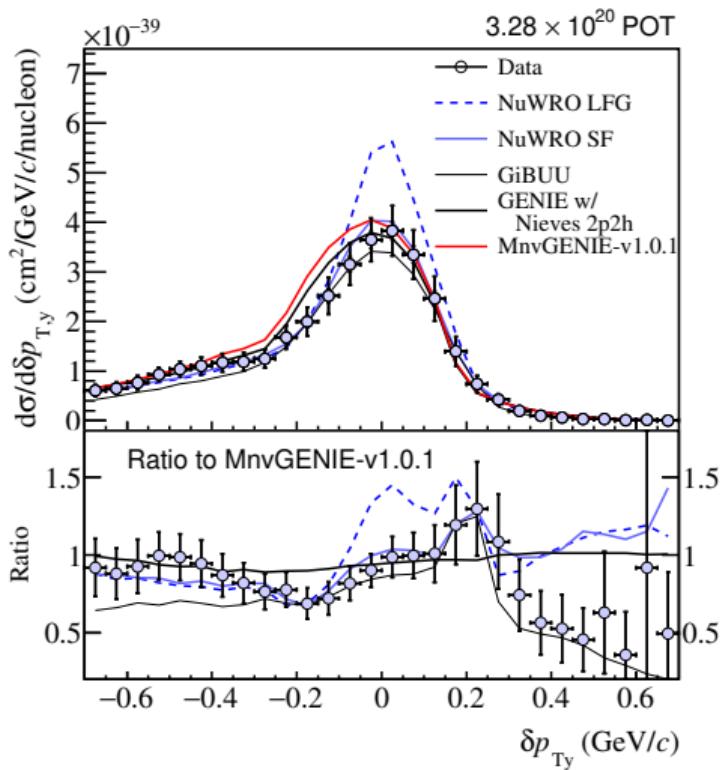
- Events with only a primary muon, at least one proton, and no mesons.
- Compare momentum imbalance between  $\mu$  and  $p$ .
- $\delta p_{Ty}$  measures whether  $\vec{p}^\mu + \vec{p}^p$  balance out along the  $\vec{p}_T^\mu$  axis.



Cai et al., 2019<sup>23</sup>

Low energy era measurement: MINER $\nu$ A developed a set of *transverse kinematics imbalance* techniques for the neutrino sample:

- Events with only a primary muon, at least one proton, and no mesons.
- Compare momentum imbalance between  $\mu$  and  $p$ .
- $\delta p_{Ty}$  measures whether  $\vec{p}^\mu + \vec{p}^p$  balance out along the  $\vec{p}_T^\mu$  axis.
- NuWro spectral function (solid blue) better describes data.

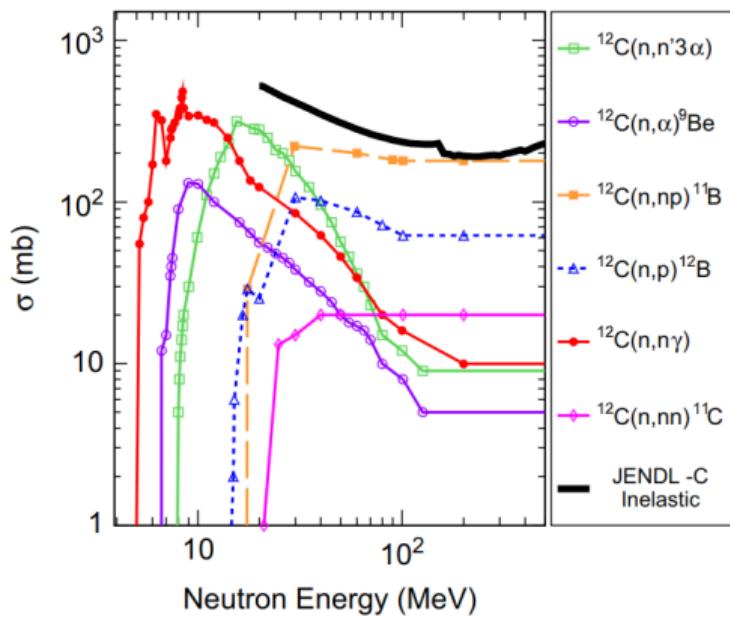


Cai et al., 2019<sup>23</sup>

Constraining MINER $\nu$ A's GEANT4 neutron model with external data.

# Neutron model from Modular Neutron Array (MoNA)

- The MoNA collaboration collected (Del Guerra, 1976<sup>21</sup>) and modeled (Kohley et al., 2012<sup>22</sup>) neutron cross section on CH.
- $^{12}C(n,np)^{11}B$  the dominant interaction channel
- We tune each channel to the MoNA cross sections based on secondary daughter particles.

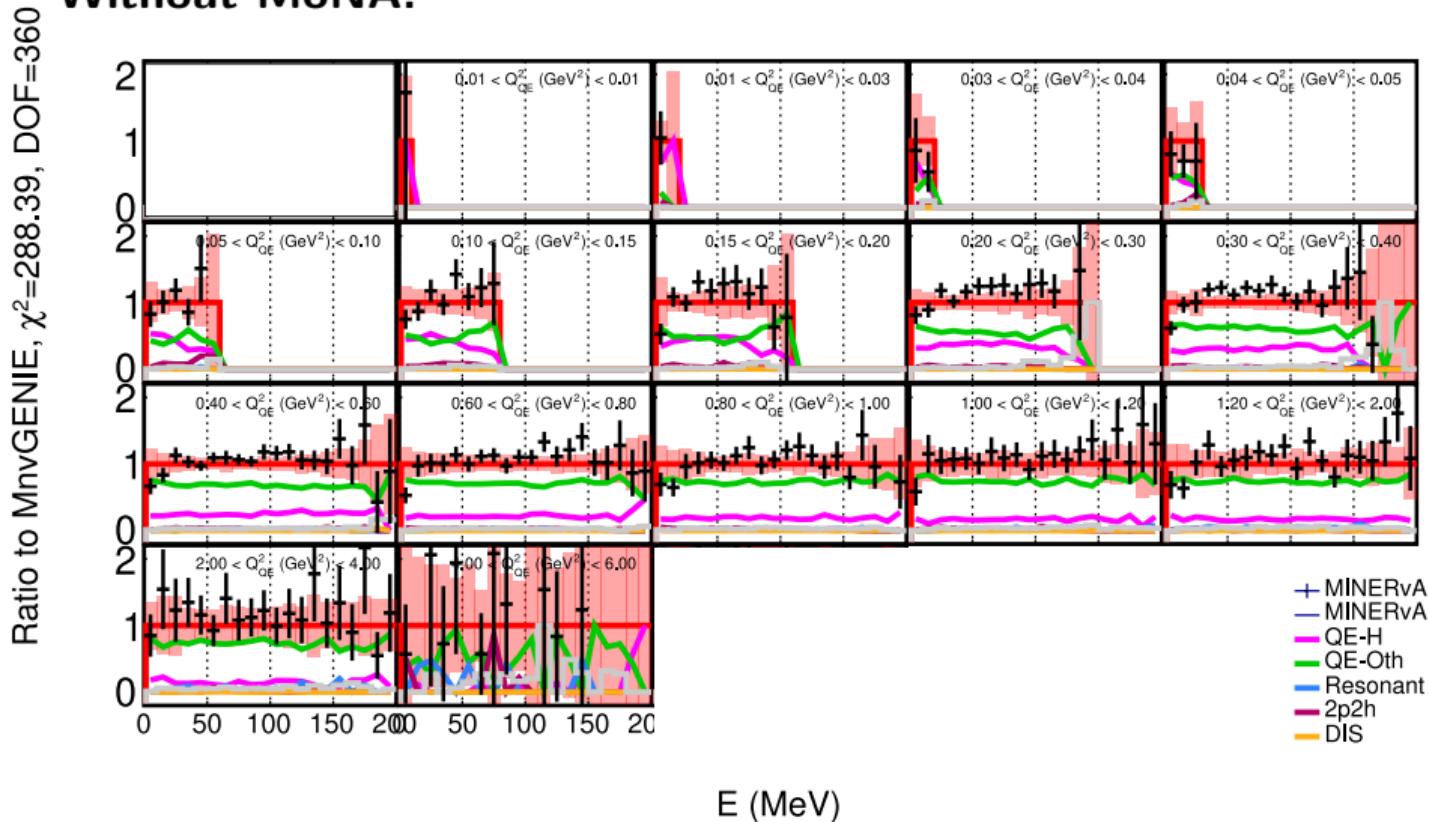


**Fig. 3.** Inelastic neutron–carbon reaction cross-sections are shown as a function of the incident neutron energy. MENATE\_R uses the six different discrete reaction channel cross-sections while the G4-Physics uses the total inelastic reaction cross-sections taken from the JENDL-HE library [37].

- We looked at “nuisance” variables to see if the MONA data improved the data/MC agreement
- These variables are not used in the fit and not tuned
- Example includes energy of neutron candidates, and distance of the candidate to vertex
- will show neutron candidate energy deposits in each  $Q_{QE}^2$  bin.

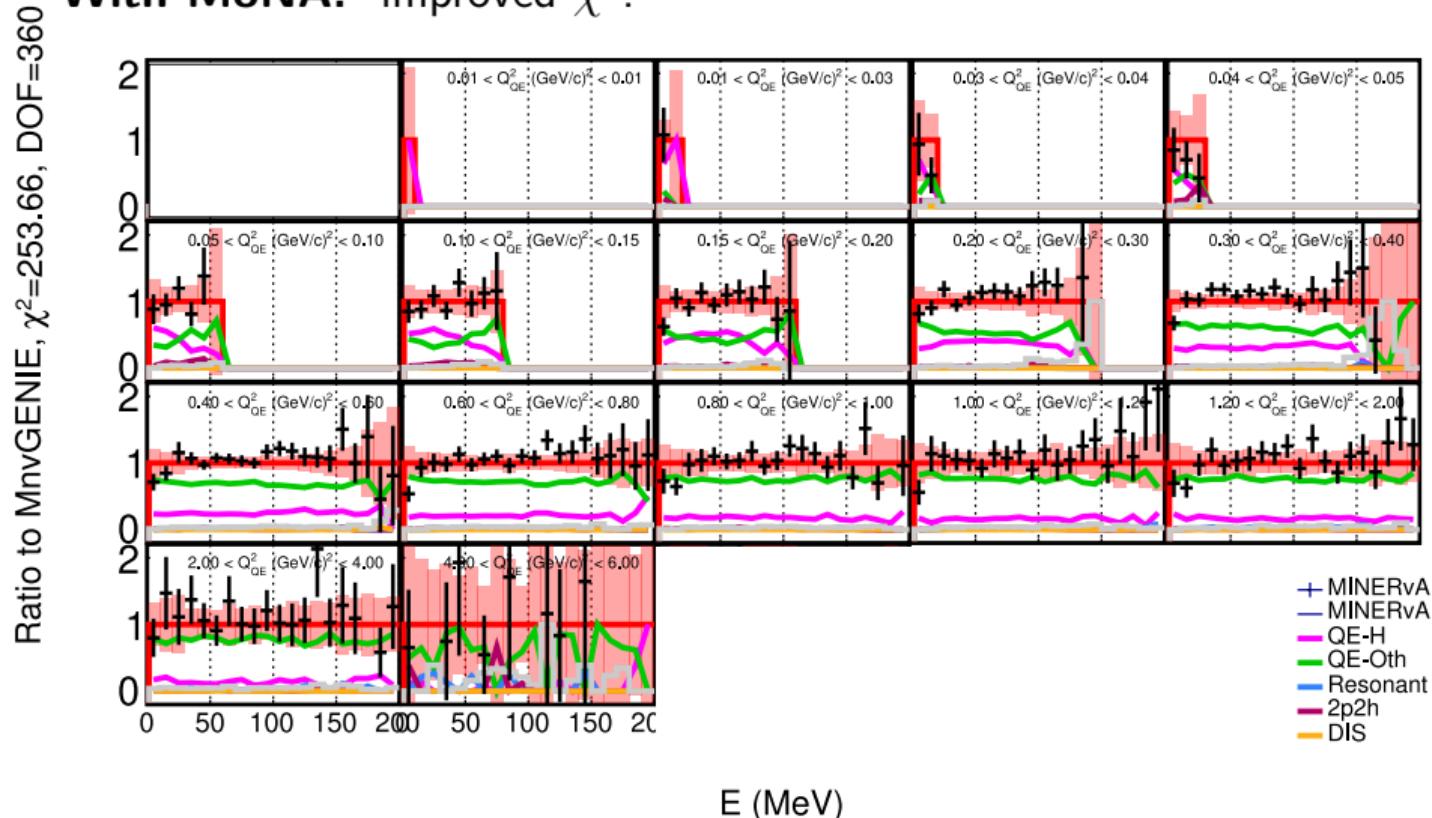
# Neutron candidate energy deposits vs $Q^2_{QE}$

## Without MoNA.



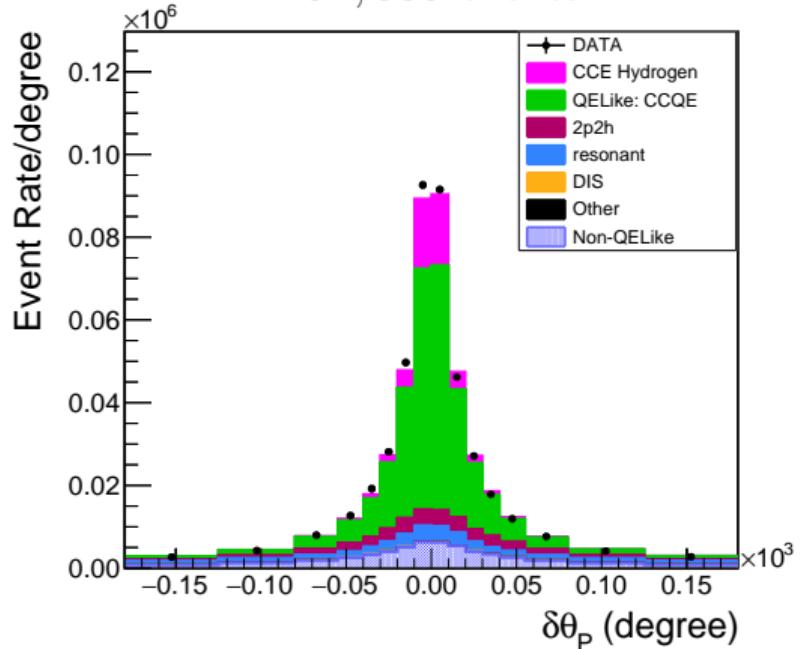
# Neutron candidate energy deposits vs $Q_{QE}^2$

## With MoNA: improved $\chi^2$ .

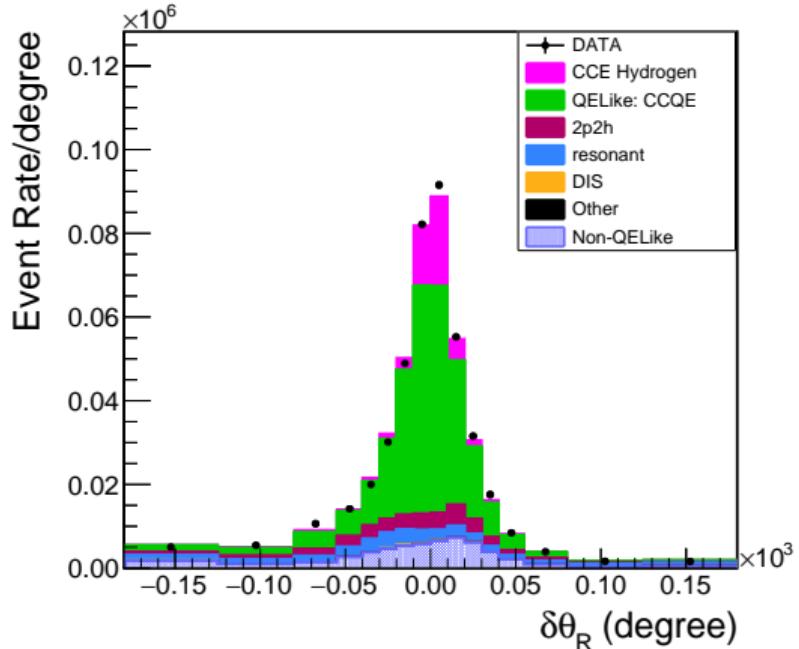


Let's return to the fun part

~ 94,000 events.



Each angular variable is not selective enough → go to two dimensions

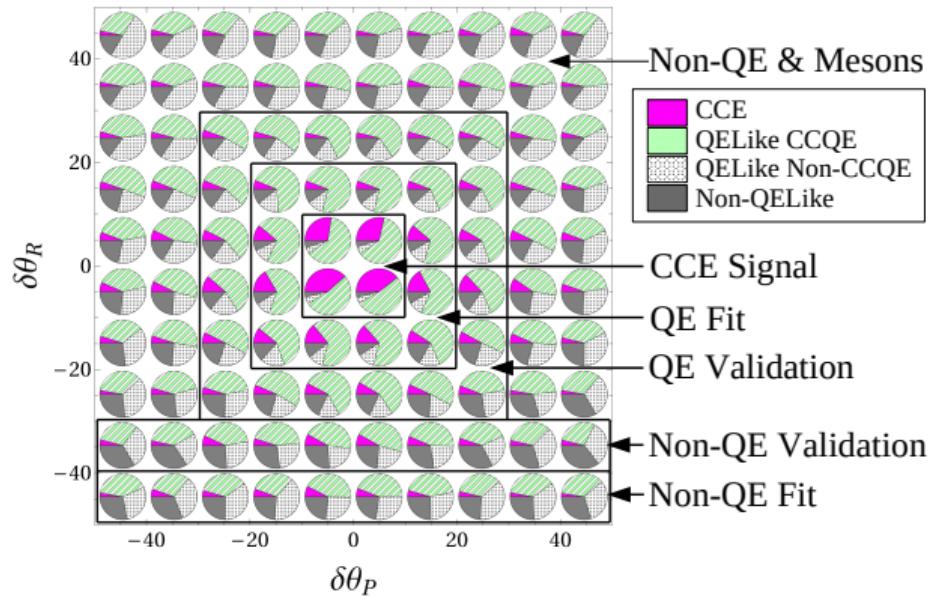


**CCE hydrogen**

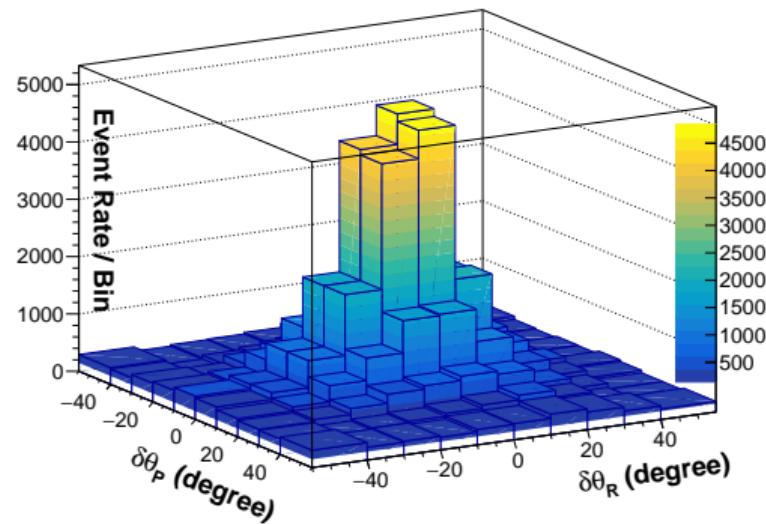
**QElike CCQE carbon**

**2p2h**

## Angular regions and MC fraction

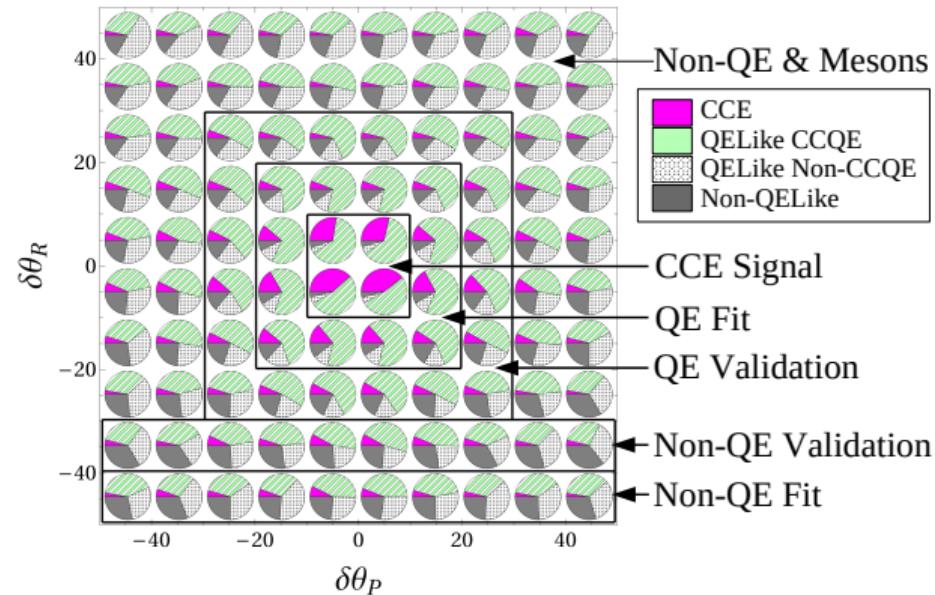


## Total data event rate



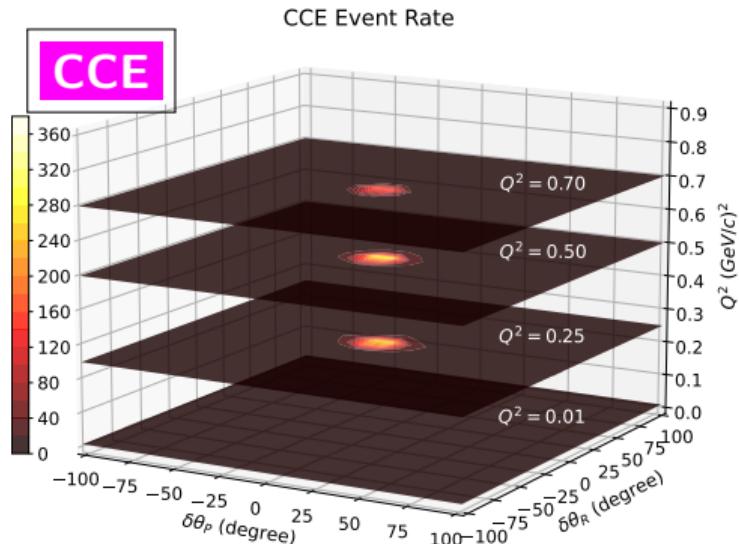
Created different angular regions – **Hydrogen signal** in the center. Outer regions are used for fit and validation – expand each region in  $Q^2$

## Angular regions and MC fraction

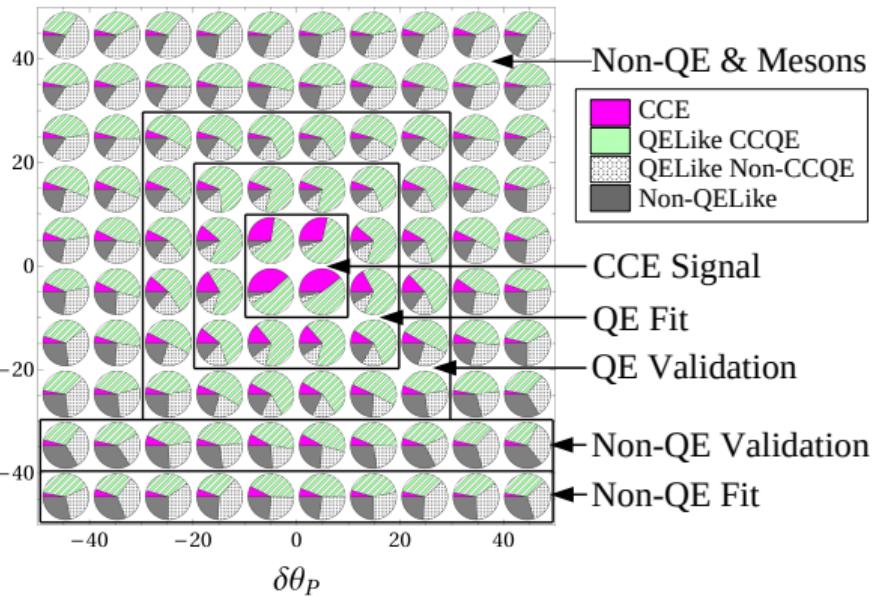


Predicted hydrogen angles – concentrated in the center.

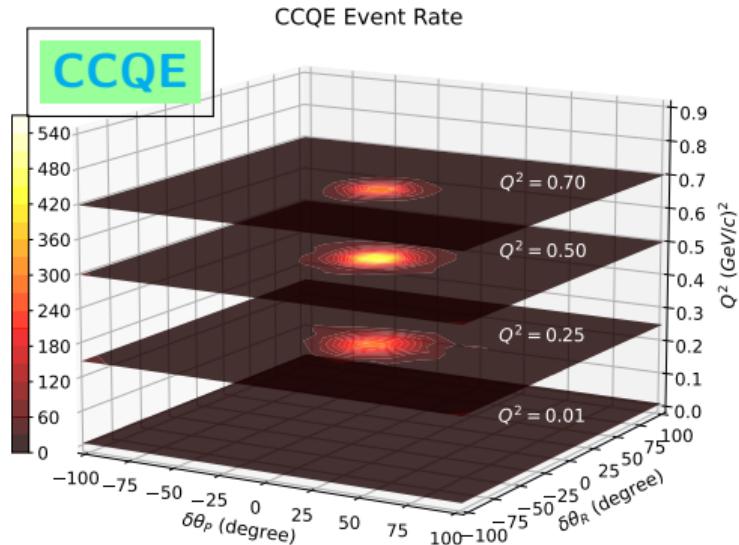
## MC events in slices of $Q^2$



## Angular regions and MC fraction

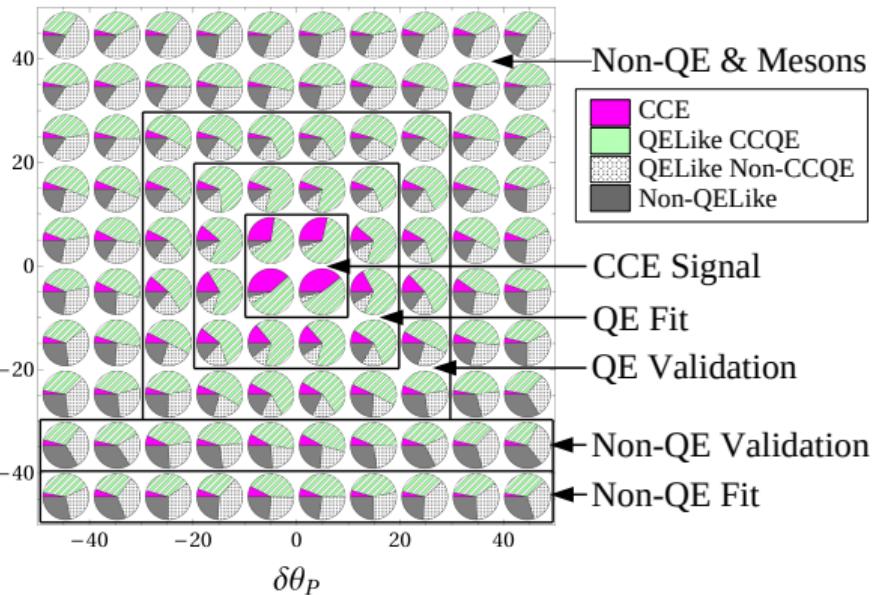


## MC events in slices of $Q^2$

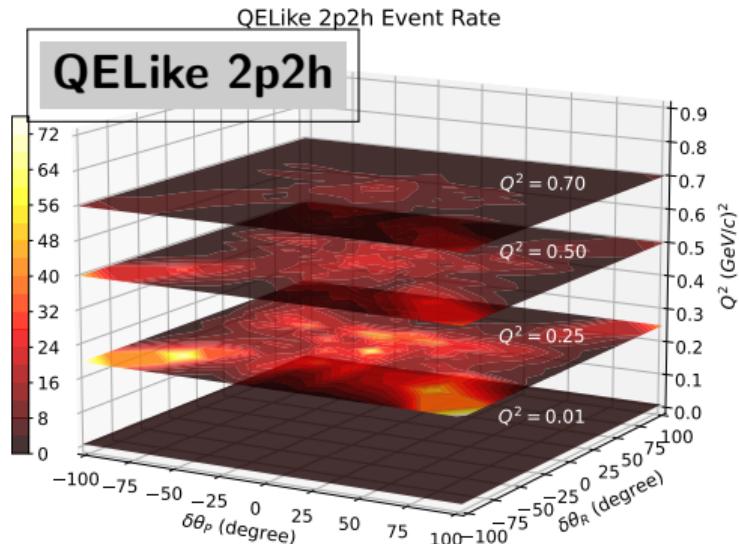


Carbon QELike (CCQE) – more spread out due to Fermi motion and final state interactions.

## Angular regions and MC fraction

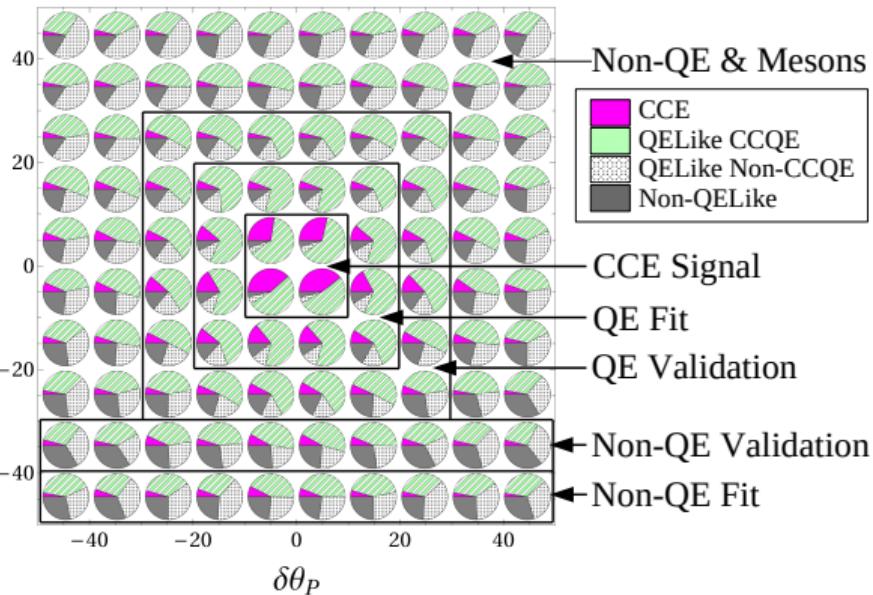


## MC events in slices of $Q^2$

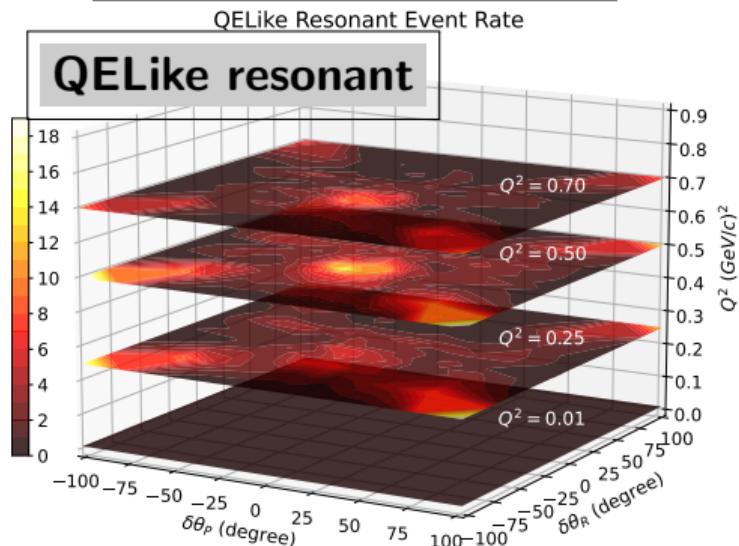


2p2h and resonant – all over the place but different.

## Angular regions and MC fraction

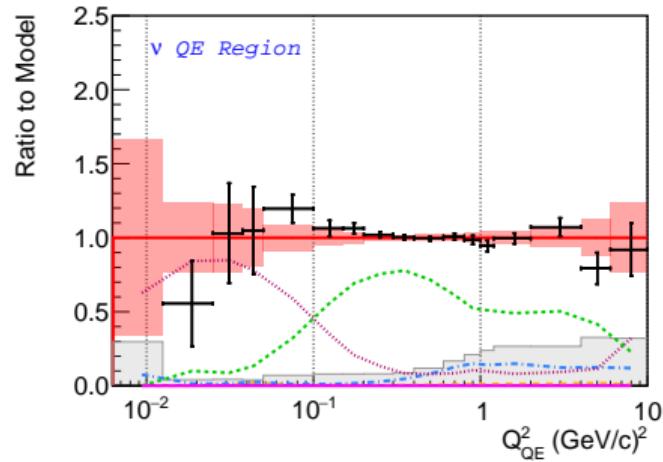
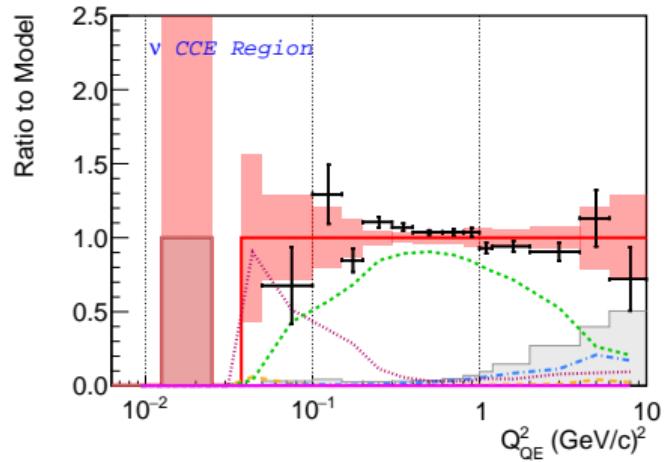


## MC events in slices of $Q^2$



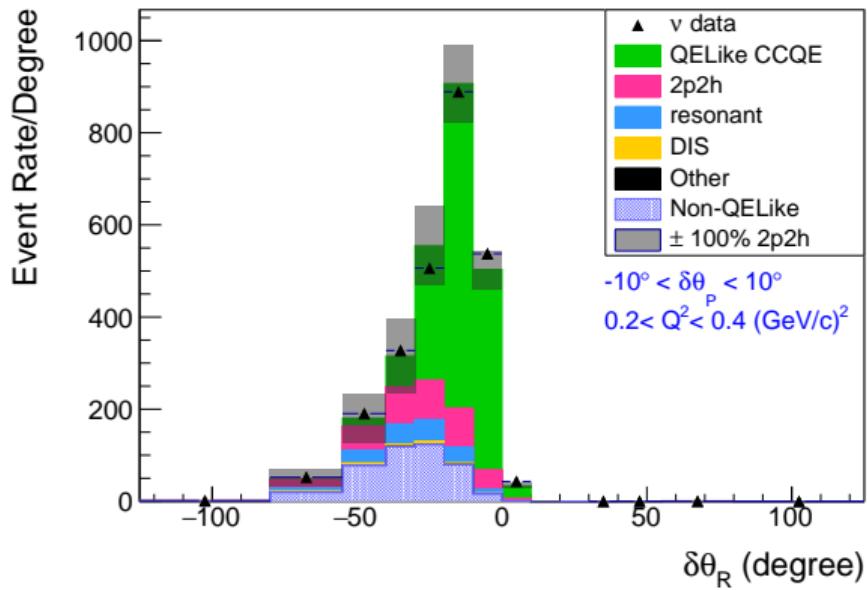
2p2h and resonant – all over the place but different.

# Validating background constraint method with neutrino sample



We select events with trackable protons in a **neutrino sample**. Different final states and available kinematics. Apply same fitting mechanism. Data and MC agree within uncertainty except  $Q^2 \in (0.2, 0.4)(\text{GeV}/c)^2$ .

# Validating background constraint method with neutrino sample



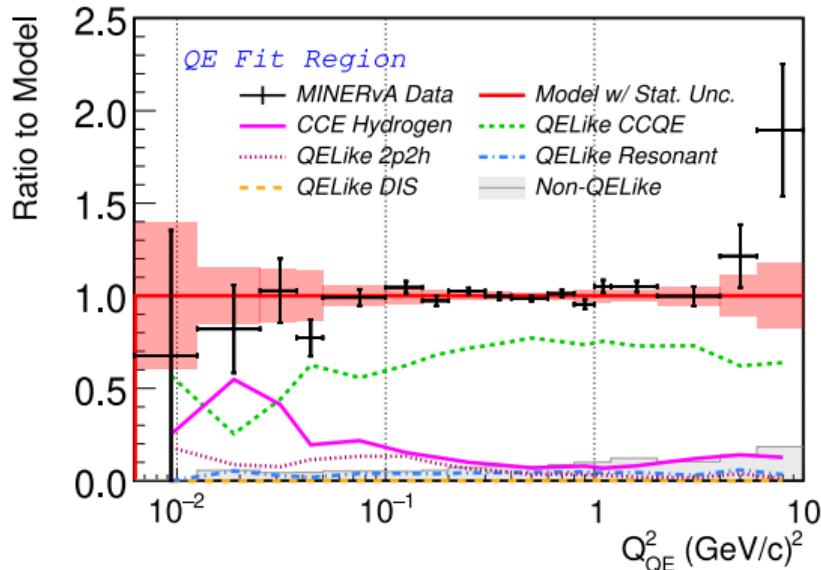
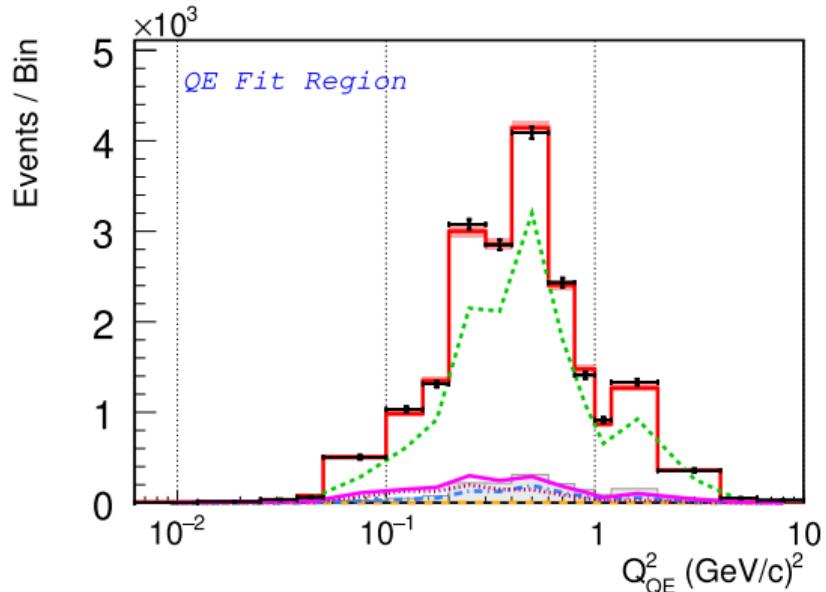
Neutrino mode:

- 100% 2p2h uncertainty covers data/simulation discrepancy.

Antineutrino mode:

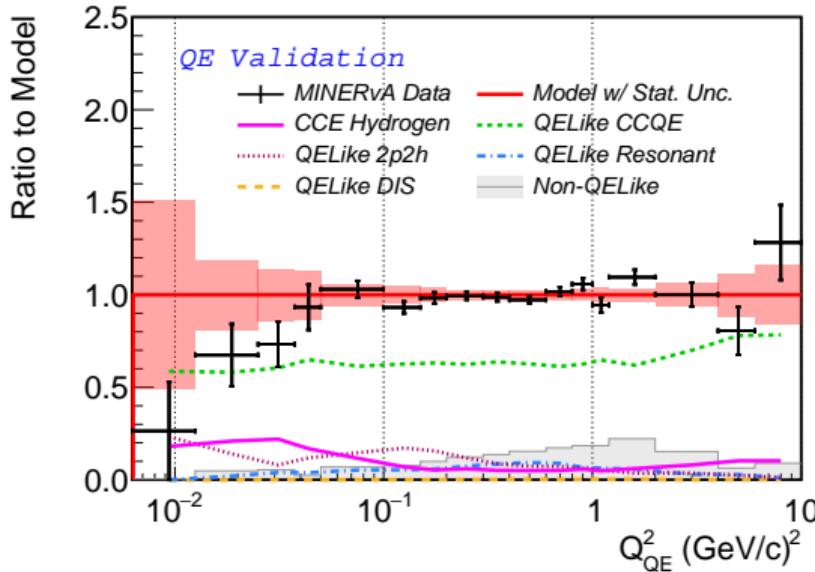
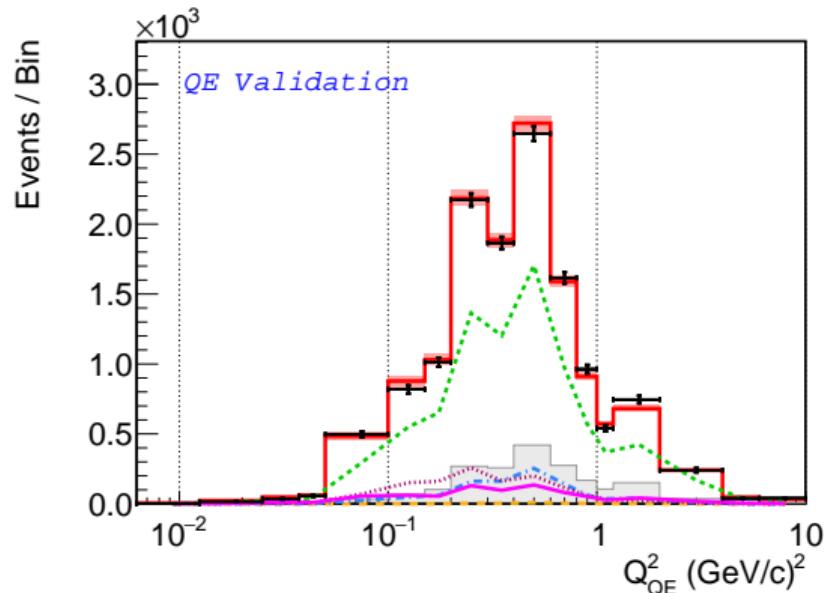
- Small 2p2h contribution.
- Sufficiently covered by total uncertainty.

# Fit using sideband region



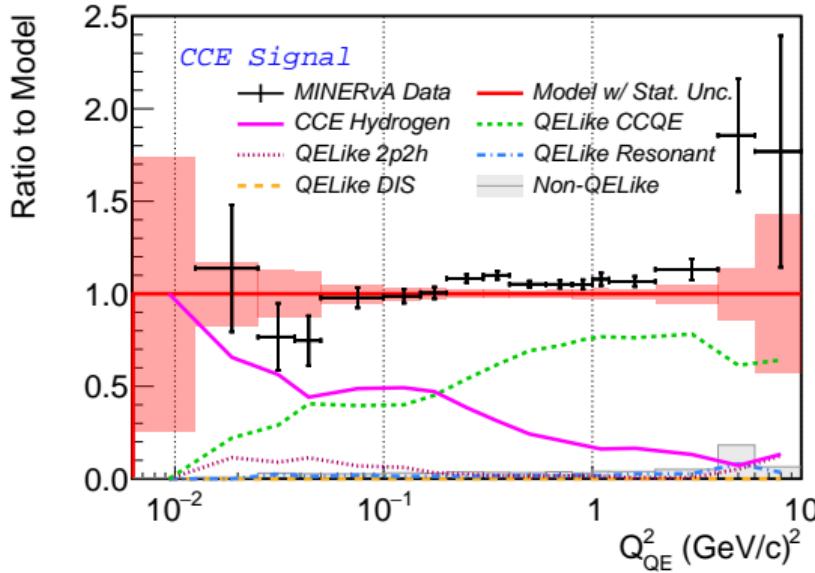
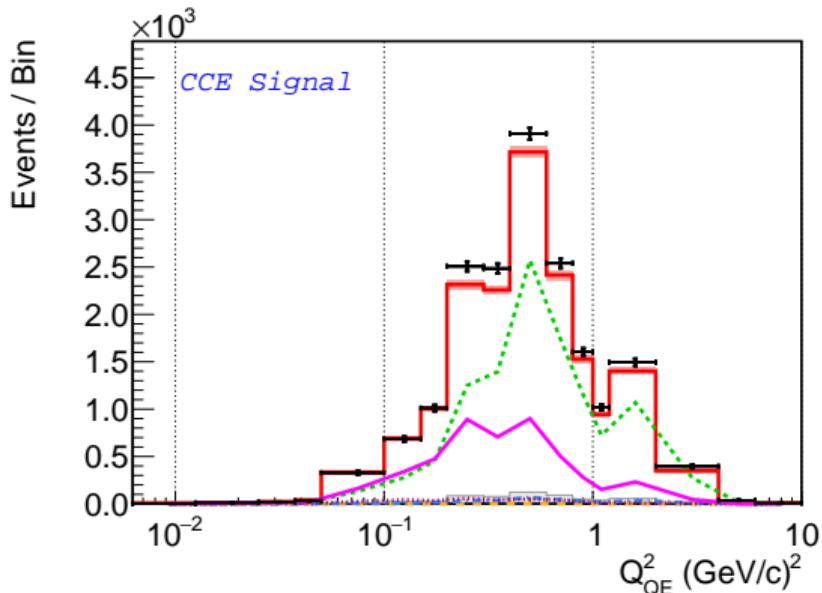
CCQE is the dominant background. Small 2p2h, resonant, and Non-QELike contributions. The fitted model are well constrained by data.

## Check fit with validation region



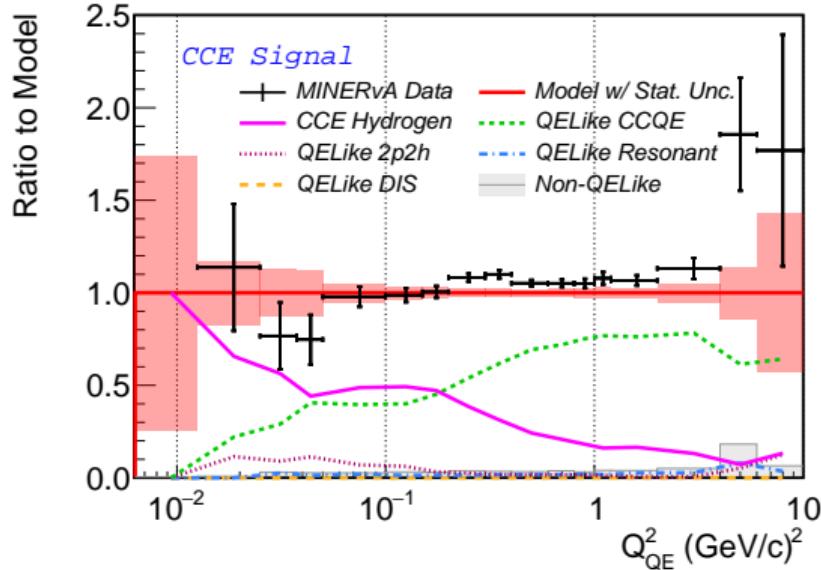
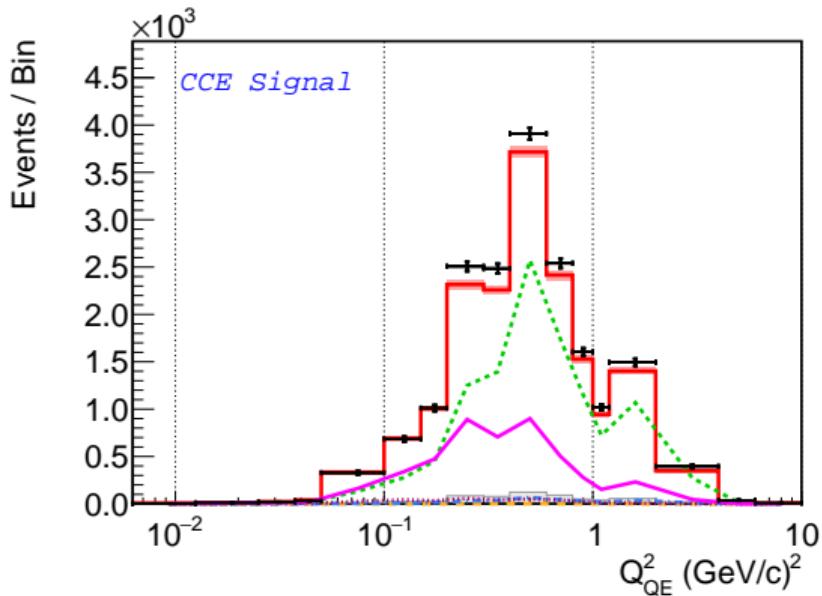
CCQE is the dominant background. Small 2p2h, resonant, and Non-QELike contributions. The fitted model are well constrained by data.

# Apply fit to signal region



Projecting the fit into the signal region. Difference between data and background is the physics. More than 5000 hydrogen events!

# Apply fit to signal region

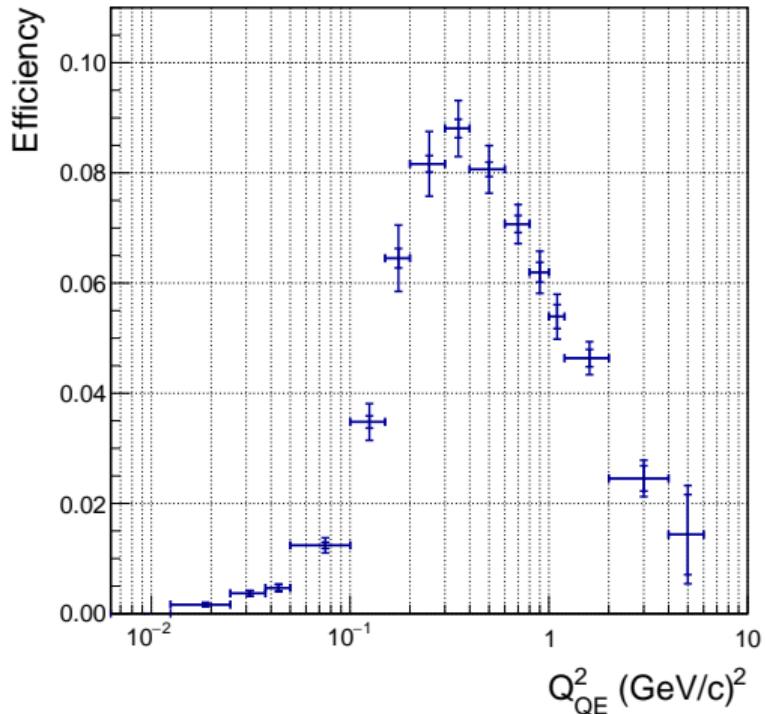


Subtract background, and correct detector smearing using D'Agostini iterative unfolding with 4 iterations.

# Efficiency

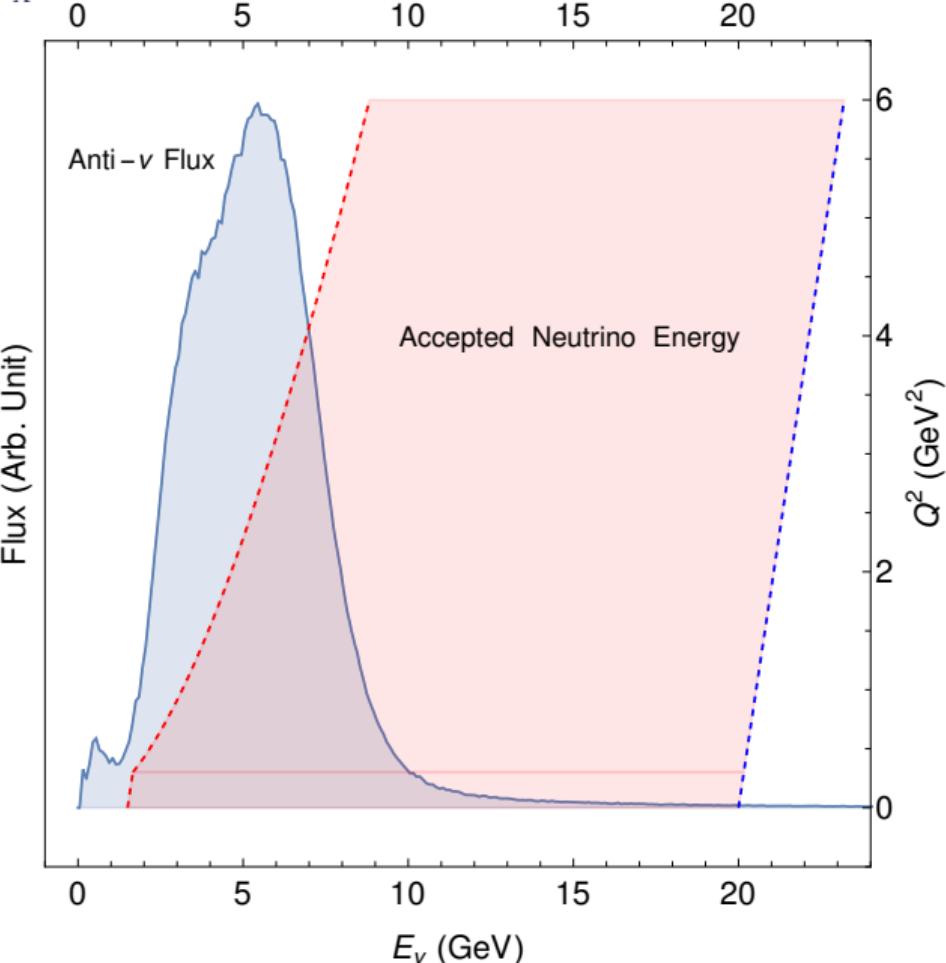
$$\epsilon = \frac{\text{signal events recorded}}{\text{total signal events produced}}$$

- Peaks at  $Q^2 \sim 0.3 \text{ (GeV}/c)^2$
- Low  $Q^2$ : Low acceptance because neutron needs to produce protons that span at least 2 planes.
- high  $Q^2$ : Reconstruction inefficiency and larger opening angles and less detector material to contain neutrons.

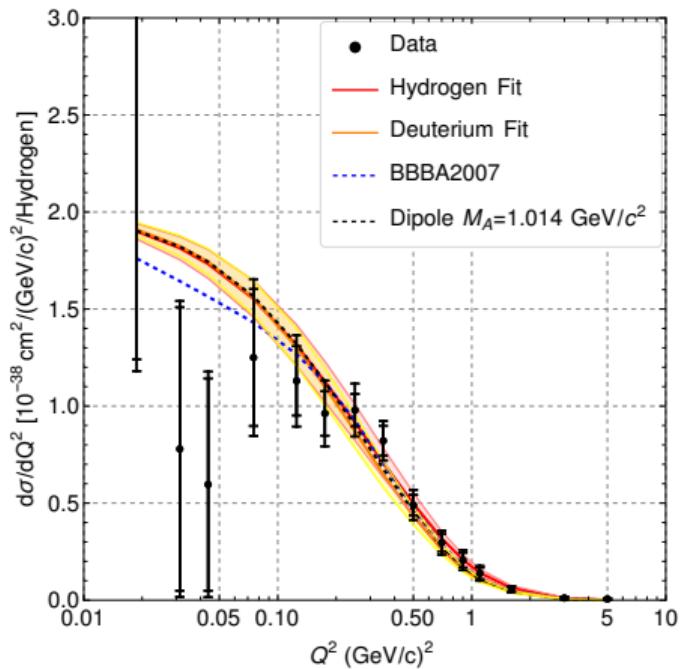


## Cross section prediction

- The single nucleon Llewelyn Smith equation convolutes with flux and muon acceptance.
  - ▶ Necessary for theory calculations.
- We use a standard vector form factor parameterization. (BBBA2005, Bradford et al., 2005<sup>24</sup>)
- $F_A$  fit assumes z-expansion form<sup>25</sup> (details in the backup).



# Cross section



Deuterium Fit: Meyer et al., 2016<sup>25</sup>  
 BBBA2007: Bodek et al. 2007<sup>26</sup>

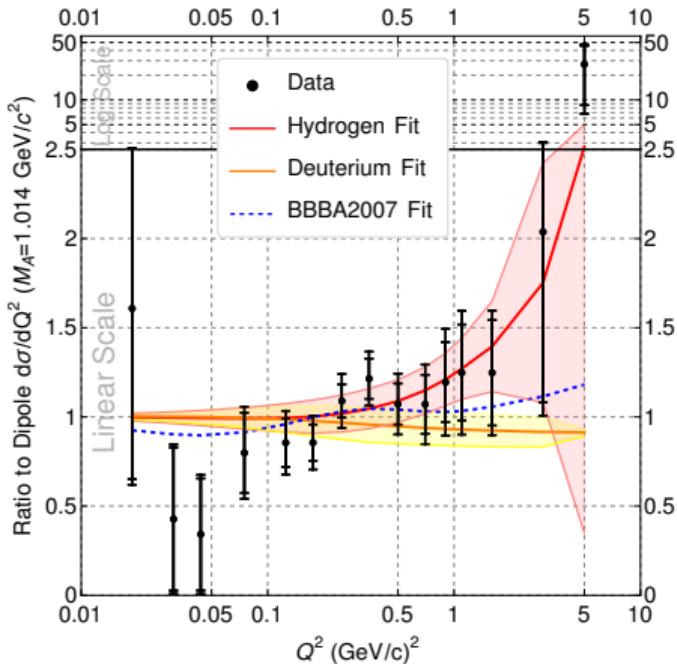
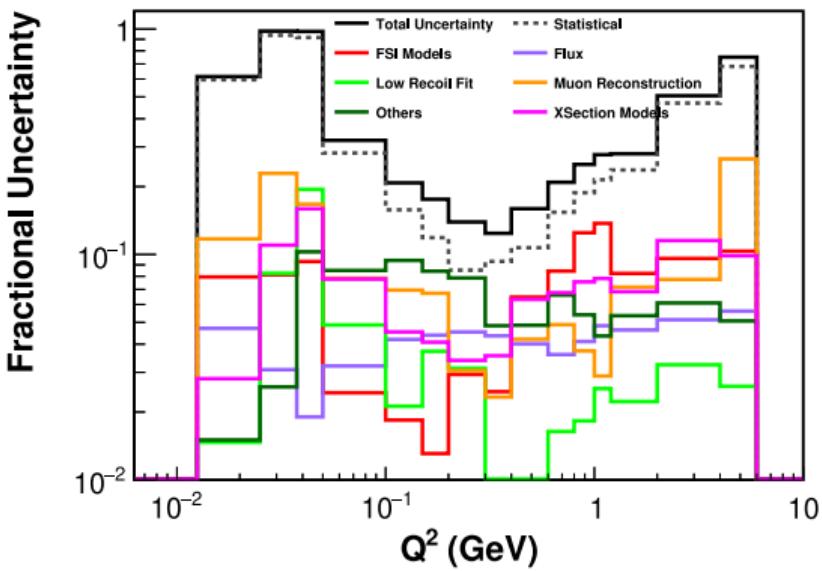


Figure: Extracted cross section and ratio to a dipole form factor.

# Systematic and statistical uncertainty

- Dominated by statistical uncertainty from background subtraction, despite enhanced signal
- Systematic uncertainties from residuals of background subtraction
- Particle responses in the “other” category, dominated by neutron systematics.



# $F_A$ fit and axial radius of the nucleon

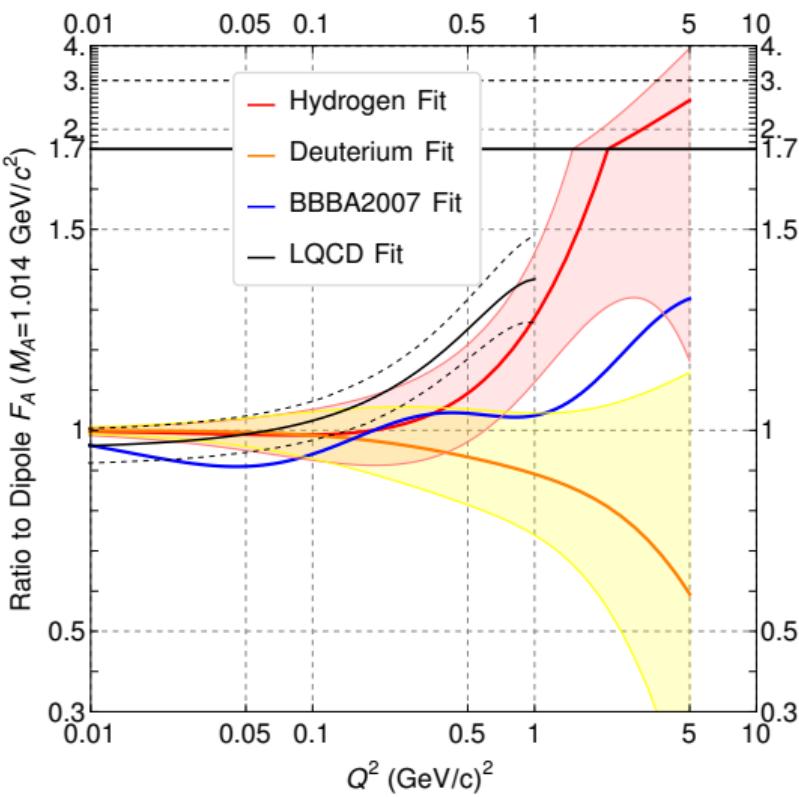
Result of  $F_A$  fit:

Calculate proton radius from  $F_A$  for  $Q^2 \rightarrow 0$ .

$$F_A(Q^2) = F_A(0) \left( 1 - \frac{\langle r_A^2 \rangle}{3!} Q^2 + \frac{\langle r_A^4 \rangle}{5!} Q^4 + \dots \right),$$

$$\frac{1}{F_A(0)} \frac{dF_A}{dQ^2} \Big|_{Q^2=0} = -\frac{1}{6} \langle r_A^2 \rangle$$

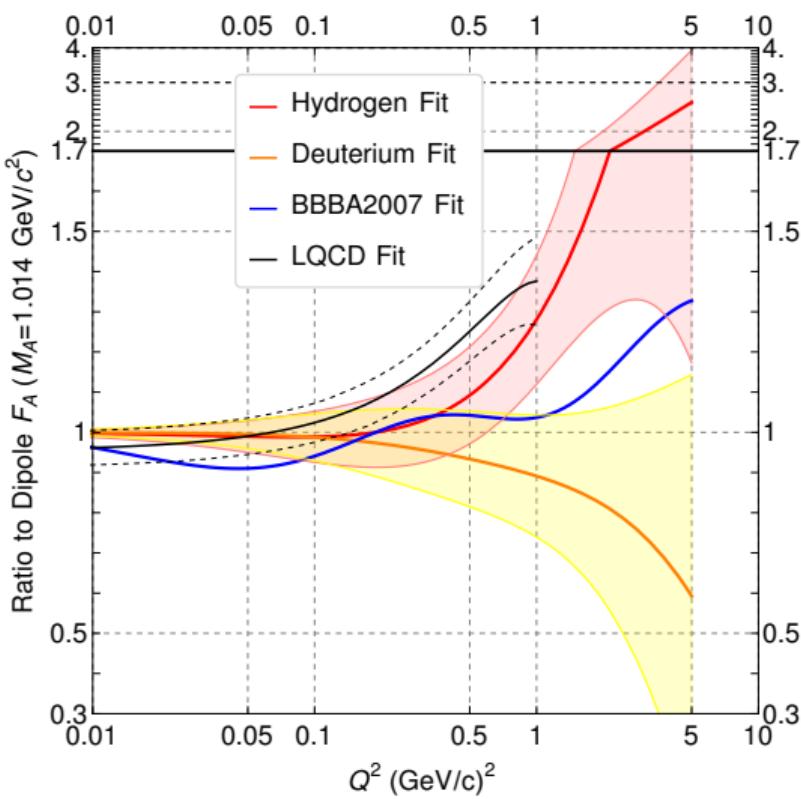
- $\langle r_A^2 \rangle = 0.53(25)\text{fm}^2$
- $\sqrt{\langle r_A^2 \rangle} = 0.73(17)\text{fm}$



# $F_A$ fit and axial radius of the nucleon

Result of  $F_A$  fit:

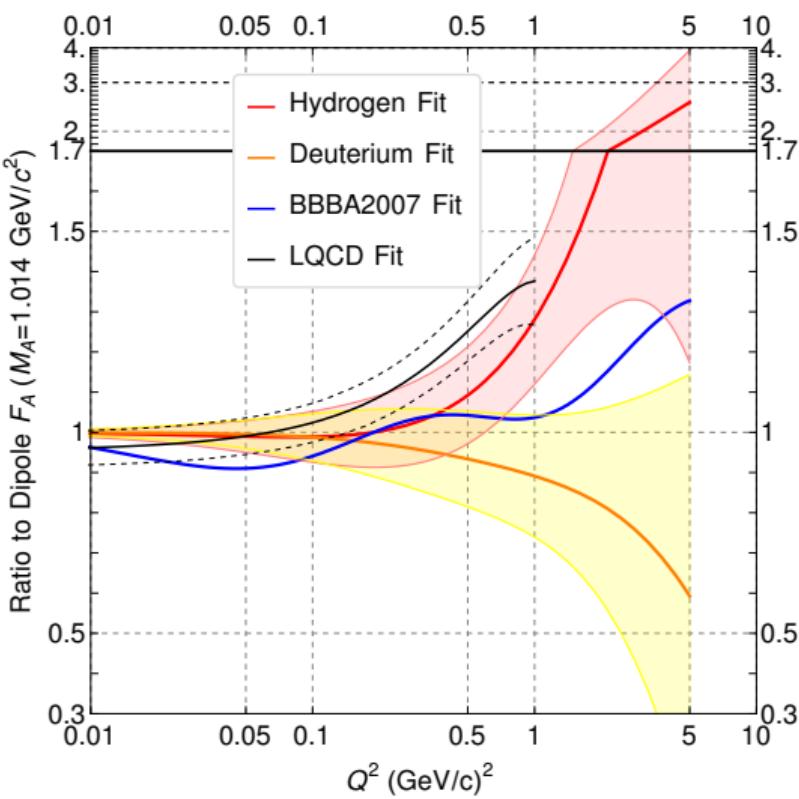
Deuterium fit applies z-expansion to bubble chamber data focused for  $Q^2 < 1 \text{ (GeV}/c)^2$ .  
 Hydrogen fit data up to  $Q^2 = 5 \text{ (GeV}/c)^2$ .  
 Tension between hydrogen and deuterium at larger  $Q^2$ .



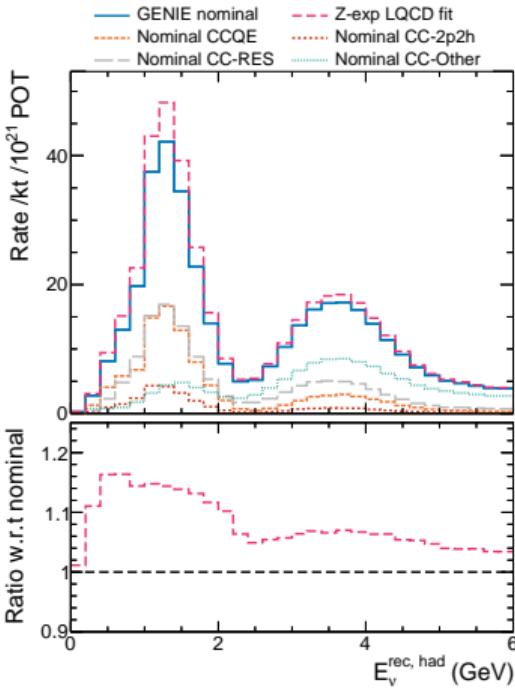
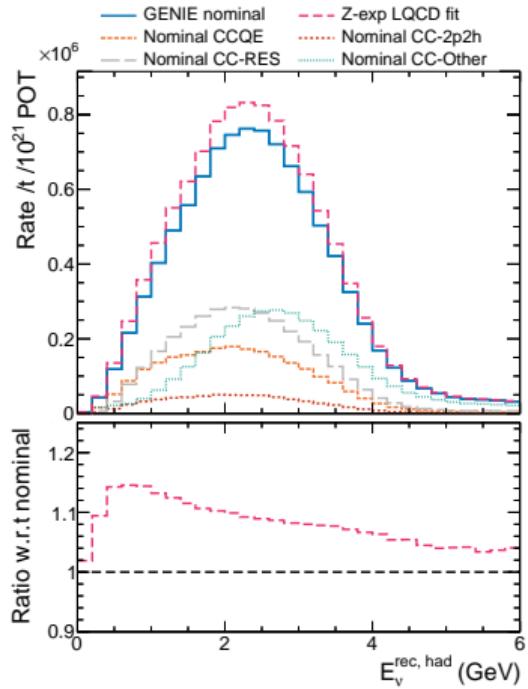
## $F_A$ fit and axial radius of the nucleon

Result of  $F_A$  fit:

A fit to lattice QCD calculation by Lin<sup>27</sup> is consistent with our fitted  $F_A$  up to  $1(\text{GeV}/c)^2$



# Nucleon form factor impacts DUNE physics



$\nu_{\mu}$  event rate in DUNE near (left) and far (right) detector.

$$E_{\nu}^{\text{rec, had}} = E_l + \sum_p E_{\text{kin}} + \sum_{\pi^{\pm}, \pi^0, \gamma} E_{\text{tot}}$$

- Solid blue: GENIEv3 10a\_02\_11a, CCQE uses dipole  $F_A$ .
- Dashed black: replace dipole  $F_A$  with z-expansion  $F_A$  fitted to LatCat LQCD.

10% ~ 20% effect.

Meyer, Walker-Loud, and Wilkinson, 2022<sup>28</sup>.

## Summary

First new nucleon  $F_A$  measurement in 30 years and the only statistically significant measurement on free nucleon.

- The result will aid in better understanding the weak nucleon structure.
- An important ingredient for DUNE → impacts cross section.
- The new techniques developed are useful for current and future experiments:
  - ▶ neutron reconstruction
  - ▶ method of background constraint

Appeared online today: *Nature* 614, 48–53 (2023)

# Thank you! The MINERvA Collaboration



# Backup

## Background constraint method

MC categories (**C**) with significant contributions in each  $Q_{\text{QE}}^2$  bin and angular region is used as template. QELike: CCQE, 2p2h, resonant. Non-QELike:  $\pi^0$ ,  $\pi^\pm$ .

$$\chi^2 = \sum_{S,i} \frac{\left( \left[ \sum_C w_{C,i} N_{C,S,i}^{\text{mc}} \right] - N_{S,i}^{\text{data}} \right)^2}{N_{S,i}^{\text{data}}} + \lambda_S \sum_C \sum_{j=1}^{N-2} (w_{C,j} + w_{C,j+2} - 2w_{C,j+1})^2.$$

$\chi^2$  term

regularization term

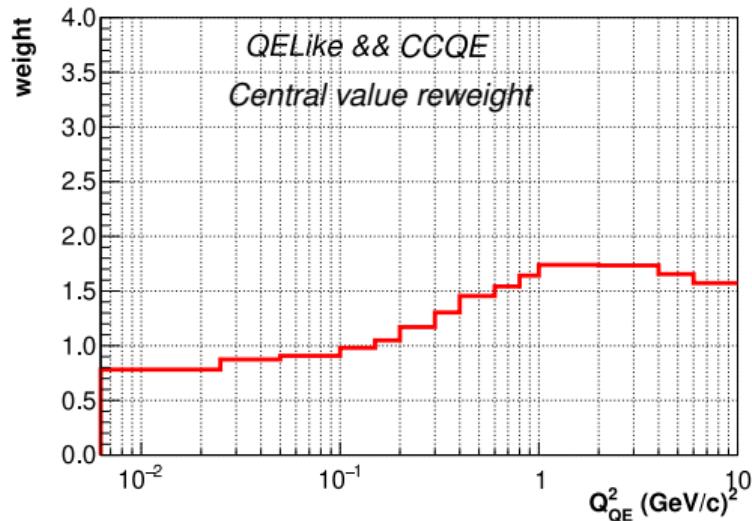
**S**: sideband – angular regions participating in fit. **i**:  $Q_{\text{QE}}^2$  bin. **w**: Weight for each category in each bin.

$\lambda_S$  controls the strength of the regularization, affects smoothness of the fit, and obtained through an L-curve style scan.

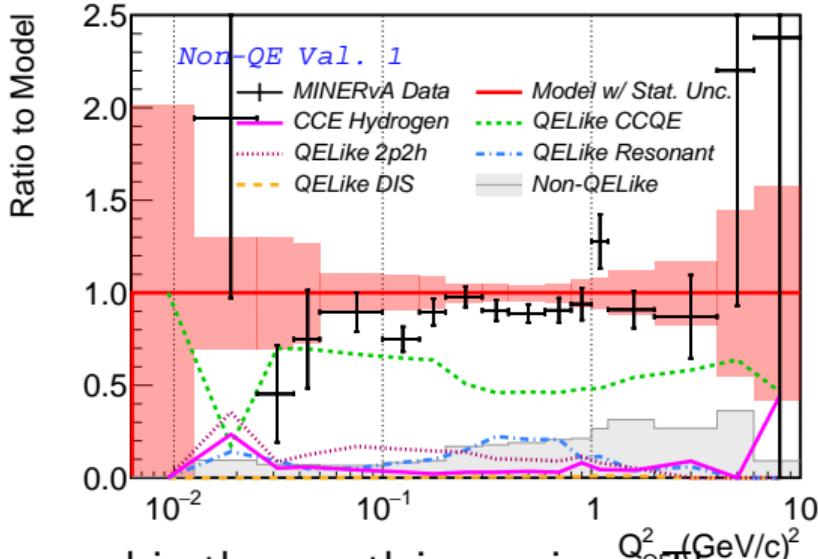
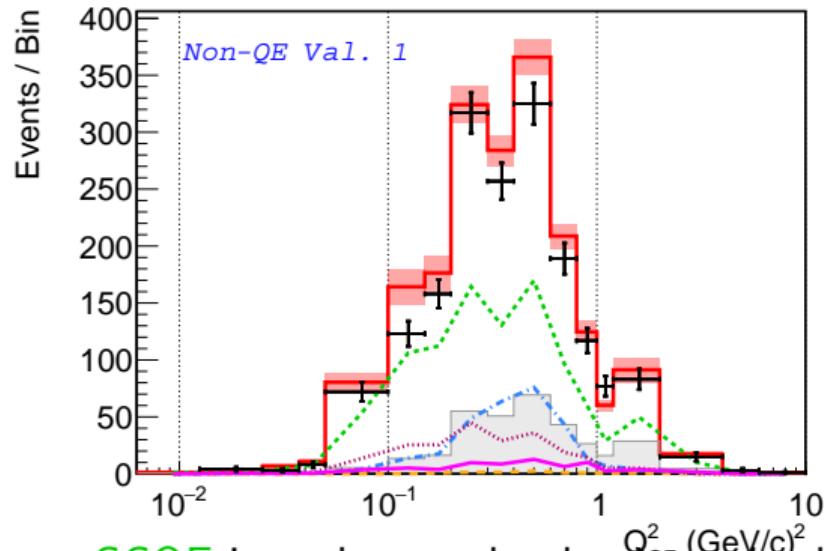
# Explanation of the Weights

The CCQE weight is affected by

- Base GENIE model
- Various model tunes:
  - ▶ RPA, NuWro SF
- Final state tunes:
  - ▶ FSI tunes

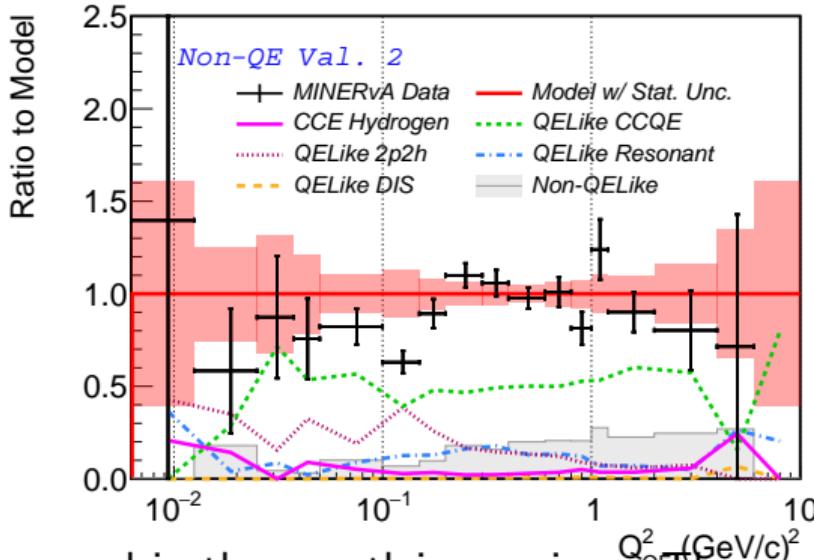
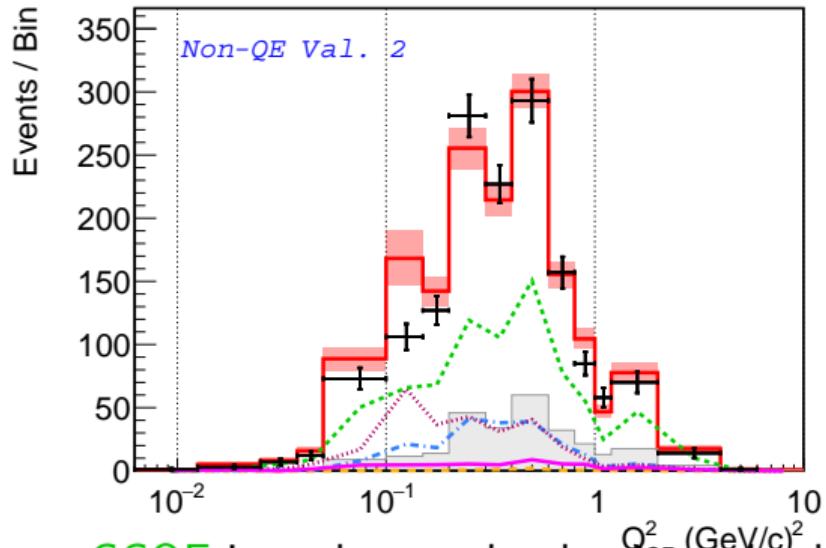


# Non-QE validation region 1 – fitted



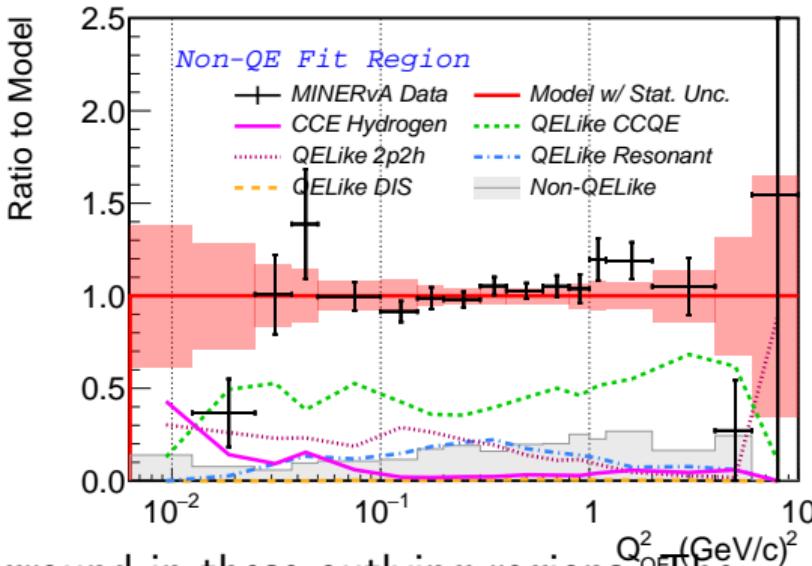
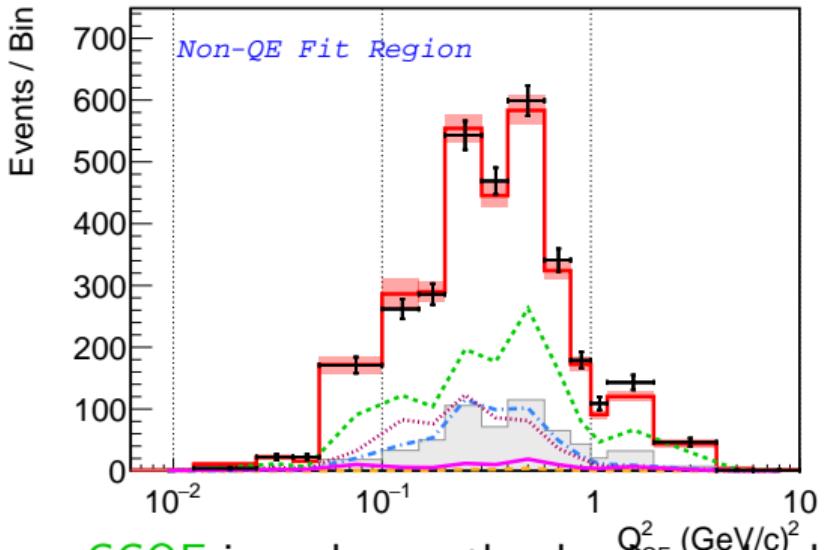
CCQE is no longer the dominant background in these outlying regions. The Non-QE validation regions are mechanically separated into two sub-regions but has smaller effect on the parameter scan due to low statistics.

## Non-QE validation region 2 – fitted



CCQE is no longer the dominant background in these outlying regions. The Non-QE validation regions are mechanically separated into two sub-regions but has smaller effect on the parameter scan due to low statistics.

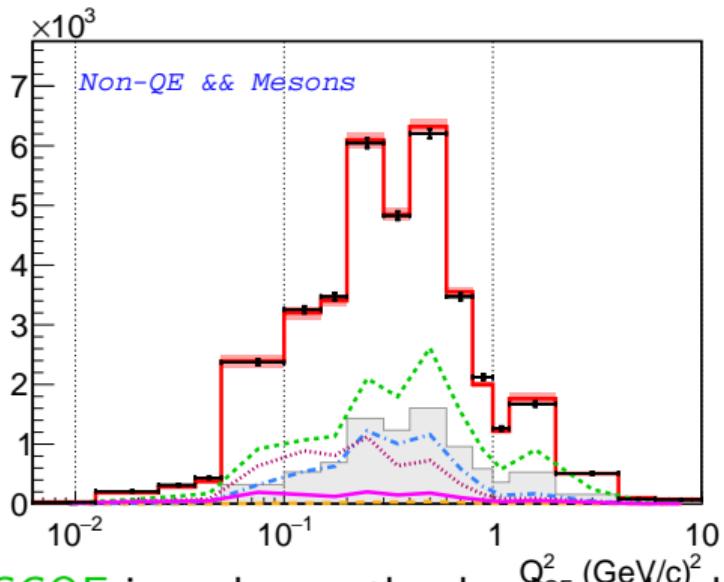
## Non-QE fit region – fitted



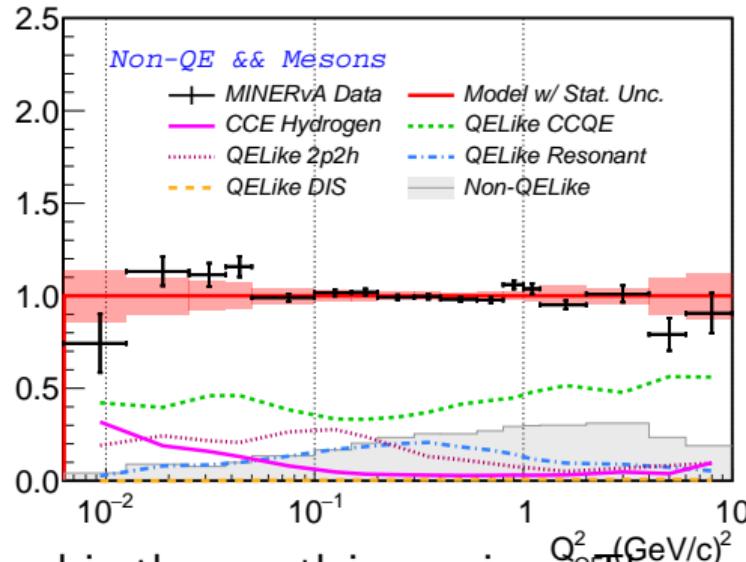
CCQE is no longer the dominant background in these outlying regions. The Non-QE validation regions are mechanically separated into two sub-regions but has smaller effect on the parameter scan due to low statistics.

## Non-QE meson fit region – fitted

Events / Bin

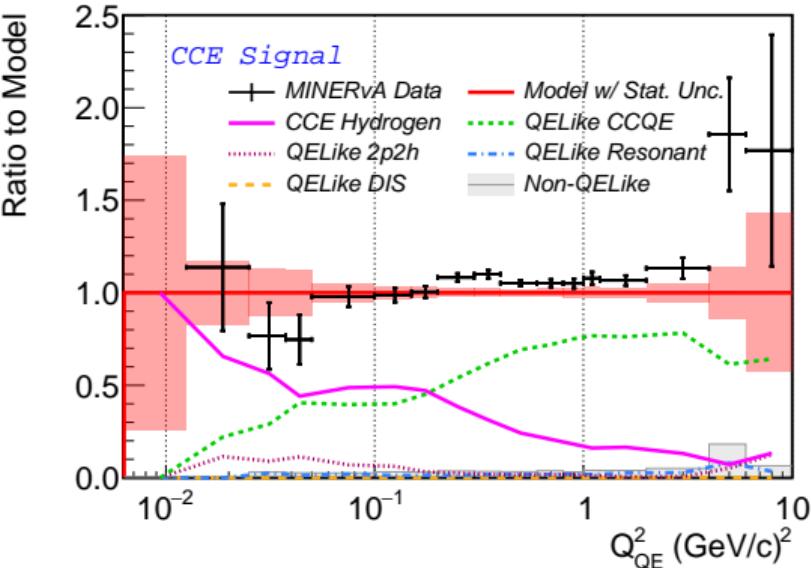
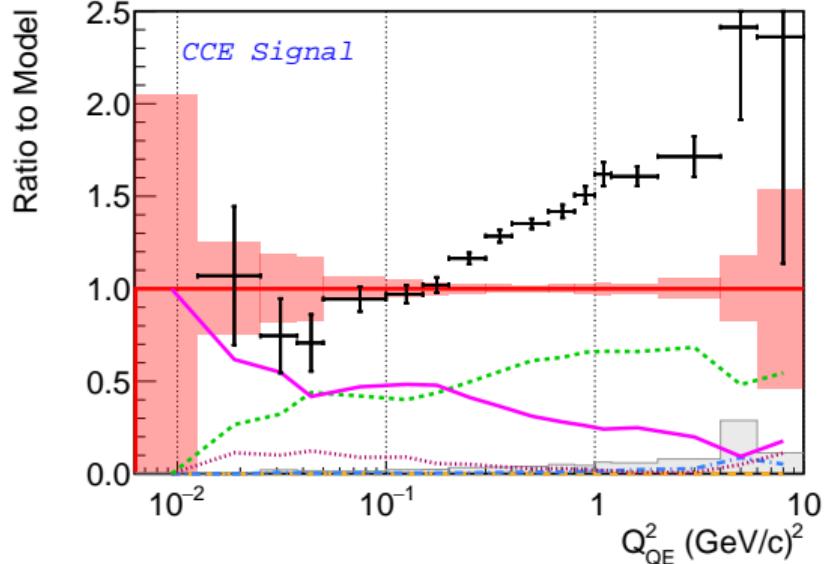


Ratio to Model



CCQE is no longer the dominant background in these outlying regions. The Non-QE validation regions are mechanically separated into two sub-regions but has smaller effect on the parameter scan due to low statistics.

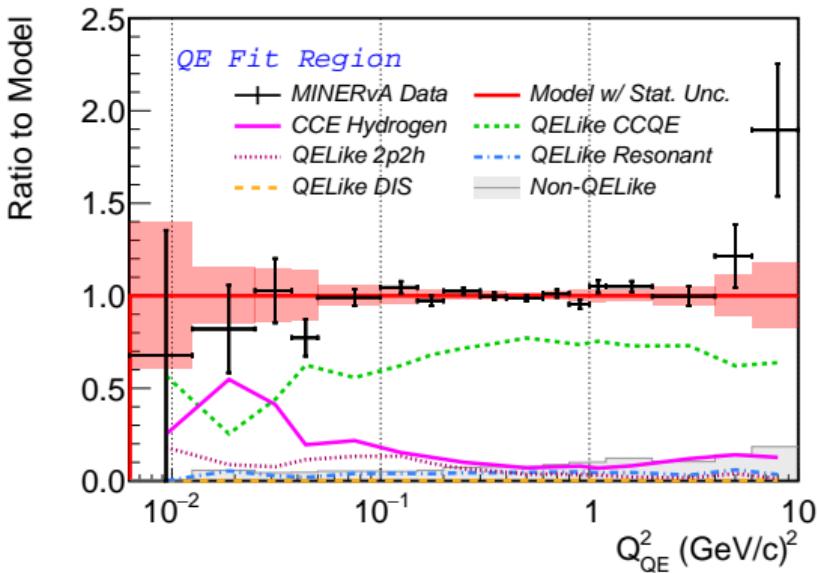
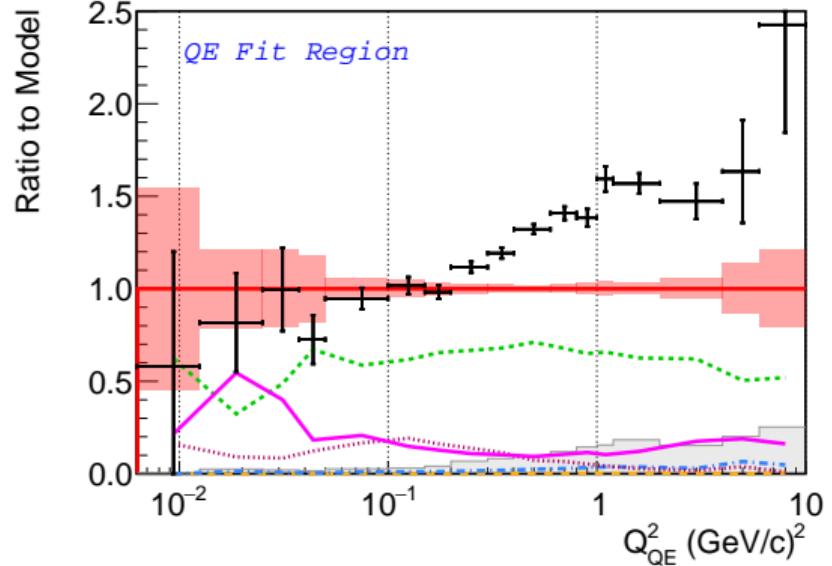
# Effects of background constraint on angular regions



CCE signal region

Ratio of data to (left) unfitted model, (right) post-fitted model.

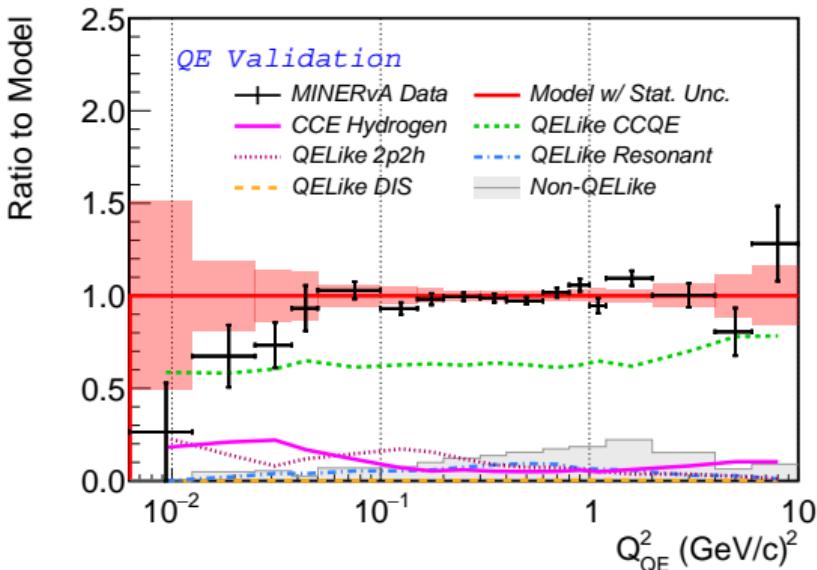
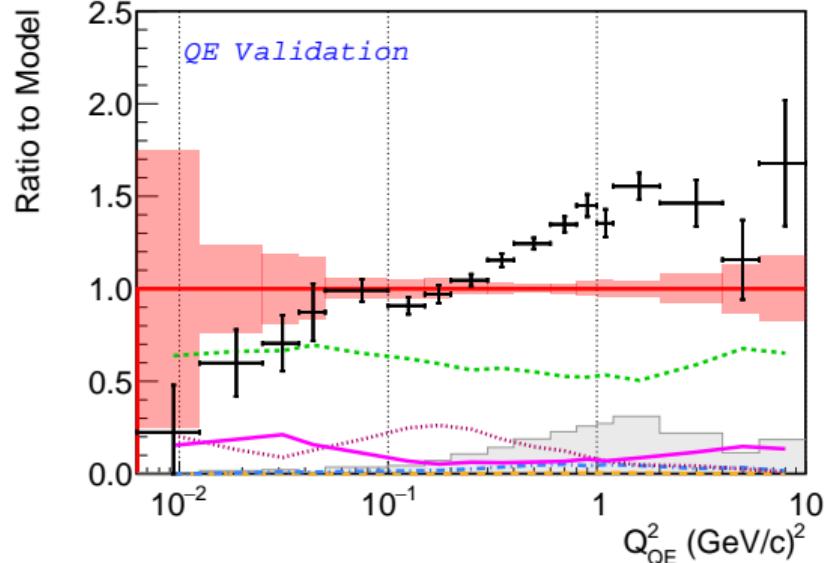
# Effects of background constraint on angular regions



QE fit region

Ratio of data to (left) unfitted model, (right) post-fitted model.

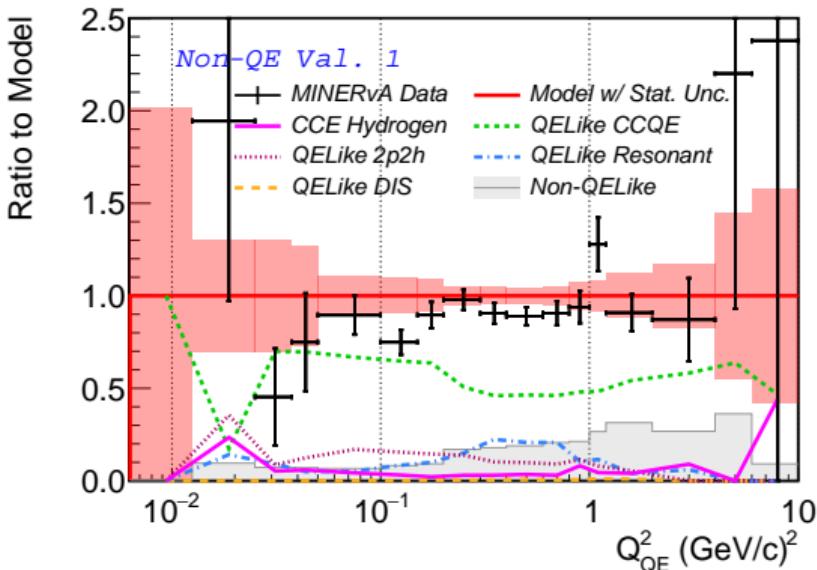
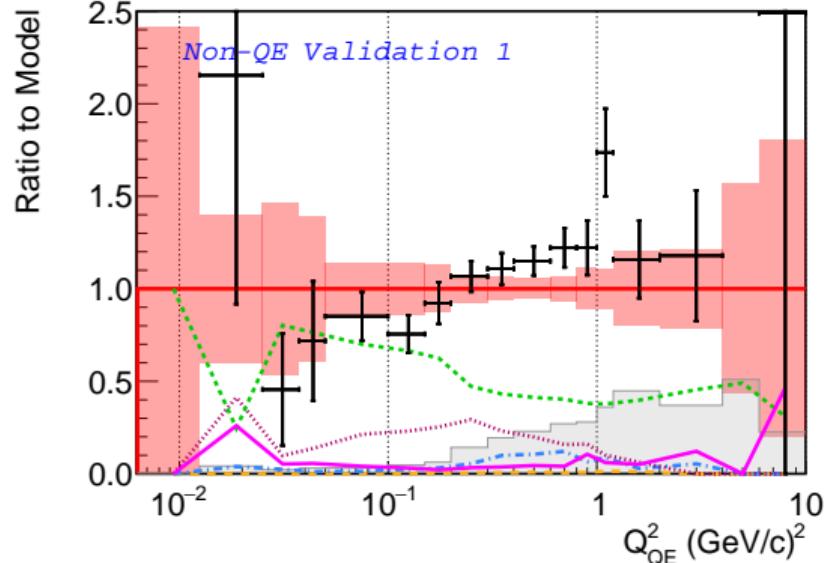
# Effects of background constraint on angular regions



QE validation region

Ratio of data to (left) unfitted model, (right) post-fitted model.

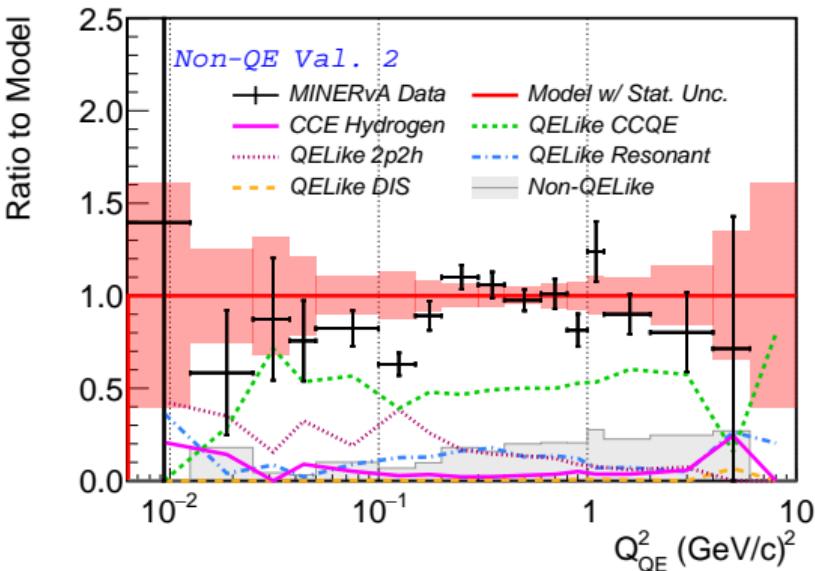
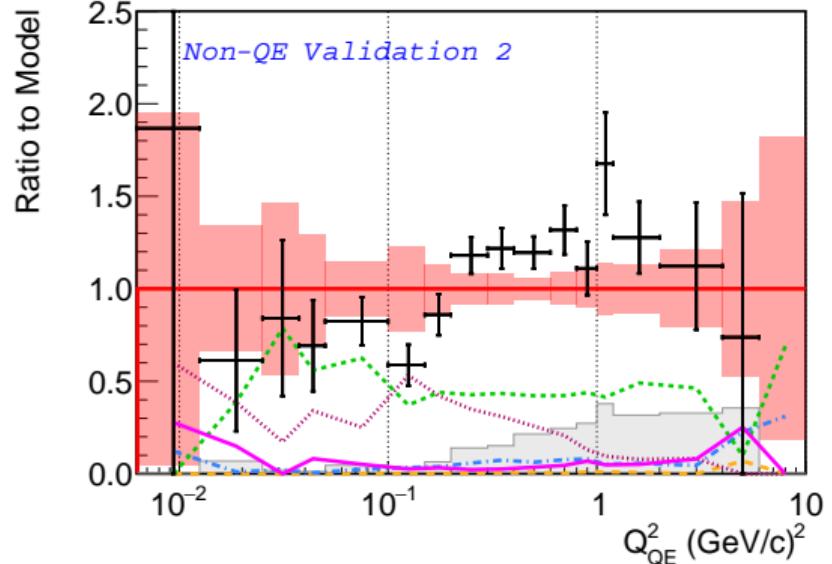
# Effects of background constraint on angular regions



Non-QE validation 1

Ratio of data to (left) unfitted model, (right) post-fitted model.

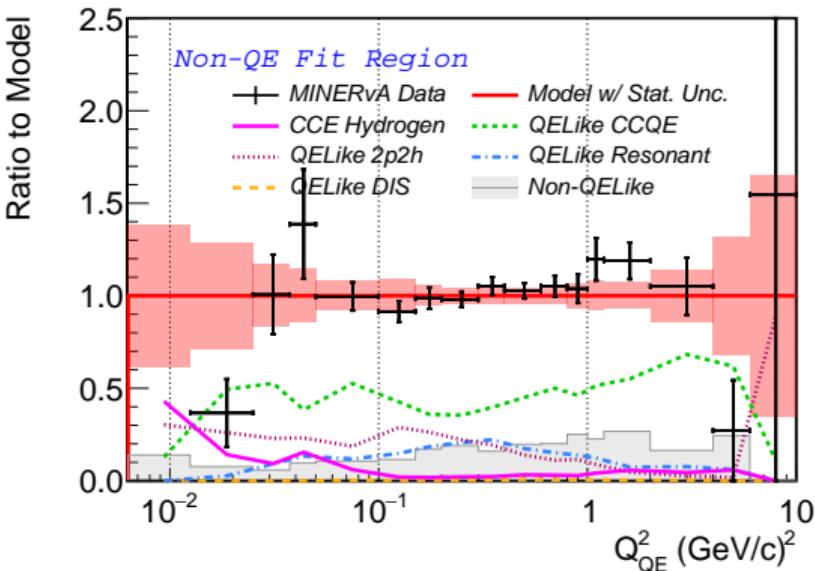
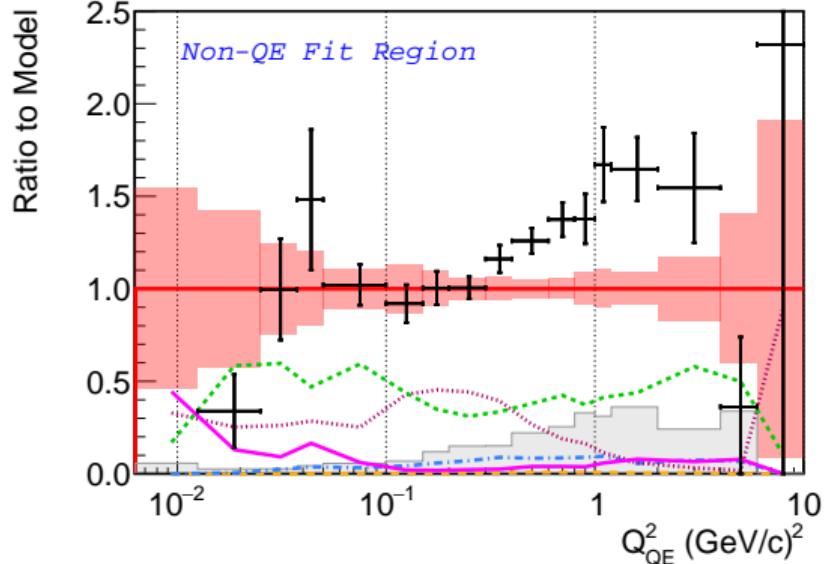
# Effects of background constraint on angular regions



Non-QE validation 2

Ratio of data to (left) unfitted model, (right) post-fitted model.

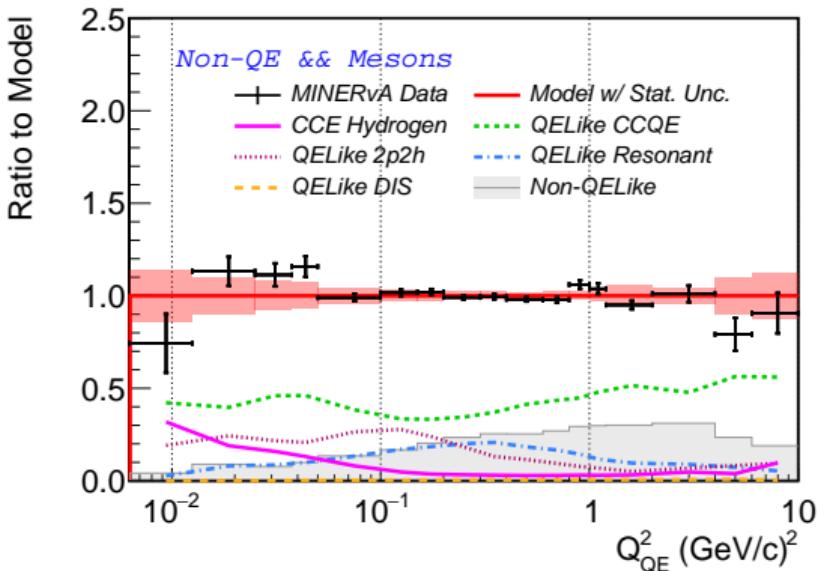
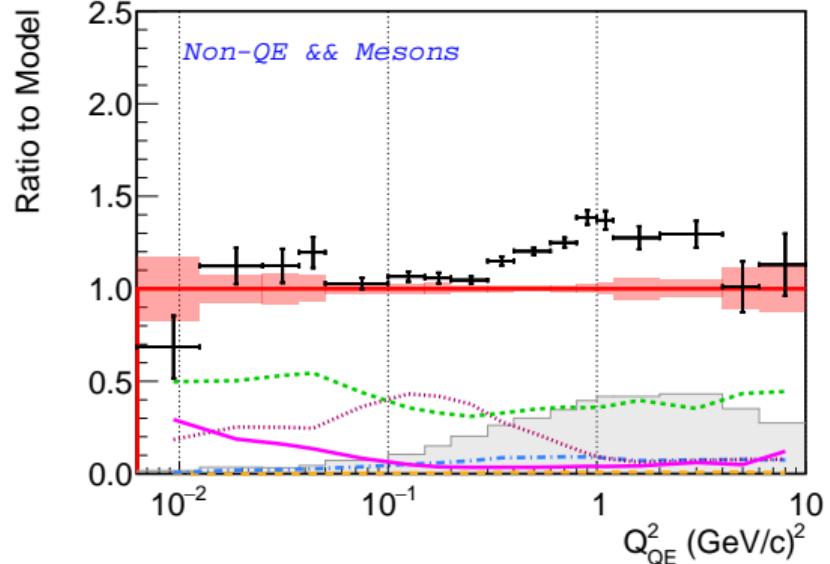
# Effects of background constraint on angular regions



Non-QE fit region

Ratio of data to (left) unfitted model, (right) post-fitted model.

# Effects of background constraint on angular regions



Non-QE and mesons fit region

Ratio of data to (left) unfitted model, (right) post-fitted model.

# MINERvA $\bar{\nu}_\mu$ -CH result – ratio of data to base model

Bashyal et al., 2022<sup>29</sup>

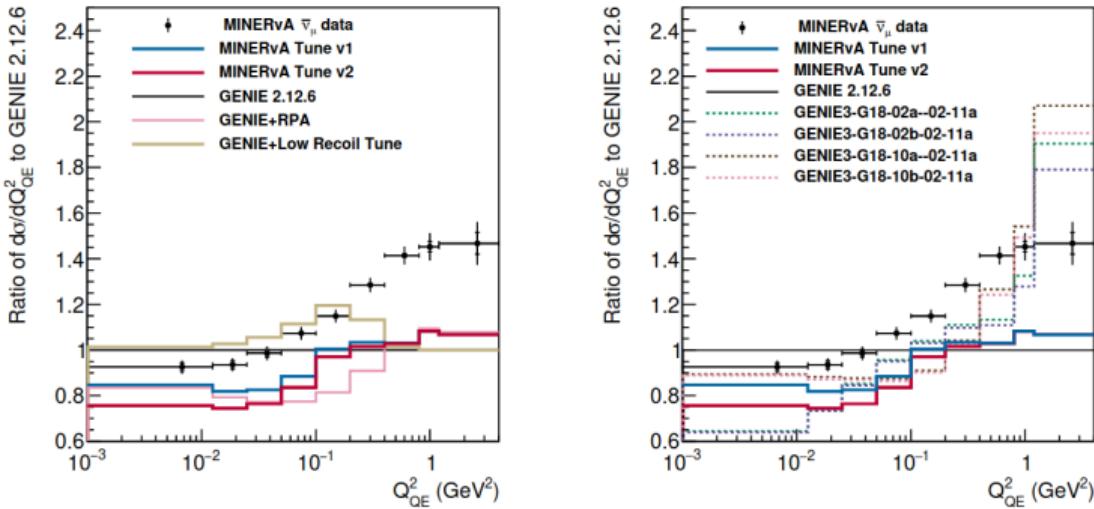
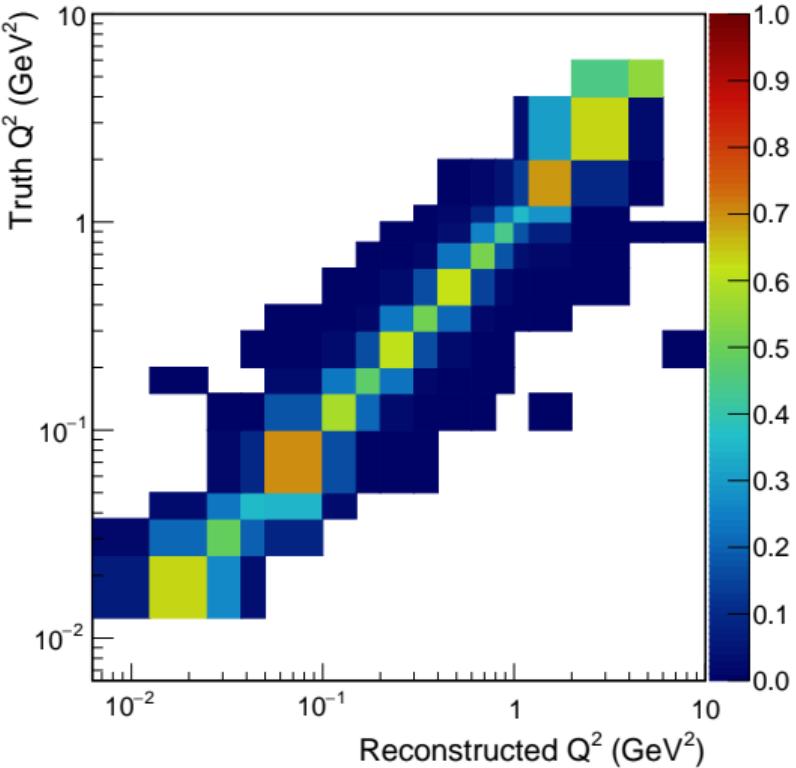


FIG. 4. Comparisons of the cross section predicted by various tunes applied on GENIE with respect to the baseline GENIE 2.12.6 (black) as a function of  $Q_{QE}^2$  (left). MINERvA Tune v1 (blue) is the standard simulation tuned to the MINERvA low energy data. MINERvA Tune v2 (red) is MINERvA Tune v1 with the non-resonant pions suppressed in the low  $Q_{QE}^2$  region [30]. The remaining curves show the effect of enabling different corrections to the base model. The plots on the right show comparisons of cross sections predictions for GENIE v3.0.6 (dotted lines) with the MINERvA tuned GENIE predictions. Inner ticks in the data are statistical and the outer ticks are the systematic uncertainties.

# Migration matrix and warping study

Row normalized migration matrix:  
Unfolding matrix obtained by

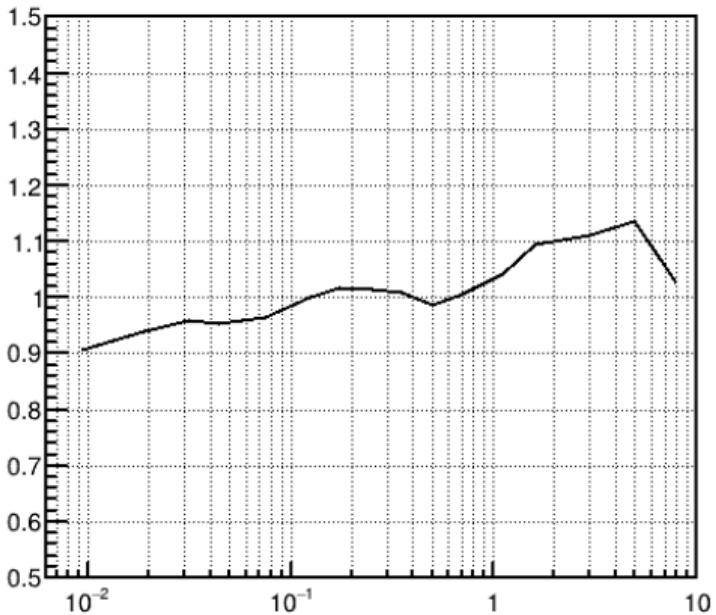
- D'Agostini iterative unfolding – stops at 4th iteration.



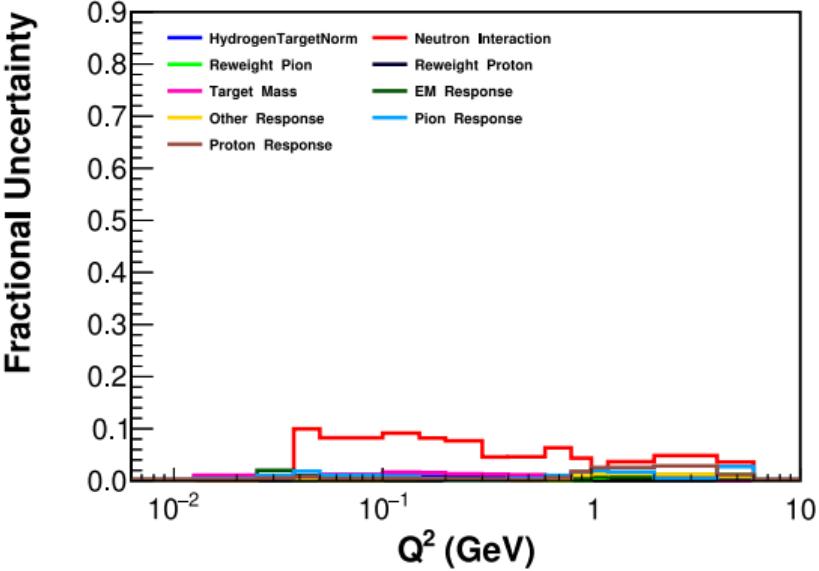
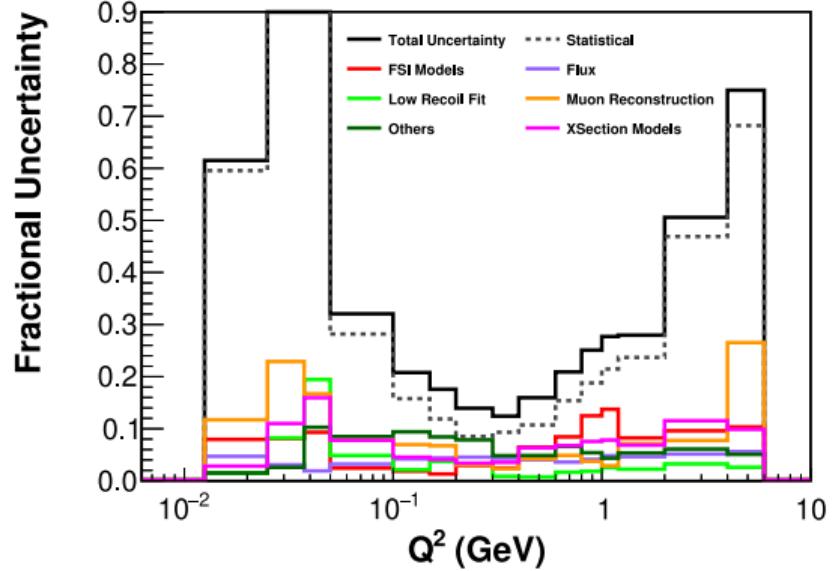
# Migration matrix and warping study

Ratio of “warped prediction” to base prediction.

- Performed warping study to check stability of regularization.
- “Warped” prediction with ratio, and
- and generated ensemble of 1000 statistical variations.
- Obtain average and median  $\chi^2$  between unfolded and truth of the ensemble for each iteration.

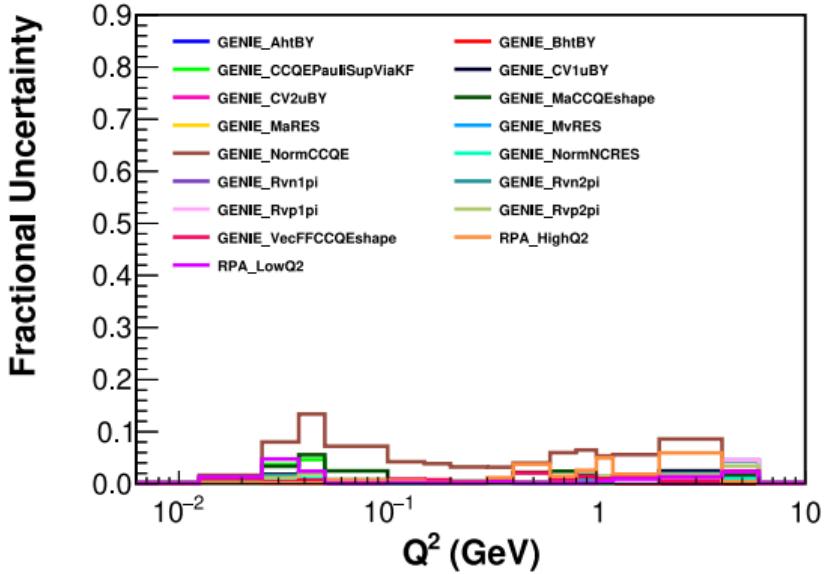
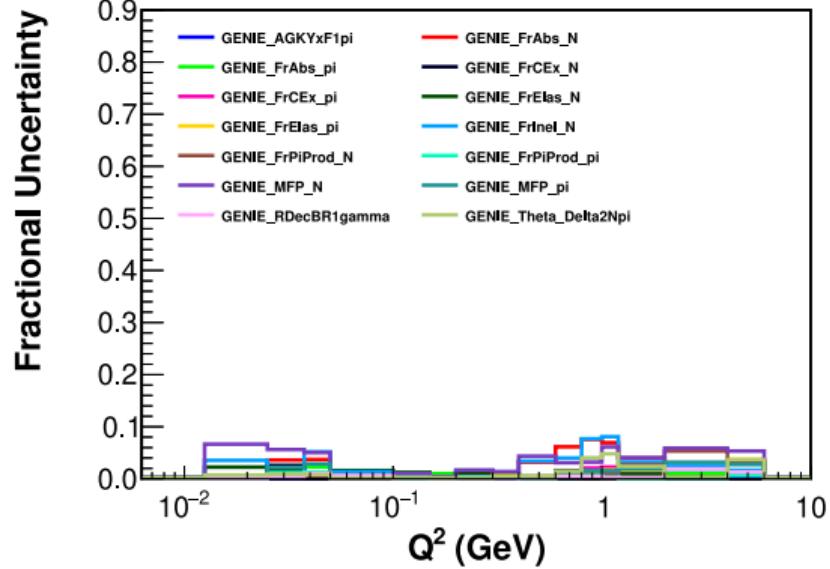


# Systematics Uncertainties



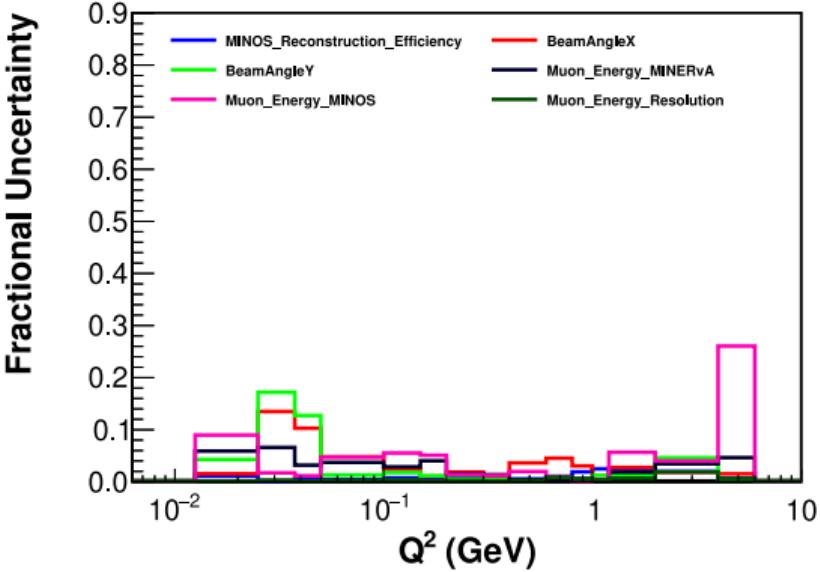
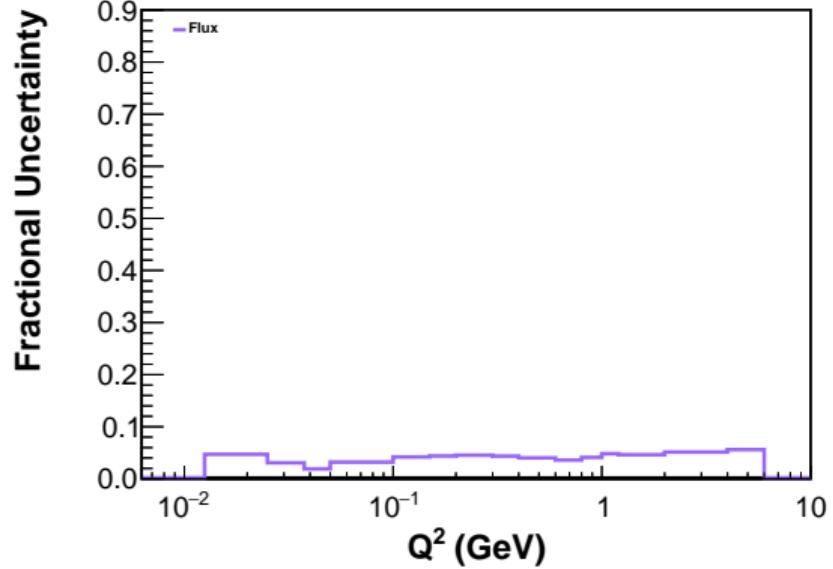
(left) all systematics, (right) others

# Systematics Uncertainties



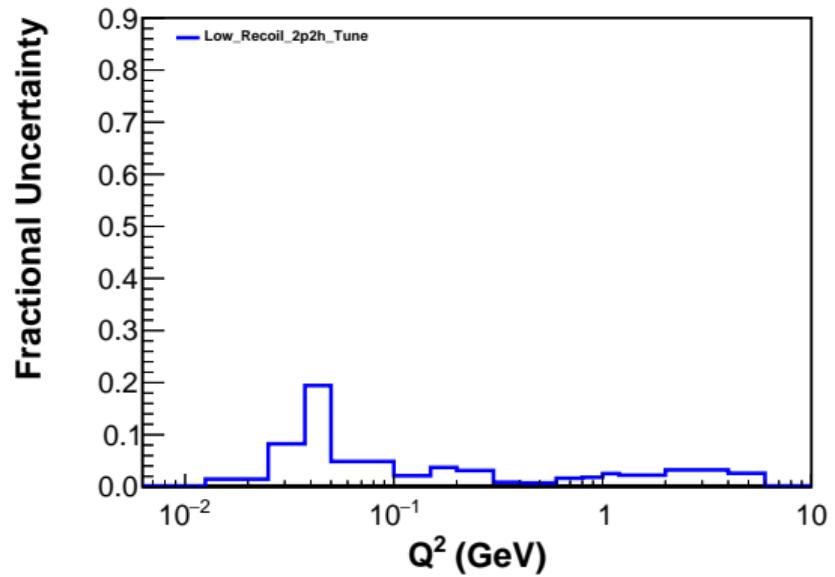
(left) FSI model, (right) cross section model

# Systematics Uncertainties



(left) flux, (right) muon reconstruction

# Systematics Uncertainties

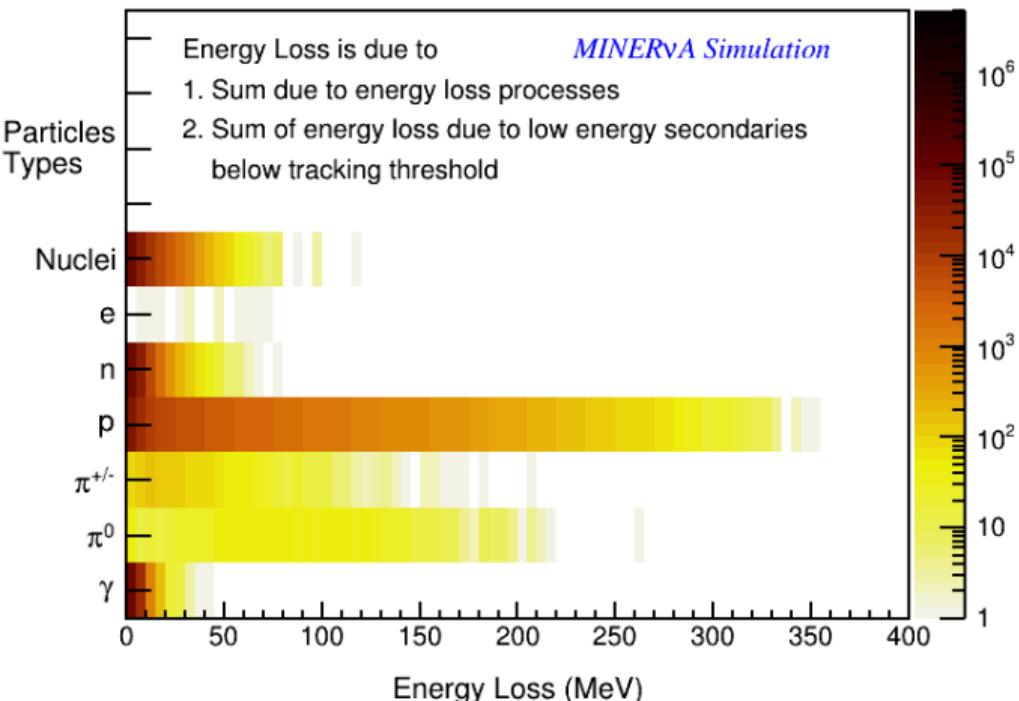


low recoil fit

# Neutron secondary scattering

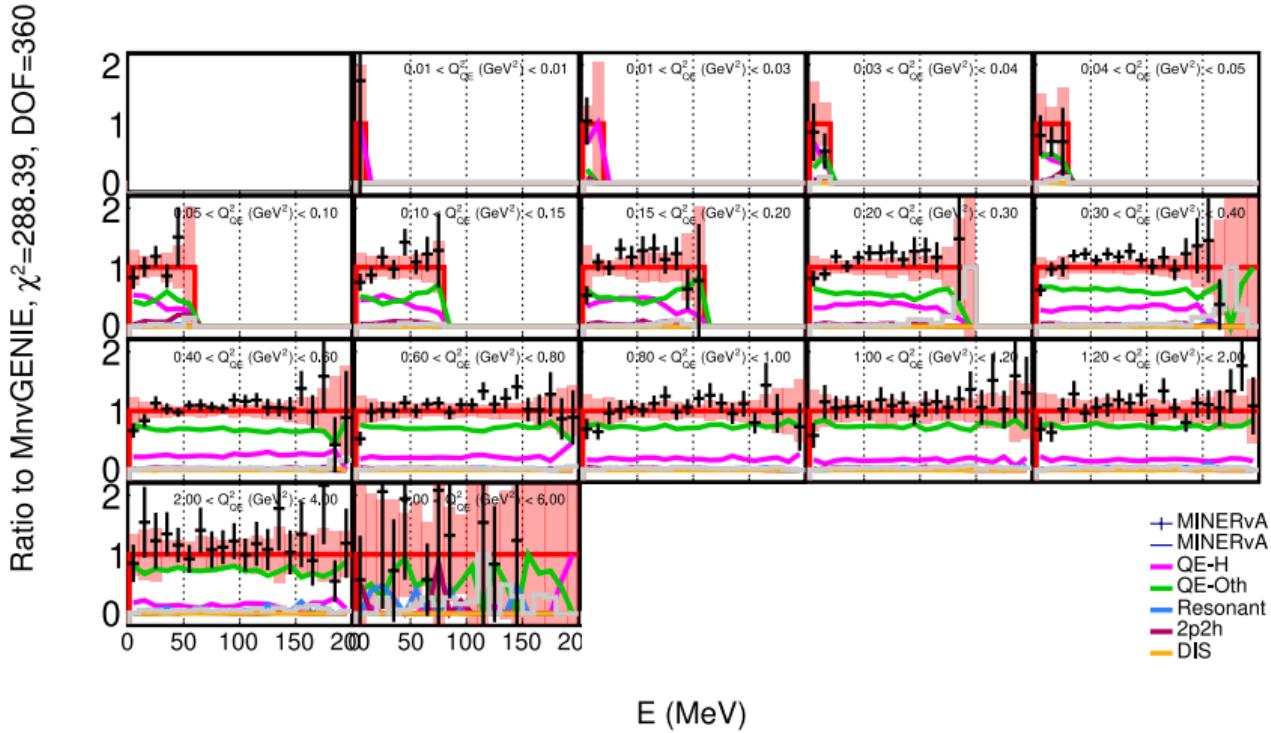
- Ejected protons responsible for majority of secondary neutron interactions.
- Inelastic scattering from carbon dominates (see previous slide)
- MINER $\nu$ A doesn't have access to the most updated GEANT4 because we need to access MINOS.

**Fragment Particles Type vs Energy Loss**



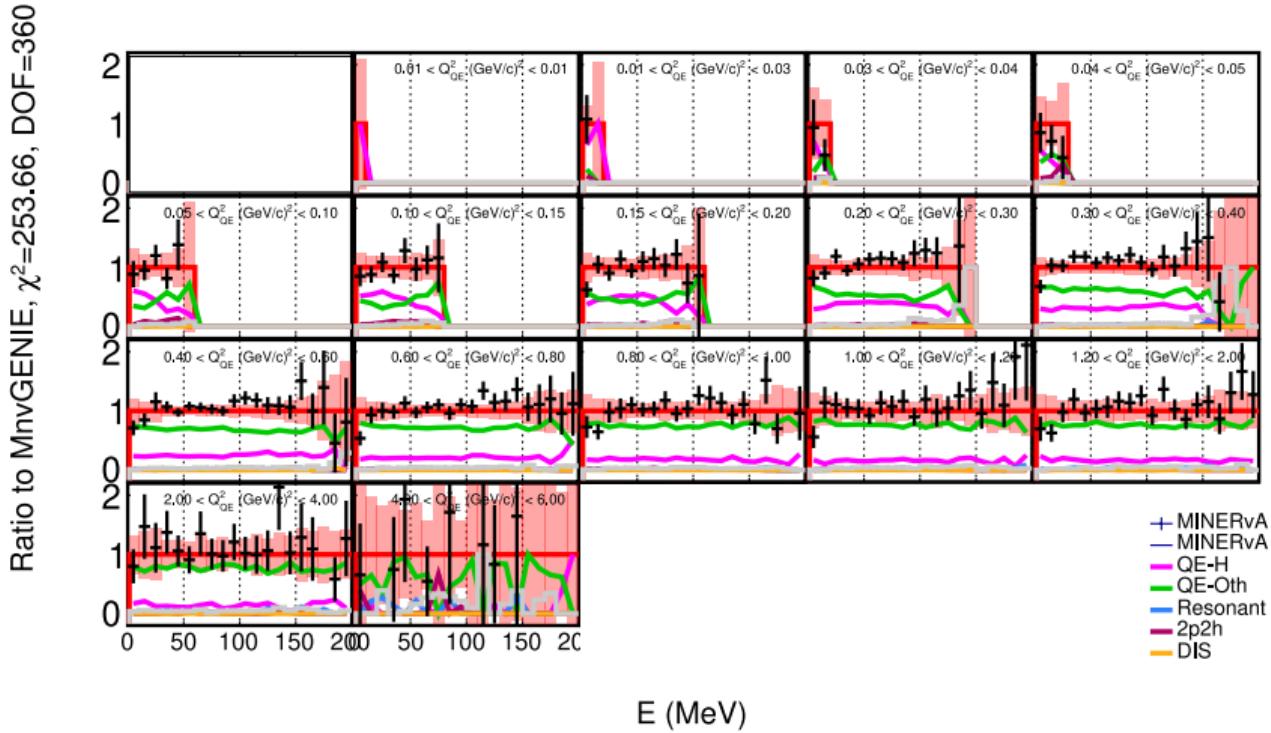
# Effect of MONA Reweighting – Blob Energy

Without MoNA



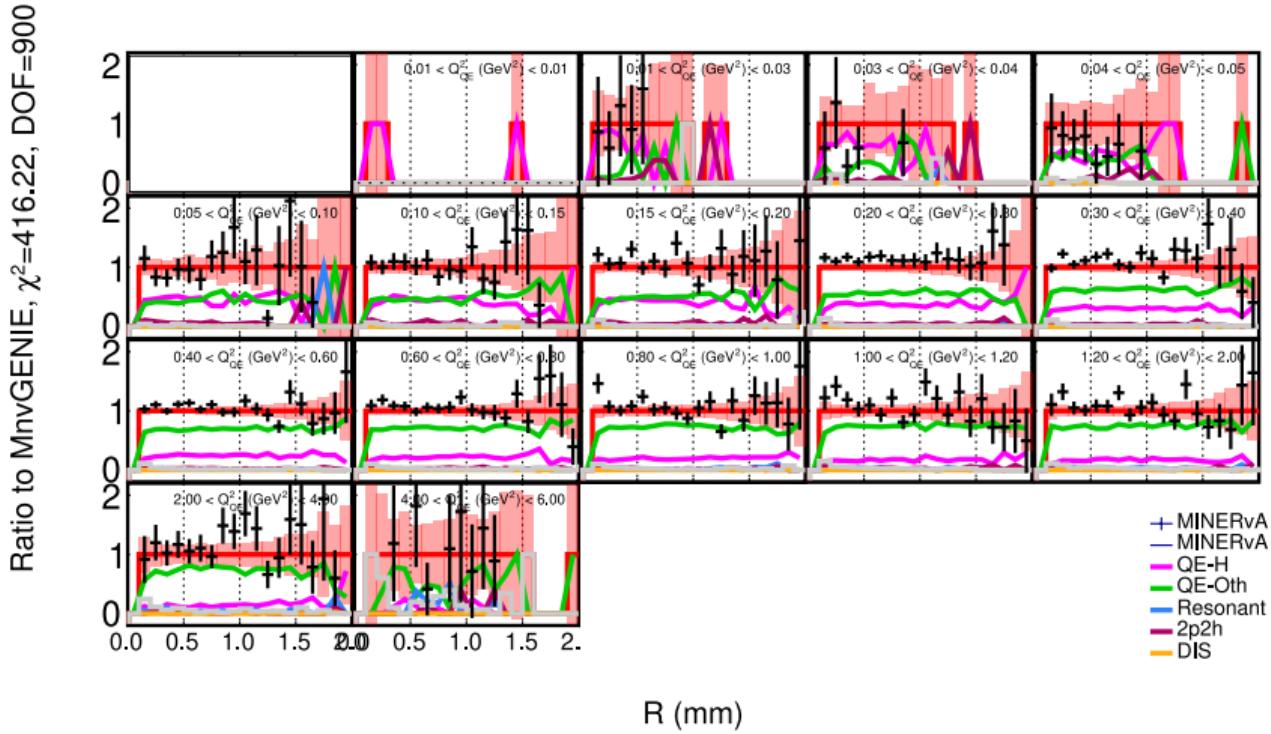
# Effect of MONA Reweighting – Blob Energy

With MoNA



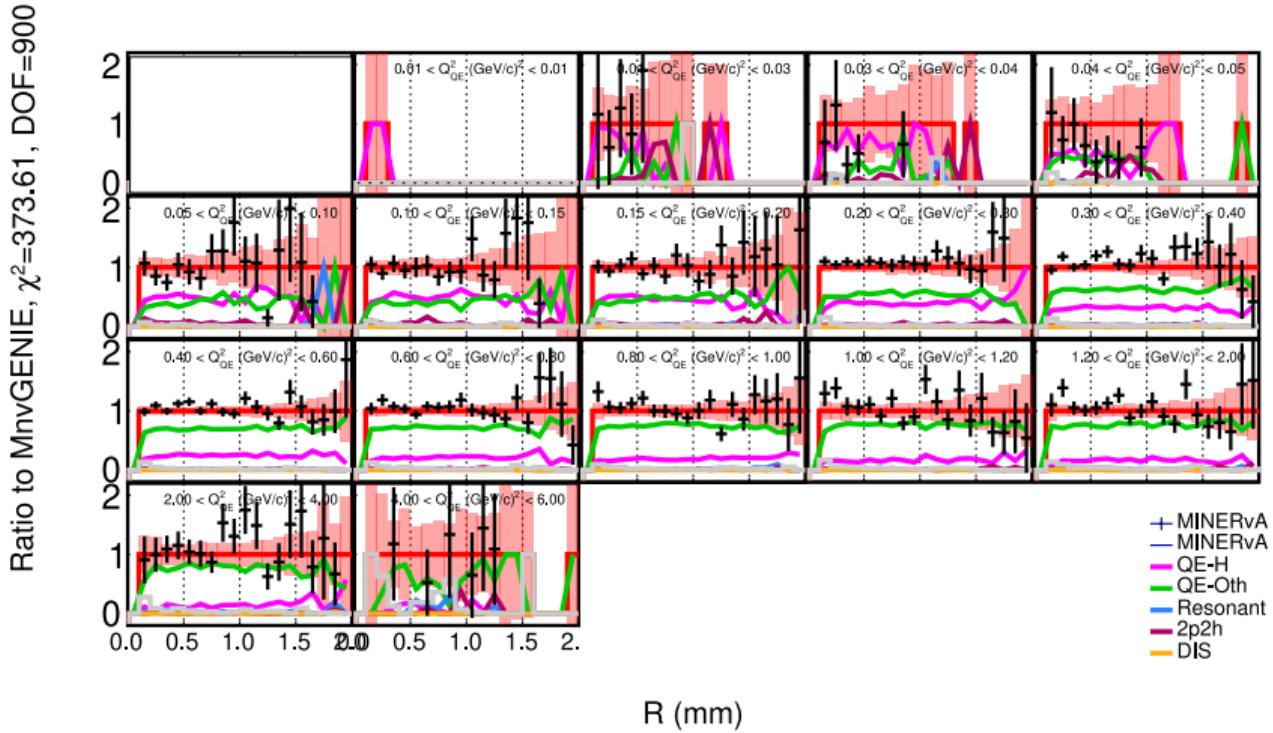
# Effect of MONA Reweighting – Blob Distance to Vertex

Without MoNA



# Effect of MONA Reweighting – Blob Distance to Vertex

With MoNA



## z-expansion parameters

z-expansion formalism and constraints on  $a_k$

$$F_A(Q^2) = \sum_{k=0}^{k_{\max}} a_k z^k$$

$$z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}}$$

$$\sum_{k=n}^{\infty} k(k-1)\dots(k-n+1)a_k = 0, n \in (0, 1, 2, 3)$$

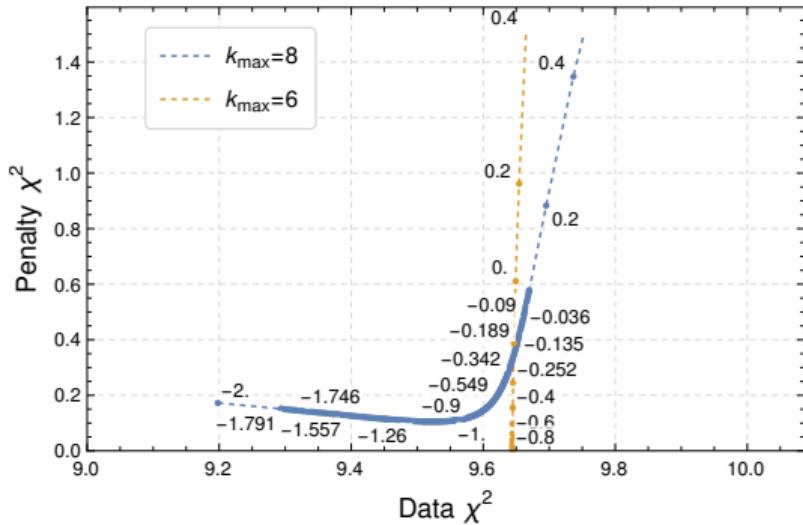
$$\chi^2 = \Delta X \cdot \text{cov}^{-1} \cdot \Delta X + \lambda \left[ \sum_{k=1}^5 \left( \frac{a_k}{5a_0} \right)^2 + \sum_{k=5}^{k_{\max}} \left( \frac{ka_k}{25a_0} \right)^2 \right]$$

$\Delta X$  = data – prediction

## z-expansion parameters II

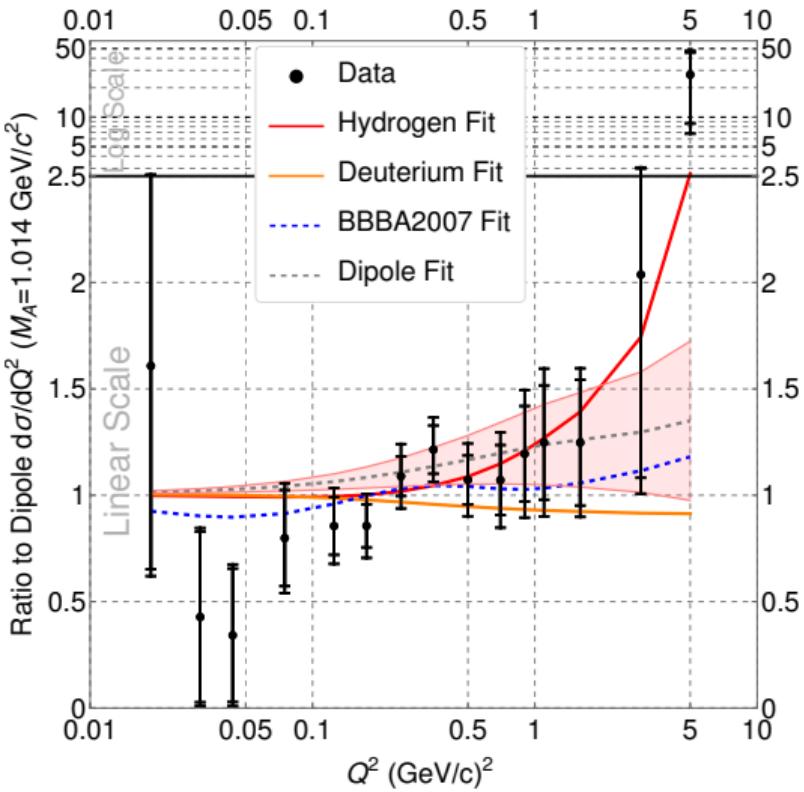
Central value fit:  $k_{\max} = 8, \lambda = 0.13$

- Scan through large range of  $\lambda$
- Data  $\chi^2$  for  $k_{\max} = 8$  can be less than  $k_{\max} = 6$
- $\lambda$  chosen at point of maximum curvature.

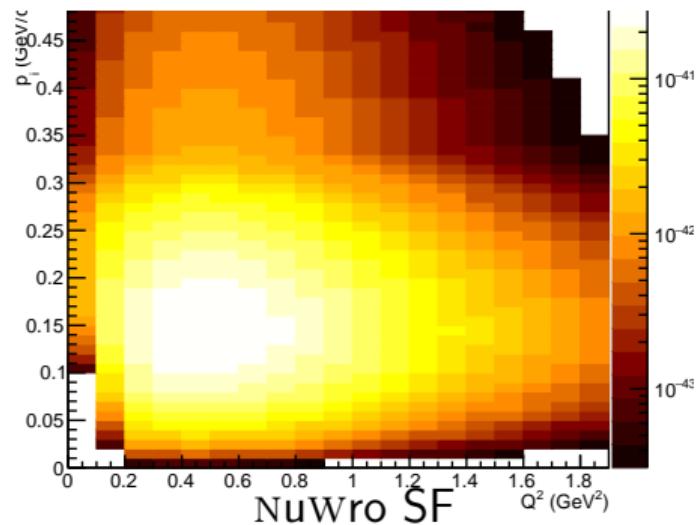
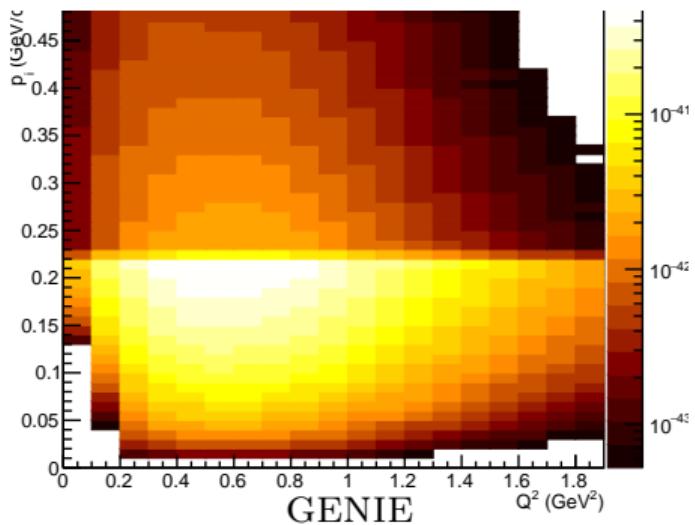


# Dipole Fit

- $M_A = 1.15(10)$  GeV
- Fit  $\chi^2 = 10.2$
- Comparable with z-expansion fit
  - ▶  $k_{\max} = 6$
  - ▶  $\lambda = 0$
  - ▶  $\chi^2 = 9.64$



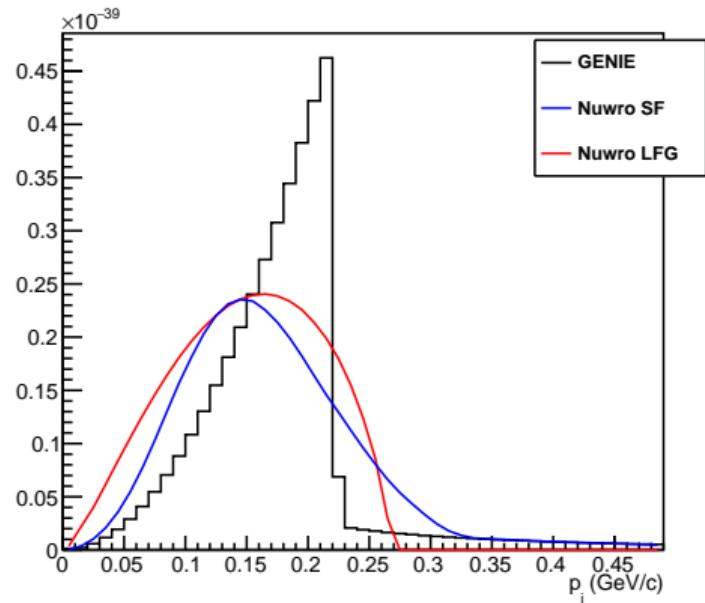
# Changing GENIE's initial state model



Transform GENIE RFG to NuWro SF  
in  $|p_i| - Q^2$  space

## Relativistic Fermi Gas (RFG), Local Fermi Gas (LFG) and Spectral Function (SF)

- RFG: non-interacting fermions in a potential well with fixed Fermi momentum  $K_F$ .
- GENIE RFG includes an additional tail.
- LFG: Fermi gas with location-dependent  $K_F$ .
- SF: a nuclear-shell model.



Initial state momentum distribution.

## $F_A$ fit and axial radius of the nucleon

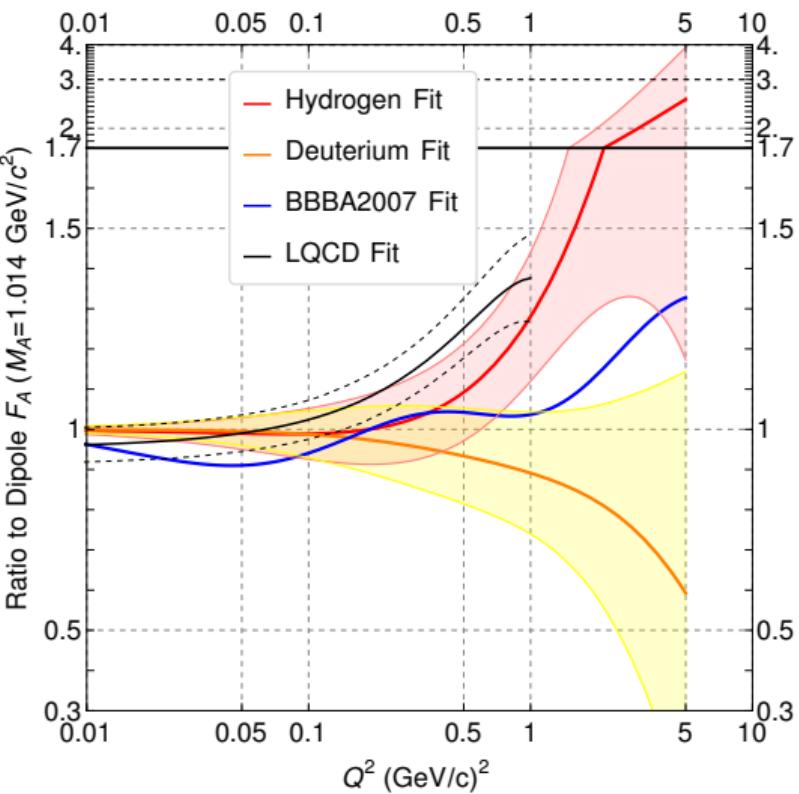
Favors larger  $F_A$  at higher  $Q^2$ .

Calculate proton radius from  $F_A$  for  $Q^2 \rightarrow 0$ .

$$F_A(Q^2) = F_A(0) \left( 1 - \frac{\langle r_A^2 \rangle}{3!} Q^2 + \frac{\langle r_A^4 \rangle}{5!} Q^4 + \dots \right),$$

$$\frac{1}{F_A(0)} \frac{dF_A}{dQ^2} \Big|_{Q^2=0} = -\frac{1}{6} \langle r_A^2 \rangle$$

- $\langle r_A^2 \rangle = 0.53(25)\text{fm}^2$
- $\sqrt{\langle r_A^2 \rangle} = 0.73(17)\text{fm}$



## $F_A$ fit and axial radius of the nucleon

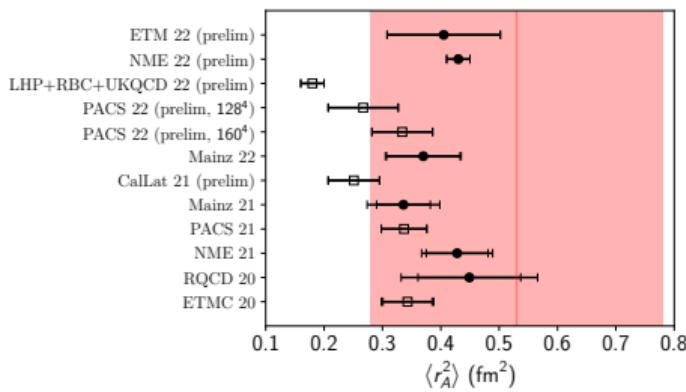
Favors larger  $F_A$  at higher  $Q^2$ .

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$$\frac{1}{F_A(0)} \frac{dF_A}{dQ^2} \Big|_{Q^2=0} = -\frac{1}{6} \langle r_A^2 \rangle$$

- $\langle r_A^2 \rangle = 0.53(25)\text{fm}^2$
- $\sqrt{\langle r_A^2 \rangle} = 0.73(17)\text{fm}$



Filled circle: full error budget.

Open square: incomplete.

Red band: this result.

Courtesy of Aaron Meyer.

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