Theory status for the muon g-2 experiment

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September 28, 2016 – Fermilab Colloquium

The anomalous magnetic moment

- Potential of particle in magnetic field

$$V = -\vec{\mu} \cdot \vec{B}$$

  with

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{S} \, ,$$

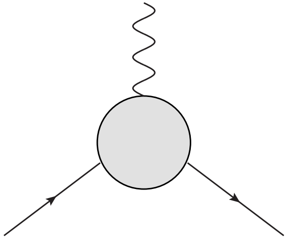
  where $\vec{S}$ is the spin of the particle.

- Relativistic QM: $g = 2$

- QFT: $g \neq 2$ in general
  Define anomalous magnetic moment $a = (g-2)/2$

The anomalous magnetic moment

- In QFT a can be expressed in terms of scattering of particle off a classical photon background



For external photon index μ with momentum q the scattering amplitude can be generally written as

$$(-ie)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right]$$

with $F_2(0) = a$.

The anomalous magnetic moment

- These anomalous moments are measured very precisely. For the electron (Hanneke, Fogwell, Gabrielse 2008)

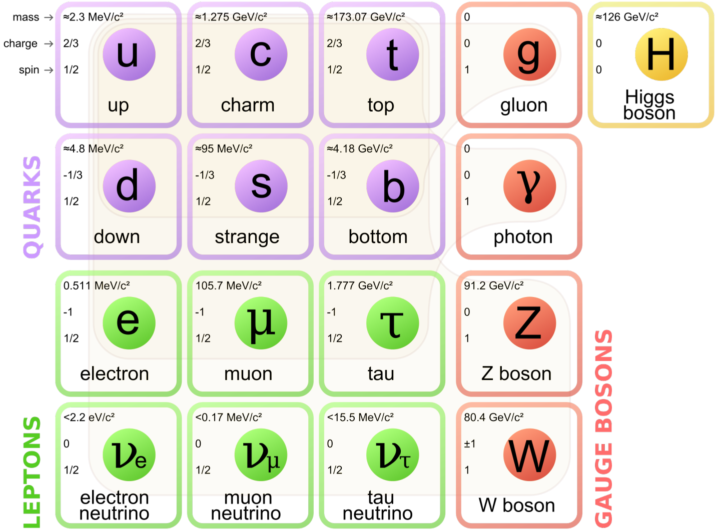
$$a_e = 0.00115965218073(28)$$

yielding the currently most precise determination of the fine structure constant

$$\alpha = 1/137.035999157(33)$$

(Aoyama, Hayakawa, Kinoshita, Nio 2015).

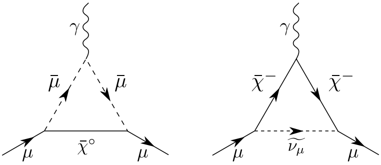
A brief reminder of the standard model



https://commons.wikimedia.org/wiki/File:Standard_Model_of_Elementary_Particles.svg

The muon anomalous magnetic moment

- In general, physics beyond the standard model (SM) contributes to a_ℓ with $a_\ell - a_\ell^{\mathrm{SM}} \propto (m_\ell^2/\Lambda_{\mathrm{NP}}^2)$ for lepton $\ell = e, \mu, \tau$ and new physics scale Λ_{NP}.
- With $\ell = \tau$ being experimentally inaccessible, $\ell = \mu$ promises good sensitivity to new physics.
- Example contributions: one-loop MSSM neutralino/smuon and chargino/sneutrino contributions to a_μ:



The muon anomalous magnetic moment

Interestingly, there is a tension of more than 3σ for a_μ:

Total SM prediction	0.0011 659 181 5 (49)
BNL E821 result	0.0011 659 209 1 (63)

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = (27.6 \pm 8.0) \times 10^{-10}$$

New experiment: Fermilab E989

Aims at a 4× reduction in experimental uncertainty!

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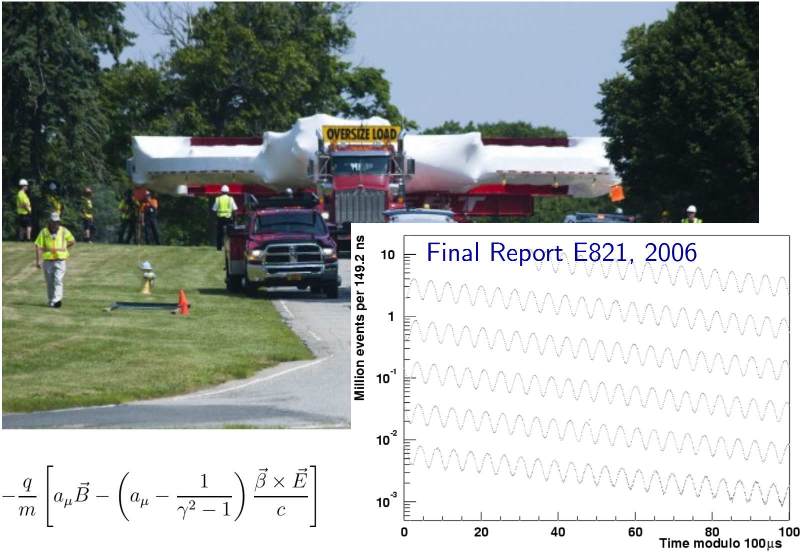
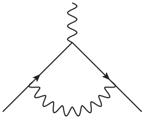
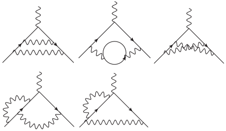


$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Theory Status – leptons and photons (QED)

1 loop  $\frac{1}{2}\ (\alpha/\pi)$ Schwinger 1948

2 loops  $0.765857426(16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann 1957; . . .

3 loops . . . $24.05050988(28)\ (\alpha/\pi)^3$

Remiddi, Laporta, . . .

4 loops . . . $130.8773(61)\ (\alpha/\pi)^4$

Kinoshita & Lindquist 1981, . . .

5 loops . . . $752.85(93)\ (\alpha/\pi)^5$

Kinoshita et al. 1990–2015, . . .; now complete

Theory Status – leptons and photons (QED)

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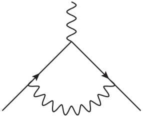
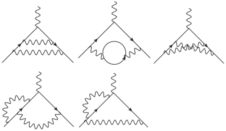


$\frac{1}{2}\ (\alpha/\pi)$ Schwinger 1948

2 loops



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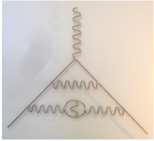

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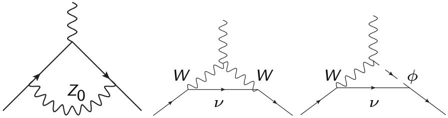
Kinoshita et al. 1990–2015, . . .; now complete

Theory Status – electroweak corrections (EW)

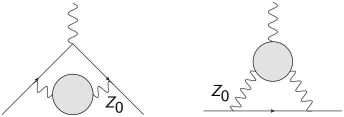
1 loop



19.5×10^{-10}

Jackiv, Weinberg 1972; . . .

1 loop + hadrons

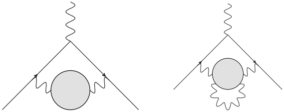


$15.4(1) \times 10^{-10}$

Kukhto et al. 1992; Czarnecki, Krause, Marciano 1995; . . .

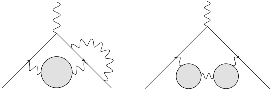
Theory Status – quarks and hadrons

HVP LO



e^+e^- scattering data analysis:
$692(4) \times 10^{-10}$ Davier et al. 2011;
$695(4) \times 10^{-10}$ Hagiwara et al. 2011;
$684(5) \times 10^{-10}$ Benayoun et al. 2015;
$687(4) \times 10^{-10}$ Jegerlehner 2015

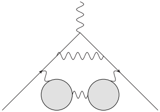
HVP NLO



. . .

$-9.8(1) \times 10^{-10}$
Krause 1996; Alemany et al. 1998; Hagiwara et al. 2011

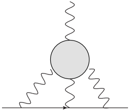
HVP NNLO



. . .

$1.24(1) \times 10^{-10}$
Kurz, Liu, Marquard, Steinhauser 2014

HLbL



Model calculations:
$8.0(4.0) \times 10^{-10}$ Knecht & Nyffeler 2002;
$13.6(2.5) \times 10^{-10}$ Melnikov & Vainshtein 2003;
$10.5(2.6) \times 10^{-10}$ Prades, de Raphael, Vainshtein 2009;
$10.2(3.9) \times 10^{-10}$ Jegerlehner 2015

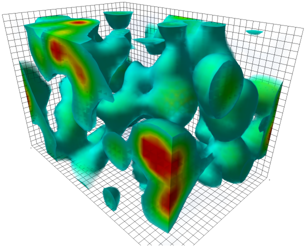
Theory status – summary

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		$\approx$ **1.6**

A reduction of uncertainty for HVP and HLbL is needed. A systematically improvable first-principles calculation is desired. Many new methods and results.

Hadronic contributions from first principles

- Simulate QFT in terms of fundamental quarks and gluons (QCD) on a supercomputer with discretized four-dimensional space-time lattice
- Hadrons are emergent phenomena of statistical average over background gluon configurations to which quarks are coupled
- In this framework draw diagrams only with respect to quarks, photons, and leptons; gluons and their effects are generated by the statistical average.



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

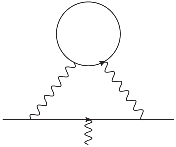
Computing resources

The RIKEN-BNL-Columbia (RBC) $g-2$ project for this year has $\approx$ 200M core hours (23k years on a single core) on Mira through the ALCC program and 70M core hours (8k years on a single core) on USQCD resources (FNAL/JLab).

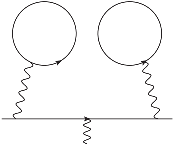
Top 10 positions of the 46th TOP500 in June 2016[13]

Rank	Rmax Rpeak (PFLOPS)	Name	Model	Processor	Interconnect	Vendor	Site Country, year
1	93.015 125.436	*Sunway TaihuLight*	Sunway MPP	SW26010	Sunway[14]	NRCPC	National Supercomputing Center in Wuxi China, 2016[14]
2	33.863 54.902	*Tianhe-2*	TH-IVB-FEP	Xeon E5–2692, Xeon Phi 31S1P	TH Express-2	NUDT	National Supercomputing Center in Guangzhou China, 2013
3	17.590 27.113	*Titan*	Cray XK7	Opteron 6274, Tesla K20X	Gemini	Cray Inc.	Oak Ridge National Laboratory United States, 2012
4	17.173 20.133	*Sequoia*	Blue Gene/Q	PowerPC A2	Custom	IBM	Lawrence Livermore National Laboratory United States, 2013
5	10.510 11.280	*K computer*	K computer	SPARC64 VIIIfx	Tofu	Fujitsu	RIKEN Japan, 2011
6	8.586 10.066	*Mira*	Blue Gene/Q	PowerPC A2	Custom	IBM	Argonne National Laboratory United States, 2013

First-principles approach to HVP LO

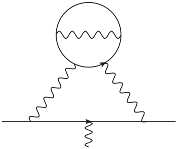


Quark-connected piece with by far dominant part from up and down quark loops, $\mathcal{O}(700 \times 10^{-10})$

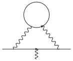


Quark-disconnected piece, $-9.6(4.0) \times 10^{-10}$

Phys.Rev.Lett. 116 (2016) 232002



QED corrections, $\mathcal{O}(10 \times 10^{-10})$

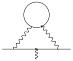


HVP quark-connected contribution

Biggest challenge to direct calculation at physical pion masses is to control statistics and potentially large finite-volume errors

Statistics: for strange and charm solved issue, for up and down quarks existing methodology less effective

Finite-volume errors are exponentially suppressed in the simulation volume but may be sizeable



HVP quark-connected contribution

Starting from the vector current

$$J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x) \gamma_\mu \Psi_f(x)$$

we may write

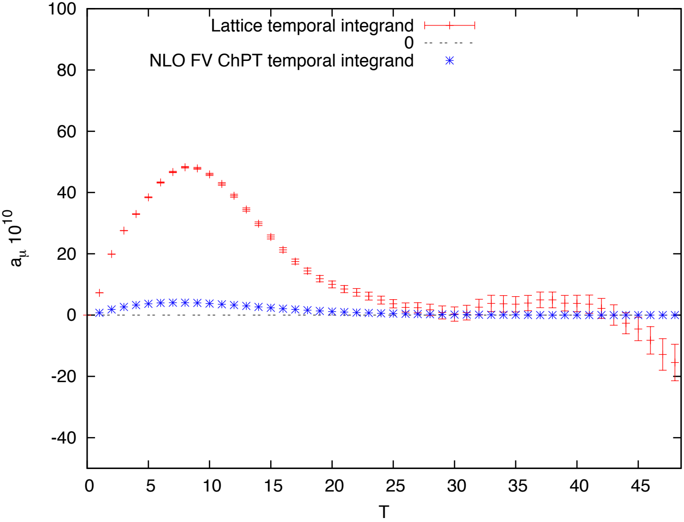
$$a_\mu^{\mathrm{HVP}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the diagram (Bernecker-Meyer 2011).

Integrand $w_T C(T)$ for the light-quark connected contribution:



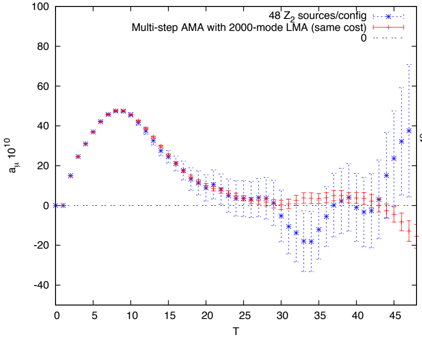
$m_\pi = 140$ MeV, $a = 0.11$ fm (RBC/UKQCD 48^3 ensemble)

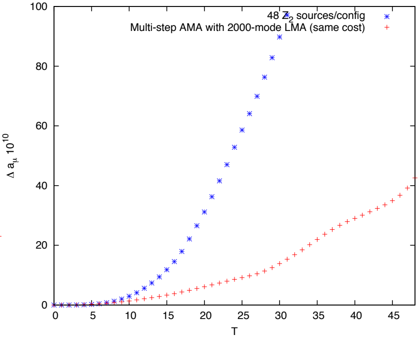
Statistical noise from long-distance region

Addressing the long-distance noise problem

- Replace $C(t)$ for large t with model; multi-exponentials for $t \geq 1.5$ fm was recently used to compute $a_\mu^{\mathrm{HVP\ LO\ CON}} = 666(6) \times 10^{-10}$ arXiv:1601.03071.

- RBC: Improved stochastic estimator (hierarchical approximations including exact treatment of low-mode space):





Complete first-principles analysis

- Currently our statistical uncertainty for a pure first-principles analysis in the continuum limit is at the $\Delta a_\mu \approx 15 \times 10^{-10}$ level

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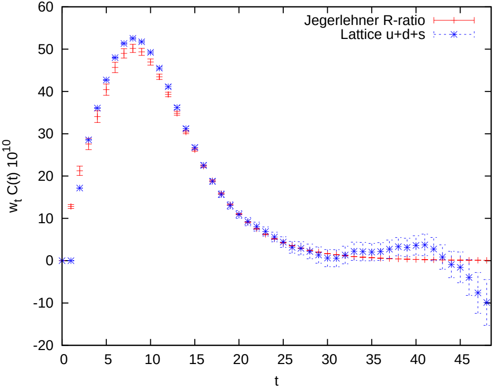
- Sub-percent statistical error likely achievable later this year

- While we are waiting for more statistics . . .

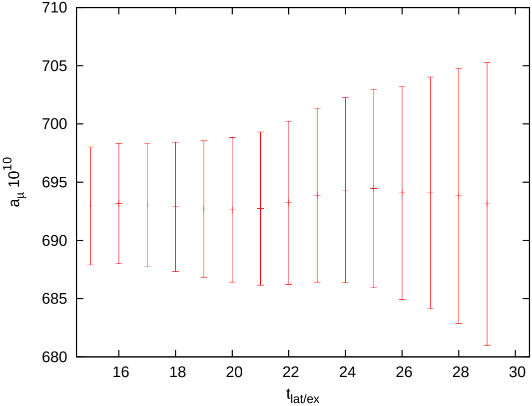
Combined lattice and dispersive analysis

We can use the dispersion relation to overlay experimental e^+e^- scattering data (Bernecker, Meyer 2011). Below the experimental result is taken from Jegerlehner 2016:



Work done with T. Izubuchi.

The lattice data is precise at shorter distances and the experimental data is precise at longer distances. We can do a combined analysis with lattice and experimental data:
$a_\mu = \sum_{t=0}^{t_{\rm lat/ex}} w_t C^{\rm lattice}(t) + \sum_{t=t_{\rm lat/ex}+1}^{\infty} w_t C^{\rm exp}(t)$
As expected a nice plateau region as a function of $t_{\rm lat/ex}$ is visible.

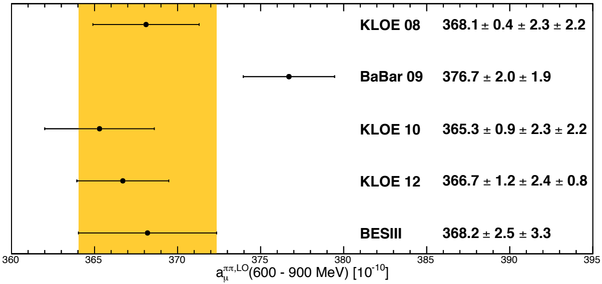


If we took $t_{\rm lat/ex} = 15a \approx 1.7$ fm, we currently have a statistical error of 0.7%

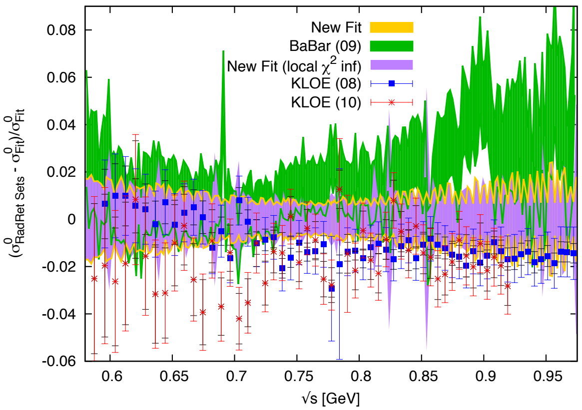
$$a_\mu^{\rm HVP, \mathit{u,d,s}} = 693(5) \times 10^{-10}\,.$$

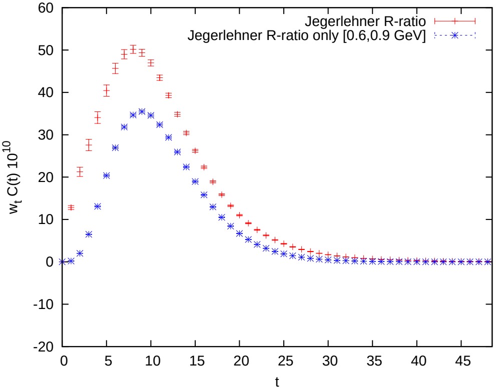
Continuum limit, charm contribution, and QED/IB correction missing. This is a promising way to reduce the overall uncertainty on a short time-scale.

BESIII 2015 update

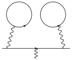


Hagiwara et al. 2011





Problematic experimental region can readily be replaced by precise lattice data. Lattice also can be arbiter regarding different experimental data sets.



HVP quark-disconnected contribution

First results at physical pion mass with a statistical signal
Phys.Rev.Lett. 116 (2016) 232002

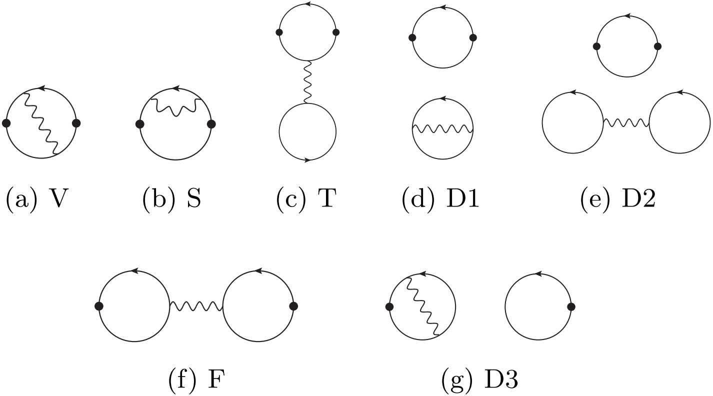
Statistics is clearly the bottleneck; calculation was a potential road-block of a first-principles calculation for a long time; due to very large pion-mass dependence calculation at physical pion mass is crucial.

New stochastic estimator allowed us to get result

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

from a modest computational investment ($\approx$ 1M core hours).

HVP QED contribution

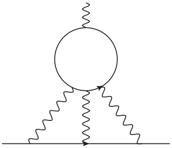


This is work in progress, to be completed in a few weeks

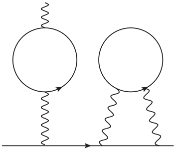
Status and outlook for the HVP LO

- New methods: improved statistical estimators both for connected light and disconnected contributions at physical point.
- For the connected light contribution the new method reduces noise in the long-distance part of the correlator by an order of magnitude compared to previous method.
- For the disconnected contributions the new method allowed for a precise calculation at physical pion mass. Phys.Rev.Lett. 116 (2016) 232002
- Combination with e^+e^- scattering data should allow for a significant improvement over current most precise estimate within the next 6 months.

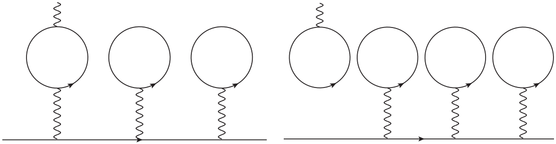
The Hadronic Light-by-Light contribution



Quark-connected piece (charge factor of up/down quark contribution: $\frac{17}{81}$)



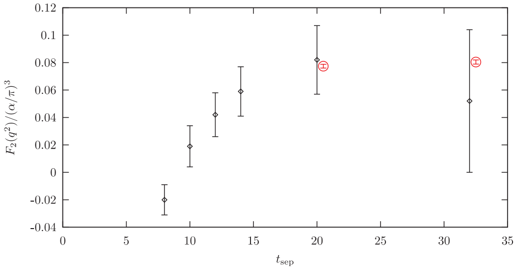
Dominant quark-disconnected piece (charge factor of up/down quark contribution: $\frac{25}{81}$)



Sub-dominant quark-disconnected pieces (charge factors of up/down quark contribution: $\frac{5}{81}$ and $\frac{1}{81}$)

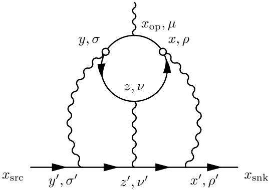
All results below are from: T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, and C.L., Phys. Rev. D 93, 014503 (2016)

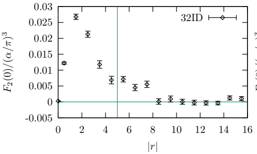
Compute quark-connected contribution with new computational strategy

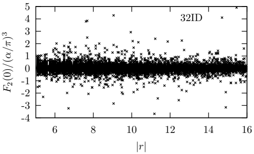


yields more than an order-of-magnitude improvement (red symbols) over previous method (black symbols) for a factor of ≈ 4 smaller cost.

New stochastic sampling method



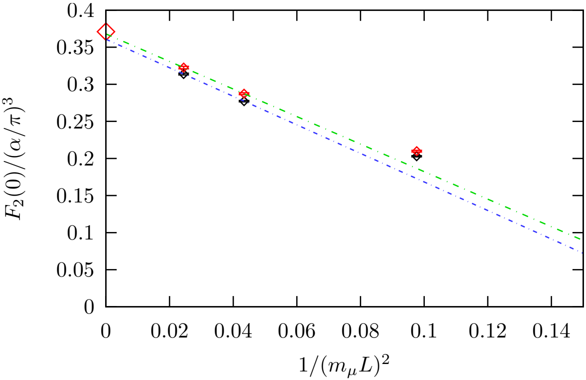




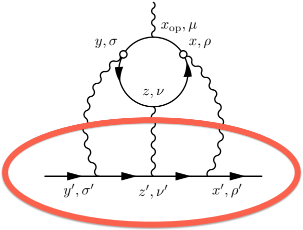
Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$, see vertical red line
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

Cross-check against analytic result where quark loop is replaced by muon loop



Finite-volume errors of the HLbL



Need to sum over all displacements on the muon line to control FV errors.

Since muon line does not couple to gluons, this can be done in a straightforward way: C.L. talk at lattice 2015 and Green et al. 2015, PRL115(2015)222003

Current status of the HLbL

- We have already below 10% statistical uncertainty on quark-connected contribution and a similar absolute uncertainty on the dominant quark-disconnected contribution.

  Results at physical pion mass to be published in the next few weeks

- Remaining systematic uncertainties: discretization and finite-volume errors

- To control discretization errors we will within the next year repeat the current computation with a second lattice spacing that sits halfway between current spacing and continuum limit.

Current status of the HLbL

- We have already below 10% statistical uncertainty on quark-c… uncertai… on.

Results at ph…

$$a_\mu^{\mathrm{cHLbL}} = \left.\frac{g_\mu - 2}{2}\right|_{\mathrm{cHLbL}} = (0.0926 \pm 0.0077)\left(\frac{\alpha}{\pi}\right)^3 = (11.60 \pm 0.96) \times 10^{-10} \quad (11)$$

$$a_\mu^{\mathrm{dHLbL}} = \left.\frac{g_\mu - 2}{2}\right|_{\mathrm{dHLbL}} = (-0.0498 \pm 0.0064)\left(\frac{\alpha}{\pi}\right)^3 = (-6.25 \pm 0.80) \times 10^{-10} \quad (12)$$

$$a_\mu^{\mathrm{HLbL}} = \left.\frac{g_\mu - 2}{2}\right|_{\mathrm{HLbL}} = (0.0427 \pm 0.0108)\left(\frac{\alpha}{\pi}\right)^3 = (5.35 \pm 1.35) \times 10^{-10} \quad (13)$$

- Remaini… finite-vo…

- To cont… repeat t… ng that sits halfway between current spacing and continuum limit.

Summary and outlook

New methods allow for a substantial reduction in uncertainty of the theory calculation of the $(g-2)_\mu$.

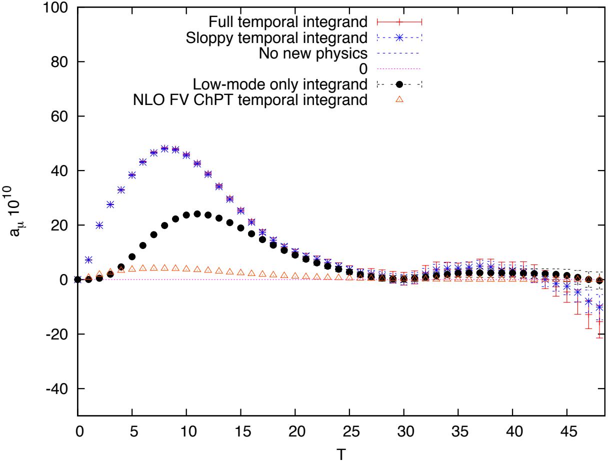
A reduction of uncertainty over the currently most precise value within the next year seems possible.

Over the next five years should allow for a reduction of uncertainty commensurate with the Fermilab E989 target precision.

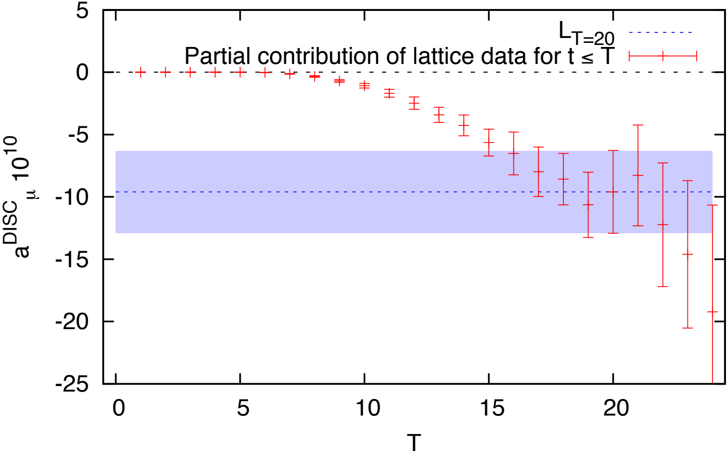
The Fermilab experiment may have first results in 2018?

Thank you

Low-mode saturation for physical pion mass (here 2000 modes):

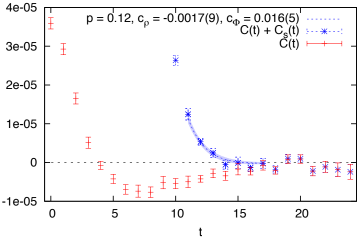


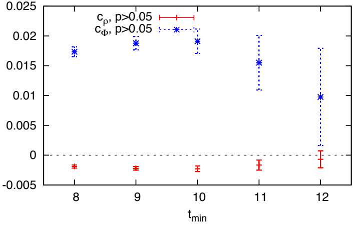
Result for partial sum $L_T = \sum_{t=0}^{T} w_t C(t)$:



For $t \geq 15$ $C(t)$ is consistent with zero but the stochastic noise is t-independent and $w_t \propto t^4$ such that it is difficult to identify a plateau region based only on this plot

Resulting correlators and fit of $C(t) + C_s(t)$ to $c_\rho e^{-E_\rho t} + c_\phi e^{-E_\phi t}$ in the region $t \in [t_{\min}, \ldots, 17]$ with fixed energies $E_\rho = 770$ MeV and $E_\phi = 1020$. $C_s(t)$ is the strange connected correlator.

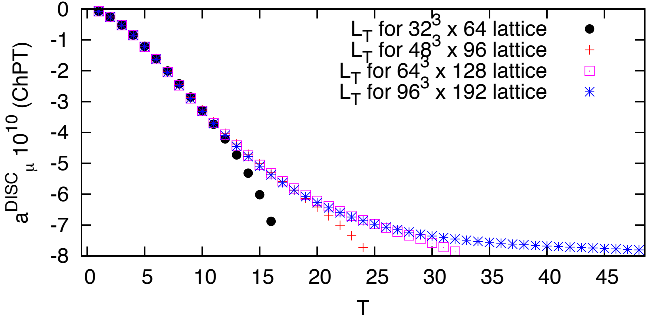




We fit to $C(t) + C_s(t)$ instead of $C(t)$ since the former has a spectral representation.

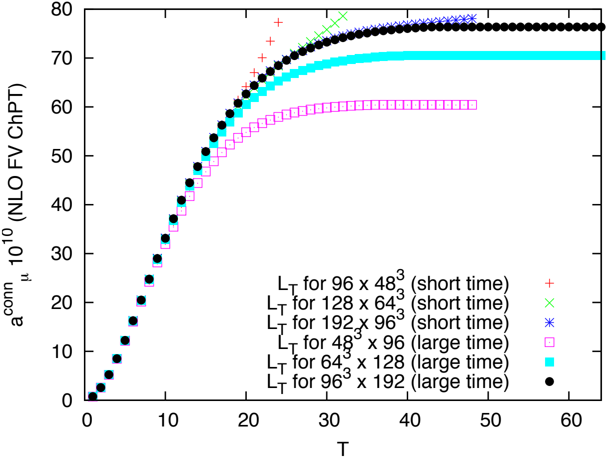
We could use this model alone for the long-distance tail to help identify a plateau but it would miss the two-pion tail

We therefore additionally calculate the two-pion tail for the disconnected diagram in ChPT:



A closer look at the NLO FV ChPT prediction (1-loop sQED):

We show the partial sum $\sum_{t=0}^{T} w_t C(t)$ for different geometries and volumes:



The dispersive approach to HVP LO

The dispersion relation

$$\begin{aligned}\Pi_{\mu\nu}(q) &= i\left(q_\mu q_\nu - g_{\mu\nu} q^2\right)\Pi(q^2)\\ \Pi(q^2) &= -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\frac{\mathrm{Im}\Pi(s)}{q^2-s}\,.\end{aligned}$$

allows for the determination of a_μ^{HVP} from experimental data via

$$a_\mu^{\mathrm{HVP\ LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2\left[\int_{4m_\pi^2}^{E_0^2} ds\,\frac{R_\gamma^{\mathrm{exp}}(s)\hat{K}(s)}{s^2} + \int_{E_0^2}^{\infty} ds\,\frac{R_\gamma^{\mathrm{pQCD}}(s)\hat{K}(s)}{s^2}\right],$$

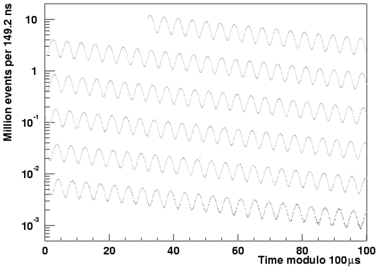
$$R_\gamma(s) = \sigma^{(0)}(e^+e^- \to \gamma^* \to \mathrm{hadrons})/\frac{4\pi\alpha^2}{3s}$$

Experimentally with or without additional hard photon (ISR: $e^+e^- \to \gamma^*(\to \mathrm{hadrons})\gamma$)

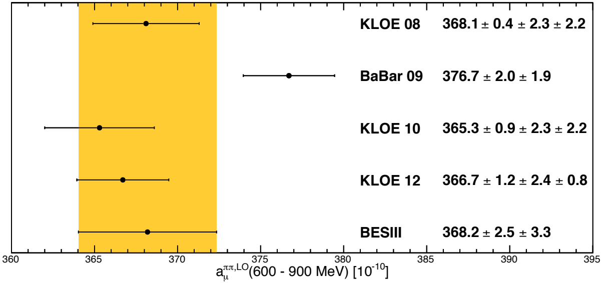
Experimental setup: muon storage ring with tuned momentum of muons to cancel leading coupling to electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

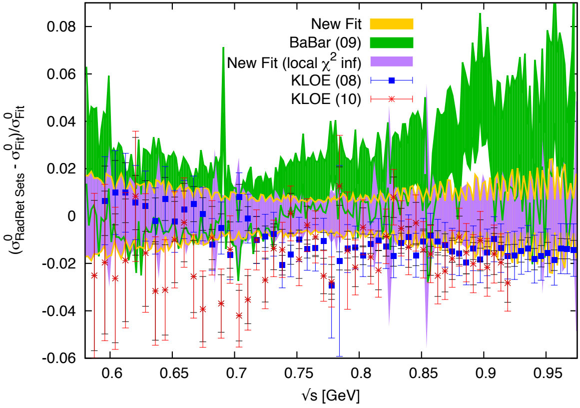
Because of parity violation in weak decay of muon, a correlation between muon spin and decay electron direction exists, which can be used to measure the anomalous precession frequency ω_a:

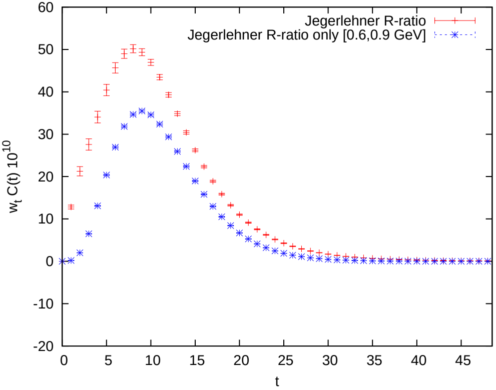


BESIII 2015 update:



Hagiwara et al. 2011:





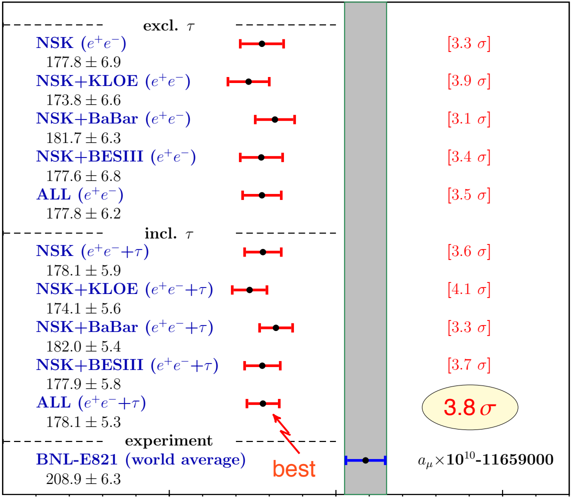
Problematic experimental region can readily be replaced by precise lattice data. Lattice also can be arbiter regarding different experimental data sets.

Jegerlehner FCCP2015 summary:

final state	range (GeV)	$a_\mu^{\mathrm{had}(1)} \times 10^{10}$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	507.55 (0.39) (2.68)[2.71]	0.5%	39.9%
ω	(0.42, 0.81)	35.23 (0.42) (0.95)[1.04]	3.0%	5.9%
ϕ	(1.00, 1.04)	34.31 (0.48) (0.79)[0.92]	2.7%	4.7%
J/ψ		8.94 (0.42) (0.41)[0.59]	6.6%	1.9%
Υ		0.11 (0.00) (0.01)[0.01]	6.8%	0.0%
had	(1.05, 2.00)	60.45 (0.21) (2.80)[2.80]	4.6%	42.9%
had	(2.00, 3.10)	21.63 (0.12) (0.92)[0.93]	4.3%	4.7%
had	(3.10, 3.60)	3.77 (0.03) (0.10)[0.10]	2.8%	0.1%
had	(3.60, 9.46)	13.77 (0.04) (0.01)[0.04]	0.3%	0.0%
had	(9.46,13.00)	1.28 (0.01) (0.07)[0.07]	5.4%	0.0%
pQCD	(13.0,∞)	1.53 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	687.06 (0.89) (4.19)[4.28]	0.6%	0.0%
total		688.59 (0.89) (4.19)[4.28]	0.6%	100.0%

Results for $a_\mu^{\mathrm{had}(1)} \times 10^{10}$. Update August 2015, incl SCAN[NSK]+ISR[KLOE10,KLOE12,BaBar,BESIII]

Jegerlehner FCCP2015 summary ($\tau \leftrightarrow e^+e^-$):



Our setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t') \mathcal{V}_j(t') \rangle_{\mathrm{SU}(3)} \tag{1}$$

where V stands for the four-dimensional lattice volume, $\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

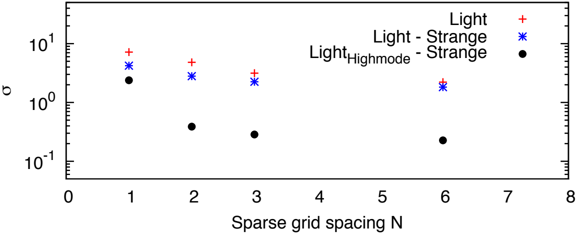
$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \tag{2}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

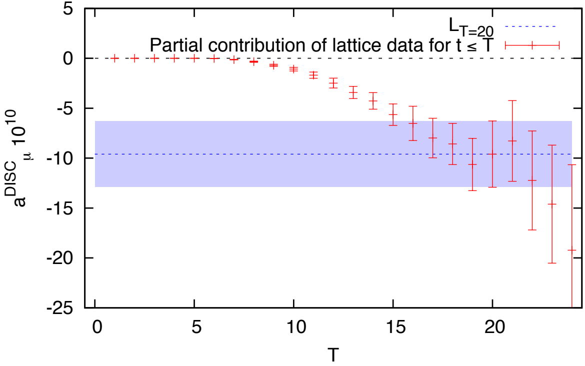
Combination of both ideas is crucial for noise reduction at physical pion mass!

Fluctuation of $\mathcal{V}_\mu$ (σ):



Since $C(t)$ is the autocorrelator of $\mathcal{V}_\mu$, we can create a stochastic estimator whose noise is potentially reduced linearly in the number of random samples, hence the normalization in the lower panel

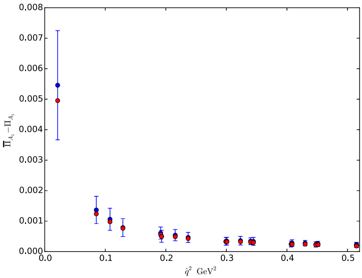
Study $L_T = \sum_{t=T+1}^{\infty} w_t C(t)$ and use value of T in plateau region (here $T = 20$) as central value. Use a combined estimate of a resonance model and the two-pion tail to estimate systematic uncertainty.

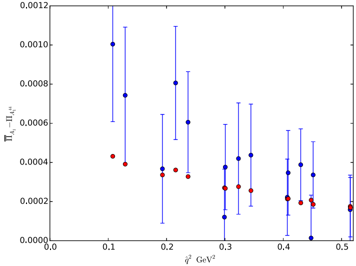


Combined with an estimate of discretization errors, we find

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}\,. \tag{3}$$

From Aubin et al. 2015 (arXiv:1512.07555v2)



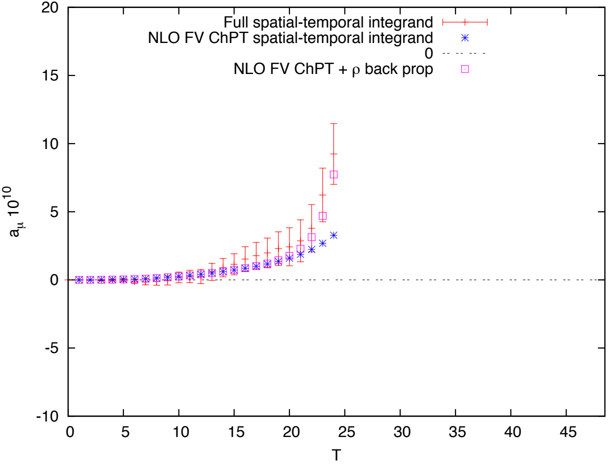


MILC lattice data with $m_\pi L = 4.2$, $m_\pi \approx 220$ MeV; Plot difference of $\Pi(q^2)$ from different irreps of 90-degree rotation symmetry of spatial components versus NLO FV ChPT prediction (red dots)

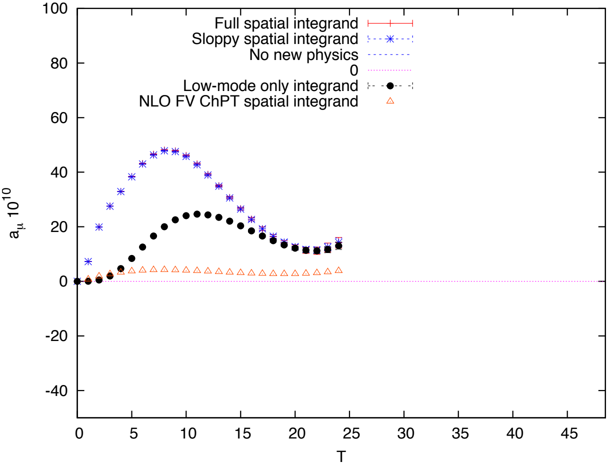
While the absolute value of a_μ is poorly described by the two-pion contribution, the volume dependence may be described sufficiently well to use ChPT to control FV errors at the 1% level; this needs further scrutiny

Aubin et al. find an $O(10\%)$ finite-volume error for $m_\pi L = 4.2$ based on the $A_1 - A_1^{44}$ difference (right-hand plot)

Compare difference of integrand of $48 \times 48 \times 96 \times 48$ (spatial) and $48 \times 48 \times 48 \times 96$ (temporal) geometries with NLO FV ChPT $(A_1 - A_1^{44})$:



$m_\pi = 140$ MeV, $p^2 = m_\pi^2/(4\pi f_\pi)^2 \approx 0.7\%$



Full spatial integrand
Sloppy spatial integrand
No new physics
0
Low-mode only integrand
NLO FV ChPT spatial integrand
$a_\mu\ 10^{10}$
T
100
80
60
40
20
0
-20
-40
0
5
10
15
20
25
30
35
40
45