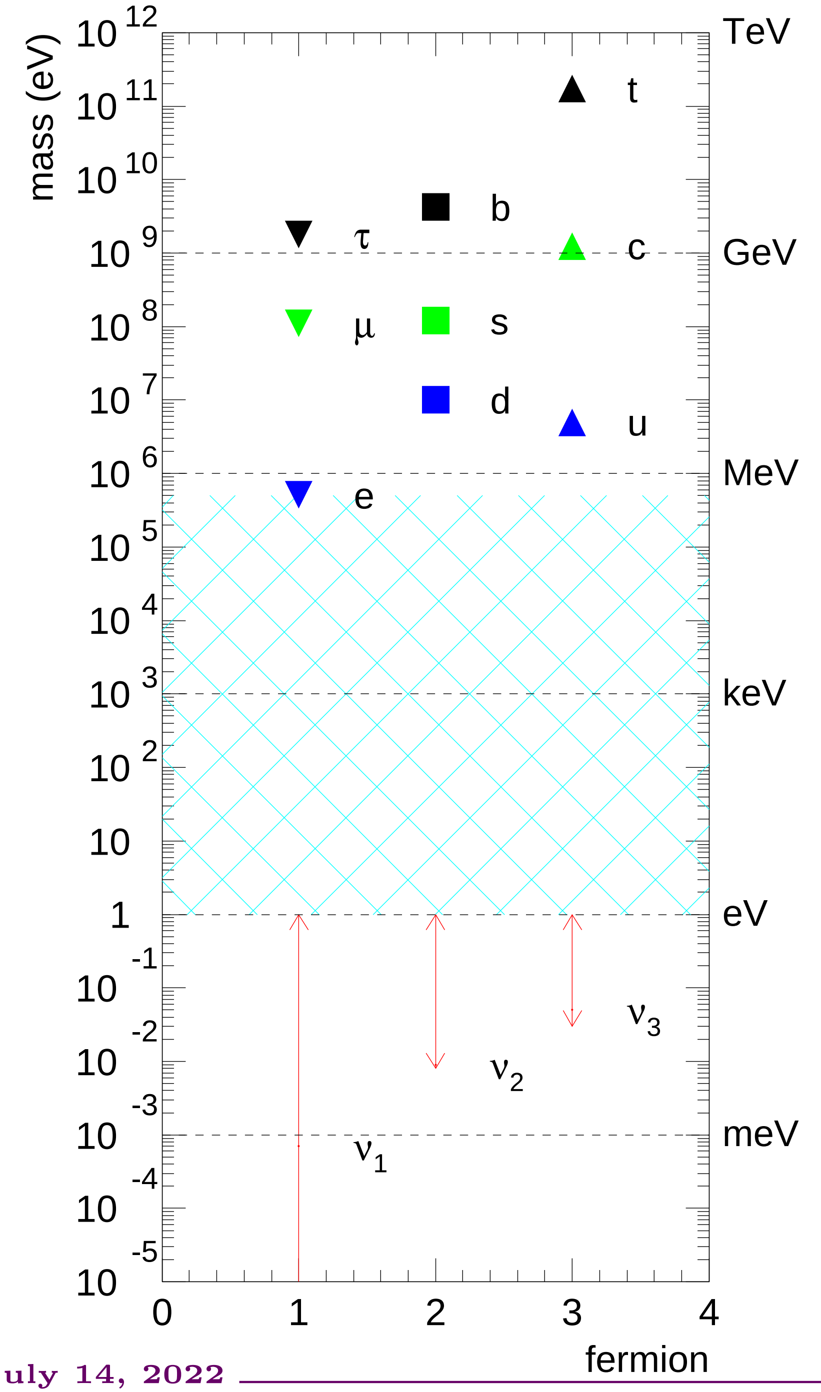
Majorana or Dirac, That is the Question

André de Gouvêa – Northwestern University

Neutrino University – Fermilab

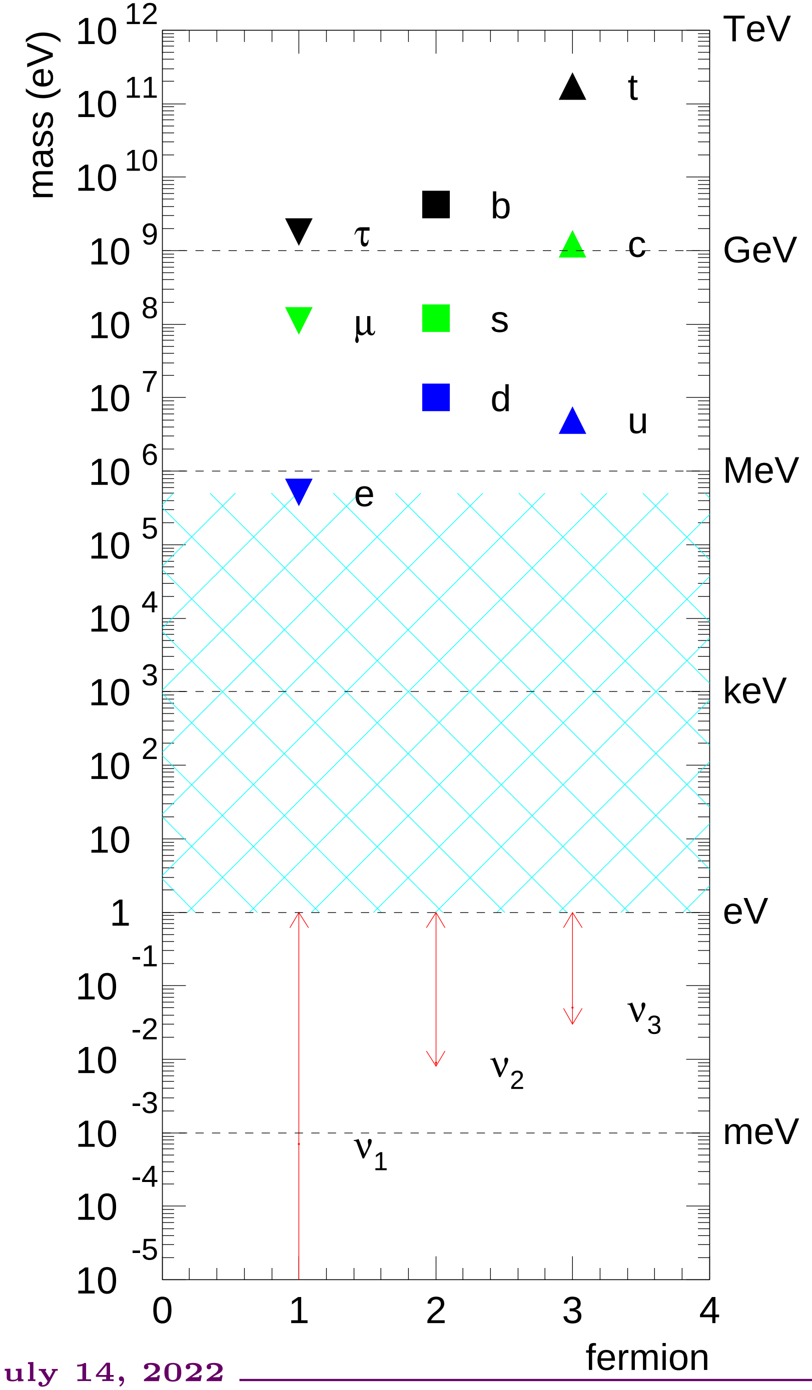
July 14, 2022



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?

⇓

NEW PHYSICS

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$\Updownarrow$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input.

Piecing the Neutrino Mass Puzzle

Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts, including ...

- **understanding the fate of lepton-number**. Neutrinoless double beta decay.
- a comprehensive long baseline neutrino program, towards precision oscillation physics.
- other probes of neutrino properties, including neutrino scattering.
- precision studies of charged-lepton properties ($g-2$, edm), and searches for rare processes ($\mu \to e$-conversion the best bet at the moment).
- collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- cosmic surveys. Neutrino properties affect, in a significant way, the history of the universe. Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?
- searches for baryon-number violating processes.

Fork on the Road: Are Neutrinos Majorana or Dirac Fermions?

[9 out of 10 theorists agree: “Best” Question in Neutrino Physics Today!]

How Many Degrees of Freedom are There in a Neutrino?

(2 versus 4)

A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$e^-_L \leftarrow \text{CP} \rightarrow e^+_R$$

$$\updownarrow \text{“Lorentz”}$$

$$e^-_R \leftarrow \text{CP} \rightarrow e^+_L$$

This is referred to as a Dirac fermion. Here, we can talk about

Parity: relates $e^\pm_R$ with $e^\pm_L$

Charge-Conjugation: relates $e^\pm_R$ with $e^\mp_R$

(Massless fermions are weird. We can make do with only “half” of them, even if they are charged.)

How Many Degrees of Freedom are There in a Neutrino? (2 versus 4)

For a massive neutral fermion (s=1/2), there are two choices: Dirac ...

$$\nu_L \leftarrow \text{CP} \rightarrow \bar{\nu}_R$$

$$\updownarrow \text{“Lorentz”}$$

$$\nu_R \leftarrow \text{CP} \rightarrow \bar{\nu}_L$$

or Majorana ...

$$\nu_L \leftarrow \text{CP} \rightarrow \nu_R$$

$$\updownarrow \text{“Lorentz”}$$

$$\nu_R \leftarrow \text{CP} \rightarrow \nu_L$$

In the Majorana case, neutrinos are their own antiparticles. This means $\nu_L = \bar{\nu}_L$ and $\nu_R = \bar{\nu}_R$. (Helicity matters!)

Why Don't We Know the Answer to 4 versus 2?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are sensitive to the Majorana versus Dirac nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

Charged-Current Weak Interactions are Purely Left-Handed (Chirality)

What does this mean? For example, In the decay of a muon at rest,

$$\mu^- \to e^- \nu_\mu \bar{\nu}_e,$$

the electrons come out almost 100% polarized:

$$|e^-\rangle \sim |L\rangle + \left(\frac{m_e}{m_\mu}\right)|R\rangle.$$

For the CP-conjugated process, we get the CP-conjugated answer: In the process

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e,$$

the positrons come out almost 100% polarized:

$$|e^+\rangle \sim |R\rangle + \left(\frac{m_e}{m_\mu}\right)|L\rangle.$$

Charged-Current Weak Interactions are Purely Left-Handed (Chirality)

When it comes to neutrino production, for example, in pion-decay at rest

$$\pi^+ \to \mu^+ \nu$$

$$|\nu\rangle \sim |L\rangle + \left(\frac{m_\nu}{m_\pi}\right)|R\rangle.$$

For the CP-conjugated process, we get the CP-conjugated answer:

$$\pi^- \to \mu^- + \mathrm{CP}(\nu)$$

the CP-conjugated neutrino state comes out almost 100% polarized:

$$|\mathrm{CP}(\nu)\rangle \sim |R\rangle + \left(\frac{m_\nu}{m_\pi}\right)|L\rangle.$$

(Remember: $m_\nu/m_\pi < 10^{-9}$)

Charged-Current Weak Interactions are Purely Left-Handed (Chirality)

The same goes for neutrino detection. Ignoring neutrino-mass effects

$$\nu_L + X \to e^- + Y$$

and the CP conjugate channel is

$$\mathrm{CP}(\nu_L) + \mathrm{CP}(X) \to e^+ + \mathrm{CP}(Y)$$

So, if we can ignore neutrino masses, left-handed neutrinos are produced together with positively-charged leptons and, when they are detected, they only know how to produce negatively-charged leptons. The opposite goes for the CP-conjugate of the neutrino: these are produced with negatively-charged leptons and, when they are detected, they only know how to produce positively-charged leptons. It does not matter if they are Dirac fermions or Majorana fermions!

Global Lepton Number Symmetry

In the massless-neutrino limit, there is a conserved global symmetry we call **Lepton Number**. If we assign the following charges to the leptons

$$L(e^-) = L(\mu^-) = L(\tau^-) = 1 = L(\nu),$$

$$L(e^+) = L(\mu^+) = L(\tau^+) = -1 = L(\mathrm{CP}(\nu)),$$

the total lepton number is always conserved.

If neutrinos are massive Majorana fermions, we can't assign them ANY quantum number, including lepton number. Hence, lepton number cannot be exactly conserved. If neutrinos are Majorana fermions, lepton number is only approximately conserved. Hence, the "smoking gun" signature of Majorana neutrinos is the observation of LEPTON NUMBER violation.

Gedanken Experiment, remembering that $m_\nu \neq 0$:

In the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \quad \text{followed by} \quad \nu_e + X \to e^+ + X, \qquad P \simeq \left(\frac{m}{E}\right)^2$$

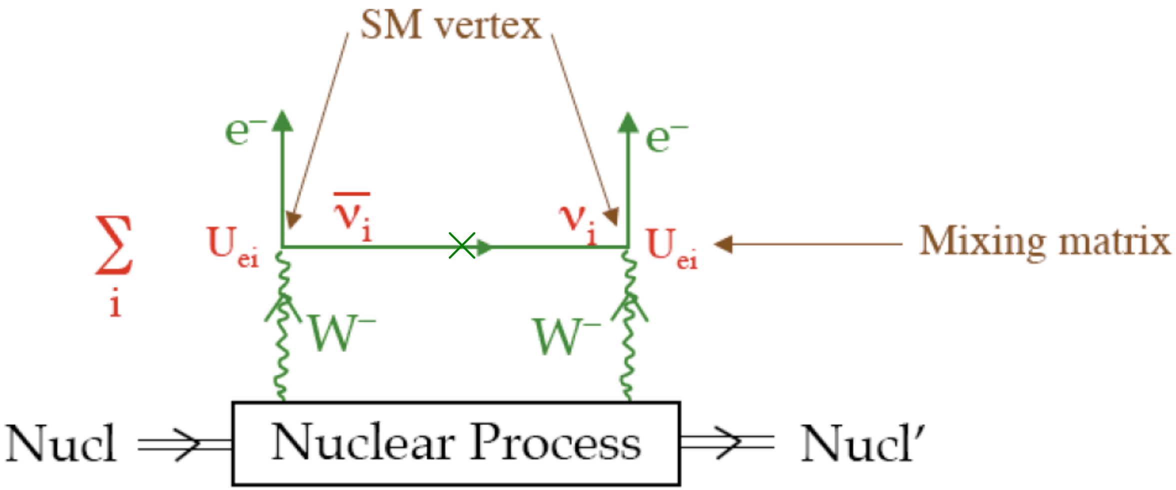
Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1\ \text{eV}/100\ \text{MeV})^2 = 10^{-18}$. VERY Challenging!

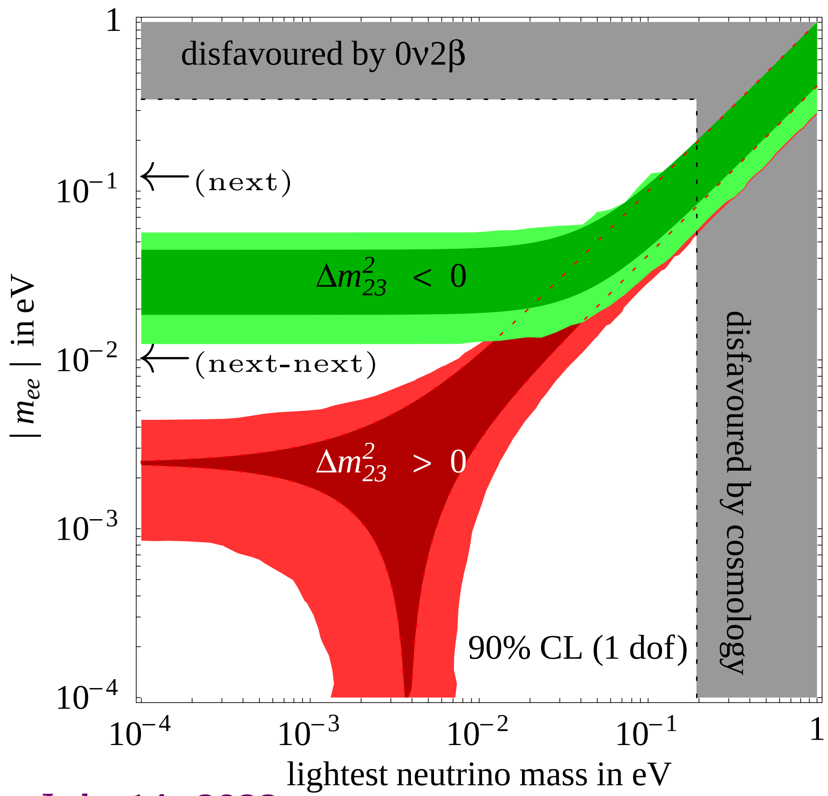
Search for the Violation of Lepton Number (or $B-L$)

Best Bet: search for

Neutrinoless Double-Beta

Decay: $Z \to (Z+2)e^- e^-$





Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

$\Leftarrow$ **no longer lamp-post physics!**

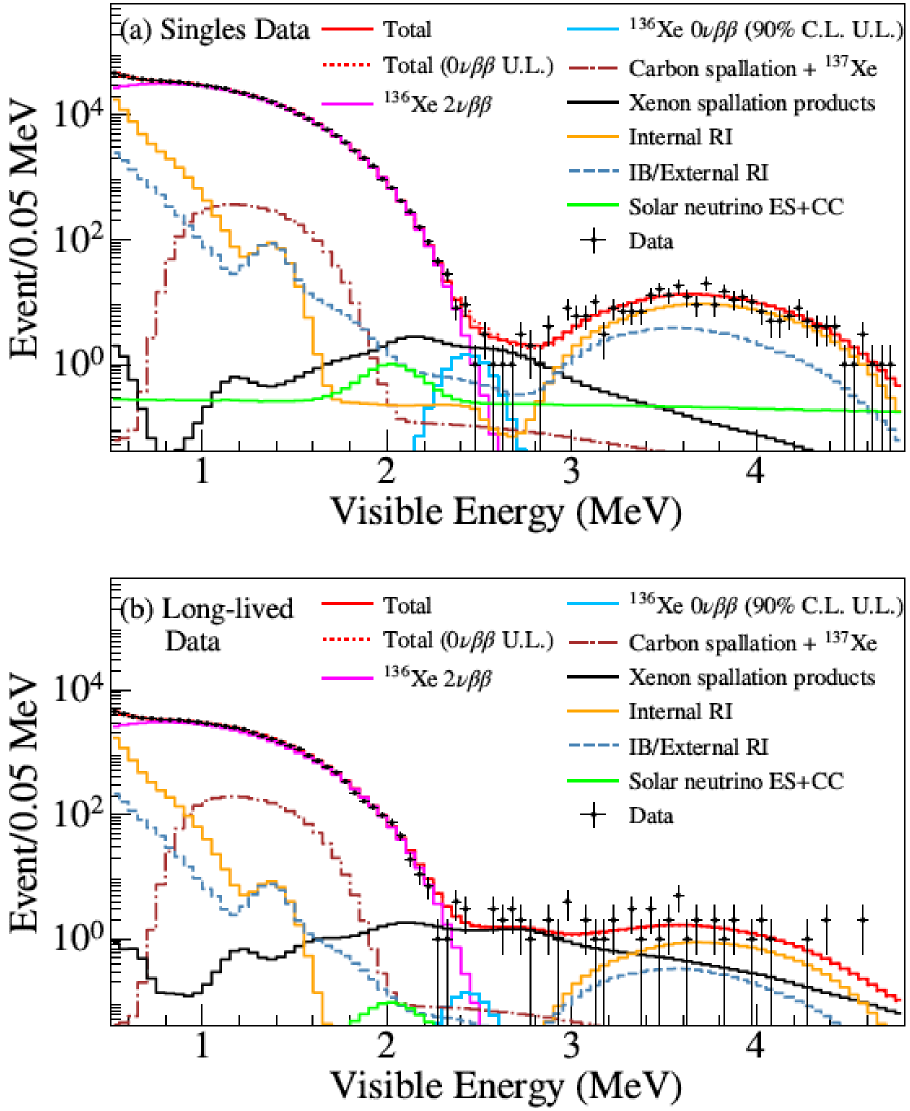


FIG. 2: Energy spectra of selected $\beta\beta$ candidates within a 1.57-m-radius spherical volume drawn together with best-fit backgrounds, the $2\nu\beta\beta$ decay spectrum, and the 90% C.L. upper limit for $0\nu\beta\beta$ decay of (a) singles data (SD), and (b) long-lived data (LD). The LD exposure is about 10% of the SD exposure.

Lots of Experimental Activity!

Moving Towards Ton-Scale Expts.

(LEGEND, CUPID, nEXO, etc)

[KamLAND-Zen Coll. (Abe *et al*), 2203.02139 [hep-ex]]

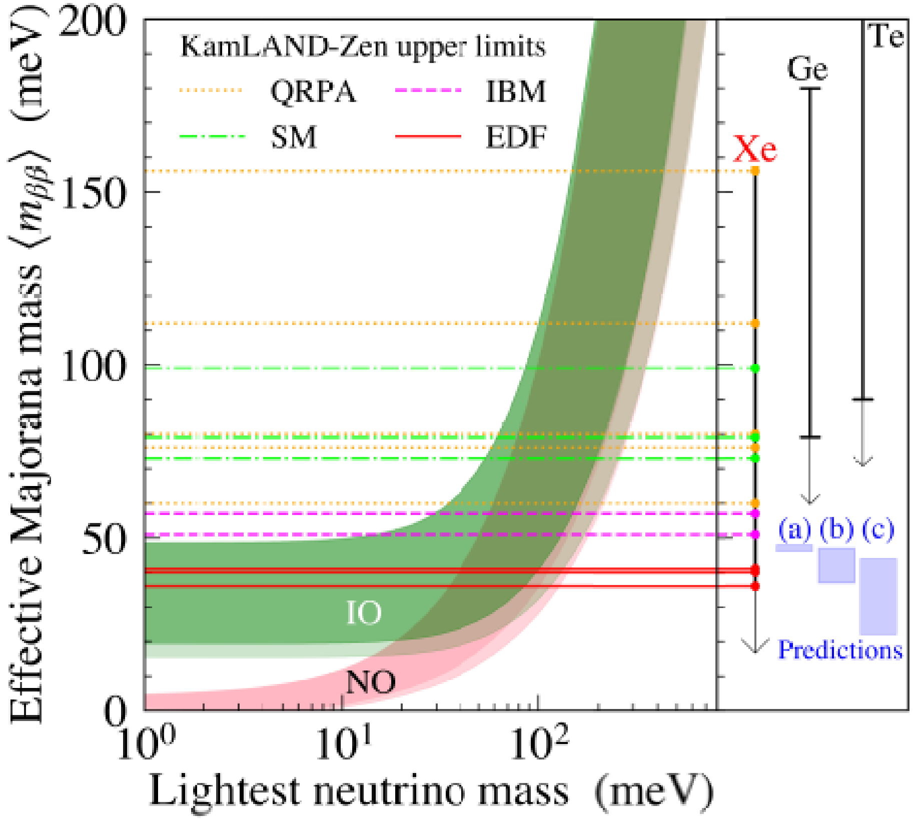


FIG. 4: Effective Majorana neutrino mass $\langle m_{\beta\beta} \rangle$ as a function of the lightest neutrino mass. The dark shaded regions are predictions based on best-fit values of neutrino oscillation parameters for the normal ordering (NO) and the inverted ordering (IO), and the light shaded regions indicate the 3σ ranges calculated from oscillation parameter uncertainties [23, 24]. The regions below the horizontal lines are allowed at 90% C.L. with ^{136}Xe from KamLAND-Zen (this work) considering an improved phase space factor calculation [25, 26] and commonly used nuclear matrix element estimates, EDF [27–29] (solid lines), IBM [30, 31] (dashed lines), SM [32–34] (dot-dashed lines), QRPA [35–39] (dotted lines). The side-panel shows the corresponding limits for ^{136}Xe, ^{76}Ge [40], and ^{130}Te [41], and theoretical model predictions on $\langle m_{\beta\beta} \rangle$, (a) Ref. [2], (b) Ref. [3], and (c) Ref. [4] (shaded boxes), in the IO region.

Lots of Experimental Activity!

Moving Towards Ton-Scale Expts.

(LEGEND, CUPID, nEXO, etc)

[KamLAND-Zen Coll. (Abe *et al*), 2203.02139 [hep-ex]]

Caveats for $0\nu\beta\beta$ as input for neutrino masses

- Indirect probe of neutrino mass;
- Only works if the neutrinos are Majorana fermions;
- Model dependent. While a nonzero rate for $0\nu\beta\beta$ implies neutrinos are massive Majorana fermions, the connection to nonzero neutrino masses can be very indirect. How do we learn that we are measuring what we think we are measuring?
- Real life is hard. Large uncertainties in translating the half-life to the effective neutrino mass (nuclear matrix elements).

What Could Go Wrong? Example...

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \to (Z+2)e^-e^-$.

Imagine neutrinos are a consequence of the Type-I seesaw but that all right-handed neutrino masses are below 100 MeV. In this case, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

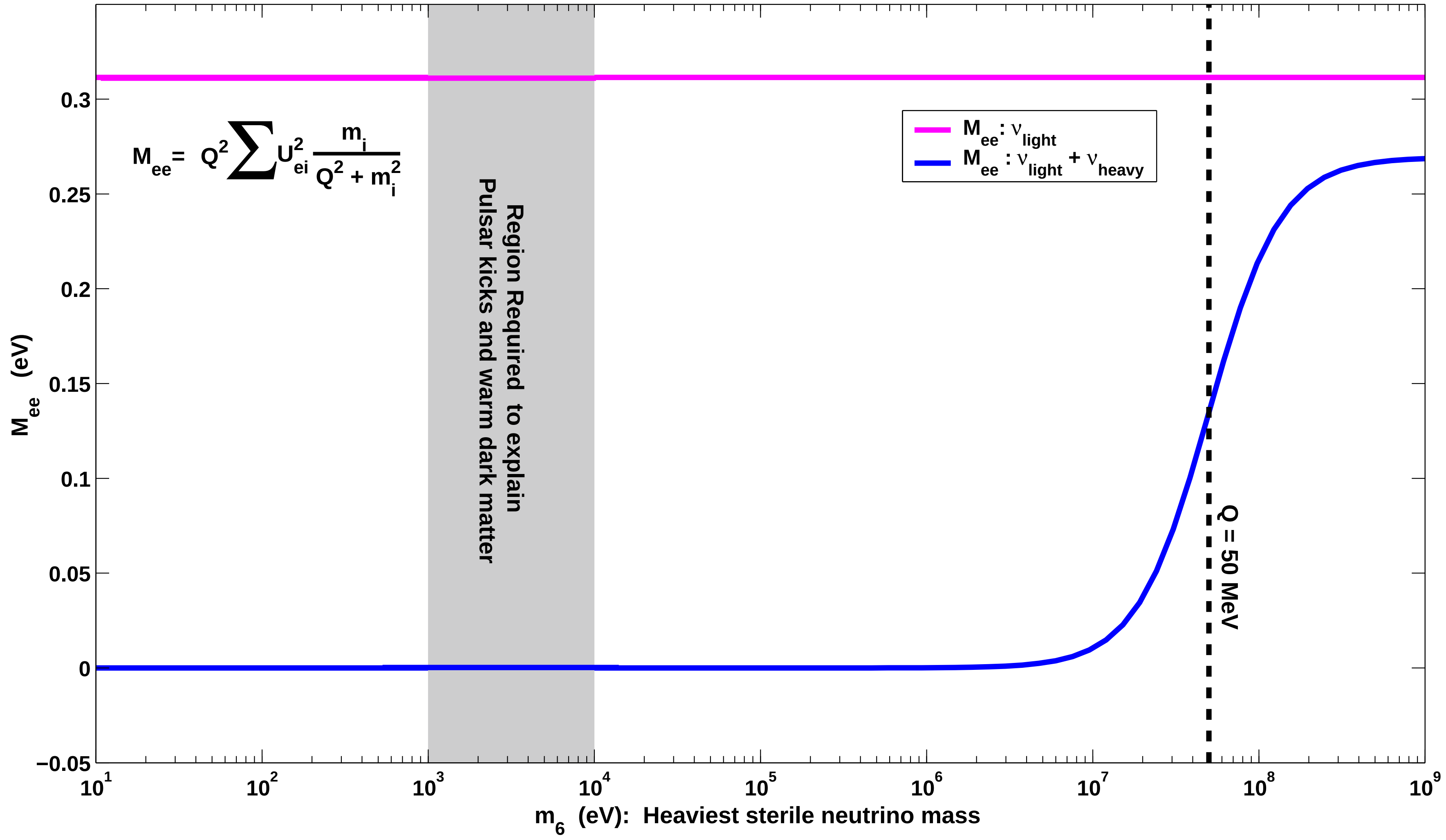
$$m_{ee} = \left|\sum_{i=1}^{6} U_{ei}^2 m_i\right| \sim \left|\sum_{i=1}^{3} U_{ei}^2 m_i + \sum_{i=1}^{3} \vartheta_{ei}^2 M_i\right|.$$

However, upon further examination, $m_{ee} = 0$ in this case! **The contribution of light and heavy neutrinos exactly cancels!** This remains true to a good approximation as long as $M_i \ll 100$ MeV.

$$\left[\ \mathcal{M} = \begin{pmatrix} 0 & \mu^{\rm T} \\ \mu & M \end{pmatrix} \ \to\ m_{ee} \text{ is identically zero!}\ \right]$$

(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]



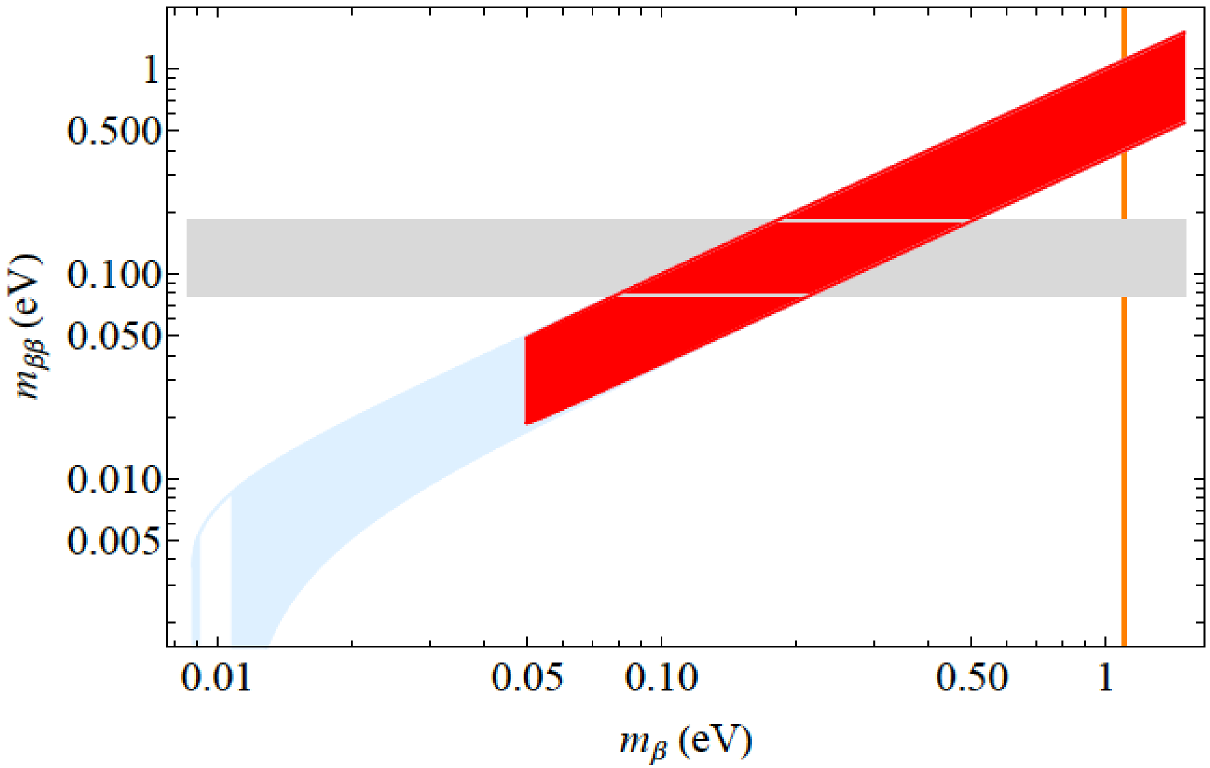


FIG. 5: $m_{\beta\beta}$ as a function of m_β, for both the normal (lighter, blue) and inverted (darker, red) mass orderings. The bands are a consequence of allowing for all possible values of the relative Majorana phases. For everything else, we use the current best-fit values of the oscillation parameters from [29]. The whited-out region inside the light-blue contour is meant to highlight the values of m_β for which $m_{\beta\beta}$ can vanish exactly. We assume the neutrinos are Majorana fermions. If neutrinos are Dirac fermions, $m_{\beta\beta} = 0$. The grey, horizontal band corresponds to the 95% CL upper bound on $m_{\beta\beta}$ from GERDA [37]. The width of the band is a consequence of uncertainties in the nuclear matrix element for the neutrinoless double-beta decay of ^{76}Ge. The vertical line corresponds to the current 90% upper bound on m_β [56].

[Formaggio, AdG, Robertson, Phys.Rept. 914 (2021)]

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to p e)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p \mu)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion		
$\sigma(\mu^- \, {}^{32}S \to e^+ \, {}^{32}Si^*)$ / $\sigma(\mu^- \, {}^{32}S \to \nu_\mu \, {}^{32}P^*)$		$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- \, {}^{127}I \to e^+ \, {}^{127}Sb^*)$ / $\sigma(\mu^- \, {}^{127}I \to$ anything)		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- \, Ti \to e^+ \, Ca)$ / $\sigma(\mu^- \, Ti \to$ capture)		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- \pi^-)/\Gamma_{total}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- \pi^-)/\Gamma_{total}$		$<3.9 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- K^-)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+ K^- K^-)/\Gamma_{total}$		$<3.3 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- K^-)/\Gamma_{total}$		$<4.8 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ K^- K^-)/\Gamma_{total}$		$<4.7 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to p \mu^- \mu^-)/\Gamma_{total}$		$<4.4 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \mu^+ \mu^-)/\Gamma_{total}$		$<3.3 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \gamma)/\Gamma_{total}$		$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \pi^0)/\Gamma_{total}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} 2\pi^0)/\Gamma_{total}$		$<3.3 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \eta)/\Gamma_{total}$		$<8.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \pi^0 \eta)/\Gamma_{total}$		$<2.7 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \Lambda \pi^-)/\Gamma_{total}$		$<7.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \Lambda \pi^-)/\Gamma_{total}$		$<1.4 \times 10^{-7}$, CL = 90%
$t_{1/2}({}^{76}Ge \to {}^{76}Se + 2\,e^-)$ $\Leftarrow$ $0\nu\beta\beta$		$>1.9 \times 10^{25}$ yr, CL = 90%
$\Gamma(\pi^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{total}$	[q]	$<1.5 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^- \mu^+ e^+)/\Gamma_{total}$		$<5.0 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^- e^+ e^+)/\Gamma_{total}$		$<6.4 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^- \mu^+ \mu^+)/\Gamma_{total}$	[q]	$<1.1 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{total}$	[q]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0 e^+ \overline{\nu}_e)/\Gamma_{total}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^- 2e^+)/\Gamma_{total}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^- 2\mu^+)/\Gamma_{total}$		$<2.2 \times 10^{-8}$, CL = 90%
$\Gamma(D^+ \to \pi^- e^+ \mu^+)/\Gamma_{total}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \rho^- 2\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^- 2e^+)/\Gamma_{total}$		$<9 \times 10^{-7}$, CL = 90%
$\Gamma(D^+ \to K^- 2\mu^+)/\Gamma_{total}$		$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^- e^+ \mu^+)/\Gamma_{total}$		$<1.9 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to K^*(892)^- 2\mu^+)/\Gamma_{total}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^- 2e^+ +$ c.c.$)/\Gamma_{total}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^- 2\mu^+ +$ c.c.$)/\Gamma_{total}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- 2e^+ +$ c.c.$)/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- 2\mu^+ +$ c.c.$)/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- 2e^+ +$ c.c.$)/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- 2\mu^+ +$ c.c.$)/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^- \pi^- e^+ \mu^+ +$ c.c.$)/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- e^+ \mu^+ +$ c.c.$)/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- e^+ \mu^+ +$ c.c.$)/\Gamma_{total}$		$<5.7 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to p e^-)/\Gamma_{total}$	[r]	$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \overline{p} e^+)/\Gamma_{total}$	[s]	$<1.1 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^- 2e^+)/\Gamma_{total}$		$<4.1 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to \pi^- 2\mu^+)/\Gamma_{total}$		$<1.2 \times 10^{-7}$, CL = 90%
$\Gamma(D_s^+ \to \pi^- e^+ \mu^+)/\Gamma_{total}$		$<8.4 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^- 2e^+)/\Gamma_{total}$		$<5.2 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^- 2\mu^+)/\Gamma_{total}$		$<1.3 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to K^- e^+ \mu^+)/\Gamma_{total}$		$<6.1 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^*(892)^- 2\mu^+)/\Gamma_{total}$		$<1.4 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^- e^+ e^+)/\Gamma_{total}$		$<2.3 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \pi^- \mu^+ \mu^+)/\Gamma_{total}$		$<4.0 \times 10^{-9}$, CL = 95%
$\Gamma(B^+ \to \pi^- e^+ \mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^- e^+ e^+)/\Gamma_{total}$		$<1.7 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^- \mu^+ \mu^+)/\Gamma_{total}$		$<4.2 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^- e^+ \mu^+)/\Gamma_{total}$		$<4.7 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^- e^+ e^+)/\Gamma_{total}$		$<3.0 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to K^- \mu^+ \mu^+)/\Gamma_{total}$		$<4.1 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to K^- e^+ \mu^+)/\Gamma_{total}$		$<1.6 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^- e^+ e^+)/\Gamma_{total}$		$<4.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^- \mu^+ \mu^+)/\Gamma_{total}$		$<5.9 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^- e^+ \mu^+)/\Gamma_{total}$		$<3.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to D^- e^+ e^+)/\Gamma_{total}$		$<2.6 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^- e^+ \mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^- \mu^+ \mu^+)/\Gamma_{total}$		$<6.9 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to D^{*-} \mu^+ \mu^+)/\Gamma_{total}$		$<2.4 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to D_s^- \mu^+ \mu^+)/\Gamma_{total}$		$<5.8 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to \overline{D}^0 \pi^- \mu^+ \mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to \Lambda^0 \mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \Lambda^0 e^+)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0 \mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0 e^+)/\Gamma_{total}$		$<8 \times 10^{-8}$, CL = 90%

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to pe)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p\mu)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion		
$\sigma(\mu^- {}^{32}S \to e^+ {}^{32}Si^*)$ / $\sigma(\mu^- {}^{32}S \to \nu_\mu {}^{32}P^*)$		$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}I \to e^+ {}^{127}Sb^*)$ / $\sigma(\mu^- {}^{127}I \to$ anything)		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- Ti \to e^+ Ca)$ / $\sigma(\mu^- Ti \to$ capture)		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-\pi^-)/\Gamma_{total}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-\pi^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to e^+\pi^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to e^+K^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \mu^+\pi^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \mu^+K^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to p\mu^-\mu^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\mu^+\mu^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\gamma)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\pi^0)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}2\pi^0)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\eta)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\pi^0\eta)/\Gamma_{total}$		
$\Gamma(\tau^- \to \Lambda\pi^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{\Lambda}\pi^-)/\Gamma_{total}$		
$t_{1/2}({}^{76}Ge \to {}^{76}Se + 2e^-)$		
$\Gamma(\pi^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^-\mu^+e^+)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^-e^+e^+)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$	[q]	$<1.1 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$	[q]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0e^+\overline{\nu}_e)/\Gamma_{total}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^-2e^+)/\Gamma_{total}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^-2\mu^+)/\Gamma_{total}$		$<2.2 \times 10^{-8}$, CL = 90%
$\Gamma(D^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \rho^-2\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^-2e^+)/\Gamma_{total}$		$<9 \times 10^{-7}$, CL = 90%
$\Gamma(D^+ \to K^-2\mu^+)/\Gamma_{total}$		$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<1.9 \times 10^{-6}$, CL = 90%

$\Gamma(B^0 \to \Lambda_c^+\mu^-)/\Gamma_{total}$	$<1.4 \times 10^{-6}$, CL = 90%
$\Gamma(B^0 \to \Lambda_c^+e^-)/\Gamma_{total}$	$<4 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to \pi^+e^-)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^+\mu^-)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^-e^+)/\Gamma_{total}$	$<4 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^-\mu^+)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to K^+e^-)/\Gamma_{total}$	$<2 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^+\mu^-)/\Gamma_{total}$	$<3 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^-e^+)/\Gamma_{total}$	$<2 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^-\mu^+)/\Gamma_{total}$	$<3 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K_S^0\nu)/\Gamma_{total}$	$<2 \times 10^{-5}$, CL = 90%
$\Gamma(\Xi^- \to p\mu^-\mu^-)/\Gamma_{total}$	$<4 \times 10^{-8}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p}2e^+)/\Gamma_{total}$	$<2.7 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p}2\mu^+)/\Gamma_{total}$	$<9.4 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p}e^+\mu^+)/\Gamma_{total}$	$<1.6 \times 10^{-5}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \Sigma^-\mu^+\mu^+)/\Gamma_{total}$	$<7.0 \times 10^{-4}$, CL = 90%

$\Gamma(D^+ \to K^*(892)^-2\mu^+)/\Gamma_{total}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^-2e^+ + c.c.)/\Gamma_{total}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^-2\mu^+ + c.c.)/\Gamma_{total}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-2e^+ + c.c.)/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-2\mu^+ + c.c.)/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2e^+ + c.c.)/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2\mu^+ + c.c.)/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^-\pi^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<5.7 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to pe^-)/\Gamma_{total}$	[r]	$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \overline{p}e^+)/\Gamma_{total}$	[s]	$<1.1 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-2e^+)/\Gamma_{total}$		$<4.1 \times 10^{-6}$, CL = 90%
		$<1.2 \times 10^{-7}$, CL = 90%
		$<8.4 \times 10^{-6}$, CL = 90%
		$<5.2 \times 10^{-6}$, CL = 90%
		$<1.3 \times 10^{-5}$, CL = 90%
		$<6.1 \times 10^{-6}$, CL = 90%
		$<1.4 \times 10^{-3}$, CL = 90%
		$<2.3 \times 10^{-8}$, CL = 90%
		$<4.0 \times 10^{-9}$, CL = 95%
		$<1.5 \times 10^{-7}$, CL = 90%
		$<1.7 \times 10^{-7}$, CL = 90%
		$<4.2 \times 10^{-7}$, CL = 90%
		$<4.7 \times 10^{-7}$, CL = 90%
		$<3.0 \times 10^{-8}$, CL = 90%
		$<4.1 \times 10^{-8}$, CL = 90%
		$<1.6 \times 10^{-7}$, CL = 90%
		$<4.0 \times 10^{-7}$, CL = 90%
		$<5.9 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^-e^+\mu^+)/\Gamma_{total}$		$<3.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to D^-e^+e^+)/\Gamma_{total}$		$<2.6 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^-e^+\mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^-\mu^+\mu^+)/\Gamma_{total}$		$<6.9 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to D^{*-}\mu^+\mu^+)/\Gamma_{total}$		$<2.4 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to D_s^-\mu^+\mu^+)/\Gamma_{total}$		$<5.8 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to \overline{D}^0\pi^-\mu^+\mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to \Lambda^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \Lambda^0e^+)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0e^+)/\Gamma_{total}$		$<8 \times 10^{-8}$, CL = 90%

Plenty of Other Probes. None can really compete, unless...

...there is something special about *ee* effects (e.g., they vanish) or ...

...the physics of lepton-number violation is more interesting.

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to pe)/\Gamma_{\rm total}$	$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p\mu)/\Gamma_{\rm total}$	$<1.8 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion	
$\sigma(\mu^- {}^{32}{\rm S} \to e^+ {}^{32}{\rm Si}^*)$ / $\sigma(\mu^- {}^{32}{\rm S} \to \nu_\mu {}^{32}{\rm P}^*)$	$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}{\rm I} \to e^+ {}^{127}{\rm Sb}^*)$ / $\sigma(\mu^- {}^{127}{\rm I} \to$ anything)	$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {\rm Ti} \to e^+ {\rm Ca})$ / $\sigma(\mu^- {\rm Ti} \to$ capture)	$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- \pi^-)/\Gamma_{\rm total}$	$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- \pi^-)/\Gamma_{\rm total}$	$<3.9 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- K^-)/\Gamma_{\rm total}$	$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+ K^- K^-)/\Gamma_{\rm total}$	$<3.3 \times 10^{-8}$, CL = 90%

$\Rightarrow$ limit on $\mu^- \to e^+$ conversion $\Leftarrow$ (next best?)

Experimental Sensitivities

KamLAND-Zen: $T_{0\nu\beta\beta} > 1.07 \times 10^{26}$ yr (90% CL; ^{136}Xe)

arXiv:1605.02889; KamLAND-ZEN Collaboration

SINDRUM II:

- $\mu^- \to e^-$ conversion: $$R^{\mathrm{Au}}_{\mu^- e^-} \equiv \frac{\Gamma(\mu^- + \mathrm{Au} \to e^- + \mathrm{Au})}{\Gamma(\mu^- + \mathrm{Au} \to \nu_\mu + \mathrm{Pt})} < 7 \times 10^{-13} \ (90\%\ \mathrm{CL})$$
- $\mu^- \to e^+$ conversion: $$R^{\mathrm{Ti}}_{\mu^- e^+} \equiv \frac{\Gamma(\mu^- + \mathrm{Ti} \to e^+ + \mathrm{Ca})}{\Gamma(\mu^- + \mathrm{Ti} \to \nu_\mu + \mathrm{Sc})} < \begin{cases} 1.7 \times 10^{-12} \ (\mathrm{GS},\ 90\%\ \mathrm{CL}) \\ 3.6 \times 10^{-11} \ (\mathrm{GDR},\ 90\%\ \mathrm{CL}) \end{cases}$$

*Eur. Phys. J. **C47**, 337 (2006); SINDRUM II Collaboration*
*Phys. Lett. **B422**, 334 (1998); SINDRUM II Collaboration*

Apples–to–apples comparison of $\mu^- \to e^-$ and $\mu^- \to e^+$?

- 1993 – simultaneous analysis!
- Apply this to future experiments

$$R^{\mathrm{Ti}}_{\mu^- e^-} < 4.3 \times 10^{-12} \ (90\%\ \mathrm{CL})$$
$$R^{\mathrm{Ti}}_{\mu^- e^+} < 4.3 \times 10^{-12} \ (90\%\ \mathrm{CL})$$

*Phys. Lett. **B317**, 631 (1993); SINDRUM II Collaboration*

Experimental Sensitivities

Upcoming experiments:

$$
\begin{aligned}
\text{DeeMe:} \quad & R^{\rm SiC}_{\mu^- e^-} > 5 \times 10^{-14} \ (90\% \ {\rm CL}), \\
\text{Mu2e:} \quad & R^{\rm Al}_{\mu^- e^-} > 6.6 \times 10^{-17} \ (90\% \ {\rm CL}), \\
\text{COMET Phase-I:} \quad & R^{\rm Al}_{\mu^- e^-} > 7.2 \times 10^{-15} \ (90\% \ {\rm CL}), \\
\text{COMET Phase-II:} \quad & R^{\rm Al}_{\mu^- e^-} > 6 \times 10^{-17} \ (90\% \ {\rm CL}), \\
\text{PRISM:} \quad & R^{\rm Al}_{\mu^- e^-} > 5 \times 10^{-19} \ (90\% \ {\rm CL}).
\end{aligned}
$$

Who could do this measurement?

- *Possibly* Mu2e and COMET Phase-I – similar to SINDRUM II
- *Probably* not DeeMe, COMET Phase-II or PRISM

$$
\begin{aligned}
\text{Mu2e:} \quad & R^{\rm Al}_{\mu^- e^+} \gtrsim 10^{-16} \\
\text{COMET Phase-I:} \quad & R^{\rm Al}_{\mu^- e^+} \gtrsim 10^{-14}
\end{aligned}
$$

Are there other ways to tell whether the neutrinos are Majorana or Dirac fermions?

The answer is a qualified ‘yes.’ However, it requires **non-relativistic neutrinos**.

The qualification is that we have to know the relevant underlying physics – new physics may spoil everything! One also has to “get lucky” sometimes. There are no “theorems” as far as I know…

Again: Why Don't We Know the Answer?

Neutrino Masses are Very Small*! [e.g. $|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle$]

In fact, except for neutrino oscillation experiments, no consequence of a nonzero neutrino mass has ever been observed in any experiment. As far as all non-oscillation neutrino experiments are concerned, neutrinos are massless fermions.

*Very small compared to what? Compared to the typical energies and momentum transfers in your experiment. Another way to think about this: neutrinos are always **ultrarelativistic** in the lab frame.

There are two ways around it:

1. Find something that only Majorana fermions know how to do [e.g. violate lepton number] or
2. **find some non-ultrarelativistic neutrinos to work with!**

The Burden of Working with Non-Ultrarelativistic Neutrinos

the weak interactions are weak and there is not a lot to measure (thing about the signal). Remember, at low energies

$$\sigma \propto E \quad \text{(or much worse)}$$

On the other hand, telling Majorana From Dirac neutrinos is “trivial.” Indeed, it is an order one effect.

Examples, or

Where Can We Find Some Non-Relativistic Neutrinos?

- The Cosmic Neutrino Background;
- Low-Energy Reactions with (Not-To-Be-Detected) Neutrinos in the Final State;
- Decaying Neutrinos.

Example: The Cosmic Neutrino Background

[see, e.g., Long, Lunardini, Sabancilar, arXiv:1405.7654]

Assuming the Standard Model of Cosmology, at least two of the three neutrinos are mostly non-relativistic today:

$$T_\nu \sim 2K \sim 2 \times 10^{-4} \text{ eV}.$$

Furthermore, it turns out that hitting a Majorana CνB with a charged-current process is easier than hitting a Dirac CνB, assuming the weak interactions. All of this is assuming one is measuring the CνB via neutrino-capture on nuclei, $\nu(Z,A) \to e^-(Z+1,A)$ (charged-current weak interaction on matter)

When you interact with a polarized (anti)neutrino at rest, it will either choose to behave like the left-chiral component or the right-chiral component, with the same probability.

In the Dirac case, the right-chiral component of the neutrino is sterile, i.e., it does not participate in the weak interactions and you can't interact with it. Furthermore, the antineutrinos have the opposite lepton number and can't be detected via $\nu(Z, A) \to e^-(Z+1, A)$.

In the Majorana case, the right-helicity component is the object we usually refer to as the antineutrino. In this case, both the left- and right-helicity components can interact via the weak interactions producing e^-. When it comes to the cosmic neutrino background being detected via $\nu(Z, A) \to e^-(Z+1, A)$, we get a hit from the neutrinos – just like in the Dirac case – but we also get a hit from the "antineutrino," with the same rate.

This means that if we ever observe the cosmic neutrino background, we can determine the nature of the neutrino. If all neutrinos were at rest, for the same neutrino (+ antineutrino, in the Dirac case) flux, we expect twice as many events in the experiment if the neutrinos are Majorana fermions. One can easily include finite temperature effects, effects related to the neutrino mass ordering, a potential primordial lepton asymmetry, etc.

Some challenges:

- We have never detected the cosmic neutrino background! (see, however, PTOLEMY [arXiv:1808.01892] for an idea that may work one day);
- We measure flux times cross-section. While we know the average neutrino number density of the universe very well from the Standard Model of Cosmology, we don't know the number density of neutrinos *here* very well [Uncertainty around 10%?].

Neutrinos Near Threshold

We looked at

$$e\gamma \to e\nu\bar{\nu}$$

at sub-eV energies, because it can be done, in principle (electron at rest, infrared photon). Best to do it in the mass basis! Using the Fermi theory...

$$\mathcal{L}_{\rm CC} + \mathcal{L}_{\rm NC} = -\sqrt{2} G_F \left(\overline{\nu}_j \gamma^\mu P_L \nu_i\right) \left[\overline{\ell}_\alpha \gamma_\mu \left(g_V^{\alpha\beta ij} \mathbb{1} - g_A^{\alpha\beta ij}\gamma_5\right) \ell_\beta\right], \tag{II.4}$$

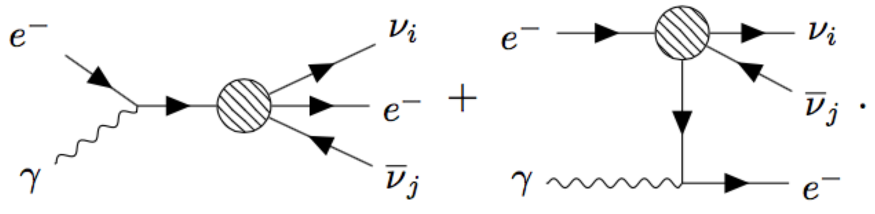
where we introduce the vector and axial couplings

$$g_V^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\left(1 - 4\sin^2\theta_W\right)\delta_{ij}\delta_{\alpha\beta}, \qquad g_A^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\delta_{ij}\delta_{\alpha\beta}. \tag{II.5}$$

Since the only charged leptons considered in this work are electrons, we will make the simplification $g^{ij}_{V,A} \equiv g^{eeij}_{V,A}$.

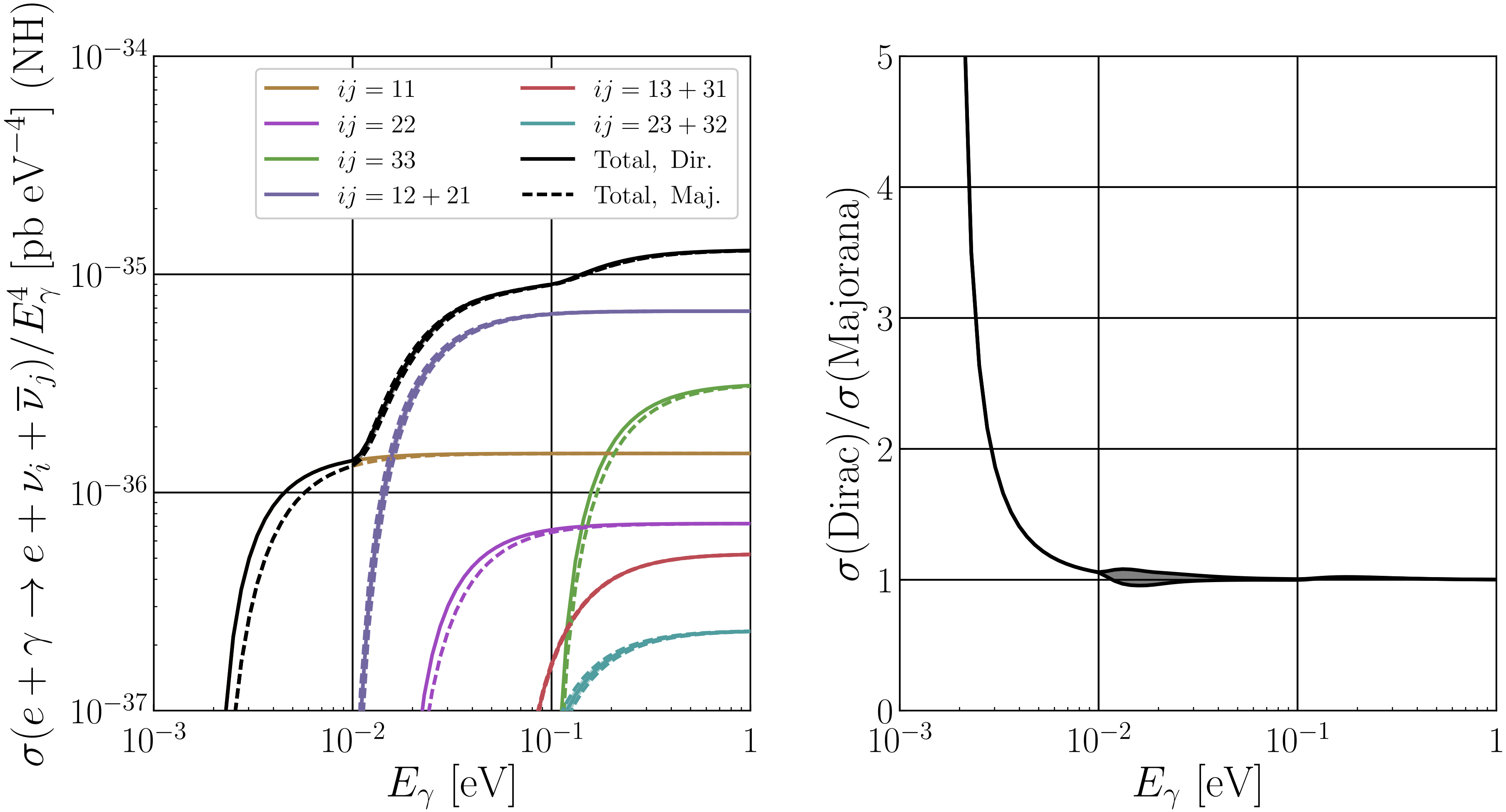
The following diagrams are relevant to the evaluation of the amplitude:

$$i\mathcal{M} \approx \tag{II.6}$$



[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]



Why Can't We Do This?

- Cross sections are small. Very, very small. For a 1 eV laser (1240 nm, near-infrared) with a power of 2 W (it exists) and a large target of electrons at rest with density 10^{23} cm^{-3} and a length of 1 m, we estimate one signal event every 10^{20} years.

- Backgrounds are ridiculous. The signal is a recoil electron and nothing else. This can be mimicked by $e + \gamma \to e + \gamma + \cdots + \gamma$ (n photons) when the photon(s) are very soft or fall within a dead-zone within the detector. Very naively,

$$\sigma_n \sim \alpha^{(n-1)} \sigma_{\rm Thomson} \left(\frac{E_\gamma^{\rm threshold}}{m_e} \right)^{2n}$$

where $\sigma_{\rm Thomson} \sim 0.7$ barn.

Neutrino Decay (Hint – Only Massive Particles Decay)

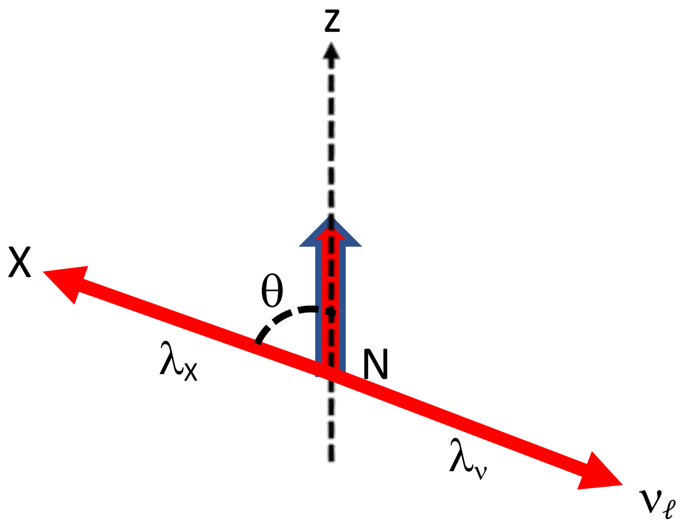
[Balantekin, AdG, Kayser, arXiv:1808.10518]

The two heavy neutrinos are expected to decay. E.g., if the neutrino mass ordering is normal, the decay modes $\nu_3 \to \nu_1 \gamma$ and $\nu_3 \to \nu_1 \nu_2 \bar{\nu}_1$ are not only kinematically allowed, they are mediated by the weak interactions once mixing is taken into account.

Dirac and Majorana neutrinos "decay differently." In particular, the number of accessible final states, and the way in which they can potentially interfere, is such that the partial widths, and the lifetimes are different – assuming the same mixing and mass parameters – if the neutrinos are Majorana or Dirac.

Obvious challenges. $\Gamma \propto (m_\nu)^n$ [n is some positive power] so the neutrino lifetimes are expected to be cosmological. Insult to injury, the $\nu \to \nu$'s decay mode is significant, which renders studying the final products of the decay a rather daunting task. Nonetheless, we proceed …

CPT invariance [at leading order]



We showed

$$\frac{d\Gamma(N \to \nu_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 + \alpha\cos\theta)$$

$$\frac{d\Gamma(\bar{N} \to \bar{\nu}_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 - \alpha\cos\theta)$$

Since $\alpha = -\bar{\alpha}$, for Majorana neutrinos we get $\alpha = 0$. This result holds for any self-conjugate boson X.

The two-body decay of a Majorana fermion into a self-conjugate final state is isotropic

A.B. Balantekin, B. Kayser, Ann. Rev. Nucl. Part. Sci. **68** (2018) 313-338 (arXiv:1805.00922)
A.B. Balantekin, A. de Gouvêa, B. Kayser, arXiv:1808.10518

A More Realistic (?) Application – Heavy Neutral Leptons

If a heavy neutral lepton ν_4 is discovered somewhere – LHC, MicroBooNE, ICARUS, DUNE, SuperB Factory, SHiP, etc – in the future, after much rejoicing, we will want to establish whether this fermion is a *Majorana* or *Dirac* fermion.

How do we do it?

- Check for lepton-number violation. What does it take?
  - A lepton-number asymmetric initial state (easy). Or an even-by-event lepton number "tag" of the neutral heavy lepton (e.g. LHC environment).
  - Charge identification capability in the detector (sometimes absent or partially absent).
- **Kinematics**. Not only are the decay widths different (not super useful, since it requires we know unknown parameters) but the kinematics are also qualitatively different, as I showed in the last slide.

ν_4 and Lepton-Number Violation at Hadron Colliders

Heavy neutrinos, when produced at a collider experiment, may also mediate lepton-number violation if they are Majorana fermions. For example,

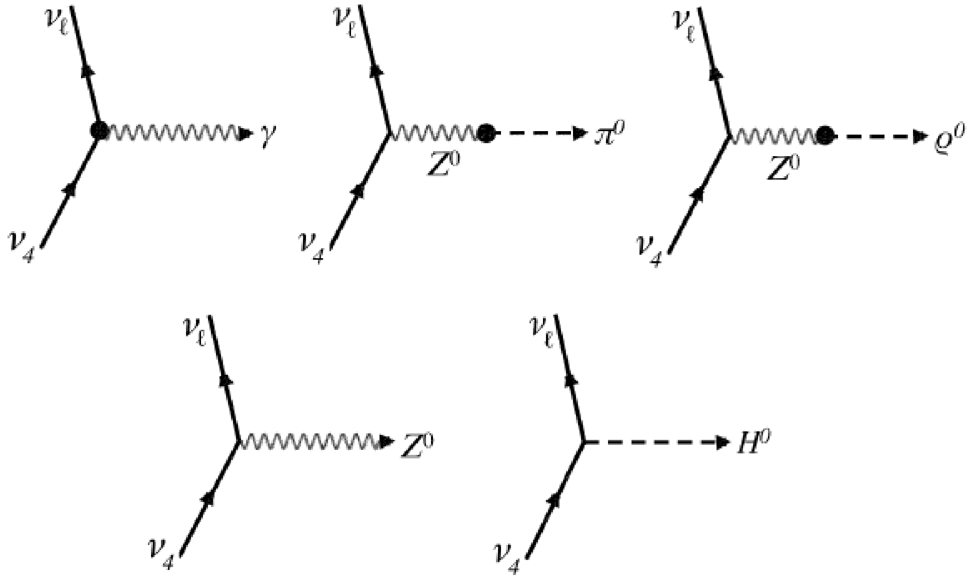
$$XW^{+(*)} \to X\ell^+\nu_4(\to \ell^- + q\bar{q}').$$

versus

$$XW^{+(*)} \to X\ell^+\nu_4(\to \ell^+ + q\bar{q}').$$

Smoking-gun if the lepton-number of X is known (e.g., zero).

Heavy Neutral Leptons – More Realistic (?) Application



All of these decays are isotropic for a Majorana parent. Dirac case ⇓ (weak interactions)

Boson	γ	π^0	ρ^0	Z^0	H^0
α	$\frac{2\Im(\mu d^*)}{\lvert\mu\rvert^2+\lvert d\rvert^2}$	1	$\frac{m_4^2-2m_\rho^2}{m_4^2+2m_\rho^2}$	$\frac{m_4^2-2m_Z^2}{m_4^2+2m_Z^2}$	1

Aside: We Can Use Charged Final States Too!

The two-body final states here all involve a neutrino and a neutral boson. Impossible to reconstruct the parent rest-frame and it requires measuring the properties of a neutral boson, which is sometimes challenging. Can we use the charged final states? E.g.

$$\nu_4 \to \mu^+ \pi^-$$

Most of the time, 'yes'! The reason is as follows. CPT invariance (at leading order) implies, for 100% polarized Majorana fermions,

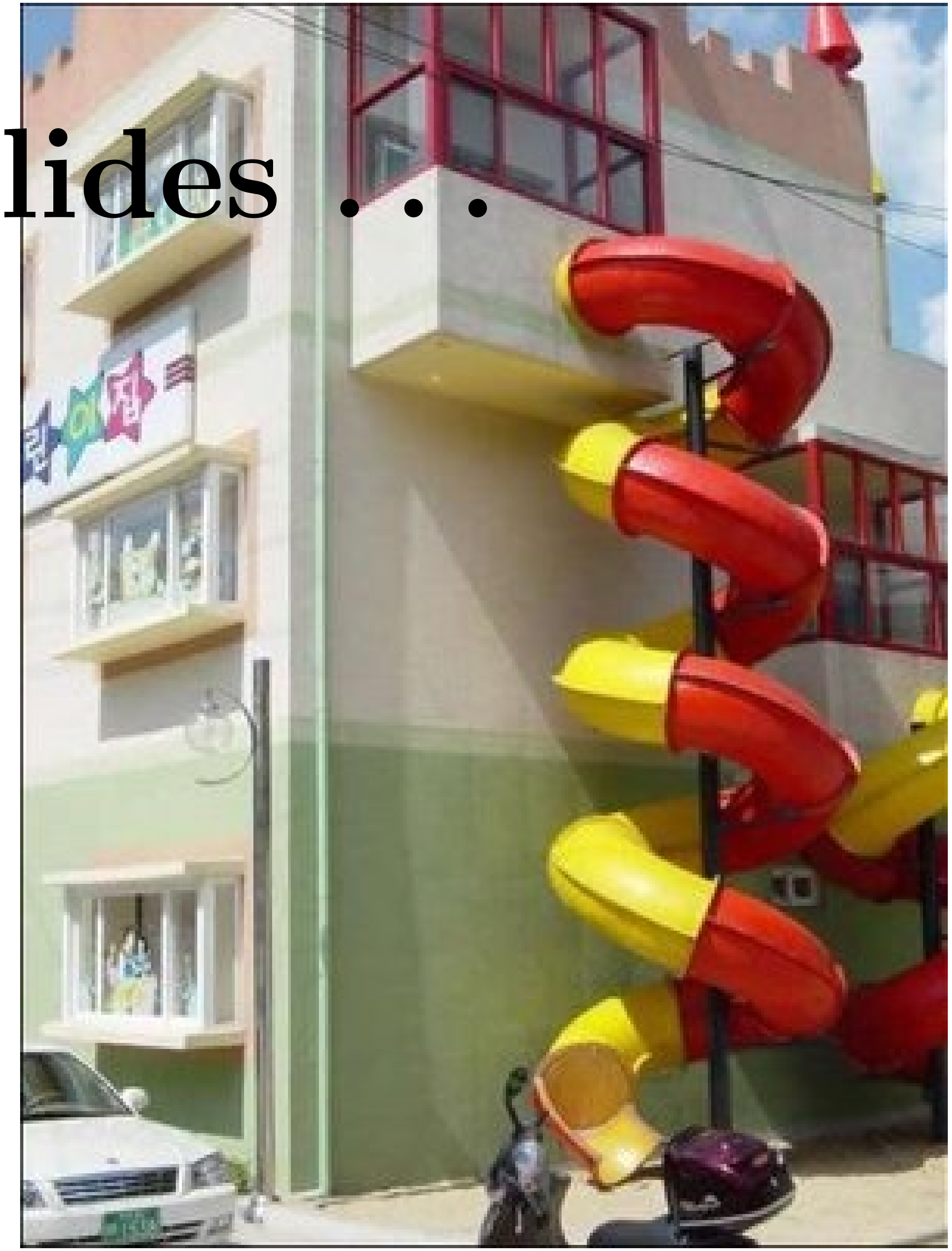
$$\frac{d\Gamma(\nu_4 \to \mu^+ \pi^-)}{d\cos\theta} \propto (1 + \alpha\cos\theta) \quad \text{while} \quad \frac{d\Gamma(\nu_4 \to \mu^- \pi^+)}{d\cos\theta} \propto (1 - \alpha\cos\theta)$$

so the **charge-blind sum of the two is also isotropic**. This is not the case for Dirac neutrinos as long as the production of neutrinos and antineutrinos is asymmetric, which is usually the case.

Quick Summary

- Majorana and Dirac Fermions are Qualitatively Different;
- However, massless Majorana and Dirac fermions are "the same" – this is a non-question!;
- Challenge for neutrinos. Since they are always ultra-relativistic, it is very difficult to address whether they are Majorana or Dirac since they are massless as far as the experiment is concerned;
- One way around it is to look for phenomena that can only occur if the neutrino is a Majorana fermion (e.g., lepton-number violation). In this case, even if the phenomenon is very rare (like $0\nu\beta\beta$), any deviation from zero would allow one to establish that the neutrinos are Majorana fermions.
- The other way is to find circumstances where the neutrinos are not ultra-relativistic. In this case, the Majorana versus Dirac differences are large. The rates, on the other hand...
- Potential application: neutral heavy leptons (new neutrino states). If they exist, we will want to known: Majorana or Dirac?

Backup Slides . . .



How many new CP-violating parameters in the neutrino sector?

If the neutrinos are Majorana fermions, there are more physical observables in the leptonic mixing matrix.

Remember the parameter counting in the quark sector:

9 $(3 \times 3$ unitary matrix)

$\underline{-5}$ (relative phase rotation among six quark fields)

4 (3 mixing angles and 1 CP-odd phase).

If the neutrinos are Majorana fermions, the parameter counting is quite different: there are no right-handed neutrino fields to "absorb" CP-odd phases:

9 (3×3 unitary matrix)

$\underline{-3}$ (three right-handed charged lepton fields)

6 (3 mixing angles and 3 CP-odd phases).

There is CP-invariance violating parameters even in the 2 family case: $4 - 2 = 2$, one mixing angle, one CP-odd phase.

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \overline{\nu^c_L}(M_\nu)\nu_L + H.c.$$

Write $U = E^{-i\xi/2} U' E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \mathrm{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \overline{\nu^c_L}(M_\nu) E^{-i\alpha} \nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

<u>on the other hand</u>

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \bar{\nu}_R (M_\nu)\nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \bar{\nu}_R (M_\nu) E^{-i\alpha/2} \nu_L + H.c.$$

ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$

Another Example of Neutrinos Near Threshold (Brief)

Atomic process: $A^* \to A\gamma$, where A (A^*) is a neutral atom (in some excited state). Now replace the γ with an off-shell Z, which manifests itself as two neutrinos:

$$A^* \to A\nu\bar{\nu}.$$

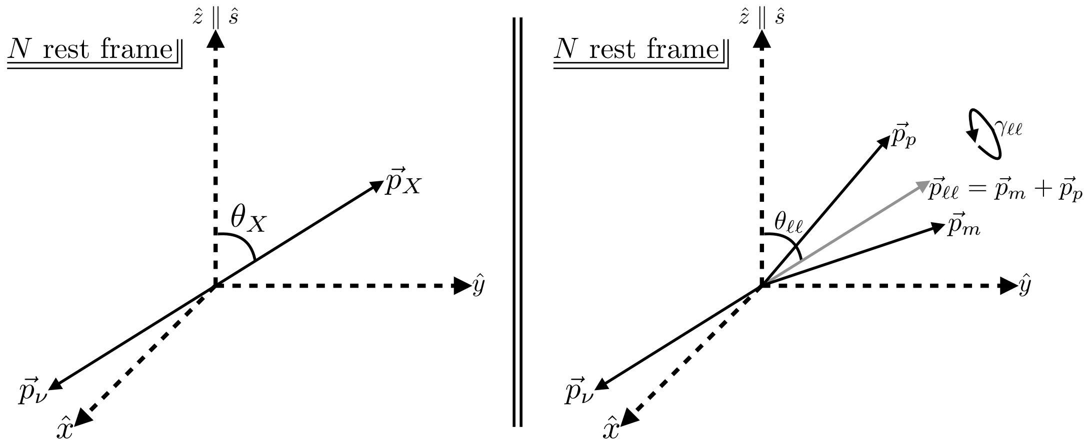
It is easy to imagine sub-eV energies and hence the neutrinos are not ultra-relativistic.

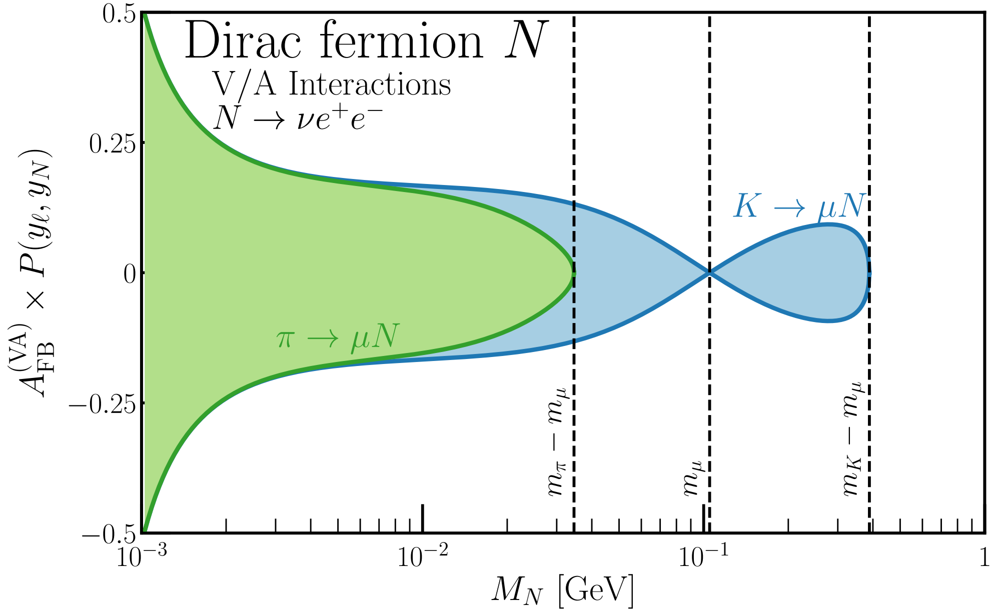
For all the details including rates – tiny – and difference between Majorana and Dirac neutrinos – large – see, for example, Yoshimura, hep-ph/0611362, Dinh *et al.*, arXiv:1209.4808, and Song *et al.* arXiv:1510.00421, and references therein.

Can this be done in practice? We have been working on it!

[AdG, Kayser, et al, 1912.07622, 2104.05719, 2105.06576, 2109.10358]

- Three-body decays (concentrating on $\nu\ell^+\ell^-$);
- Heavy neutrino production mechanism (choice is meson decay at rest);
- Consider different models for heavy neutrino decay, including the weak interactions (four-fermion interactions that preserve lepton number).





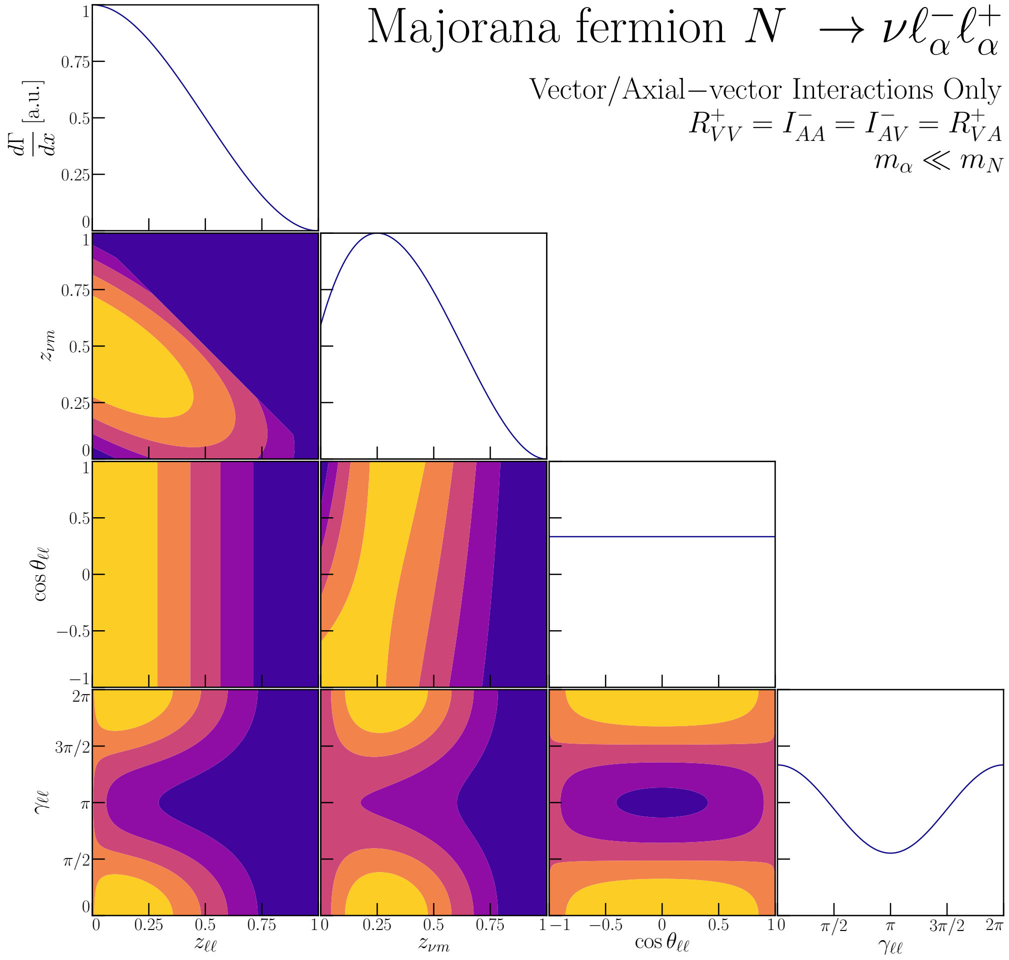
[AdG et al, arXiv: 2109.10358]

Majorana fermion $N \to \nu \ell_\alpha^- \ell_\alpha^+$

Vector/Axial−vector Interactions Only

$R_{VV}^+ = I_{AA}^- = I_{AV}^- = R_{VA}^+$

$m_\alpha \ll m_N$

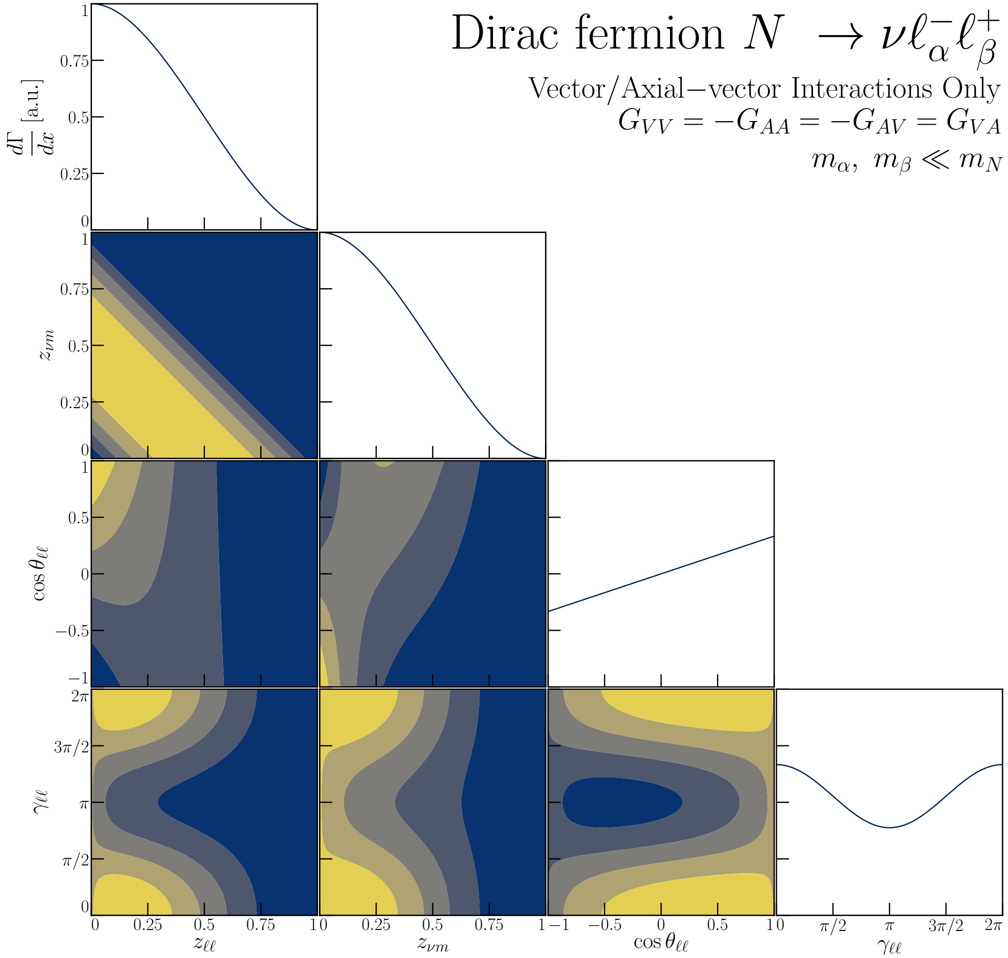


Dirac fermion $N \to \nu \ell_\alpha^- \ell_\beta^+$

Vector/Axial−vector Interactions Only

$G_{VV} = -G_{AA} = -G_{AV} = G_{VA}$

$m_\alpha,\ m_\beta \ll m_N$



[AdG et al, arXiv: 2104.05719]

ν_4 at the Z-pole

[Blondel, AdG, Kayser, arXiv: 2105.06576]

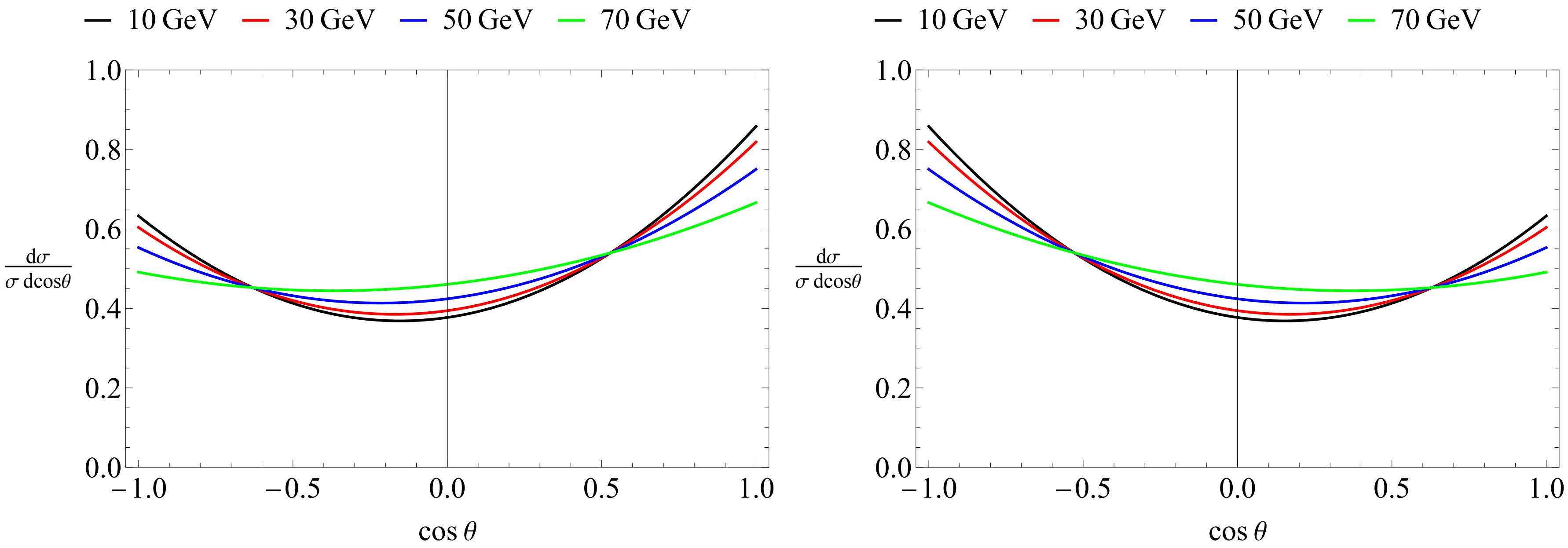
For $m_4 < M_Z$, heavy neutrinos can be produced in the decay of Z-bosons. For example,

$$e^+e^- \to Z \to \bar{\nu}_{\text{light}}\nu_4 \ (\text{or } \nu_{\text{light}}\bar{\nu}_4)$$

followed by ν_4 decay.

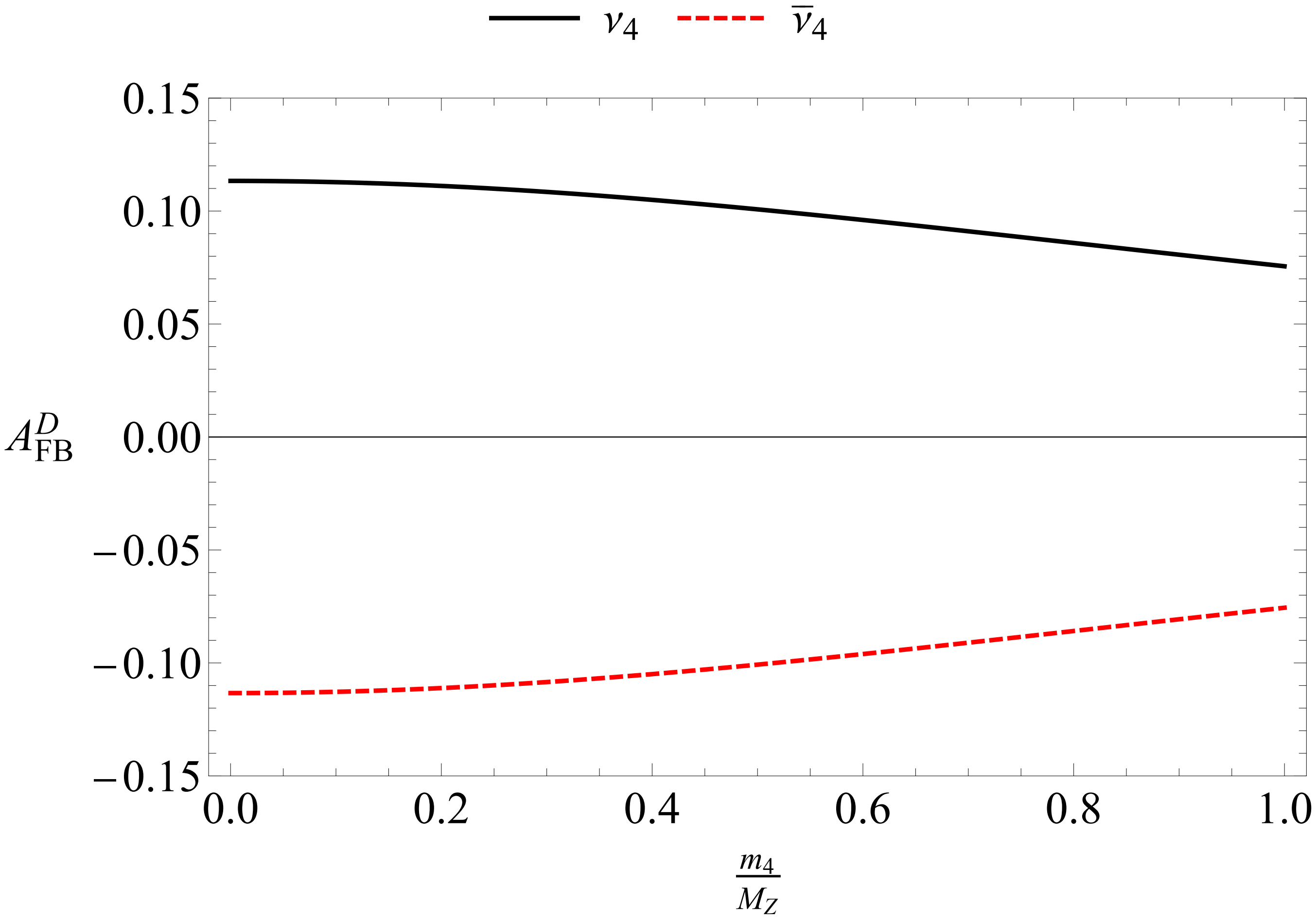
- Why do we care about this? Best constraints on m_4 around 10s of GeV from LEP! A Tera-Z-like experimental setup would be sensitive to seesaw-related heavy neutrinos. Unique opportunity.
- Challenge: Heavy neutrinos produced with light ones. We can't tell whether lepton-number is violated (at least in an event-by-event basis) since we don't get to the detect the light neutrinos.
- Main Message: If m_4 is large enough, Majorana and Dirac ν_4 are produced differently and decay differently!

[Related study for the ILC, P. Hernández, et al, arXiv:1810.07210.]

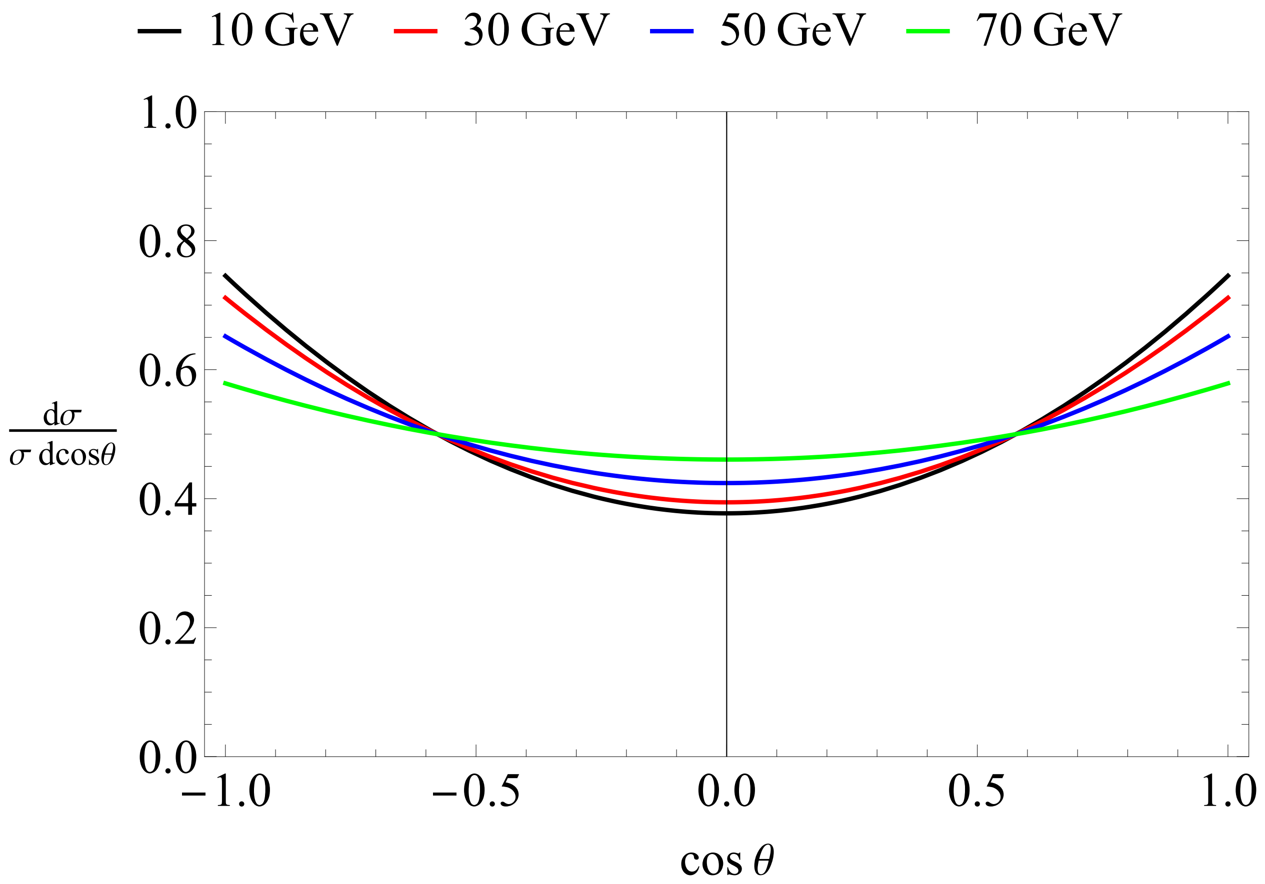


Normalized differential cross-section for $e^+e^- \to Z \to \nu_4\bar{\nu}_{\rm light}$ (left) and $e^+e^- \to Z \to \bar{\nu}_4\nu_{\rm light}$ (right) as a function of the direction of the heavy (anti)neutrino $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Dirac fermions.

[θ is defined relative to the direction of the e^--beam.]



The Forward-Backward Asymmetry A^D_{FB} of heavy neutrino or antineutrino production in $e^+e^- \to Z \to \nu_4\bar{\nu}_{\rm light}$ or $e^+e^- \to Z \to \bar{\nu}_4\nu_{\rm light}$ as a function of the heavy neutrino mass. The neutrinos are assumed to be Dirac fermions.



Normalized differential cross-section for $e^+e^- \to Z \to \nu_4\nu_{\rm light}$ as a function of the direction of the heavy neutrino, $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Majorana fermions.

The Forward-Backward Asymmetry A^M_{FB} vanishes exactly if ν_4 are Majorana fermions.

When the ν_4 decays via charged-current interactions, Dirac ν_4's decay like this:

$$\nu_4 \to \ell^- + X \quad \text{and} \quad \bar{\nu}_4 \to \ell^+ + X^*$$

so the ℓ^- inherits the angular distribution of the ν_4 while ℓ^+ inherits the angular distribution of the $\bar{\nu}_4$.

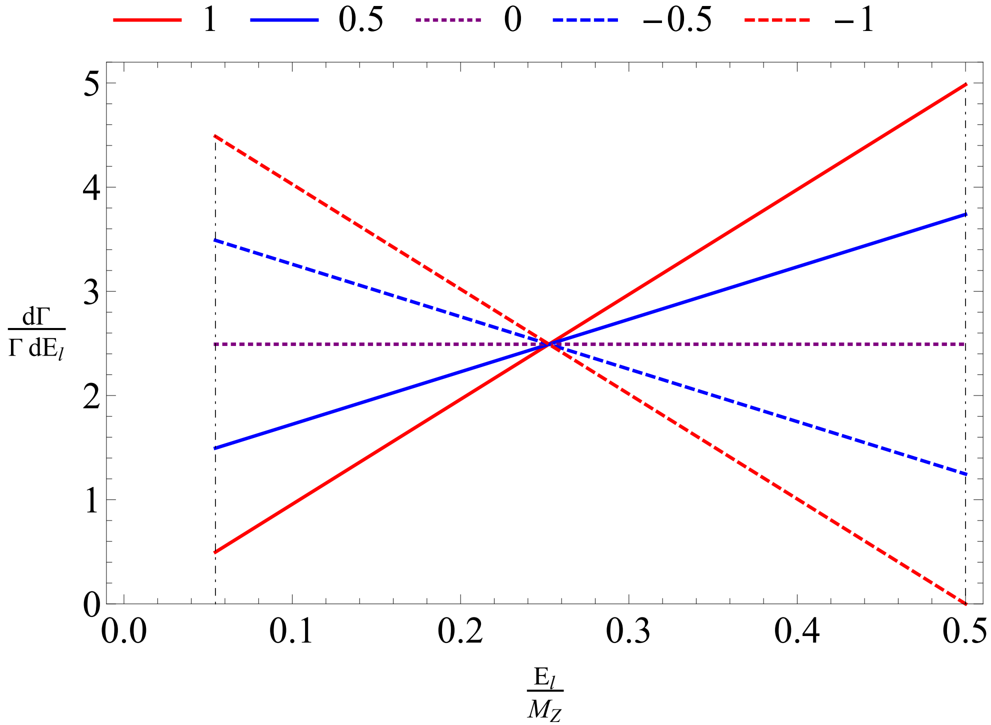
Meanwhile, Majorana ν_4's decay like this:

$$\nu_4 \to \ell^- + X \quad \text{or} \quad \nu_4 \to \ell^+ + X^*$$

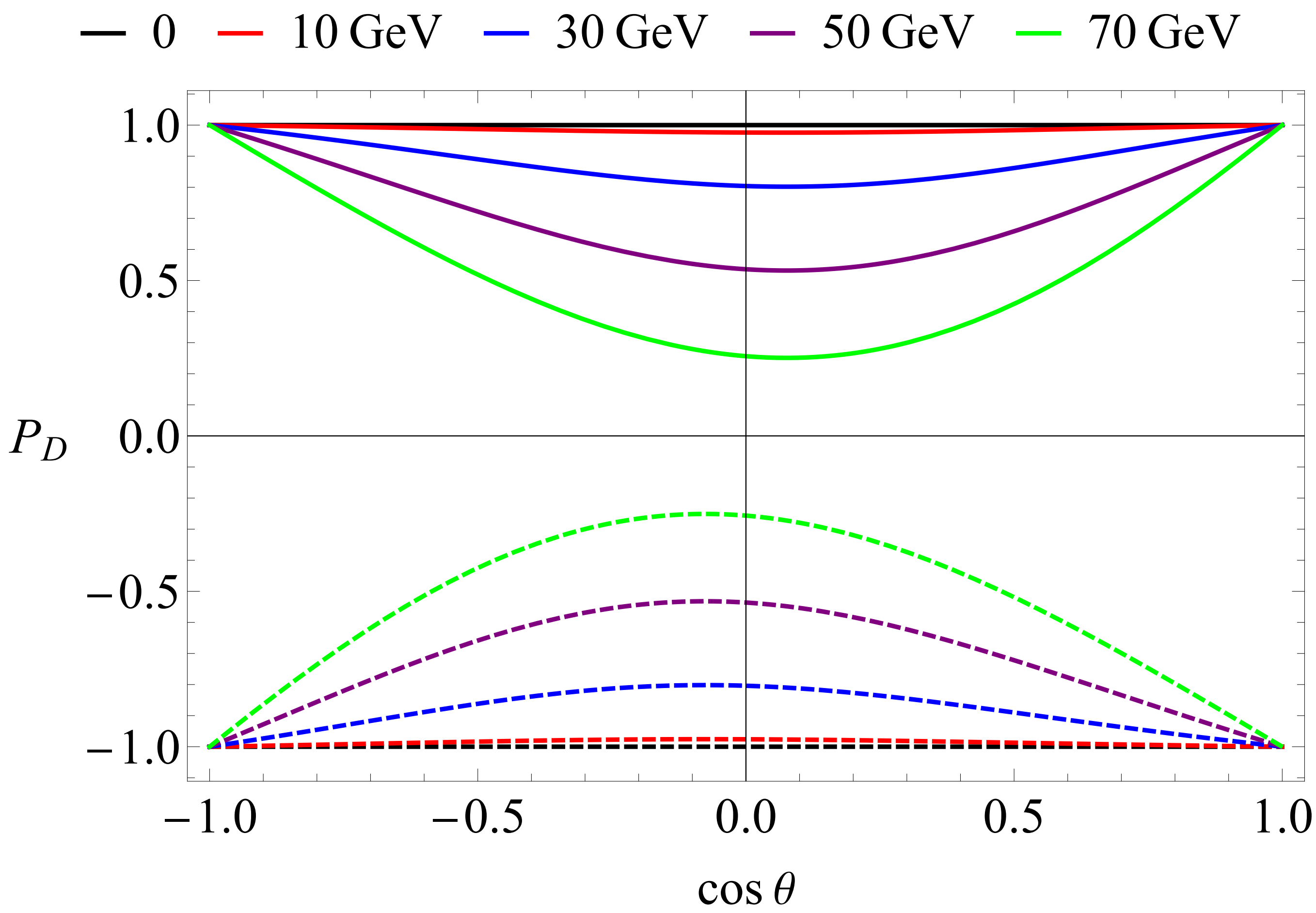
with equal probability. Both the ℓ^+ and the ℓ^- have the same angular distribution and no forward-backward asymmetry.

And there is more…

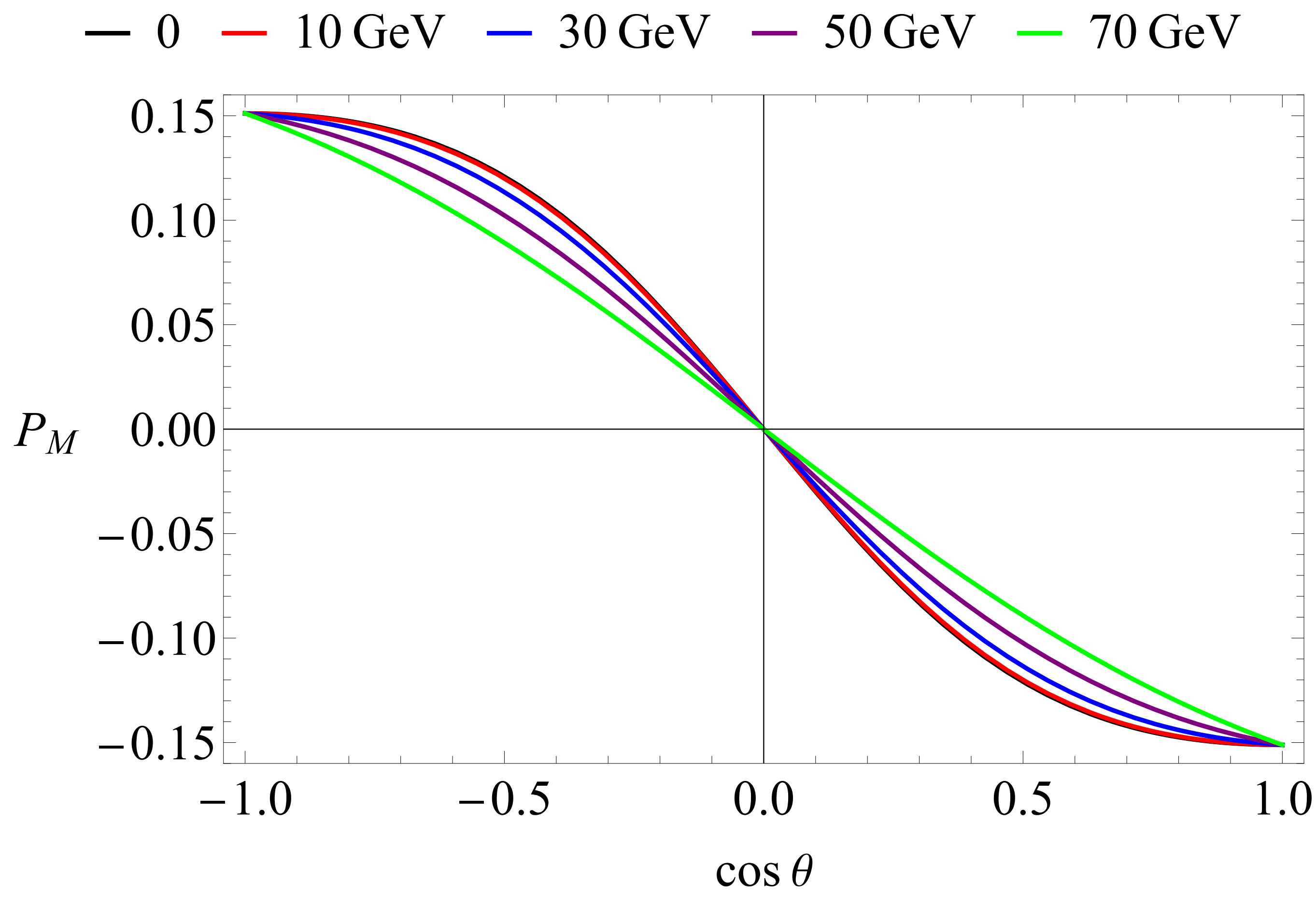
. . . the energy distribution of the daugther-charged-leptons depends on the polarization of the parent ν_4 (and $\bar{\nu}_4$ if ν_4 is a Dirac fermion). For example,



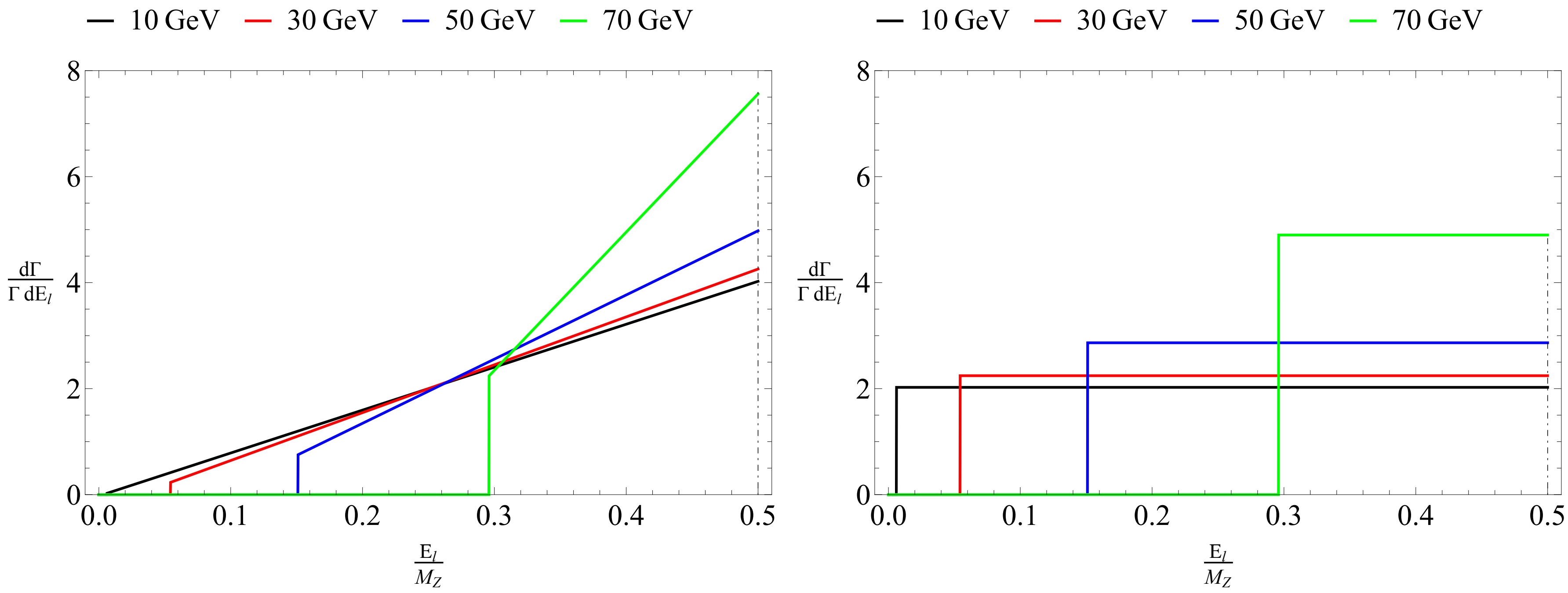
Normalized differential decay widths of $\nu_4 \to \ell^\pm \pi^\mp$ as a function of the energy of the charged-lepton, for ν_4 produced in Z-decay-at-rest. The different curves correspond to different values of $\alpha_\pm P \in [-1, 1]$ and $m_4 = 30$ GeV. $\alpha_\pm$ is the decay-asymmetry parameter and is a property of the physics responsible for the decay. For the SM, $\alpha_+ = +1 = -\alpha_-$



The polarizarion P_D of heavy neutrinos (dashed lines) or antineutrinos (solid lines) produced in $e^+e^- \to Z \to \nu_4\bar{\nu}_{\rm light}$ or $e^+e^- \to Z \to \bar{\nu}_4\nu_{\rm light}$ as a function of the direction of the heavy (anti)neutrino $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Dirac fermions.



The polarizarion P_M of heavy neutrinos produced in $e^+e^- \to Z \to \nu_4\nu_{\rm light}$ as a function of the direction of the heavy neutrino $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Majorana fermions. [Range of P_M values much smaller than range of P_D values (previous slide).]



Normalized differential decay widths of $\nu_4 \to \ell^- \pi^+$ as a function of the energy of the charged-lepton, averaged over the heavy-neutrino production angle, for ν_4 produced in Z-decay-at-rest assuming the heavy neutrinos are Dirac (left) and Majorana (right) fermions. The different curves correspond to different values of m_4. The same curves apply, both in the left-hand and in the right-hand panels, to the $\ell^+\pi^-$ final-states.